A bit of history ....

Momentum sum rule

fraction of p

early SLAC (+2) expts. (~1970)

$$\sum_q \int_0^1 x\left(q(x) + \bar{q}(x)\right) dx \simeq 0.5 \quad (\text{not } 1)$$

First (indirect) evidence for gluon

$$\int_0^1 x\, g(x)\, dx \simeq 0.5$$

Quarks not free. Interact. Need to improve parton model. How? Need the theory of q & g interactions.

Evidence for SU(3) of colour

- Originally need for hadron spectroscopy
  (1964 on)

q = col. triplet

hadron = col. singlet

$q\bar{q} = 3 \otimes \bar{3} = 1 \oplus 8$

$qqq = 3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus \ldots$

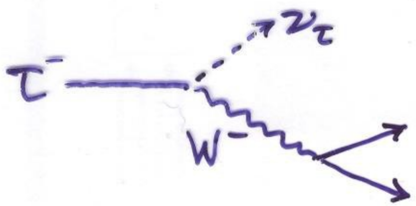
$\varepsilon_{abc}\, q_a q_b q_c$

$$\Delta^{++}\left(j_z = \tfrac{3}{2}\right) = \underbrace{u\!\uparrow u\!\uparrow u\!\uparrow}$$

fermion $\qquad \psi_{\text{space}}\, \psi_{\text{spin}}\, \psi_{\text{flavour}} \times \psi_{\text{colour singlet}}$

Antisym $\quad = \quad$ Sym $\quad \times$ Antisym

- Now "see" colour everywhere e.g. τ decay



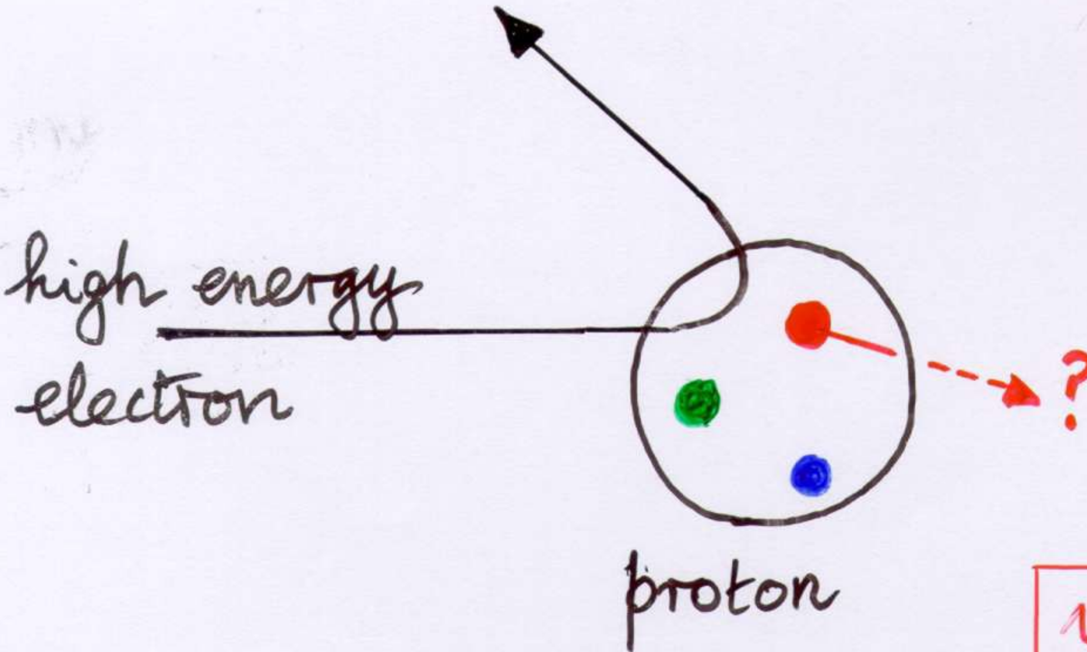
	e^-	μ^-	$\bar{u}$
	$\bar{\nu}_e$	$\bar{\nu}_\mu$	d
branching ratio ~	20%	20%	60%

3 colours

$SU(3)_c$ gauge thy

Explain confinement?

~ BIG PUZZLE ~



Quark acts as if free

yet not seen, (confined in colourless hadron)

Puzzle “explained” by QCD non-abelian gauge theory

$$\psi \rightarrow \psi\, e^{i\theta}$$ GLOBAL

Probability $|\psi|^2$ unchanged

physics "

Symmetry under ~~phase~~ gauge transformation

$$\psi \rightarrow \psi\, e^{i\theta(x)}$$ LOCAL

γ

need photon to reconcile phase differences

→ QED

massless

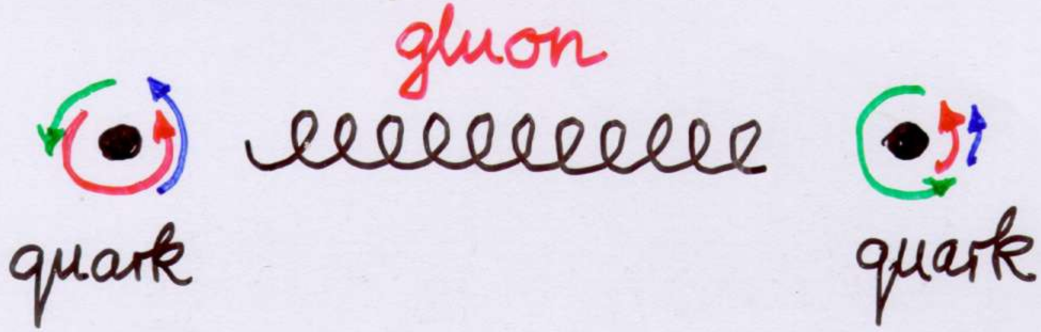
Empirical Laws → Maxwell's equations → QED → local gauge symmetry

Empirical Laws	Maxwell's equations	QED	local gauge symmetry
Coulomb's law Biot-Savart law Faraday's law Ampere's law	for $\vec{E}, \vec{B}$	photon: massless spin 1	U(1)

← ← ← Impose gauge symmetry on $\psi_{electron}$

$$\boxed{\text{ELECTROMAGNETIC FORCE} \longleftrightarrow \text{U(1) GAUGE SYMMETRY}}$$

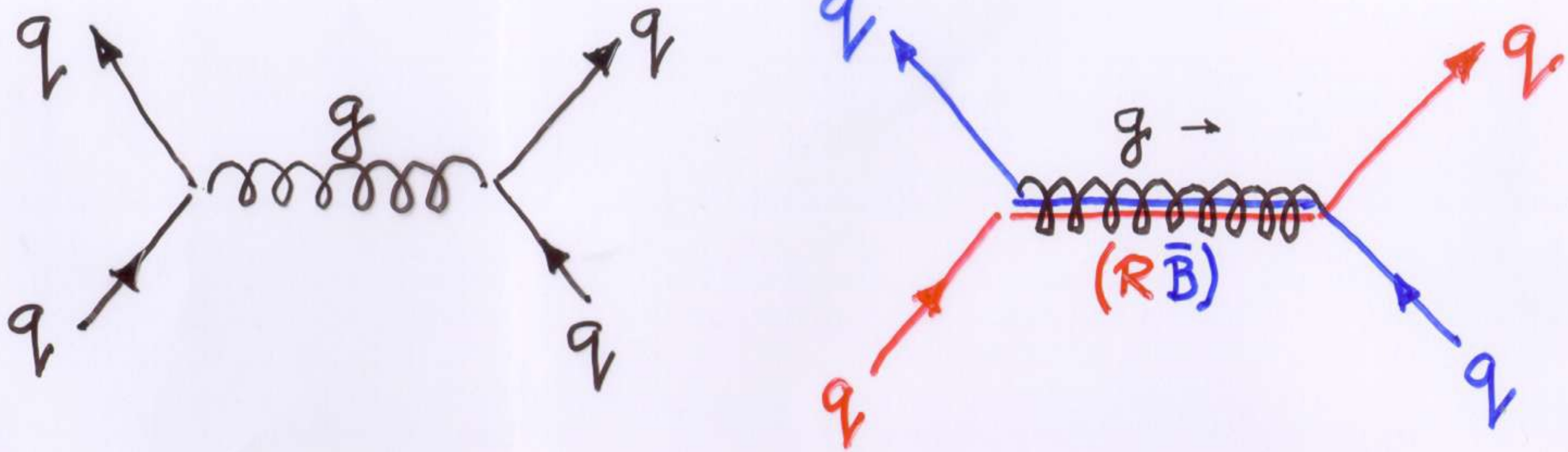
Quarks have 3 colour charges



To impose gauge symmetry we need 8 coloured gluons → QCD

massless

$SU(3)_{colour}$



8 possibilities: 8 coloured gluons $R\bar{B}$, $R\bar{G}$, ...

9th is colourless (colour singlet) : $\frac{1}{\sqrt{3}}\left(R\bar{R}+B\bar{B}+G\bar{G}\right)$

Coupling constant renormalisation

no scale in $\mathcal{L}_{QCD}$

Consider a dimensionless QCD observable R

Naive prediction for energies $Q \gg m_i$: $R \to$ const. indep of Q

Not true in renormalizable field theory like QCD (or QED)

Scale enters when calculate $R = \sum_n c_n \alpha_s^n$ since encounter (loop) Feynman diagrams which diverge logarithmically.

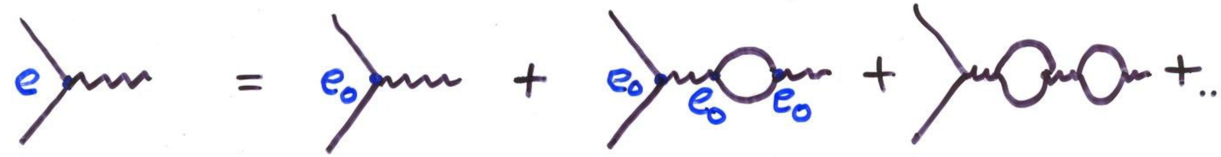
Need to renormalize (reparametrize) the theory

→ introduces a renormalization scale μ

→ $R\left(\log \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$

Running coupling

QED



physical or effective charge — bare charge — bare charge screened

At large Q^2

$$\alpha(Q^2) = \alpha_0 \left\{ 1 + \frac{\alpha_0}{3\pi} \log \frac{Q^2}{M^2} + \left(\frac{\alpha_0}{3\pi} \log \frac{Q^2}{M^2} \right)^2 + \dots \right\}$$

$$= \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \log \frac{Q^2}{M^2}}$$

Loops each give infinite contribution

M = cut-off on loop momentum

To eliminate dep. on M, introduce renormalisation scale μ

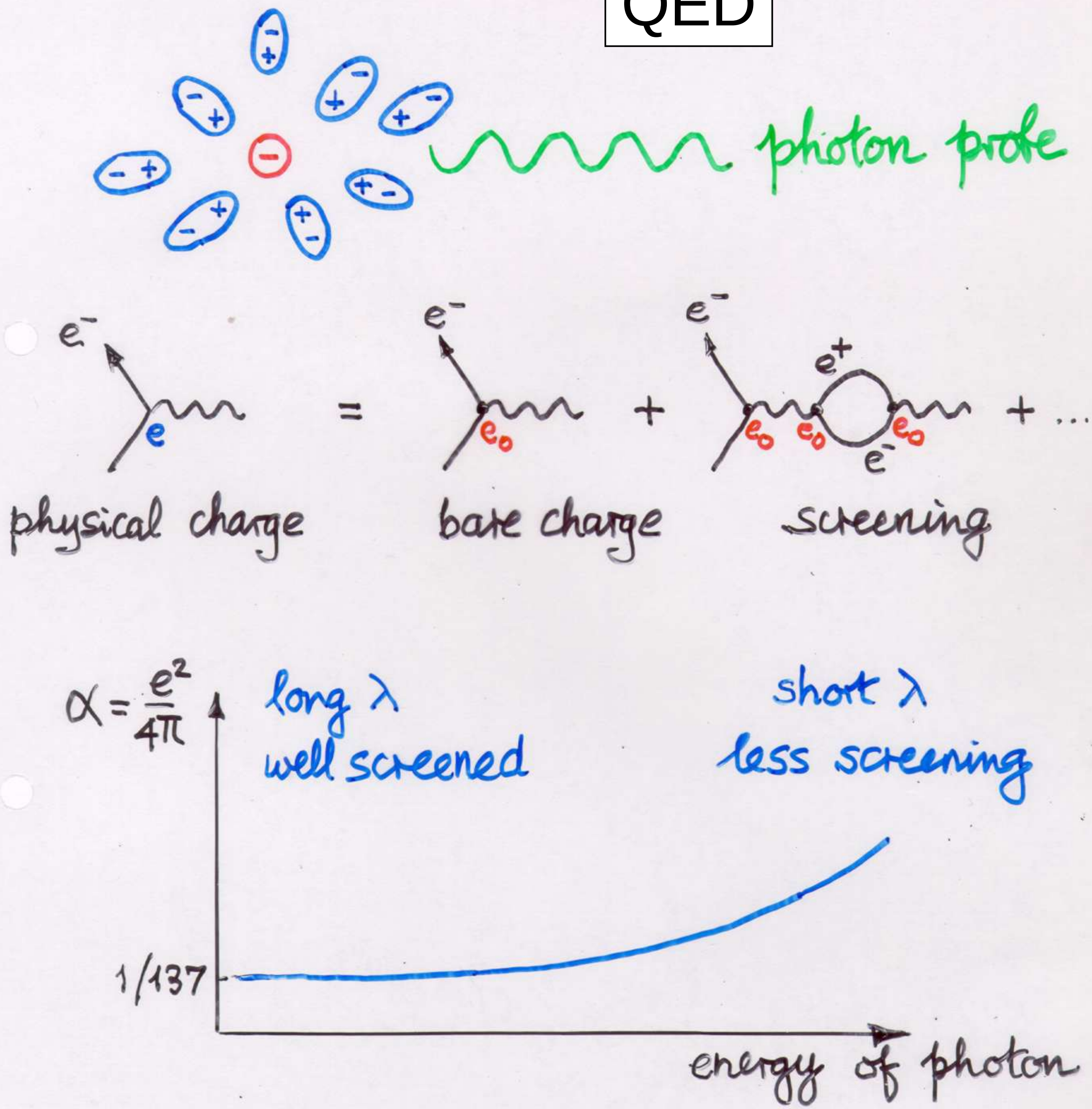
$$\left.\begin{aligned} \frac{1}{\alpha(Q^2)} &= \frac{1}{\alpha_0} - \frac{1}{3\pi} \log \frac{Q^2}{M^2} \\ \frac{1}{\alpha(\mu^2)} &= \frac{1}{\alpha_0} - \frac{1}{3\pi} \log \frac{\mu^2}{M^2} \end{aligned}\right\} \text{subtract}$$

$$\boxed{\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log \frac{Q^2}{\mu^2}}}$$

infinities removed at price of ren. scale μ

QED predicts running, expt. the absolute value.

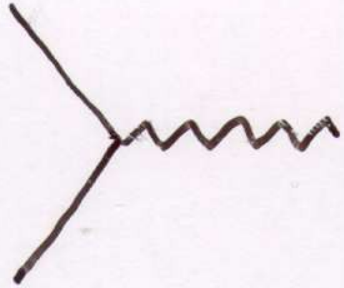
QED



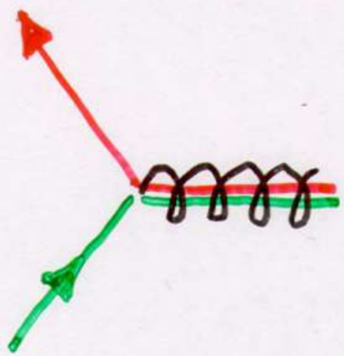
Vacuum is polarised

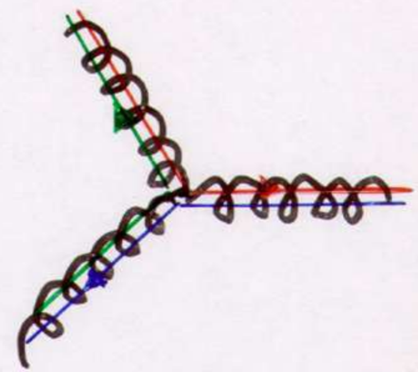
QED coupling runs

QED



QCD





VERY
INTERESTING

Note: UV divergences of 

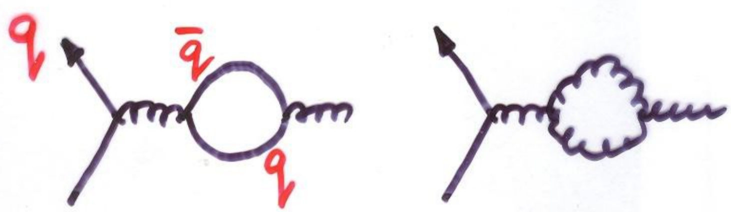
cancel via Ward identities (basic prop. of gauge FT's)

Just as well – so renormalized charge for $e, \mu, \ldots$ remains equal

QCD

Running of α_s

like $-\frac{1}{3\pi}$ of QED   New



$$b_0 = -\frac{n_f}{6\pi} + \frac{33}{12\pi}$$

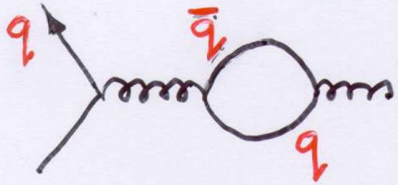
screening   antiscreening

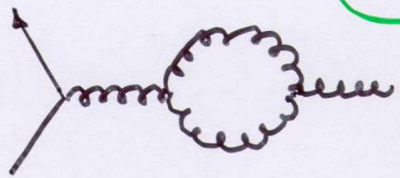
$$\boxed{\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0\, \alpha_s(\mu^2) \ln \frac{Q^2}{\mu^2}}}$$

Running of QCD coupling α_s

~ QED

NEW

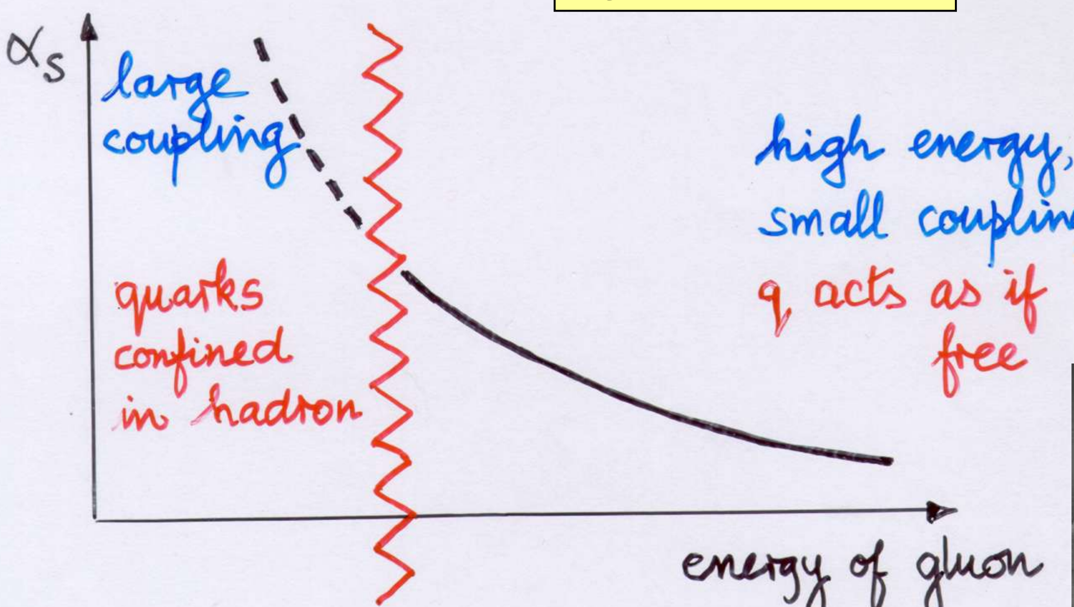




colour screening

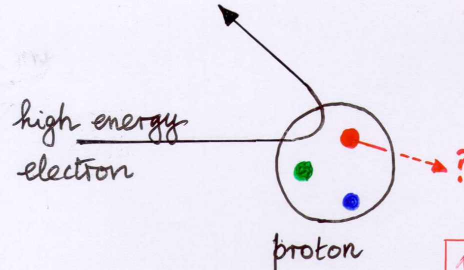
colour antiscreening

QCD -- 1973



A 'dream' theory !

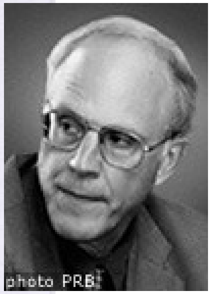
~ BIG PUZZLE ~

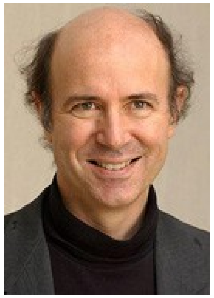


Quark acts as if free

yet not seen, (confined in colourless hadron)

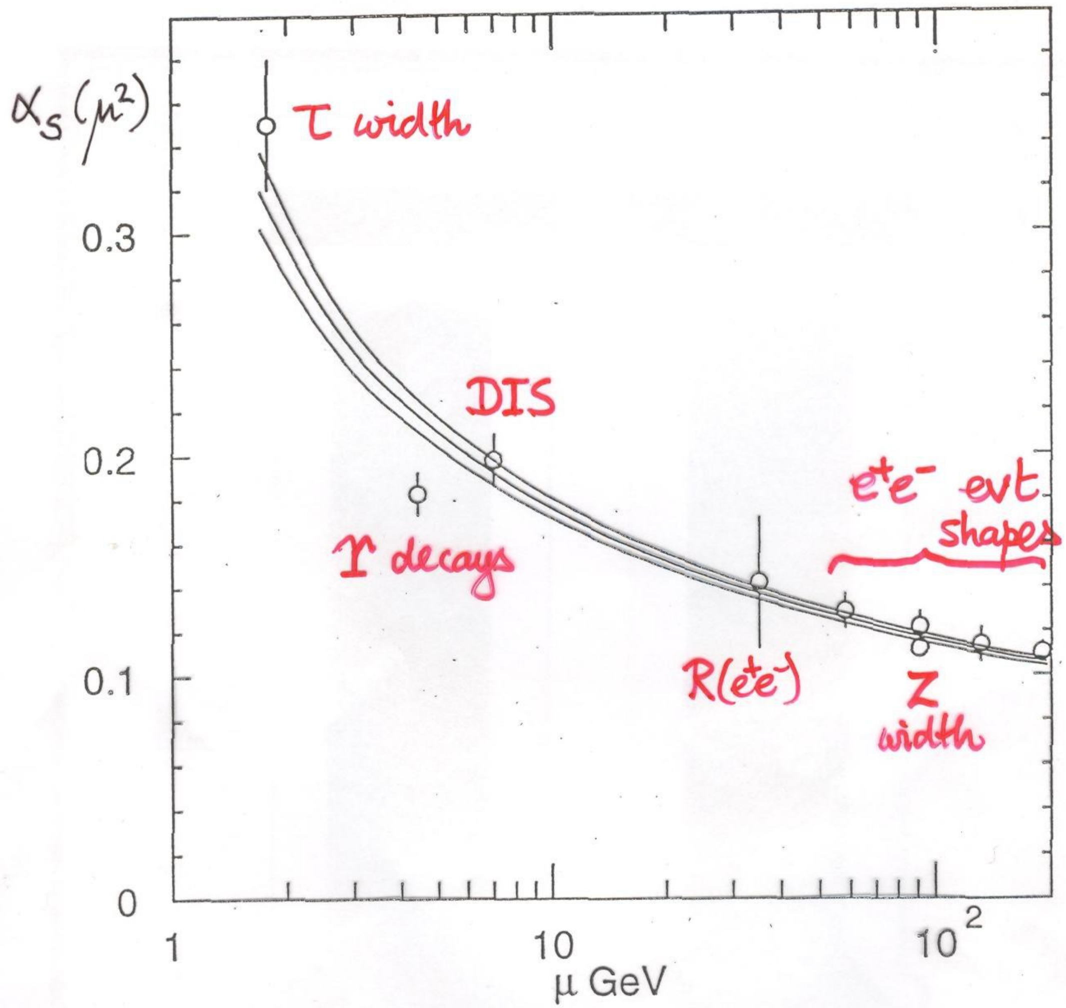
Gross Wilczek Politzer



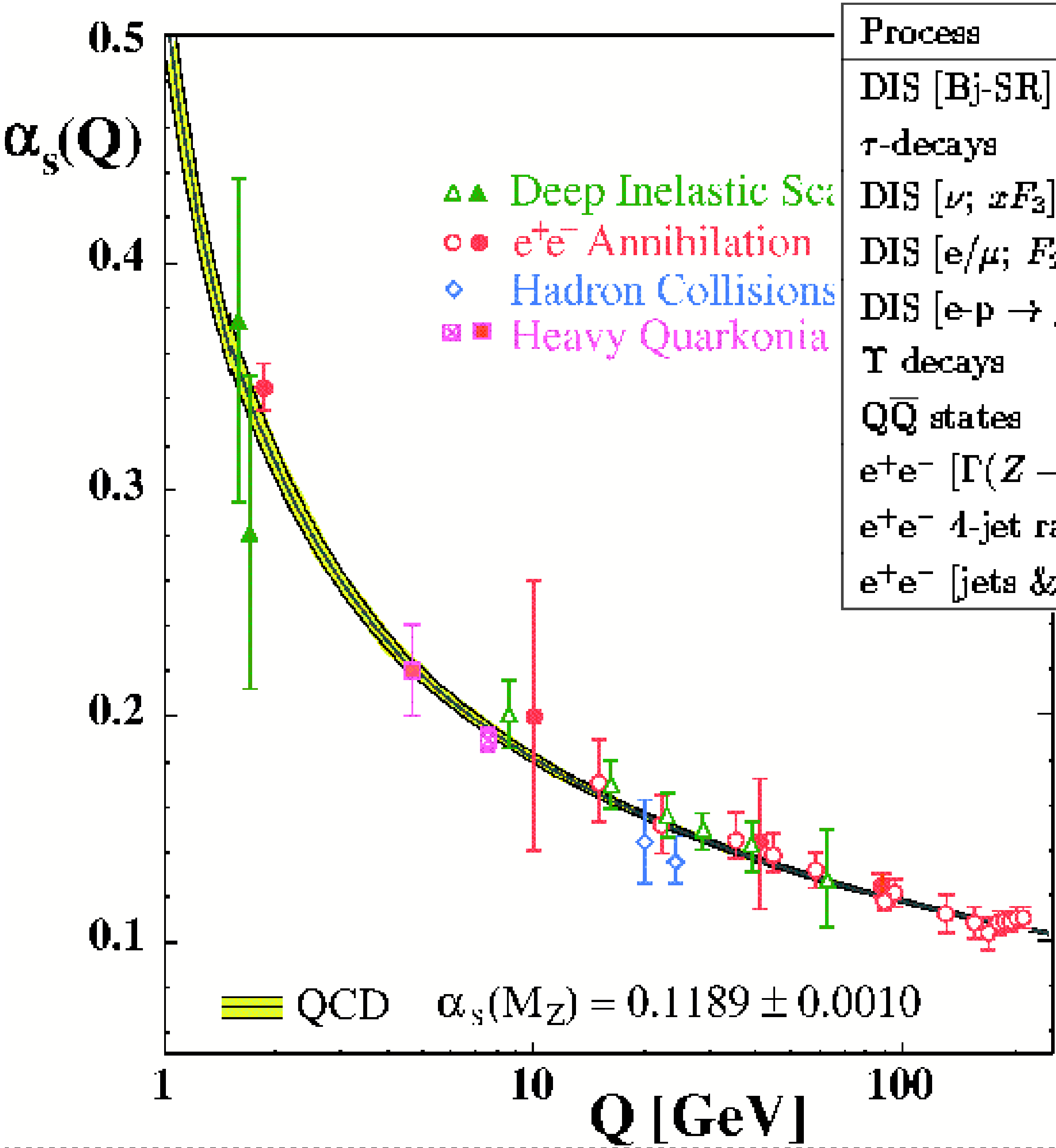


Nobel prize -- 2004

$$\frac{Z, \tau \to \text{hadrons}}{Z, \tau \to \text{leptons}} = R_{EW}\left(1 + \delta_{QCD} + \delta_{NP}\right)$$



curves correspond to $\alpha_s(M_Z^2) = 0.117_2 \pm 0.002$



Process	Q [GeV]	$\alpha_s(M_{Z^0})$
DIS [Bj-SR]	1.58	$0.121\ {}^{+\,0.005}_{-\,0.009}$
τ-decays	1.78	0.1215 ± 0.0012
DIS [ν; xF_3]	2.8 - 11	$0.119\ {}^{+\,0.007}_{-\,0.006}$
DIS [e/μ; F_2]	2 - 15	0.1166 ± 0.0022
DIS [e-p $\rightarrow$ jets]	6 - 100	0.1186 ± 0.0051
Υ decays	4.75	0.118 ± 0.006
$Q\overline{Q}$ states	7.5	0.1170 ± 0.0012
e^+e^- [$\Gamma(Z \rightarrow had)$]	91.2	$0.1226\ {}^{+\,0.0058}_{-\,0.0038}$
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022
e^+e^- [jets & shps]	189	0.121 ± 0.005

<u>Note</u>: UV divergences of 

cancel via Ward identities (basic prop' of gauge FT's)

Just as well – so renormalized charge for $e, \mu, \ldots$ remains equal

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Ward identities of QED → Slavnov-Taylor identities of QCD

$$\alpha_s(q\bar{q}g)_{\text{with loops}} = \alpha_s(ggg)_{\text{with loops}}$$

(gauge theory)

equality preserved by renormalisation

dimensionless $R \xrightarrow{\text{renormalisation}} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$

but R cannot depend on renorm scale μ : RGE $\quad \frac{\partial R}{\partial \log \mu^2} = 0$

can show this gives $R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = R(1, \alpha_s(Q^2))$

$\underbrace{\text{running of } \alpha_s}$ determines Q dep of R

$$\frac{\partial \alpha_s}{\partial \log \mu^2} \equiv \beta(\alpha_s)$$

$\boxed{\text{1 loop } \beta \text{ function}}$ so far only summed LL (i.e. $\left(\alpha_s \log \frac{Q^2}{\mu^2}\right)^n$)

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0 \alpha_s(\mu^2) \log \frac{Q^2}{\mu^2}}$$

$$b_0 = -\frac{n_f}{6\pi} + \frac{33}{12\pi}$$

$$\frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s(\mu^2)} + b_0 \log \frac{Q^2}{\mu^2}$$

$$0 = -\frac{1}{\alpha_s^2(\mu^2)} \frac{d\alpha_s}{d\log\mu^2} - b_0 \quad \Rightarrow \quad \boxed{\beta(\alpha_s) = -b_0 \alpha_s^2}$$

$\left[\frac{dg}{d\log\mu^2} \sim g^3 \right.$
$\left. \therefore \frac{d\alpha_s}{d\log\mu^2} \sim g^4 \sim \alpha_s^2 \right]$