LECTURES ON THE GAUGE/GRAVITY DUALITY

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New Trends in High Energy Physics and QCD, IIP-UFRN, Natal, Oct. 2014

OUTLINE OF THE LECTURES

LECTURE 1: What is Holography?

LECTURE 2: The Holographic Dictionary

LECTURE 3: Applications to the Quark-gluon plasma

*Please take a look at the excellent review A. Adams et al., 2012

LECTURE I: What is Holography ???

OUTLINE OF THIS LECTURE

1- Overview

2- Holography as seen by the Einsteinians

3- Holography as seen by the QFTonians

4- Summary

USP

1- Overview

Holography is a duality between QFT and gravity

HOLOGRAPHY

Maps quantum many body physics to classical dynamics of black hole horizons in one higher dimension

Replaces quasiparticles with geometry as the effective d.o.f.

When QFT is strongly coupled, new weakly coupled d.o.f. in the gravity theory emerge.

Emergent fields in the theory of gravity live in a dynamical spacetime with an extra dimension.

This extra dimension plays the role of an energy scale in the QFT with the motion along the extra dimension providing a geometric representation of the QFT's renormalization group (RG) flow.

Quantum many body physics problems, such as thermodynamics and transport phenomena, become equivalent to problems in classical gravity.

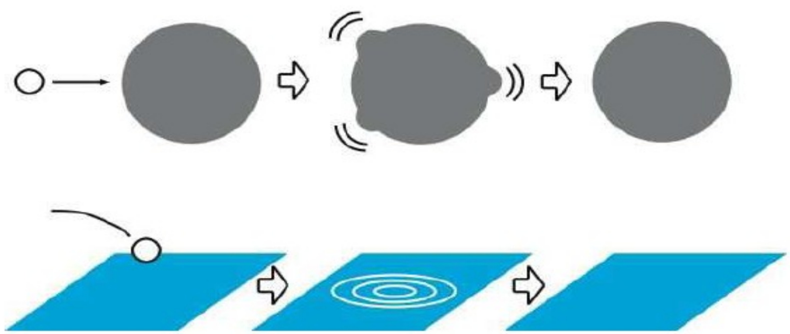
Universal black hole phenomena are mapped into universal behavior in QFT's

Example:

KSS, PRL 2005

Universality of black hole horizons → **HOLOGRAPHY** → Universality of transport coefficients in QFT

Black hole in the higher dimensional theory of gravity



4-dimensional QCD-like theory in equilibrium

The dissipation of sound waves in the gauge theory and the dissipation of these horizon disturbances are controlled by the same parameter

Shear viscosity/entropy density =

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B} \sim 0.08$$

(much more on this later)

- Holography gives us a completely new way to think about QFTs.

- Holography gives us a recipe for using classical gravity in d+1-dimensions to compute quantum amplitudes that satisfy the expected consistency conditions of a d-dimensional QFT (locality, causality, Poincare invariance, ...).

When is Holography applicable????

Holography becomes useful (or under control) when:

I) The coupling of the QFT, λ , is $\lambda \gg 1$

II) The number of d.o.f. /volume, N, is very large, i.e., N >> 1.

$\lambda \gg 1$

d-dimensional QFT
with d.of./vol $\rightarrow \infty$

→ HOLOGRAPHY →

$\sim 1/\lambda \ll 1$

d+1-dimensional
classical gravity

In this case we can take the gravitational description as the definition of the QFT !!!!

- Holography can then be used to define strongly coupled many-body systems (replacing quasi-particles by geometry as the main guiding principle).

- While supersymmetry and conformal invariance were of fundamental importance for the original discovery of holography, the duality itself does not depend on these concepts.

- In these lectures I will take the point of view that holography is a property shared among QFTs and quantum gravity.

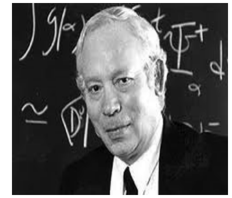
- Applications of holography nowadays can be found in heavy ion collisions, hadronic physics, condensed matter physics, and etc.

Beautiful … it all sounds too good to be true.

The question you should be asking yourself now is:

Why should any of this be true?????

Can one really get gravitons out of gauge theories with spin 1 and spin 1/2 fields?




An unexpected bump on the road … the Weinberg - Witten theorem !!!

LIMITS ON MASSLESS PARTICLES

Steven WEINBERG and Edward WITTEN
Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Received 6 August 1980

We show that in all theories with a Lorentz-covariant energy–momentum tensor, such as all known renormalizable quantum field theories, composite as well as elementary massless particles with $j > 1$ are forbidden. Also, in all theories with a Lorentz-covariant conserved current, such as renormalizable theories with a symmetry that commutes with all local symmetries, there cannot exist composite or elementary particles with nonvanishing values of the corresponding charge and $j > 1/2$.

Theorem 1. A theory that allows the construction of a Lorentz-covariant conserved four-vector current J^{μ} cannot contain massless particles of spin $j > 1/2$ with nonvanishing values of the conserved charge $\int J^0 \, d^3x$.

Theorem 2. A theory that allows the construction of a conserved Lorentz covariant energy–momentum tensor $\theta^{\mu\nu}$ for which $\int \theta^{0\nu} \, d^3x$ is the energy–momentum four-vector cannot contain massless particles of spin $j > 1$.

Thus we conclude that all these theories have no massless bound states with $j > 1$, and quantum chromodynamics has no flavor nonsinglet massless bound states with $j > 1/2$.

Can gravitons be like a massless bound state of spin 1 fields in a QCD-like theory? NO!!!!!

“There are trivial truths and the great truths.
The opposite of a trivial truth is plainly false.
The opposite of a great truth is also true”.

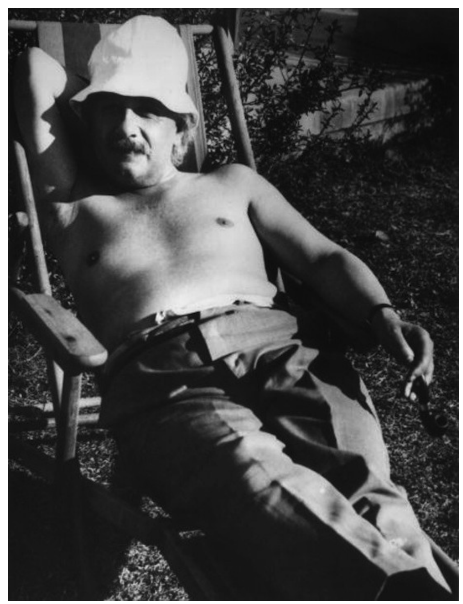
- Note that in holography the theory of gravity and the QFT are defined in spacetimes of different dimensionalities. This is the way out of Weinberg-Witten's theorem (also we will be dealing with an infinite number of d.o.f.). We are safe to proceed!

- In the following I will present a few hand waving arguments that will be used to “convince” you that holography should indeed exist in a certain limit!

Holography as seen by the Einsteinians

Imagine that there is a planet in a d+1 dimensional universe called Einsteinia* where its inhabitants, who call themselves Einsteinians, know EVERYTHING about general relativity in d+1 dimensions but somehow they are not yet aware about the existence of quantum field theory (they cannot solve a single exercise in Peskin-Schroeder's book).

Photo of a typical Einsteinian



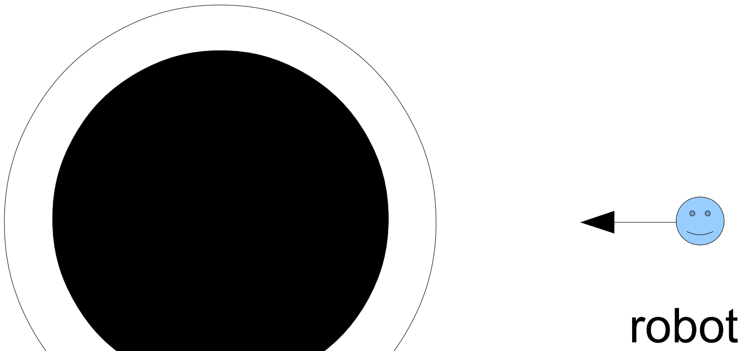
*Not to be otherwise confused with the plural form of the chemical element with Z=99.

The Einsteinians worked for years in an experiment called

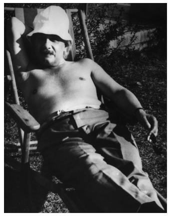
LHC = Large Horizon Collider

which basically consisted in throwing robots towards the horizon of a black hole and study their fate.

They first send a robot which has a device that flashes light once a second (I think this argument is originally due to Susskind).



Black hole horizon

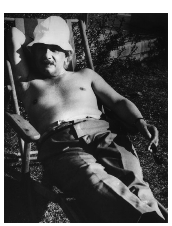


Einsteinian observer at infinity

- According to the robot, it will reach the horizon in a finite proper time (as measured by its internal clock) and it will have emitted light a finite number of times.

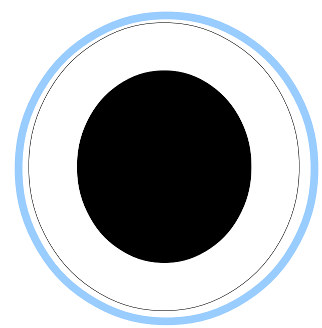
- However, for the Einsteinian at infinity, the story is different. The light emitted by the falling robot becomes increasingly redshifted and the flashes reach him at ever greater intervals. As the robot approaches the horizon, the redshift diverges and the light from the robot after it crossed never reaches the Einsteinian.

- Actually, the Einsteinian would say that the robot did not fall through the horizon. For him, the robot gradually slowed down as it approached the horizon and it was compressed into a thin membrane on the surface of the black hole horizon (it is like this squeezed robot had stopped interacting with gravity at all).



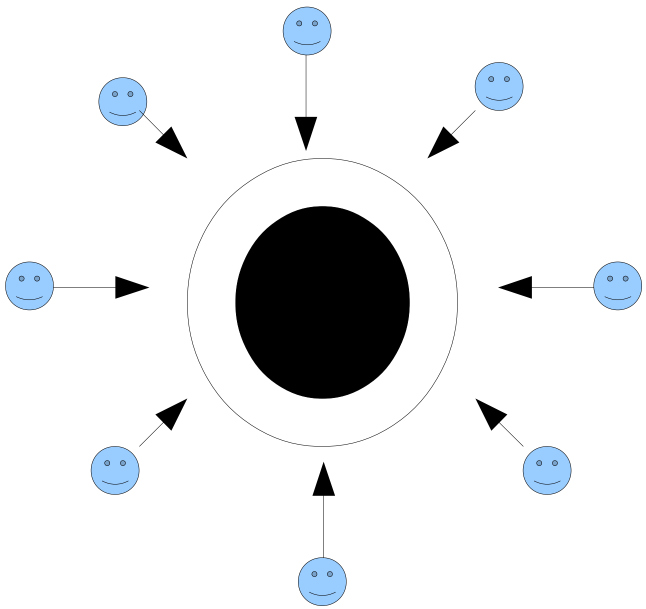
Einsteinian observer at infinity sees a

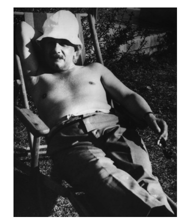
“robot membrane”



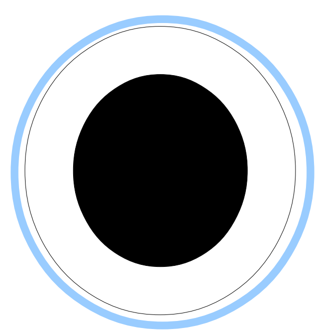
Black hole horizon

Now imagine that they send a bunch of robots at once in a spherical manner



What will  say?

As the robots reach the horizon, they will be compressed into a very thin membrane above the black hole horizon.



One may call this a “stretched” horizon.

- The mass of the black hole increases by the amount of matter and energy given by the robots. However, to the Einsteinian, this shell of matter never actually crosses the horizon.

- As each new bit of matter reaches this stretched horizon, it generates a disturbance in this membrane which spreads in waves through the stretched horizon as if it were a fluid flowing in the absence of gravity defined in d-dimensions, not in d+1 dimensions!

- With the Large Horizon Collider, the Einsteinians back in Einsteinia have discovered this new thing called “fluid” whose dynamics is only d-dimensional and it does not seem to couple to gravity!!!!!

- Now, what do the robots see? The robots will continue to interact with each other and with the gravitational field of the black hole as they fall through it. For them, everything can be naturally modeled using GR in d+1 dimensions (before they reach the singularity, of course)!!!!

- The observer at infinity (Einsteinian) and the in-falling observer (robot) cannot observe contradictory events unfold. Therefore, the Einsteinians would conclude that:

HOLOGRAPHY = The gravitational dynamics of a d+1 black hole must be somehow equivalent to the dynamics of a d-dimensional “fluid” in the absence of gravity !!!!

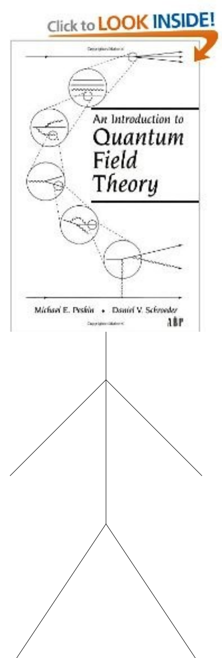
This is how they would be forced to think about field theories without gravity and they would, eventually, be able to understand this NEW AND EXCITING thing, ordinary field theory in flat space, using their well known toolkit from GR.

Ok, now, let's visit another planet called … QFTonia !!!!

Welcome to QFTonia!

The inhabitants of QFTonia, called QFTonians, know a lot about QFT in flat spacetime but they are not aware of a way to understand why “things fall” (they do not know how to solve a single exercise from Misner-Thorne-Wheeler's famous gravitation book).

Photo of a typical QFTonian



PS: All the QFTonians absolutely love the renormalization group.

A brief reminder about the Renormalization Group (RG)

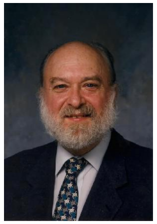
Consider a system on a lattice with spacing “a” and Hamiltonian given by

$$H = \sum_{x,i} J_i(x)\mathcal{O}^i(x)$$

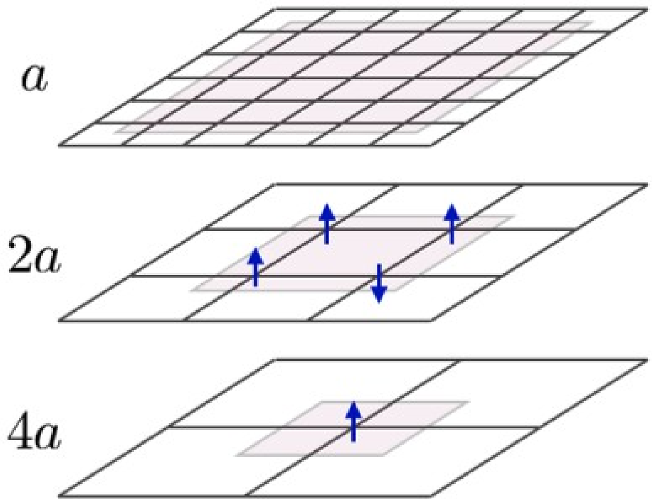
$\mathcal{O}^i(x)$ Operators located at each site x

$J_i(x)$ Coupling constants/sources for the operators

Given this we would like to understand the physics of the ground state and the low energy excitations around it as a function of the microscopic coupling constants computed at the lattice scale “a”. We all know that this is in general very hard to do.



Two very smart QFTonians, let's call them Kadanoff and Wilson, figured out a way to attack this problem: the renormalization group !!!!



$$H = \sum_i J_i(x, a)\, \mathcal{O}^i(x)$$

$$H = \sum_i J_i(x, 2a)\, \mathcal{O}^i(x)$$

$$H = \sum_i J_i(x, 4a)\, \mathcal{O}^i(x)$$

The basic idea is to iteratively coarse grain the lattice, making the lattice spacing larger, so that at each step a single site represents the average of multiple sites from the previous lattice. Every time we do that, we readjust the couplings to preserve the physics of the ground state and the low energy excitations around it.

This is why the couplings $J_i(x)$ are replaced by the scale dependent couplings $J_i(x, u)$ defined at the scale “u”.

One can show that the resulting flow of the couplings with the scale “u” can be described by the equation

$$u\frac{\partial}{\partial u}J_i(x,u) = \beta_i(J_j(x,u),u)$$

β_i are the beta functions.

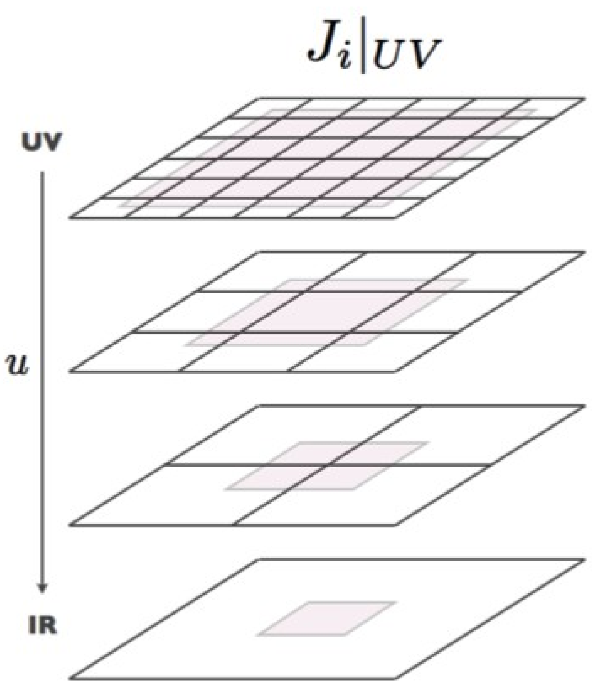
- Fixed points of the RG flow correspond to scale invariant (or conformal field theories).
QCD has a trivial fixed point in the UV (asymptotic freedom – no interactions).
Some condensed matter systems have non-trivial fixed points (i.e., strongly coupled conformal field theories).

One can in fact define a scale dependent QFT by going to its fixed point in the UV and add relevant operators to the Hamiltonian to induce a running of the RG flow towards the IR.

There is no free lunch in QFTonia, however, since β_i are extremely hard to compute in general ...

However, perhaps rearranging the previous plot may give us some insight ...

A. Adams et al., 2012



The extra parameter "u" in $J_i(x, u)$

can be "thought" of as a type of coordinate so then this scale dependent coupling looks like a field defined on a lattice with one extra dimension.

Taking this analogy a bit further, is there a conventional local field theory in this higher dimensional spacetime that can be used to find the beta functions of the original theory?

The QFTonians want to find a simple QFT defined in (d+1) dimensions for the

bulk fields $\Phi_i(x, r)$ that are related to the couplings of the original d-dimensional theory in the UV, i.e.,

$$\Phi_i(x, a) = J_i(x)$$

As a consequence, the bulk field $\Phi_i(x, r)$ must have the same charges, tensor structure, and other quantum numbers possessed by the couplings $J_i(x)$

Therefore, by classifying the different operators in the d-dimensional theory according to their Lorentz structure they would see that

d-dimensional theory			d+1-dimensional theory	
Scalar operator	$\mathcal{S}(x)$	➡	$\Phi(x,r)$	Bulk scalar field
Current operator	$\mathcal{C}^{\mu}(x)$	➡	$A_{\mu}(x,r)$	Bulk spin 1 field
Tensor operator	$\mathcal{T}^{\mu\nu}$	➡	$g_{\mu\nu}(x,r)$	Bulk spin 2 field

Such that $\Phi(x,a)\mathcal{O}(x) = J(x)\mathcal{O}(x)$ is added to the d-dimensional Hamiltonian and so forth.

But what is the Lagrangian of this d+1 dimensional theory of bulk fields ???
This is the point where the idea of an effective theory comes to the rescue!!!

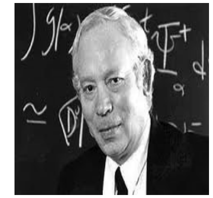
Note that any QFT defined by a perturbation from a fixed point must include among its operators the energy-momentum tensor $T^{\mu\nu}$

which is the conserved Noether charge associated with the spacetime symmetry group.

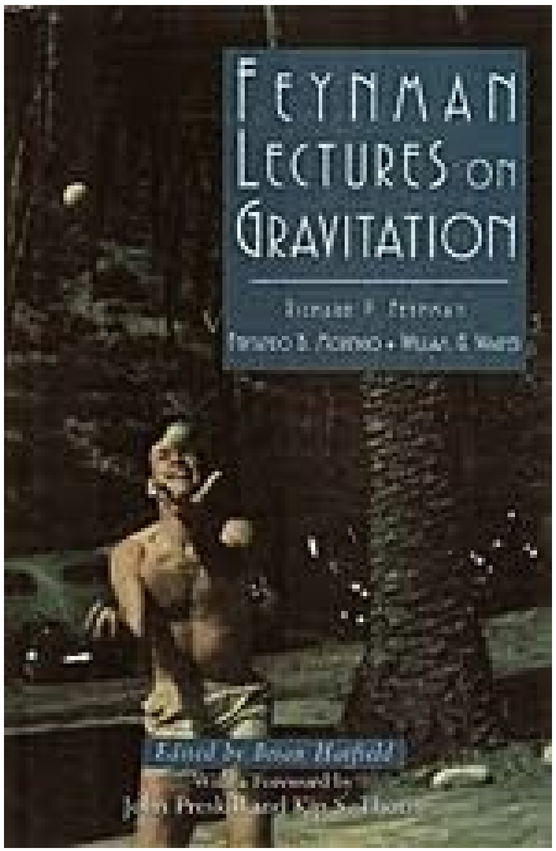
Therefore, the QFTonians would say that there must be a spin 2 field $g_{\mu\nu}(x,r)$

in the bulk QFT. However, any dynamical theory of a massless spin 2 field must be, at low energies, given by GR !!!! This is how the QFTonians would then discover GR in d+1-dimensions starting from QFT in a d-dimensional flat spacetime!!!!

In fact ...



Any theory for an interacting massless spin 2 field must be equivalent to GR at low energies (or decouple becoming a topological theory) !!!!



Indeed, let's see how that works!

Remember, the QFTonians know field theory but they have never seen the Einstein-Hilbert action.

They then try to write down the simplest theory they can for a spin 2 fluctuating field

$$h_{\mu\nu}$$

to try to quantize it (as they did it with QED, for example) by introducing creation and annihilation operators

$$a^{\dagger}_{\mu\nu}, a_{\mu\nu}$$

These operators would obey the commutation rules

$$[a_{\mu\nu}, a^{\dagger}_{\rho\sigma}] \sim \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}$$

$$\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, \ldots, +1)$$

However, this would imply that $a^{\dagger}_{0i}|0\rangle$ possesses negative norm!!!

A smart QFTonian would immediately jump up and say: Oh, but that is easy to fix! We have done just that in QED!!!!

Indeed, if you remember your old QFT lectures about the canonical quantization of a spin 1 field, you would remember that this problem with states of negative norm also appears and that gauge invariance in that case came to the rescue.

A very similar mechanism would also work here, a QFTonian would say.

Indeed, a very smart QFTonian, let's call him Richard, would soon figure out that imposing a type of gauge transformation of this kind for the spin 2 field

$$h_{\mu\nu} \longrightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

for any function $\xi_\mu(x)$, the unphysical states would decouple from the spectrum.

Then, they would also rapidly realize that the simplest non-interacting theory involving the field that preserves diffeomorphism invariance is

$$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x \left[\partial_\mu h^\rho{}_\rho \partial_\nu h^{\mu\nu} - \partial^\rho h^{\mu\nu} \partial_\mu h_{\rho\nu} + \frac{1}{2} \partial_\rho h_{\mu\nu} \partial^\rho h^{\mu\nu} - \frac{1}{2} \partial_\mu h^\nu{}_\nu \partial^\mu h^\rho{}_\rho \right]$$

where M_{pl} is an energy scale.

What happens when we include interactions? The first thing one must realize is that this symmetry

$$g_{\bar{\mu}\bar{\nu}} = g_{\alpha\beta} \frac{\partial x^\alpha}{\partial \bar{x}^\mu} \frac{\partial x^\beta}{\partial \bar{x}^\nu} \qquad h_{\mu\nu} \longrightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

must also be present in the interacting theory. They would be led to define diffeomorphism invariance and then the simplest theory that fulfills this symmetry and contains 2 derivatives of the field

$$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\, R + \ldots$$

➡ General Relativity !!!!!

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha{}_{\mu\nu} - \partial_\mu \Gamma^\alpha{}_{\alpha\nu} + \Gamma^\alpha{}_{\mu\nu} \Gamma^\beta{}_{\alpha\beta} - \Gamma^\alpha{}_{\beta\mu} \Gamma^\beta{}_{\alpha\nu} \qquad \Gamma^\lambda_{\mu\nu}(g) = \frac{1}{2} g^{\lambda\kappa} \left(\partial_\mu g_{\kappa\nu} + \partial_\nu g_{\kappa\mu} - \partial_\kappa g_{\mu\nu} \right)$$

Therefore, the QFTonians would discover GR by studying the RG flow of QFTs !!!

- These QFTonians would then say that there is a duality at play in here: one has two equivalent descriptions of the same physical phenomena.

- In fact, on one hand one could use the original lattice theory with lattice spacing “a” and sources $J_i(x)$ which satisfy first order beta-function equations.

- On the other hand, they could also say that one could use this “recently discovered” GR theory that uses d+1 dimensional bulk fields $\Phi_i(x, r)$ that are in direct correspondence with the d-dimensional field theory sources $J_i(x)$

- In other words, there are two ways to compute the correlation functions of the system: one may work in the lattice theory and solve the QFT problem or one may work in the gravitational description and solve a GR problem.

Ok, but there are a few things that the QFTonians still think are weird …

First, the entropy computed in the two descriptions must be the same.

For the original QFT in d-dimensions, the entropy will be extensive, i.e., it will scale with the d-dimensional volume of the system

$$S_{QFT} \sim V_d$$

However, since the bulk theory lives in d+1-dimensions, the entropy of this bulk theory must not scale with the d+1-dimensional volume but with its boundary, a d-dimensional area.

Well, this is a basic feature about the entropy of black holes, isn't it? So, the QFTonians have started to discover fundamental features of these “newly” found very weird objects (which we call black holes) !!!!

- There is another aspect of this duality that may be bothering the QFTonians …

- The beta function equations in the boundary lattice theory are first-order in the RG scale “r”. Physically, this means that when you specify the couplings in the d-dimensional theory at one scale, this completely determines the coupling at all other scales (i.e., this fixes the RG trajectory).

- On the other hand, the gravitational system in d+1-dimensions possesses 2 solutions due to the second order nature of the Einstein's equations. How can these two descriptions now be equivalent ????

- Somehow the gravitational system must choose only one of these solutions as physical … again, this is nothing but the physics of black holes at work !!!!

Imagine that you are solving a simple wave equation near a black hole …

- There are two solutions to the second order wave equation.

- One can show that (in Euclidean signature) that only the in-falling solution towards the horizon is regular at the horizon while the out-going solution isn't.

- This reflects the fact that things that fall into a black hole horizon cannot escape afterwards (classically).

- Therefore, while the gravitational field equations are indeed 2^{nd} order, the presence of a black hole horizon effectively adds a second boundary condition and, thus, we only need to specify a single boundary condition to completely determine the solution.

- Since the location of the horizon encodes the thermodynamic variables (temperature, entropy) of the system, the physical meaning of this second boundary condition is to specify the state of the dual QFT.

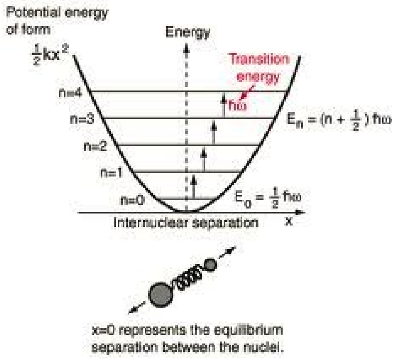
- Therefore, we see that the QFTonians would have discovered GR and the fundamental properties of black holes through holography.

- Alternatively, the Einsteinians would have then discovered the existence of QFT in flat space by just using their knowledge of GR and the physics of black holes.

- As remarked by Stroeminger (I think):

Black holes are the harmonic oscillators of the 21st century!!!

20th century



21th century

Summary

"Give me a classical black brane and I will give you a quantum many-body strongly coupled system".

- There should be a connection between QFT in d-spacetime dimensions and quantum gravity in d+1-dimensions.

- The fields in the gravitational system are exactly paired with sources in the dual QFT.

- The extra spatial coordinate in the gravitational bulk theory plays the role of an RG scale for the QFT.

- In the next lecture we will define this holographic duality more precisely and discuss how to describe QFTs at finite temperature within this setup.

LECTURE II: The Holographic Dictionary

OUTLINE OF THIS LECTURE

1- Guessing the gravity dual of a CFT

2- Holography and thermodynamics

3- On shell gravity action as the generating functional of QFT

4- Summary

- When you are learning new concepts, it is always better to simplify the subject as much as you can to extract the main idea first and then work out the details later on.

- Although our previous argument for the existence of the holographic mapping seemed to work for very general QFTs, let's start with the simplest kind you can imagine in 4 dimensions.

- The simplest (interacting, Poincare invariant, though still generic) QFT that one can think of is the one that comes up at an RG fixed point. At a fixed point the QFT beta function vanishes and the theory is scale invariant (and in general also conformal invariant).

Conformal Transformations

Remember that under general coordinate transformations   x → x'

$$g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x') = g_{\lambda\rho}(x)\,\frac{\partial x^{\lambda}}{\partial x'^{\mu}}\,\frac{\partial x^{\rho}}{\partial x'^{\nu}}$$

The conformal group is then defined as the group that leaves the metric invariant up to an arbitrary space dependent factor

$$g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x') = \Omega(x)\, g_{\mu\nu}(x)$$

If the 4d Lorentz group is $SO(1,3)$ the conformal group is $SO(2,4)$

Conformal Group

Generators: $P_\mu = -i\partial_\mu, \qquad M_{\mu\nu} = i\,(x_\mu\partial_\nu - x_\nu\partial_\mu)$ = Poincare group

Scale transformations

$$x^\mu \to C\,x^\mu$$

$$D = -i\,x\cdot\partial$$

Special conformal transf.

$$x^\mu \to \frac{x^\mu + a^\mu x^2}{1 + 2x^\mu a_\mu + a^2 x^2}$$

$$K_\mu = -i[x^2\partial_\mu - 2x_\mu(x\cdot\partial)]$$

Algebra

$$[M_{\mu\nu}, P_\rho] = -i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu)$$

$$[M_{\mu\nu}, K_\rho] = -i(\eta_{\mu\rho}K_\nu - \eta_{\nu\rho}K_\mu)$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i\eta_{\mu\rho}M_{\nu\sigma} \pm permutations$$

$$[P_\mu, K_\nu] = 2iM_{\mu\nu} - 2i\eta_{\mu\nu}D$$

$$[D, K_\mu] = iK_\mu \qquad [D, P_\mu] = -iP_\mu$$

$$[D, M_{\mu\nu}] = 0$$

So, what do we need then?

- List of local operators in the CFT $\mathcal{O}_i$ labeled by their Lorentz structure, their charges, and UV scaling dimensions Δ_i defined at the fixed point.

- To generate the RG flow of interest, we perturb the system away from this fixed point by turning on appropriate sources J_i with which we construct the generating functional

$$Z_{\mathrm{QFT}}[J_i(x)] = \langle e^{\int dx^d J_i(x)\mathcal{O}_i(x)} \rangle \quad \longrightarrow \quad \langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{\delta^n \ln Z_{\mathrm{QFT}}[J(x)]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\Big|_{J_i=0}$$

At the fixed point then the geometry in the bulk theory should be very special to encode all the symmetries present in the QFT conformal group.

Indeed, the bulk geometry that describes the 4d CFT state must have:

- At least 5 dimensions (where “r” is the extra holographic coordinate).
- 4d translations, rotations, and boosts as isometries for each “r” slice.
- Rotations and boosts that mix up “r” with the other coordinates.
- Scaling transformations $(r, \vec{x}, t) \to (\alpha r, \alpha \vec{x}, \alpha t)$

The only bulk geometry that fulfills all of these requirements is AdS5.

Maldacena, 1998

AdS5: $ds^2 = \frac{L^2}{r^2}\left[-dt^2 + d\vec{x}^2 + dr^2\right]$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu}$$

Einstein's equations

$$\Lambda = -\frac{d(d-1)}{2L^2}$$

Negative cosmological constant

L

AdS radius

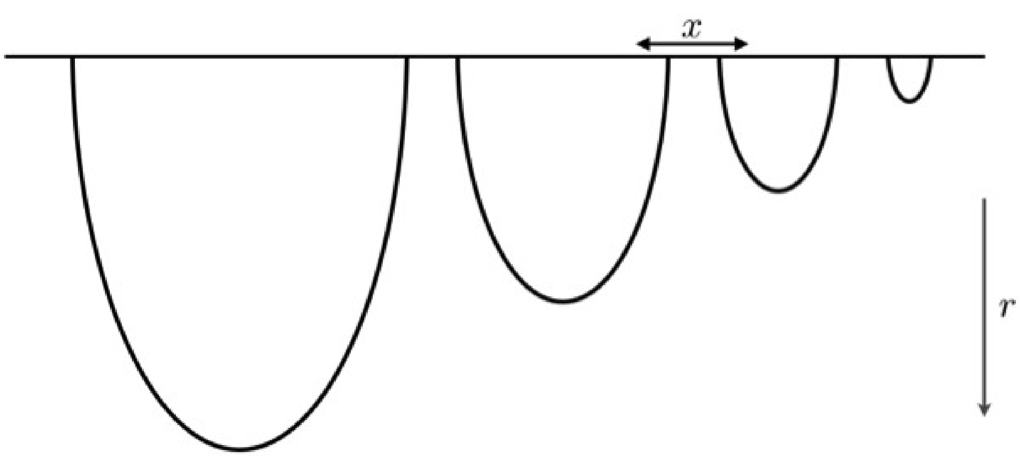
$R \sim \frac{1}{L^2}$

Ricci scalar

- The full isometry of AdS5 is identical to the conformal group in D=4.

- You can think about the negative curvature in AdS as a “closed box” for gravity that is however still homogeneous and isotropic (there is no center to which objects can return).

- In fact, if you are inside AdS sitting at an arbitrary point and send a photon, that photon will run away to the boundary at infinity and return to you in a finite amount of time. Nothing gets out of the AdS box.



Boundary $r \to 0$

UV physics in the gauge theory

IR physics in the gauge theory ~ far from boundary in the bulk

Another cool thing about AdS space is that it is a solution to the equations of motion of a generic Wilsonian action for the metric

$$I_{\text{Gravity}} = \frac{1}{16\pi G_N} \int d^{d+1}x \sqrt{-g} \left(-2\Lambda + R + c_2 R^2 + c_3 R^3 + \ldots\right)$$

Higher order derivatives

G_N Gravitational constant

$g = det(g_{\mu\nu})$

$R \sim \partial^2 g$ Ricci scalar

GR consists of keeping only the terms with at most 2 derivatives, which gives the EOM

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu}$$

The AdS metric solves the Einstein equation with radius L determined by

$$\Lambda \sim 1/L^2$$

- One can show that AdS remains to be a solution even when higher order corrections are included.

- Also, since $R \sim 1/L^2$ note that our Wilsonian action for quantum gravity

$$I_{\text{Gravity}} = \frac{1}{16\pi G_N}\int d^{d+1}x\,\sqrt{-g}\,\left(-2\Lambda + R + c_2 R^2 + c_3 R^3 + \ldots\right)$$

becomes classical GR when $L/\ell_{planck},\ \ L/\ell_{string} \gg 1$

Is there a gauge theory in D = 3+1 which is conformally invariant (after quantization)?

YES!!!! $\mathcal{N} = 4$ SU(Nc) Supersymmetric Yang-Mills

- 16 supercharges + extra 16 due to conformal invariance.
- SU(4) R-symmetry (rotates the scalars and the fermions).
- Global SO(6) symmetry.
- Field content: massless

A^a_μ	ψ	ϕ^I	$I = 1, \ldots, 6$
1 Gauge boson	4 fermions	6 Scalars	

All in the adjoint representation of SU(Nc)

$$\mathcal{L} = \frac{1}{g^2} Tr \left[F^2 + (D\phi)^2 + \bar{\psi} \not{D} \psi + \sum_{I,J} \left(\phi^I \phi^J\right)^2 + \bar{\psi} \Gamma^I \phi^I \psi \right]$$

The cool thing about this theory is that it comes up quite naturally in string theory …

Black holes in AdS

- If pure empty AdS gives the ground state for a CFT in the vacuum, finite temperature CFTs should correspond to AdS spacetimes with black holes (black branes – translational invariant in “x” and “t” but not in “r”).

The simplest asymptotically AdS-Schwarzchild d+1-dimensional, black brane is given by

$$ds^2 = \frac{L^2}{r^2}\left[-f(r)\,dt^2 + d\vec{x}^2 + \frac{1}{f(r)}dr^2\right]$$

Also solution of

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\Lambda g_{\mu\nu}$$

Blackening factor: $f(r) = 1 - \frac{r^d}{r_H^d}$.

Horizon $r_H \to f(r_H) = 0$

boundary is at $r \to 0$ and $\lim_{r\to 0} f(r) = 1$

Black holes in AdS

Within a semi-classical (in Euclidean signature) regime (G_N → 0)

$$Z = \int \mathcal{D}g_{\mu\nu}\, e^{-S^E_{gravity}(g_{\mu\nu})} \sim e^{-S^E_{gravity}}\Big|_{Saddle\ Points}$$

One such saddle point is the analytical continuation of the AdS black brane metric (Euclidean signature)

$$\tau = it$$

$$ds^2_\star = \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^i dx^i\right)$$

Black holes in AdS

The fact that f(r) → 0 at the horizon places a constraint on this Euclidean spacetime. For the space to be regular at the horizon (and hence a genuine saddle point of the action) we must periodically identify (absence of a conical singularity)

$$\tau \sim \tau + \frac{4\pi}{|f'(r_H)|}$$

period

$$\beta = \frac{4\pi}{|f'(r_H)|}$$

What does this mean for the dual quantum field theory?

The basic object for this discussion is the bulk metric $g_{\mu\nu}$

At the boundary we have $g_{(0)\mu\nu}$

$$g_{\mu\nu}(r) = \frac{L^2}{r^2} g_{(0)\mu\nu} + \cdots \quad \text{as} \quad r \to 0 .$$

The d-dimensional background metric in the field theory is $ds^2 = d\tau^2 + d\vec{x}^2$

where τ is periodic with period $\beta = \frac{4\pi}{|f'(r_H)|}$

This implies that the volume integrals in the field theory are like

$$\int_0^\beta d\tau \int d^{d-1}\vec{x}$$

Just like thermal field theory !!!

$$T = \frac{d}{4\pi r_H}$$

Hawking temperature black brane = Temperature na QFT

The other thermodynamic quantities are evaluated using the partition function

$$Z \sim e^{-S_E} \sim e^{-FV_d/T}$$

where F is the system's free energy density.

One could show that the Hawking temperature, energy density,
and entropy density of this black brane are

$$G_N = \ell_p^{d-1} \qquad T = \frac{d}{4\pi\, r_{\rm H}}\,, \qquad \epsilon = \frac{d-1}{16\pi\, r_{\rm H}^d}\left(\frac{L}{\ell_p}\right)^{d-1}\,, \qquad s = \frac{1}{4\, r_{\rm H}^{d-1}}\left(\frac{L}{\ell_p}\right)^{d-1}$$

$$G_N = \ell_p^{d-1} \qquad T = \frac{d}{4\pi\, r_{\rm H}}\,, \qquad \epsilon = \frac{d-1}{16\pi\, r_{\rm H}^d}\left(\frac{L}{\ell_p}\right)^{d-1}\,, \qquad s = \frac{1}{4\, r_{\rm H}^{d-1}}\left(\frac{L}{\ell_p}\right)^{d-1}$$

- What do these expressions tell us? They tell us that if you throw energy into the black brane, this moves the horizon closer to the boundary at “r → 0”, which leads to an increase in temperature.

- In other words, the specific heat of this system is positive, which is necessary for thermodynamic equilibrium.

- This is very different than black holes in asymptotically flat spaces where the specific heat is negative, which means that it cannot achieve thermodynamical equilibrium in the absence of external sources.

- Black branes in asymptotically AdS spaces behave as heat baths. This means that our intuition about what is a CFT at finite T is correct.

- Since AdS works as a "box", any radiation that the black brane evaporates away goes back into the horizon at finite time, which means that detailed balance can be achieved and, consequently, thermal equilibrium.

- Also, from our Wilsonian point of view for the bulk gravity theory, we know that GR should appear when

$$L/\ell_p \gg 1$$

- Remembering that the entropy density "s" goes with the area of the horizon and

$$s = \frac{1}{4\, r_{\mathrm{H}}^{d-1}} \left(\frac{L}{\ell_p} \right)^{d-1}$$

Classical GR in the bulk appears when the number of QFT degrees of freedom goes to infinity (horizon area must be infinitely large in Planck units for us to see a coherent classical gravitational field instead of quantum gravity).

Charged black branes in AdS

We all remember the first law of thermodynamics

$$d\epsilon = T ds + \mu\, d\rho.$$

How do we include a chemical potential μ ? If the black brane-based description of the 4d CFT in thermal equilibrium is true, we should be able to somehow include a conserved charge in this black brane.

This is done by considering the solutions of the d+1-Einstein-Maxwell action

$$I_{ME} = \frac{1}{16\pi G_N}\int d^d x\sqrt{-g}\left[-2\Lambda + R - \frac{L^2}{4e^2}F^2\right]$$

$F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$

$F^2 = F^{\mu\nu}F_{\mu\nu} = (\vec{E}^2 - \vec{B}^2)$

$E^i = -F^{0,i} \quad B^i = -\epsilon^{ijk}F_{jk}$

We look for black brane solutions of the EOM of the form

$$ds^2 = L^2 \frac{-f(r)dt^2 + d\vec{x}^2 + \frac{1}{f(r)}dr^2}{r^2}, \qquad A = A_t(r)dt$$

The result is

$$f = 1 - M\,r^d + Q^2\,r^{2(d-1)} \qquad A_t(r) = \mu\left(1 - \left(\frac{r}{r_H}\right)^{d-2}\right)$$

$$T = \frac{d}{4\pi r_H}\left(1 - \frac{d-2}{d}Q^2 r_H^{2d-2}\right) \qquad \mu = \frac{2Q}{\mathcal{C}}\,r_H^{d-2} \text{ and } \mathcal{C} = \sqrt{\frac{2(d-2)}{d-1}}$$

$$\epsilon = M\,\frac{d-1}{16\pi}\left(\frac{L}{\ell_p}\right)^{d-1}, \qquad s = \frac{1}{4\,r_H^{d-1}}\left(\frac{L}{\ell_p}\right)^{d-1}, \qquad \rho = Q\,\frac{d-1}{8\pi\mathcal{C}}\left(\frac{L}{\ell_p}\right)^{d-1}.$$

$$d\epsilon = T\,ds + \mu\,d\rho.$$

Since $$T = \frac{d}{4\pi r_{\mathrm{H}}}\left(1 - \frac{d-2}{d} Q^2 r_{\mathrm{H}}^{2d-2}\right)$$ one can find a charge Q at which

$$Q \to Q_* = \sqrt{\frac{d}{d-2}\frac{1}{r_{\mathrm{H}}^{d-1}}}.$$

Temperature vanishes but entropy density doesn't !!!!

- The ground state of this system has large degeneracy and it plays an interesting role in the holographic descriptions of phase transitions.

- Also, the microcanonical counting of this type of ground state entropy in quantum gravity (a more symmetrical version of this one) was solved by Stroeminger and Vafa in 1996 using string theory and D-branes.

- Now we have an idea of how to include temperature and density in holography.

Holographic dictionary: The effective action as a functional of boundary conditions

Consider a scalar field of mass “m”

$$I_\Phi \propto \int d^{d+1}x \sqrt{-g} \left(-\frac{1}{2} (\partial\Phi)^2 - \frac{m^2}{2} \Phi^2 + \dots \right)$$

In the (d+1) AdS black brane background

$$ds^2 = \frac{L^2}{r^2} \left[-f(r)\, dt^2 + d\vec{x}^2 + \frac{1}{f(r)} dr^2 \right]$$

Take the plane wave Ansatz $\Phi(x,r) = \Phi(r) e^{ik\cdot x}$

and the EOM becomes: $r^2 f\, \Phi''(r) - r\left[r f' - (d-1) f \right] \Phi'(r) - \left[k^2 r^2 + m^2 L^2 \right] \Phi(r) = 0$

Let's study the solutions of this equation at the boundary and at the horizon.

Near the boundary at r → 0, where f → 1, the solution becomes

$$\Phi \sim \phi_{d-\Delta}(k)\, r^{d-\Delta} + \phi_{\Delta}(k)\, r^{\Delta} + \ldots\,;$$

With scaling dimension: $\Delta(\Delta - d) = m^2 L^2$

Breitenloner-Freedman (BF) bound: $m^2 \geq -d^2/4L^2$;

Differently than in flat space, a small negative mass squared does not lead to an instability in AdS space.

- Again, the fact that AdS space is like a “closed box” makes a big difference. This “tachyon instability” is removed by the extra “harmonic potential” generated by the curvature of AdS space.

- Note that, as expected, when L → infinity, Ricci scalar R → 0, and $m^2 \geq 0$.

- Thus, as long as the BF bound is preserved, solving the radial evolution implies that we need to know 2 constants of motion:

normalizable ϕ_Δ $\phi_{d-\Delta}$ non-normalizable

The non-normalizable mode must be fixed at the boundary to have a well defined variational problem.

Ex: Scalar field in T=0 AdS5: $ds^2 = \frac{L^2}{r^2}\left[-dt^2 + d\vec{x}^2 + dr^2\right]$

In this case one can find the solutions of the wave equations analytically:

Ex:5

$$\Phi(t,x,r) = \left[\phi_{\rm reg}\, r^{\frac{d}{2}}\, K_\nu\left(\kappa r\right) + \phi_{\rm irreg}\, r^{\frac{d}{2}}\, I_\nu\left(\kappa r\right)\right] e^{-i(\omega t - kx)}$$

Integration constants

$\nu = \Delta - \frac{d}{2}$, $\kappa = \sqrt{\omega^2 + \vec{k}^2}$, K_ν and I_ν are modified Bessel functions

Since $I_\nu(kr) \sim e^{kr}$ near the “horizon” at r → infinity, regularity at the horizon implies that $\phi_{irreg} = 0$

Now, let's see what happens at the boundary ...

Remember that at the boundary r → 0

$$\Phi \sim \phi_{d-\Delta}(k)\, r^{d-\Delta} + \phi_{\Delta}(k)\, r^{\Delta} + \ldots$$

We can now expand the full solution left over ~ $r^{d/2} K_\nu(kr)$ near r → 0 to find

$$\phi_\Delta = \frac{\Gamma(\frac{d}{2}-\Delta)}{2^{\Delta-\frac{d}{2}}\Gamma(\Delta-\frac{d}{2})}\,(\omega^2+k^2)^{\Delta-\frac{d}{2}}\,\phi_{d-\Delta}$$

Since we already fixed the non-normalizable mode at the boundary and imposed the regularity at the horizon, the value of ϕ_Δ is determined.

Suppose that we want to study the partition function of the gravity theory in AdS by integrating all the bulk fields $\Phi(x,r)$.

We then specify a boundary condition for each of the bulk fields at the boundary of AdS

$$\phi_{d-\Delta}(x) = \lim_{r\to 0} r^{\Delta-d}\Phi(x,r)$$

This means that the partition function in AdS is a functional of the boundary conditions for all bulk fields that are “switched on”

$$Z_{\mathrm{AdS}}[\phi_{d-\Delta}(x)] \equiv Z_{\mathrm{AdS}}[\Phi[\phi_{d-\Delta}(x)]]$$

*Note that one has to perform the sum over all the possible spacetimes that are asymptotically AdS.

Holographic dictionary

Boundary QFT				Bulk Gravity
Operator	$\mathcal{O}(x)$	$\longleftrightarrow$	$\Phi(x,r)$	Field
Spin	$s_{\mathcal{O}}$	$\longleftrightarrow$	s_Φ	Spin
Global Charge	$q_{\mathcal{O}}$	$\longleftrightarrow$	q_Φ	Gauge Charge
Scaling dimension	$\Delta_{\mathcal{O}}$	$\longleftrightarrow$	m_Φ	Mass
Source	$J(x)$	$\longleftrightarrow$	$\Phi(x,r)\|_\partial$	Boundary Value (B.V.)
Expectation Value	$\langle\mathcal{O}(x)\rangle$	$\longleftrightarrow$	$\Pi_\Phi(x,r)\|_\partial$	B.V. of Radial Momentum
Global Symmetry Group	G	$\longleftrightarrow$	G	Gauge Symmetry Group
Source for Global Current	$\mathcal{A}_\mu(x)$	$\longleftrightarrow$	$A_\mu(x,r)\|_\partial$	B.V. of Gauge Field
Expectation of Current	$\langle\mathcal{J}^\mu(x)\rangle$	$\longleftrightarrow$	$\Pi^\mu_A(x,r)\|_\partial$	B.V. of Momentum
Stress Tensor	$T^{\mu\nu}(x)$	$\longleftrightarrow$	$g_{\mu\nu}(x,r)$	Spacetime Metric
Source for Stress-Energy	$h_{\mu\nu}(x)$	$\longleftrightarrow$	$g_{\mu\nu}(x,r)\|_\partial$	B.V. of Metric
Expected Stress-Energy	$\langle T^{\mu\nu}(x)\rangle$	$\longleftrightarrow$	$\Pi^{\mu\nu}_g(x,r)\|_\partial$	B.V. of Momentum
# of Degrees of Freedom Per Spacetime Point	N^2	$\longleftrightarrow$	$\left(\frac{L}{\ell_P}\right)^{d-1}$	Radius of Curvature In Planck Units
Characteristic Strength of Interactions	λ	$\longleftrightarrow$	$\left(\frac{L}{\ell_s}\right)^{d}$	Radius of Curvature In String Units
QFT Partition Function with Sources $J_i(x)$	$Z_{\mathrm{QFT}_d}[J_i]$	$\longleftrightarrow$	$Z_{\mathrm{QG}_{d+1}}[\Phi_i[J_i]]$	QG Partition Function in AdS w/ $\Phi_i\|_\partial = J_i$
QFT Partition Function at Strong Coupling	$Z^{\lambda,N\gg1}_{\mathrm{QFT}_d}[J_i]$	$\longleftrightarrow$	$e^{-I_{\mathrm{GR}_{d+1}}[\Phi[J_i]]}$	Classical GR Action in AdS w/ $\Phi_i\|_\partial = J_i$
QFT n-Point Functions at Strong Coupling	$\langle\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle$	$\longleftrightarrow$	$\left.\frac{\delta^n I_{\mathrm{GR}_{d+1}}[\Phi[J_i]]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\right\|_{J_i=0}$	Classical Derivatives of the On-Shell Classical Gravitational Action
Thermodynamic State		$\longleftrightarrow$		Black Hole
Temperature	T	$\longleftrightarrow$	T_H	Hawking Temperature $\sim$ Mass
Chemical Potential	μ	$\longleftrightarrow$	Q	Charge of Black Hole
Free Energy	F	$\longleftrightarrow$	$I_{\mathrm{GR}}\|_{(\text{on-shell})}$	On-Shell Bulk Action
Entropy	S	$\longleftrightarrow$	A_H	Area of Horizon

Mathematical Definition of the Holographic Duality

$$Z_{\mathrm{QFT}}[J_i] \;=\; Z_{\mathrm{QG}}[\Phi[J_i]]$$

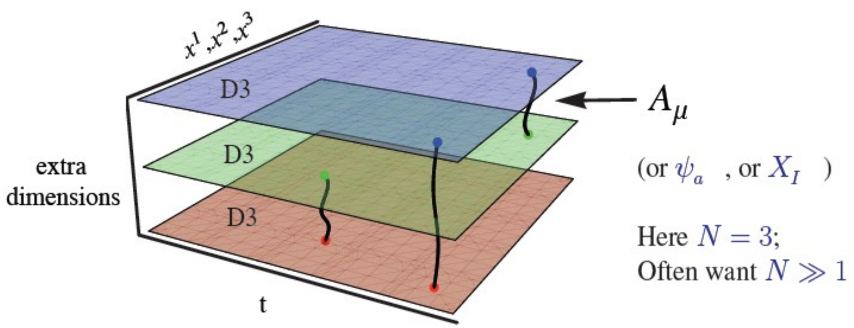
$Z_{\mathrm{QFT}}[J_i]$ Partition function of the QFT as a function of the sources

$Z_{\mathrm{QG}}[\Phi[J_i]]$ Partition function of the gravitational theory in AdS

The bulk fields play the role of the coupling constants of the QFT that are now promoted to dynamical fields on the higher dimensional spacetime where the extra dimension is the RG scale.

STANDARD EXAMPLE

$\mathcal{N} = 4$ SU(Nc) Supersymmetric Yang-Mills



Fields in the adjoint rep. of SU(Nc)

- 16 + 16 supercharges
- SU(4) R-symmetry
- SO(6) global symmetry

$\beta = 0$ **CFT !!!!**

Maldacena, 1998: This gauge theory is dual to Type IIB string theory on AdS5 x S5

Strongly-coupled, large Nc gauge theory	Weakly-coupled, low energy string theory
$N_c \to \infty$	$g_s \to 0$
$\lambda = L^4/\ell_s^4 \to \infty$	$\ell_s/L \to 0$
t'Hooft coupling in the gauge theory	

Entropy density: The Stefan-Boltzmann limit for N=4 SYM

$$s_{free} = \frac{2}{3}\pi^2 N_c^2 T^3 \qquad \lambda \to 0$$

What about the limit $N_c \to \infty \qquad \lambda \gg 1$?

Bekenstein-Hawking formula: $s_{BH} = \frac{a}{4G_{10}}$

$$A = a\,V_3 = \int d^3\vec{x} \int_{S_5} d^5\Omega \sqrt{-det\, g_{\mu\nu}} \;\Rightarrow\; a = r_H^3\,\pi^3\,L^2$$

$$\boxed{s_{BH} = \frac{\pi}{2} N_c^2\, T^3 = \frac{3}{4} s_{free}}$$

Isn't that neat ?

$$N^2 = \left(\frac{L}{\ell_p}\right)^{d-1}, \qquad \lambda = \left(\frac{L}{\ell_s}\right)^{d},$$

So, in order to have a weakly curved, GR-like, bulk description, the QFT must be strongly interacting (or have large anomalous dimensions for non-protected operators) and have infinitely many d.o.f.

Indeed, consider Euclidean signature and take the large N, strongly coupled limit of the QFT. In this case one can express the quantum gravity partition function as a sum over saddle points and focus on the dominant saddle

$$Z_{\rm QFT}[J] \simeq e^{-I_{\rm GR}[\Phi[J]]} \quad \Rightarrow \quad I_{\rm GR}[\Phi[J]] = \frac{1}{16\pi G_N}\int d^{d+1}x\,\sqrt{-g}\,(R - 2\Lambda + \mathcal{L}_m(\Phi))$$

$$\boxed{\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{\delta^n I_{\rm GR}[\Phi[J_i]]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\Big|_{J_i=0}}$$

On-shell gravity action as a functional of the boundary values of the fields gives all the correlation functions in the QFT.

Ex: Computation of 1-point function

Bulk scalar field (dual to a QFT operator of dimension Δ)

Euclidean signature action $$S = \frac{1}{2}\int_{r<r_H} d^{d+1}x\,\sqrt{g}\,\left[(\partial\Phi)^2 + m^2\Phi^2\right]$$

Duality: $$\left\langle exp\left[-\int d^d x\,\phi_{d-\Delta}(x)\,\mathcal{O}(x)\right]\right\rangle_{QFT} = e^{-S\left[r^{\Delta-d}\Phi(r\to 0)=\phi_{d-\Delta}\right]}$$

1-point function: $$\langle\mathcal{O}(x)\rangle_{\phi_{d-\Delta}} = \lim_{r\to 0} r^{d-\Delta}\,\Pi(x,r)$$

$$\Pi = \sqrt{g}\,g^{rr}\partial_r\Phi$$

Canonical momentum in the r direction

One can then show that, just as $\phi_{d-\Delta}(x) = J(x)$

then the 1-point function of the operator is

$$\langle \mathcal{O}(x) \rangle = \frac{2\Delta - d}{L} \phi_\Delta(x)$$

So, the non-renormalizable mode $\phi_{d-\Delta}$ is the source in the QFT for the associated operator $\mathcal{O}$ and the normalizable term ϕ_Δ determines the expectation value of the operator in the QFT.

Let's now recall a few general features of Green's functions

Consider a given (bosonic) operator $\hat{\mathcal{O}}(x)$

In Minkowski spacetime (signature -,+,+,+) $x^0 = t$ we have

The retarded Green's function (2-point function):

$$G^R(k) = -i \int d^4x \, e^{-ik\cdot x} \, \theta(t) \, \langle [\hat{\mathcal{O}}(x), \hat{\mathcal{O}}(0)] \rangle$$

The advanced Green's function (2-point function):

$$G^A(k) = i \int d^4x \, e^{-ik\cdot x} \, \theta(-t) \, \langle [\hat{\mathcal{O}}(x), \hat{\mathcal{O}}(0)] \rangle$$

Wightman correlator:

$$G(k) = \frac{1}{2} \int d^4x \, e^{-ik\cdot x} \langle \hat{\mathcal{O}}(x)\hat{\mathcal{O}}(0) + \hat{\mathcal{O}}(0)\hat{\mathcal{O}}(x) \rangle$$

Feynman correlator:

$$G^F(k) = -i \int d^4x \, e^{-ik\cdot x} \, \langle T\hat{\mathcal{O}}(x)\hat{\mathcal{O}}(0) \rangle = \frac{1}{2}[G^R(k) + G^A(k)] - iG(k)$$

From G^R you can find the other ones $G^R(k)^* = G^R(-k) = G^A(k)$

Ex: $G^F(k) = \mathrm{Re}\, G^R(k) + i \coth\frac{\omega}{2T}\,\mathrm{Im}\, G^R(k)$

Also, invariance under parity implies that: $Im\, G^{R,A}(\omega) = -Im\, G^{R,A}(-\omega)$

$$Re\, G^{R,A}(\omega) = Re\, G^{R,A}(-\omega)$$

However, despite of all these different functions in Minkowski space, in Euclidean spacetime (+,+,+,+), we can use Euclidean time ordering to define

$$G^E(k_E) = \int d^4x_E\, e^{-ik_E\cdot x_E}\langle T_E\hat{\mathcal{O}}(x_E)\hat{\mathcal{O}}(0)\rangle$$

$$G^R(2\pi i T n, \boldsymbol{k}) = -G^E(2\pi T n, \boldsymbol{k})$$

$$G^A(-2\pi i T n, \boldsymbol{k}) = -G^E(-2\pi T n, \boldsymbol{k})$$

$$G^R(0, \boldsymbol{k}) = G^A(0, \boldsymbol{k}) = -G^E(0, \boldsymbol{k})$$

Rotational invariance

Euclidean 2-point function

We can either take two derivatives of the partition function or simply remember that in linear response theory, an infinitesimal source J(x) generates a response which is linearly proportional to the source

$$J(k) = \phi_{d-\Delta}(k) = \lim_{r\to 0} r^{\Delta-d}\Phi(k,r)$$

$$G_E(k) \equiv \frac{\langle \mathcal{O}(k)\rangle}{J(k)} = \lim_{r\to 0} r^{2(\Delta-d)}\frac{\Pi(k,r)}{\Phi(k,r)} \quad \Rightarrow \quad G_E(k) = \frac{2\Delta-d}{L}\frac{\phi_\Delta(k)}{\phi_{d-\Delta}(k)}$$

Remember that $\phi_{d-\Delta}$ and ϕ_Δ are not independent because of the condition of regularity of the solution at the horizon.

Ex: For pure AdS (T=0) one can show that $G_E(k) = \frac{2\Delta-d}{L}\frac{\Gamma(\frac{d}{2}-\Delta)}{2^{\Delta-\frac{d}{2}}\Gamma(\Delta-\frac{d}{2})}\left(k^2\right)^{\Delta-\frac{d}{2}}$

which is precisely the expected form for the 2-point function of a scalar operator of dimension Δ in a CFT.

Son and Starinets (2002) showed that for the calculation of real time correlators one must be a bit more careful with the boundary conditions at the horizon.

- For retarded Green's functions one must impose in-falling boundary conditions at the horizon, which in this case for the scalar field would give

$$G_R(\omega,\vec{k}) = \lim_{r\to 0} r^{2(\Delta-d)} \frac{\Pi_{r,\mathrm{in}}(\omega,\vec{k},r)}{\Phi_{\mathrm{in}}(\omega,\vec{k},r)} = \frac{2\Delta-d}{L}\frac{\phi_{\Delta,\mathrm{in}}(\omega,\vec{k})}{\phi_{d-\Delta,\mathrm{in}}(\omega,\vec{k})};$$

- For advanced Green's functions, one must impose out-going boundary conditions at the horizon.

- With these conditions we can compute all the different Green's functions that describe real time linear response behavior (that is how we “beat” lattice) 😁

Summary of the 2st lecture

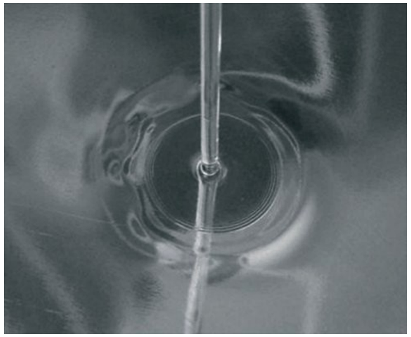
- Conformal invariance in the 4d QFT is mapped into AdS space in the bulk. finite temperature and density QFTs can be constructed using bulk geometries that are asymptotically AdS.

- There is a well defined holographic dictionary at the level of partition functions that allows one to map bulk to QFT information and compute the correlators of operators in the QFT via gravity (both in Euclidean and Minkowski signatures).

- The duality is a type of strong-weak duality with one side becoming a strongly coupled quantum mechanical system whenever the dual is weakly-coupled and semi-classical.

LECTURE III: Hydrodynamics and Holography

OUTLINE OF THIS LECTURE



1- Introducing the perfect fluids in nature

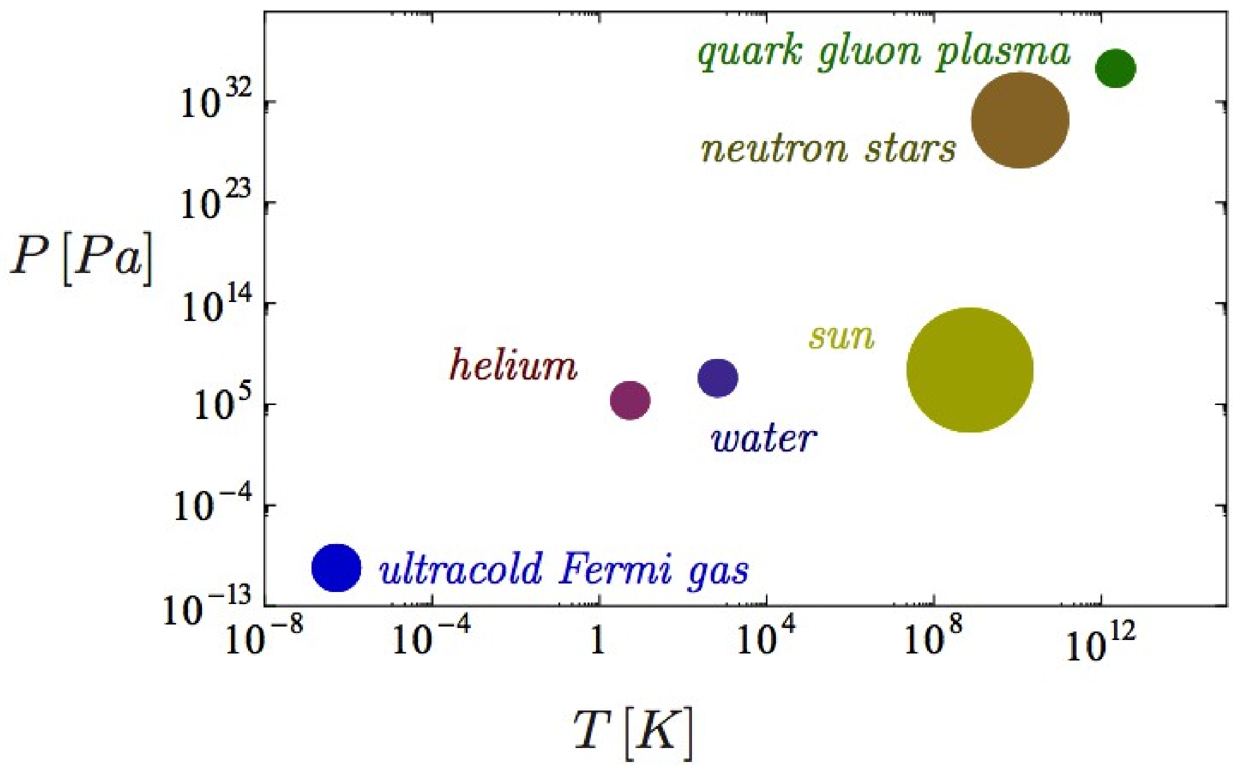
2- Conformal relativistic hydrodynamics

3- Universality of the shear viscosity to entropy density ratio in holography

4- Summary

Adams et al, ArXiv:1205.5180

Temperature and Pressure Scales of Extreme Quantum Matter



and are connected. You will see that holography may provide the natural connection between these completely distinct quantum systems.

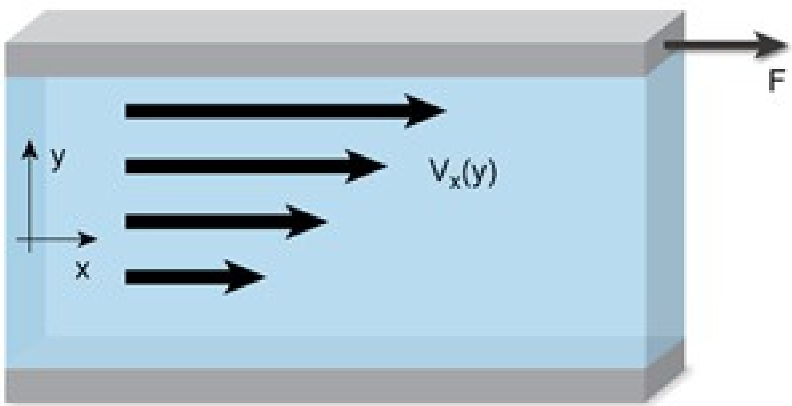
HOLOGRAPHY

Quark-Gluon Plasma		Cold Atoms
Tc ~ 150 MeV ~ 10^12 K	$$\frac{\eta}{s} \sim \frac{1}{4\pi}$$	Tc ~ 500 nK
"Life time" ~ 15 fm/c ~ 10^-23 s		"Life time" ~ 1 ms
Size ~ 15 fm ~ 10^-14 m		Size ~ 100 microns

Strongly coupled, nearly perfect fluid behavior !!!!

Shear Viscosity in Non-Relativistic Fluids

Physics 101



Friction Force

$$\frac{F}{A} = \eta\, \partial_y V_x(y)$$

Good fluidity = Large Reynolds Number

$$Re = \left(\frac{\rho}{\eta}\right) v\, L \gg 1$$

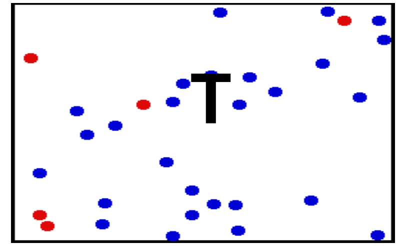
ρ = fluid's mass density

V = characteristic flow velocity

L = characteristic length scale of flow

η = shear viscosity

Shear viscosity roughly measures the ability to transport momentum from one part of the fluid to the other.



According to kinetic theory, for a dilute gas with one particle species we have

$$\eta \sim \frac{1}{3} n \langle p \rangle \ell_{\rm mfp}$$

n = fluid density

$\langle p \rangle$ = average momentum

$\ell_{\rm mfp}$ = mean free path

- In terms of the cross section, σ, we have that $\ell_{\rm mfp} = 1/(n\sigma)$ or

$$\eta \sim T/\sigma$$

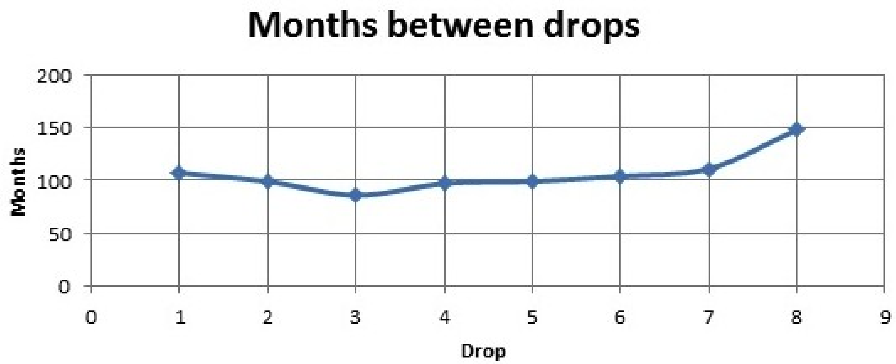
(density independent)

→ Ideal gas ($\sigma \to 0$) has $\eta \to \infty$

Ex: “Very bad” fluid

Australian Pitch drop experiment (1927-2014)

Only 8 drops have been observed during this time!!!



Timeline

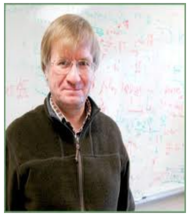
Date	Event	Duration (Months)	Duration (Years)
1927	Experiment set up		
1930	The stem was cut		
December 1938	1st drop fell	96–107	8.0–8.9
February 1947	2nd drop fell	99	8.3
April 1954	3rd drop fell	86	7.2
May 1962	4th drop fell	97	8.1
August 1970	5th drop fell	99	8.3
April 1979	6th drop fell	104	8.7
July 1988	7th drop fell	111	9.3
28 November 2000	8th drop fell	148	12.3

Ignobel Prize of Physics in 2005 !!!!

Miklos Gyulassy

+

Pawel Danielewicz



1985

- Quantum mechanics says that momentum and position cannot be determined simultaneously with arbitrary precision … The smallest mean free path is

$$p\,\ell_{\rm mfp} \gtrsim \hbar \quad \text{and} \quad s \sim k_B\, n \;\Rightarrow\; \frac{\eta}{s} \gtrsim \frac{\hbar}{k_B}$$

Uncertainty principle — Kinetic theory — Viscosity bound

- Water near the triple point: $\eta/s \simeq 2\,\hbar/k_B$

- Superfluid : $\eta/s \simeq 2\,\hbar/k_B$

1 atm, T ~ 2 K

Can we find a system where this quantity is close to this "bound" ?????

Coldest, nearly perfect fluid created in laboratory …

- Optically trapped $^6\mathrm{Li}$ atoms at very low T : 2 different hyperfine states

- Interactions between the "spin up particle" and the "spin down particle" can be tuned via a Feshbach resonance (bound state with zero binding energy).

- 2-body scattering length diverges → **"infinite coupling"**

$$\frac{\eta}{s} \simeq (0.1 - 0.5)\,\frac{\hbar}{k_B}$$

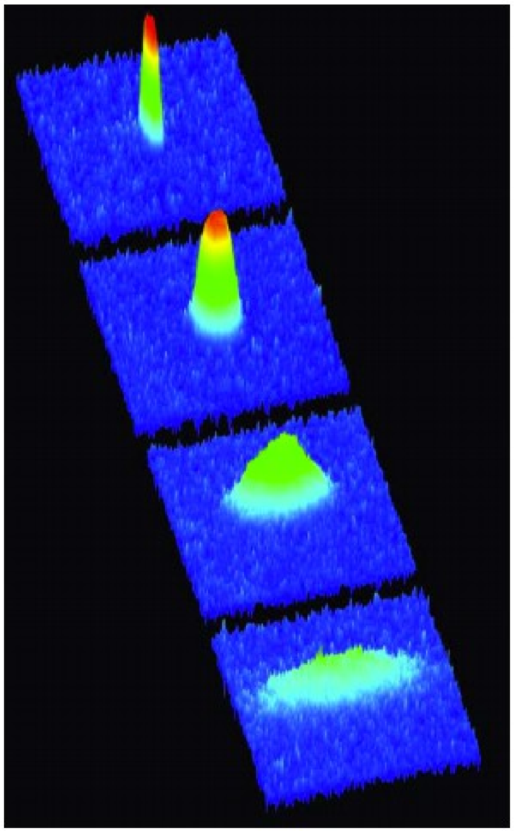
B. Clancy, L. Luo, J. E. Thomas, 2007.
J. E. Thomas, 2009.

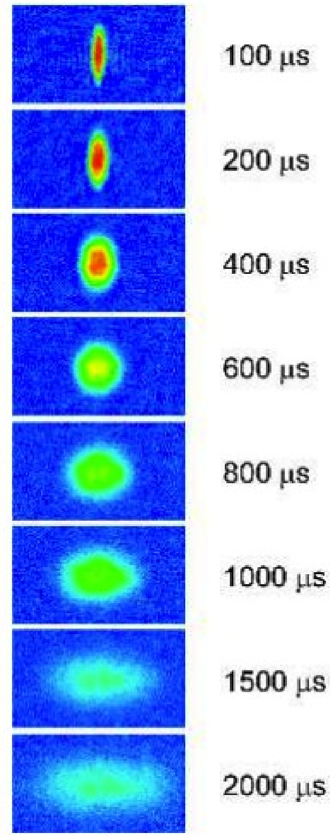
Mean free path comparable to the bound from the uncertainty principle!

The system does not admit a quasiparticle description.

Nearly Perfect (Non-Relativistic) Fluid Behavior !!!!!!

Elliptic Flow!!!





Typical time scale ~ 1 mili second

Temperature ~ nanokelvin

System size ~ 100 microns

O'Hara et al., Science 298, 2179 (2002).

Towards the Hottest (and the Tiniest) Most Perfect Fluid Ever Made:

The Strongly-Coupled Quark-Gluon Plasma

Natural units for high energy nuclear physics $\hbar = k_b = c = 1$

Typical length: Proton radius ~ fermi $1\, fm = 10^{-15}\, m$

Typical energy scale: $150\,\mathrm{MeV} = 2 \times 10^{12}\, K$

Typical time scale: fm/c ~ $10^{-24} s$

Let me say a few words about QCD and the strongly coupled quark gluon plasma …

The strongly coupled Quark-Gluon Plasma

Ultrarelativistic Heavy Ion Collisions: The idea is to create in the lab the quark-gluon plasma that existed in the early universe by colliding heavy ions at ultrarelativistic energies.

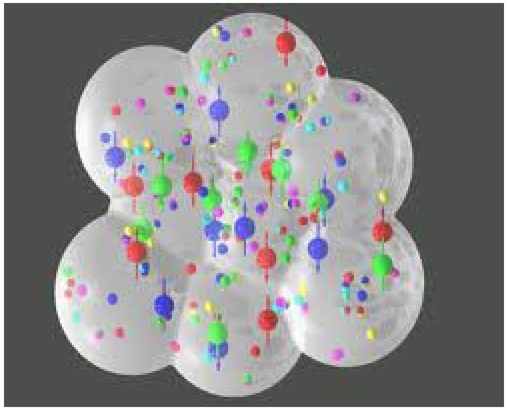
This plasma lasts for only ~$10^{-23}s$ and it is currently produced at

Relativistic Heavy Ion Collider (**RHIC**), New York

Large Hadron Collider (**LHC**), Geneva

These collisions produce in the end a large number of hadrons that are carefully measured by the detectors ...

The idea is that immediately after the collision the hadrons from the initial ions largely overlap and, thus, in that small region of space the vacuum is very different than the usual one and quarks and gluons can become the effective degrees of freedom of the system.



Quark-gluon “soup”

Temperature $\sim 4 \times 10^{12} K$

(center of the sun $T \sim 10^7 K$)

This is for sure the hottest and tiniest (only ~10 fm across) medium ever made in the lab !!!

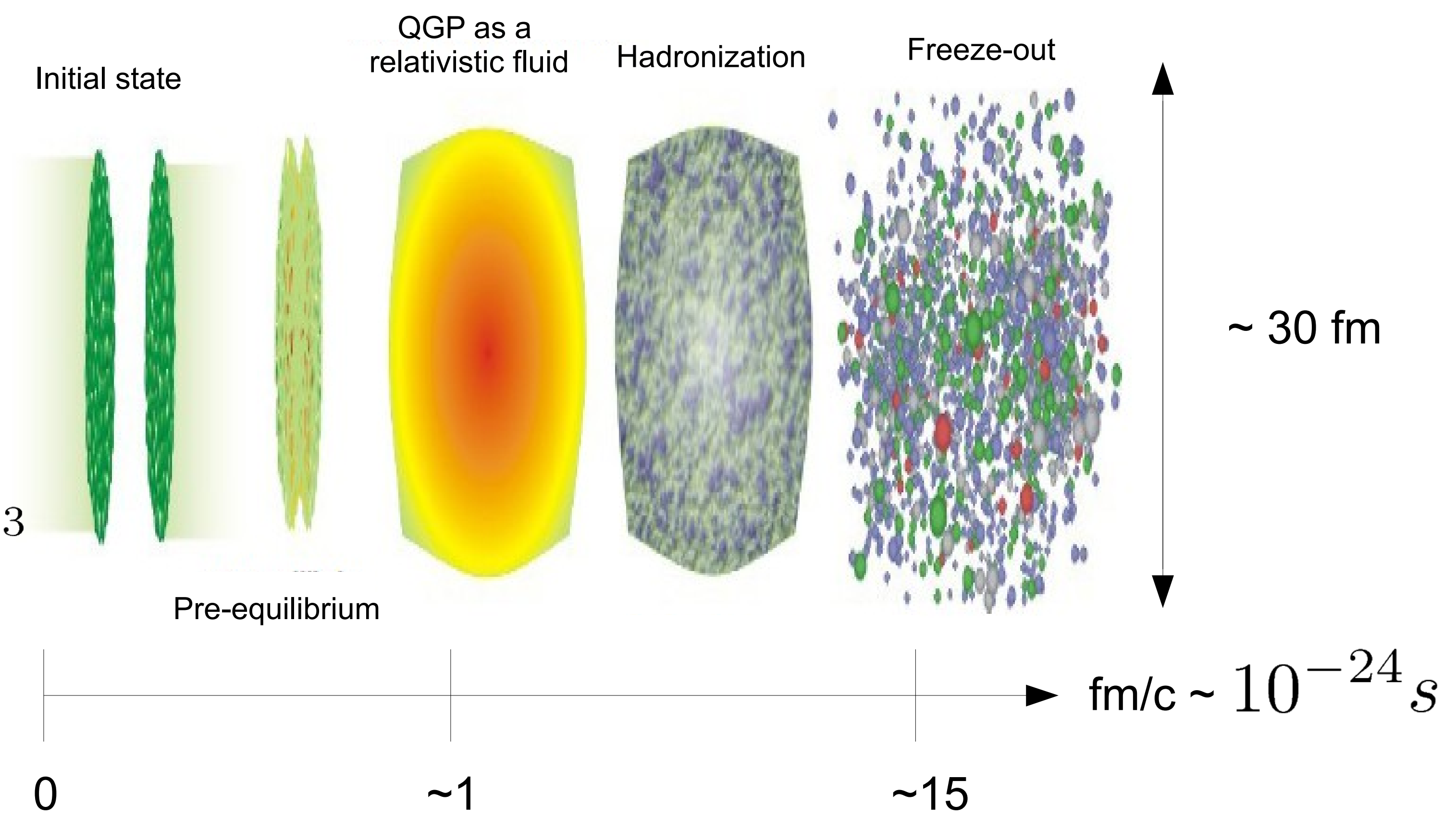
Heavy Ion Collisions in a Nutshell

fermi

$$1\, fm = 10^{-15}\, m$$

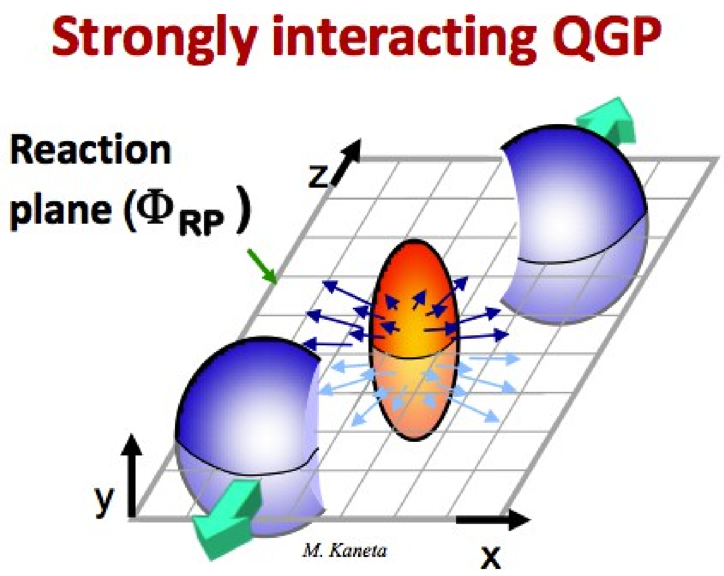
Au, Cu, Pb, U nuclei traveling with gamma factor per nucleon

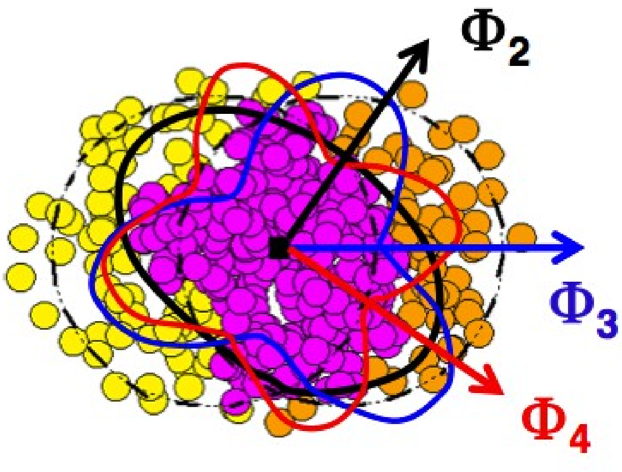
$$\gamma \sim 10^2 - 10^3$$



$$T \sim 500 MeV$$

Hadron Flow Anisotropy





$$\frac{dN}{d\phi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\phi - \Phi_n)]$$

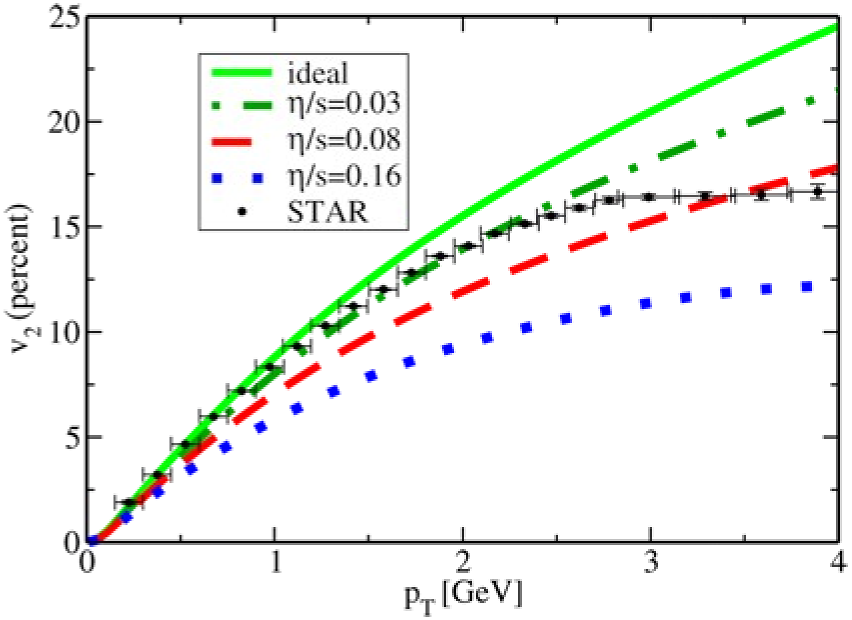
$$v_n = \langle \cos[n(\phi - \Phi_n)] \rangle$$

“Nearly Perfect Fluidity” of Quarks and Gluons: RHIC's biggest discovery !!!!!

Very small $\frac{\eta}{s} \sim \frac{1}{4\pi}$

System behaves as a strongly coupled perfect liquid!

No quasiparticle description

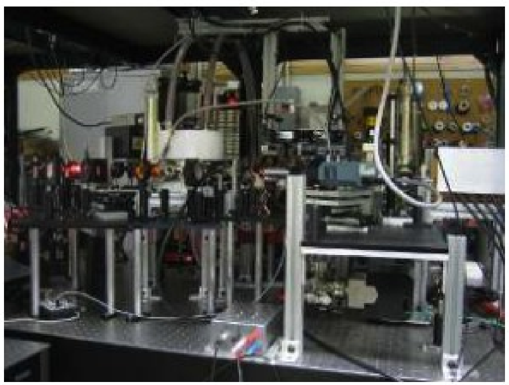


P. Romatschke, U. Romatschke, 2007

- Relativistic fluid dynamics in the form of **relaxation time equations** is the standard tool used in these calculations … A lot of theoretical work still needs to be done in this case – putting together fluid dynamics + relativity + entropy generation is not at all trivial.

To get an idea of the devices used in the detection of perfect fluids ...

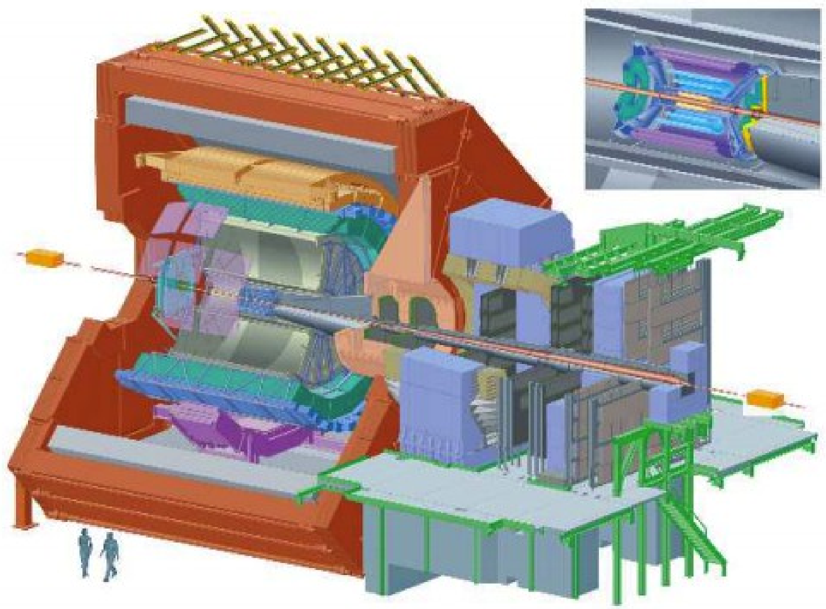
Ultracold atom apparatus



Characteristic length scale ~ 2.5 meters

Cost ~ 10^6 US dollars

ALICE detector at LHC



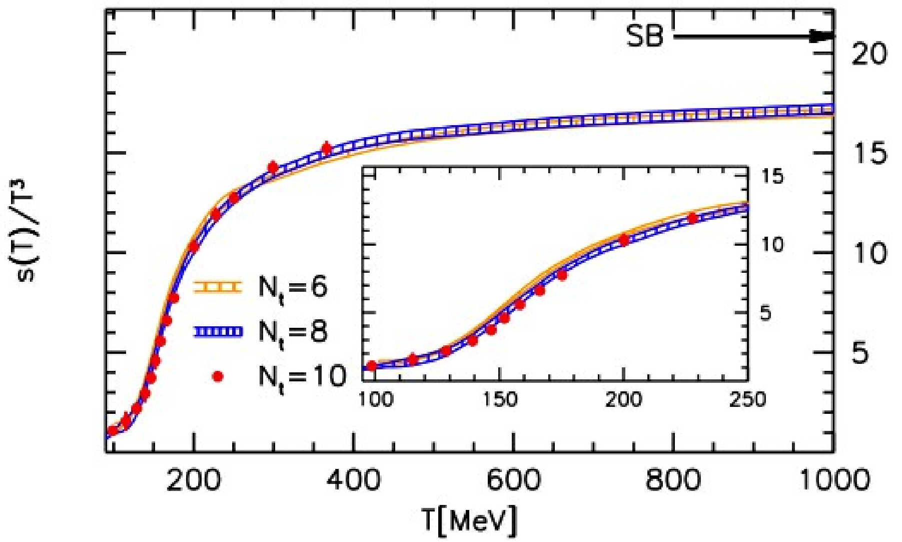
Length scale ~ 25 meters

Cost ~ 10^10 US dollars

Can η/s be computed directly from QCD?

- Equilibrium quantities such as the entropy density can be fully computed non-perturbatively using lattice QCD.

Fodor et al, JHEP 1011 (2010) 077



Can η/s be computed directly from QCD?

- The shear viscosity transport coefficient can be computed via linear response using the Kubo formula (we will see soon why)

Retarded correlator

$$G_R^{\mu\nu\alpha\beta}(x-x') = i\,\theta(x-x')\langle[\hat{T}^{\mu\nu}(x),\hat{T}^{\alpha\beta}(x')]\rangle_T$$

Its first derivative at the origin gives the shear viscosity (Kubo formula)

$$\eta = i\,\partial_\omega G_R^{xyxy}(\omega,\mathbf{0})\Big|_{\omega=0}$$

**Minkowski spacetime correlator => Lattice QCD not the best tool in this case.

Can η/s be computed directly from QCD?

- When the **temperature is large enough**, asymptotic freedom guarantees that perturbative QCD methods can be applied and for a weakly coupled QGP one finds

$$\eta \sim \frac{T^3}{g^4 \ln 1/g} \quad \longrightarrow \quad \frac{\eta}{s} \sim 1$$

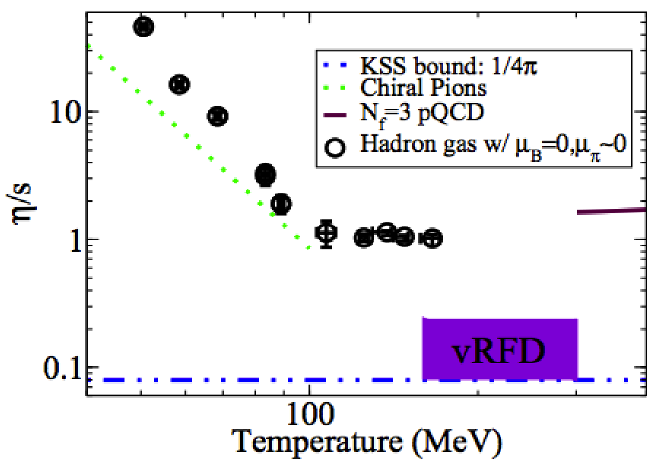
$g \ll 1$

An order of magnitude larger than the “perfect fluid” value

Clearly, in the strongly interacting region T ~ 200 MeV, these perturbative estimates are not be applicable.

Can η/s be computed directly from QCD?

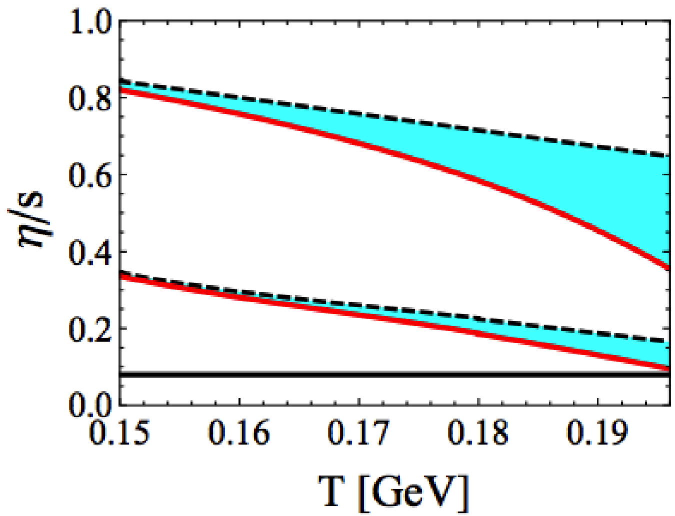
- When the **temperature is low enough**, hadronic models provide useful guidance to understand this quantity



N. Demir, S. Bass, PRL 102 (2009) 172302

Inclusion of Hagedorn resonances near Tc

$$\rho_{HS}(m) = A\,\frac{e^{m/T_H}}{(m^2+m_0^2)^{\frac{5}{4}}}$$



$\sim 1/4\pi$

J. Noronha-Hostler, JN, C. Greiner, PRL 103 (2009) 172302

- None of the approaches based on conventional methods of quantum field theory have been able to explain this perfect fluid nature of the quark gluon plasma from T ~ 150-400 MeV.

- What we need in this case is a new theoretical framework that can be used to compute transport properties of strongly coupled theories.

- The only well defined method where such calculations can be performed, exactly, is holography.

- Before we see that, let us talk about hydrodynamics for a bit.

Shear Viscosity in Non-Relativistic Fluids

Conservation of mass + Conservation of momentum $\rho \mathbf{v}$

Body forces assumed to vanish here

$$\rho\left(\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}\right) + \nabla P = \nabla \cdot \hat{\mathbf{T}}$$

Ideal fluid (inviscid) part   Dissipative part

The Navier-Stokes equations come about when we **ASSUME** that the stress tensor is

$$T_{ij} = \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{v}\right) + \zeta\,\delta_{ij}\nabla\cdot\mathbf{v}$$

ζ = bulk viscosity (neglected in the following)

This is called a Newtonian fluid ...

Shear Viscosity in Non-Relativistic Fluids

- Note that, while the general form of the equations for an arbitrary stress tensor are always valid (non-relativistically), the specific form of the stress tensor used in Navier-Stokes equations isn't.

- The proportionality between T_{ij} and $\left(\partial_i v_j + \partial_j v_i + \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{v}\right)$ is experimentally verified for several fluids but there are (non-relativistic) fluids where this relation does not hold (non-Newtonian fluids = ketchup, toothpaste, and etc).

- The Navier-Stokes equations are for the conservation of momentum what the diffusion equation (derived using Fick's law) is for the continuity equation.

$$\partial_t\rho + \nabla\cdot\mathbf{J} = 0$$

Continuity equation

$$\mathbf{J} = -D\,\nabla\rho \quad \longrightarrow \quad \partial_t\rho - D\nabla^2\rho = 0$$

Assumption: Fick's law

Diffusion equation

D = diffusion coefficient

Relativistic Hydrodynamics as an Effective Theory

Following

Landau

Energy-momentum conservation

$$\nabla_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - p\Delta^{\mu\nu} + \pi^{\mu\nu}$$

?????

Perfect fluid (inviscid) part

Dissipative part

Spatial projector

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

This tensor must be symmetric, traceless***, and transverse

Flow velocity

$$u_\mu u^\mu = 1$$

$$u_\mu \pi^{\mu\nu} = 0$$

*** Bulk viscosity is not taken into account here for simplicity.

Relativistic Hydrodynamics as an Effective Theory

Knudsen number: $$K_n \sim \frac{\ell_{micro}}{L_{macro}} \ll 1$$

$$\partial u \, , \partial \varepsilon \sim 1/L_{macro}$$

For dilute systems

$$\ell_{micro} \sim \ell_{MFP} \sim \frac{1}{n\sigma}$$

Gradient expansion
(or Knudsen series expansion) →

Assumption:
Dynamical variables → $\varepsilon \, , u^{\mu}$

$$\pi^{\mu\nu} = \sum_{i=0}^{N} \lambda_i \, A_i^{\mu\nu}$$

$$A_i^{\mu\nu} \sim \partial^i \varepsilon \, , \partial^i u \sim (1/L_{macro})^i$$

$$\lambda_i \sim \ell_{micro}^i$$

Traceless, symmetric, transverse

Relativistic Hydrodynamics as an Effective Theory

At zeroth order in Kn: $\pi^{\mu\nu} = 0$ → **Relativistic perfect fluid**

At first order in Kn: $\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$ → **Relativistic Navier-Stokes**

$$\sigma_{\mu\nu} = \nabla_{<\mu}u_{\nu>}$$

What about $\mathcal{O}(K_n^2)$???

Here things get a bit more complicated because of the equations of motion

$\nabla_\mu T^{\mu\nu} = 0$ + leading order expression $\pi^{\mu\nu} \sim \eta\sigma^{\mu\nu}$ → $(\partial\varepsilon)^2 \sim K_n^3$

The Rise of  Invariance

Baier, Romatschke, Son, Starinets, Stephanov, 2007

Conformal fluid: $T^{\mu}_{\mu} = 0 \quad \Longrightarrow \quad \varepsilon = 3p$

Hydrodynamics should be conformally invariant in this case. In fact,

$$g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x) e^{-2\Omega(x)} \quad \begin{cases} u_\mu u^\mu = 1 \rightarrow u^\mu(x) \rightarrow e^{\Omega(x)} u^\mu(x) \\ T^{\mu\nu}(x) \rightarrow e^{6\Omega(x)} T^{\mu\nu}(x) \\ \pi^{\mu\nu}(x) \rightarrow e^{6\Omega(x)} \pi^{\mu\nu}(x) \end{cases}$$

The Rise of Weyl Invariance

So, now one just needs to include all the different terms of $\mathcal{O}(K_n^2)$

that are symmetric, traceless, transverse, and that transform homogeneously under Weyl with weight = 6. BRSSS (2007) showed that there are 5 independent structures at this order. The full dissipative tensor is given by

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \sum_{i=1}^{5} 2\eta\, b_i\, O_i^{\mu\nu}.$$

$$O_1^{\mu\nu} = D\sigma^{\langle\mu\nu\rangle}$$

$$O_2^{\mu\nu} = R^{\langle\mu\nu\rangle} + 2u_\alpha R^{\alpha\langle\mu\nu\rangle\beta}u_\beta,$$

$$O_3^{\mu\nu} = \sigma_\lambda^{\langle\mu}\sigma^{\nu\rangle\lambda}, \quad O_4^{\mu\nu} = \sigma_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda}, \quad O_5^{\mu\nu} = \Omega_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda},$$

$$D_\alpha T = \nabla_\alpha T - T\,\dot{u}_\alpha + \frac{u_\alpha\,\theta\,T}{3}$$

$$D_\alpha u^\nu = \nabla_\alpha u^\nu - u_\alpha \dot{u}^\nu - \frac{\Delta_\alpha^\nu \theta}{3}$$

$$\Omega^{\mu\nu} = (D^\mu u^\nu - D^\nu u^\mu)/2$$

$$\sigma^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\nabla_\perp^\alpha u^\beta = \frac{D^\mu u^\nu + D^\nu u^\mu}{2}$$

$$D\sigma^{\langle\mu\nu\rangle} = \dot{\sigma}^{\langle\mu\nu\rangle} + \sigma^{\mu\nu}\theta/3$$

The Rise of Weyl Invariance

There are 5 new transport coefficients. This increase in the number of transport coefficients is expected when you go farther and farther away from local equilibrium. In the case of strongly coupled N=4 SYM theory at large Nc, all the 6 coefficients have been computed (BRSSS 2007, Moore and Saremi 2010, Arnold et al, 2011).

However, there are some problems … these equations violate causality and the instabilities that appear in this case make numerical applications based on these equations not viable.

The main problem in this case is that these equations are still basically diffusion equations for energy and momentum.

They imply that any pressure gradient is AUTOMATICALLY converted into gradients of flow. This can lead to problems in a relativistic theory.

Key Insight from Israel and Stewart (1978)

Also Kadanoff and Martin, 1965

Dissipative tensor $\pi_{\mu\nu}$ treated as an independent dynamical variable …

$$\tau_\pi \, \dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} = 2\,\eta\,\sigma^{\mu\nu} + \ldots$$

that now obeys a relaxation equation with relaxation coefficient $\tau_\pi \sim \ell_{MFP}$

The equations can now be causal and linearly stable if the relaxation time is sufficiently large !!!!

For a detailed discussion, see Pu, Koide, Rischke, 2010

The relaxation time gives the typical time after which $\pi_{\mu\nu}$ becomes similar to its asymptotic Navier-Stokes solution.

When is the relaxation time going to be **IMPORTANT**?

$$\tau_\pi \, \dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} = 2\,\eta\,\sigma^{\mu\nu} + \ldots$$

IMPORTANT $\rightarrow$ whenever the time variation of the flow or $\pi_{\mu\nu}$ is comparable to the microscopic scale timescale present in τ_π

- Not relevant for water under normal conditions. Very relevant for non-relativistic micro-fluids.

- I expect that this could be relevant for realistic studies of the QGP dynamics in heavy ion collisions … (more on that later)

So, now the idea is to follow the effective field theory approach of BRSSS and obtain the correct equations of motion for a conformal

Transient fluid $\equiv$ relativistic fluid under flow conditions where

$$\pi^{\mu\nu} \neq \pi^{\mu\nu}_{NavierStokes}$$

Note that in this transient regime, power counting in ℓ_{micro}/L_{macro}

is not straightforward because in this case $\pi^{\mu\nu}$ is an independent variable that follows a differential equation and not an algebraic equation.

The idea is that in the transient regime the dynamical variables are (at least)

$$\varepsilon\,,\,u^{\mu}\,,\,\pi^{\mu\nu}$$

In the transient regime, new terms are possible

$$D\pi^{\langle\mu\nu\rangle},\ D^2\pi^{\langle\mu\nu\rangle},\cdots$$

$$\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\,\alpha},\ \pi_\alpha^{\langle\mu}\Omega^{\nu\rangle\,\alpha},\ \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\,\alpha},\cdots$$

The most general equation compatible with the symmetries is

$$\cdots + \chi_2 D^2\pi^{\langle\mu\nu\rangle} + \chi_1\, D\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu}$$

$$= 2\eta\sigma^{\mu\nu} + e_1\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\,\alpha} + e_2\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\,\alpha} + e_3\pi_\alpha^{\langle\mu}\Omega^{\nu\rangle\,\alpha} - \sum_{i=1}^{5} 2\eta\, c_i\, O_i^{\mu\nu} + \cdots$$

The right-hand side, which is a source term for the nonlinear differential equation obeyed by the dissipative stress, can be organized in terms of a power series in Knudsen number and the “inverse Reynolds number”

$$\mathrm{Re}^{-1} \sim |\pi^{\mu\nu}\pi_{\mu\nu}|^{1/2}/P_0$$

→ Independent small variable << 1 in the transient regime

For instance, $e_1 \pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\,\alpha} \sim \mathcal{O}(\mathrm{Re}^{-1}\mathrm{Kn})$ while $e_2 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\,\alpha} \sim \mathcal{O}(\mathrm{Re}^{-2})$

At order $\mathcal{O}(\mathrm{Re}^{-2}, \mathrm{Re}^{-1}\mathrm{Kn}, \mathrm{Kn}^2)$ the EOM for the conformal fluid are

$$\tau_1 D\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + e_1 \pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\,\alpha} + e_2 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\,\alpha} + e_3 \pi_\alpha^{\langle\mu}\Omega^{\nu\rangle\,\alpha} - \sum_{i=1}^{5} 2\eta\, c_i\, O_i^{\mu\nu},$$

$$D\pi^{\langle\mu\nu\rangle} = \dot{\pi}^{\langle\mu\nu\rangle} + 4\pi^{\mu\nu}\theta/3$$

10 independent transport coefficients

Asymptotic limit: Use leading order $\pi^{\mu\nu} \sim 2\eta\sigma^{\mu\nu}$ back in the eq. above

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \sum_{i=1}^{5} 2\eta\, b_i\, \mathcal{O}_i^{\mu\nu} + \mathcal{O}(\mathrm{Kn}^3).$$

We recover the Kn^2 theory in the appropriate limit!

$b_1 = \tau_1 + c_1$, $b_2 = c_2$, $b_3 = c_3 - e_1 - e_2$, $b_4 = c_4 - e_3$, and $b_5 = c_5$

The nonlinear **relaxation equation** found at $\mathcal{O}(\mathrm{Re}^{-2}, \mathrm{Re}^{-1}\mathrm{Kn}, \mathrm{Kn}^2)$

is identical to the one found from the Boltzmann equation within the moments method (Denicol et al., 2012).

At order $\mathcal{O}(\mathrm{Re}^{-2}, \mathrm{Re}^{-1}\mathrm{Kn}^2, \mathrm{Kn}^2)$ the EOM become

$$\chi_2 D^2\pi^{\langle\mu\nu\rangle} + \chi_1 D\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + e_1\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\,\alpha} + e_2\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\,\alpha} + e_3\pi_\alpha^{\langle\mu}\Omega^{\nu\rangle\,\alpha}$$

$$-\sum_{i=1}^{5} 2\eta\, c_i\, O_i^{\mu\nu} + \sum_{i=1}^{5} f_i\, \pi_\rho^{\langle\mu} O_i^{\nu\rangle\,\rho} - \xi\, D^{\langle\mu} D_\lambda \pi^{\nu\rangle\,\lambda}.$$

17 independent transport coefficients

$$D^2\pi^{\langle\mu\nu\rangle} \longrightarrow \ddot{\pi}^{\langle\mu\nu\rangle} - 2u_\rho\dot{\pi}^{\rho\langle\mu}\dot{u}^{\nu\rangle} + \frac{20}{9}\pi^{\mu\nu}\theta^2 + 3\theta\,\dot{\pi}^{\langle\mu\nu\rangle} + \frac{4}{3}\pi^{\mu\nu}\dot{\theta}\,.$$

Again, this transient theory reduces to the Kn^2 theory derived by BRSSS in the asymptotic limit.

So, how do we truncate the left-hand side?? $\chi_n D^n \pi^{\langle\mu\nu\rangle} \sim \mathcal{O}(Re^{-1}, K_n^2)$

As it occurs in any differential equation, the coefficients that come with the "time" derivatives give the microscopic scales in the theory. The shorter the time, the more derivatives (and hence coefficients) one has to include on the left-hand side in the transient regime.

These timescales included in χ_n give the non-hydrodynamical modes of the system (which are complex numbers).

$$\lim_{k\to 0} \omega^{(n)}_{non-hydro}(k) = constant \qquad \lim_{k\to 0} \omega_{hydro}(k) \sim k$$

The structure of non-hydrodynamical modes depends on the magnitude of the coupling ...

Denicol, JN, Niemi, Rischke, PRD 2011

- How does one compute η and the new transport coefficient τ_{π} ????

- Several interesting attempts over the years ...

Israel and Stewart, 1978 = 14-moment approximation for the Boltzmann equation
Baier, Romatschke, Son, Starinets, Stephanov, 2007 = Kubo formulas + gradient expansion
Koide, Kodama, 2009 = Projection operator method

Progress has been achieved in this matter …

G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

General theory for the calculation of any linear transport coefficient (valid at both weak and strong coupling) of a relativistic fluid (~ a relativistic extension of Kubo's classical paper). You can ask me about the details later but you need to compute

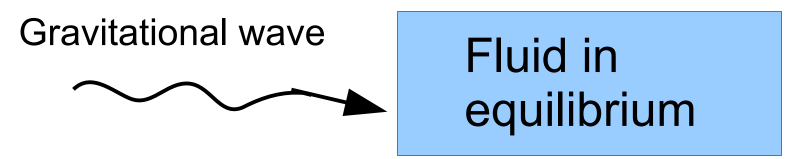
$$G_R^{\mu\nu\alpha\beta}(x-x') = i\,\theta(x-x')\langle[\hat{T}^{\mu\nu}(x), \hat{T}^{\alpha\beta}(x')]\rangle_T$$

Retarded 2-point function of the energy-momentum tensor

Determine the analytical structure of Fourier transform $G_R^{\mu\nu\alpha\beta}(\omega, k)$

Linear Response via Metric Disturbances

G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011 (based on seminal papers by Son et al.)



Metric disturbances around Minkowski

$$g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$$

Variation of the energy-momentum tensor

$$\delta T^{\mu\nu} = T^{\mu\nu}(\eta^{\alpha\beta} + h^{\alpha\beta}) - T^{\mu\nu}(\eta^{\alpha\beta})$$

Linear response theory (a la Kubo)

$$\delta T^{\mu\nu}(x) = \frac{1}{2}\int d^4x' \, G_R^{\mu\nu\alpha\beta}(x-x')\, h_{\alpha\beta}(x')$$

Retarded 2-point function

$$G_R^{\mu\nu\alpha\beta}(x-x') = i\,\theta(x-x')\langle[\hat{T}^{\mu\nu}(x), \hat{T}^{\alpha\beta}(x')]\rangle_T$$

Linear Response via Metric Disturbances

G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

A very specific disturbance is chosen where only $h_{xy}(t,z) \neq 0$
so then the equations decouple ...

$$\delta T^{xy}(t,z) = \int dt'\, dz'\, G_R^{xyxy}(t-t', z-z')\, h_{xy}(t',z')$$

The traditional form for the energy-momentum tensor is used

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}$$

To linear order in the metric disturbances we obtain …

$$\delta\pi^{xy}(t,z) = P_0\, h^{xy}(t,z) + \int dt'\, dz'\, G_R^{xyxy}(t-t', z-z')\, h_{xy}(t',z')$$

If the Green's function first pole is **purely imaginary** then the equation of motion for the dissipative current is

$$\tau_\pi \partial_t \delta\pi^{xy} + \delta\pi^{xy} = D_0 h_{xy} + D_1 \partial_t h_{xy} + D_2 \partial_t^2 h_{xy} + \mathcal{O}\left(\partial_t^3 h_{xy}, \partial_z^2 h_{xy}\right)$$

G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

And the coefficients are

Given by the first pole !!!

$$\tau_\pi = \frac{1}{i\omega_1(\mathbf{0})} ,$$

$$D_0 = \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} = -P_0 + \tilde{G}_R^{xyxy}(\omega, \mathbf{0})\Big|_{\omega=0} \equiv -P_0 + P_0 = 0 ,$$

$$D_1 = i\partial_\omega \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} + \tau_\pi D_0 = i\partial_\omega \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} \equiv \eta ,$$

Kubo formula

$$D_2 = -\frac{1}{2}\partial_\omega^2 \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} + D_1\tau_\pi - D_0\tau_\pi^2 \equiv -\frac{1}{2}\partial_\omega^2 \tilde{G}_R(\omega, \mathbf{0})\Big|_{\omega=0} + \eta\tau_\pi$$

These formulas are valid for any relativistic fluid at any value of the coupling !!!!!

Ok, so according to linear response theory, we only need the retarded correlator

$$G_R^{\mu\nu\alpha\beta}(x-x') = i\,\theta(x-x')\langle[\hat{T}^{\mu\nu}(x),\hat{T}^{\alpha\beta}(x')]\rangle_T$$

Its first derivative at the origin gives the shear viscosity (Kubo formula)

$$\eta = i\,\partial_\omega G_R^{xyxy}(\omega,\mathbf{0})\Big|_{\omega=0}$$

The relaxation time coefficient is given by the single pole nearest the origin (DNNR formula)

$$\tau_\pi = \frac{1}{i\,\omega_1(\mathbf{0})}$$

This uniquely defines the lowest order linear transport coefficients of any relativistic fluid !!!!

Application at weak coupling: Boltzmann equation

$$p_\mu \partial^\mu f = \mathcal{C}[f] \longrightarrow \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \ldots$$

- It was proven that the poles (at zero wavelength) are purely imaginary and, thus, the truncation to a relaxation equation is always possible.

- Low energy limit of relativistic dilute gases **ALWAYS** gives relaxation-type equations.

Simplest example:

Dilute 1-component, relativistic gas

Massless/classical limits

constant cross section

Denicol, JN, Niemi, Rischke, PRD 2011

In this case, $\eta = 4T/3\sigma$

Inverse of the Pole $\rightarrow$ $\tau_\pi = \dfrac{5}{3n_0\sigma}$

$$\frac{\eta}{\tau_\pi} = \frac{4}{5}P$$

After DNNR (2011), we finally know:

- What the (linear) equations of motion of any relativistic fluid dynamics are.

- The formulas that correctly define the transport coefficients at any value of the coupling

A big question however remains: How are we going to compute

$$G_R^{\mu\nu\alpha\beta}(\omega, k)$$

in a strongly-coupled system where there are no quasiparticles such as the QGP formed in heavy ion collisions ????

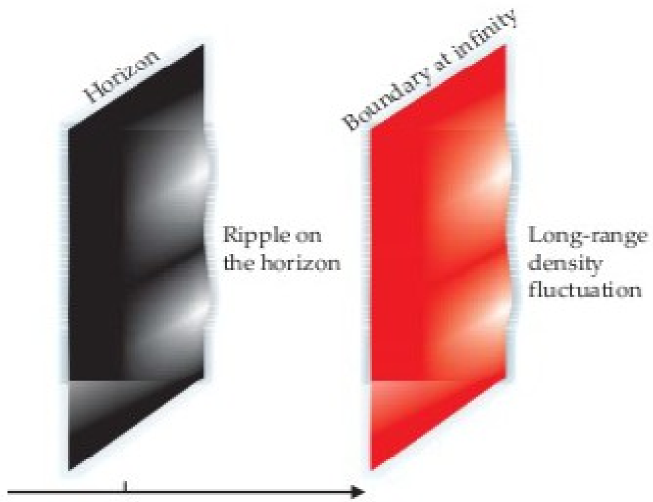
In general, lattice QCD cannot help us much here (we need a retarded correlator) ...

HOLOGRAPHY COMES TO THE RESCUE !!!!!

Black branes are solutions of the equations of motion of higher dimensional theories of gravity. These objects possess temperature (via Hawking) and they are used in the description of a thermal equilibrium state in the 4D gauge theory.

Plasma's temperature

= Black brane temperature



Near equilibrium fluctuations in the plasma ~ black brane horizon fluctuations !!!!

Application of Holography

In 2005

Pavel Kovtun

Dam Son

Andrei Starinets

asked (and answered) the following question:

"What would be the value of the shear viscosity to entropy density ratio of a strongly coupled gauge theory???? "

Universality of the shear viscosity in supergravity

The retarded correlator is obtained from the on-shell gravity action gives via the duality

$$\phi(r) \equiv \delta h_y^x(r) \quad \rightarrow \quad \Box\phi = 0 \quad \rightarrow \quad G_R^{\mu\nu\alpha\beta}$$

Massless scalar field coupled to gravity in the bulk

Retarded correlator in the gauge theory

$\eta \sim Area$ = Total graviton absorption cross section when $\omega \rightarrow 0$

$s \sim Area$ Bekenstein's area law

Thus, one can see that

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

PERFECT FLUIDITY!!!

Kovtun, Son, Starinets; Buchel, Liu, 2005

Ok, so let's start with the Kubo formula for the shear viscosity

$$\eta = -\lim_{\omega,k\to 0} \frac{\mathrm{Im}\, G_R(\omega, k)}{\omega}$$

$$G_R(t, x) = \langle T_{xy}(t, x) T_{xy}(0, 0)\rangle\, \Theta(t)$$

- The operator in this case is the T_{xy} component of the energy-momentum tensor in the QFT.

- This should be associated with the bulk metric component g_{xy}

- Disturbing the metric $g_{xy} \to g_{xy} + \delta h_{xy}$ in the bulk action one finds

defining $\phi = \delta h^x_y$

$$I_\phi = -\frac{1}{16\pi G_N} \int d^{d+1}x\, \sqrt{g}\, \frac{1}{2} (\partial\phi)^2$$

- Note that the energy-momentum tensor is always a dimension 4 operator and, thus the dual field in the bulk must be massless (for this component it is then a massless scalar).

- Note also that we are limiting ourselves at this point to consider only GR-like bulk theories, i.e., the bulk action contains at most two derivatives of the fields.

- Also, note that we are assuming that the plasma is isotropic.

- We then obtain

$$G_R(k) = \lim_{r\to 0} \frac{\Pi_{\text{in}}(\omega, \vec{k}, r)}{\Phi_{\text{in}}(\omega, \vec{k}, r)} \qquad \eta = -\lim_{\omega, k\to 0} \lim_{r\to 0} \frac{\Pi_{\text{in}}(\omega, \vec{k}, r)}{i\omega\Phi_{\text{in}}(\omega, \vec{k}, r)}$$

Ok, so how does the alleged universal behavior for η show up?

First, write the equations of motion for the field in Hamiltonian form

$$\Pi = -\sqrt{-g}\, g^{rr} \partial_r \phi \qquad \partial_r \Pi = \sqrt{-g}\, g^{rr} k^2 \phi$$

Now we take the low frequency limit $k_\mu \to 0$ with $\omega\phi$ and Π fixed

$$\partial_r \Pi = 0 + \mathcal{O}(k_\mu \omega \phi) \qquad \partial_r(\omega\phi) = 0 + \mathcal{O}(\omega\Pi)$$

Therefore, the equations become trivial in this limit and the shear

$$\eta = -\lim_{\omega,k\to 0} \lim_{r\to 0} \frac{\Pi_{\rm in}(\omega, \vec{k}, r)}{i\omega \Phi_{\rm in}(\omega, \vec{k}, r)}$$

can be computed at any value of “r” !!!!

The remarkable observation is that things simplify enormously at the horizon !!!!

Indeed, consider the general 5d metric consistent with the symmetries

$$ds^2 = -g_{00}dt^2 + g_{xx}d\vec{x}^2 + g_{rr}dr^2$$

where $g_{00}(r_H) = g^{rr}(r_H) = 0$

EOM: $\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) = 0$

Ansatz: $\phi(r,\omega) = g_\omega(r)\phi_0(\omega)$

$\vec{k} = 0$

we then obtain

$$\left(\sqrt{g_{00}g^{rr}g_{xx}^3}\partial_r\right)^2 g_\omega(r) + \omega^2 g_{xx}^3\, g_\omega(r) = 0$$

Now, here comes the magic!

"Universality of low-energy absorption cross-sections for black holes"
Das, Gibbons, Mathur, PRL 1997

Using that $$T = \frac{1}{4\pi}\sqrt{g_{00}'(r_H)\,g^{rr\prime}(r_H)}$$

NEAR THE HORIZON
the equation becomes

$$g_\omega''(r) + \frac{g_\omega'(r)}{r - r_H} + \left(\frac{\omega}{4\pi T}\right)^2 \frac{g_\omega(r)}{(r - r_H)^2} = 0$$

which is now universal since it does not depend on the details of the geometry !!!!!

Near horizon solutions ~ $(r - r_H)^{-i\omega/4\pi T}$ $\quad$ $(r - r_H)^{i\omega/4\pi T}$

In-falling at horizon $\quad$ out-going at horizon

The correct boundary condition for the retarded correlator is the in-falling solution.

In this case the canonical momentum at the horizon is

$$\Pi(r_H,\omega) = \frac{-i\omega\, g_{xx}^{3/2}(r_H)\phi_0(\omega)g_\omega(r_H)}{16\pi G_5}$$

which means that $\eta = \dfrac{g_{xx}^{3/2}(r_H)}{16\pi G_5}$

However, since the entropy density computed using the horizon area is

$$s = \frac{g_{xx}^{3/2}(r_H)}{4G_5} \quad \Rightarrow \quad \boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

PERFECT FLUIDITY!!!

Kovtun, Son, Starinets; Buchel, Liu, 2005

UNIVERSAL LOWER BOUND FOR ALL THE SUBSTANCES IN NATURE???

Kovtun, Son, Starinets, 2005.
Buchel, Liu, 2005

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

See also,
Danielewicz, Gyulassy, 1985.

Is this universal at least in string theory? In some limits, yes! However, deviations from these limits can lead to extra terms that can violate the bound. For instance,

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2G_5} \left(R + \frac{12}{L^2} \right) + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right]$$

Brigante et al., 2008
Petrov and Kats, 2007

$$\lim_{N_c \to 1} c_i \ll 1$$

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\gamma)$$

$$\gamma = 4c_3\, G_5/L^2$$

When $N_c \to \infty$ $\lambda \gg 1$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Universal for all gravity dual theories in the supergravity approximation Buchel, Liu, 2004; KSS, 2005

- Conformal and non-conformal theories
- Non-zero chemical potential
- Presence of fundamental matter

λ - corrections for $\mathcal{N}=4$ SYM / Type IIB → $\alpha'^3 R^4$

added to the bulk action

Gubser, Klebanov, Tseytlin, 1998

$$\frac{L^2}{\alpha'} = \sqrt{\lambda}$$

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{135\zeta(3)}{8(2\lambda)^{3/2}} + \dots\right) \geq \frac{1}{4\pi}$$

Buchel, Liu, Starinets, 2005

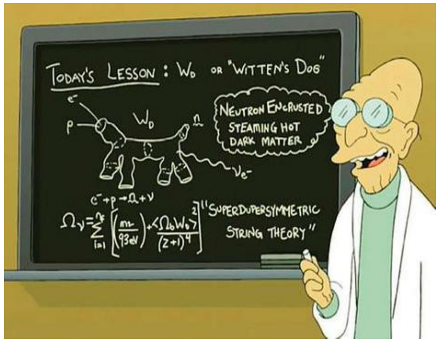
SUMMARY OF THE 3rd LECTURE

- Strongly coupled systems (defined at VERY different length scales) can behave like fluids in which the shear viscosity nears the uncertainty principle lower bound.

- The gauge/gravity duality may be the most natural tool to understand the perfect fluidity of strongly coupled systems that do not admit a standard quasiparticle description.

- According to holography, any strongly interacting quantum many-body system at finite density and temperature with sufficiently many d.o.f /volume is predicted to behave at low energies as a perfect fluid.

LECTURE IV: Getting stringy and etc…



OUTLINE OF THIS LECTURE

1- Some comments about string theory

2- Large N gauge theories and string theory

3- Holography and the decoupling limit

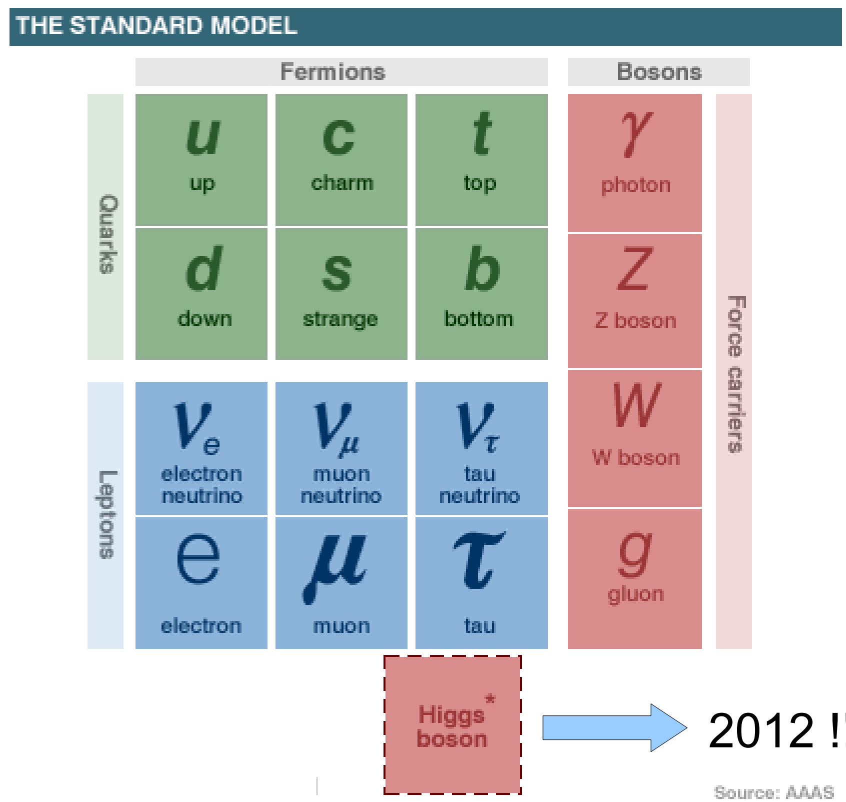
4- Deconfinement from the top-down

5- Summary

Standard Model of Particle Physics

Everything??????

Current status ...



2012 !!!!

Gravity ????

Dark matter ???

Dark energy ???

Supersymmetry???

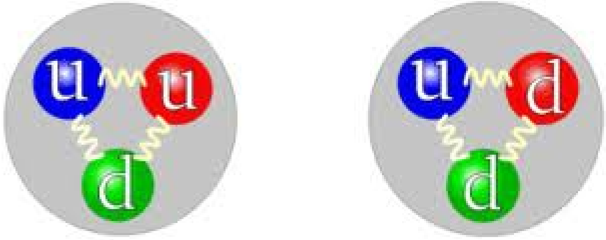
The Theory of Strong Interactions

One of the fundamental pieces within the standard model

In development since 1960's … the fundamental theory of hadrons

Baryon = 3 quarks

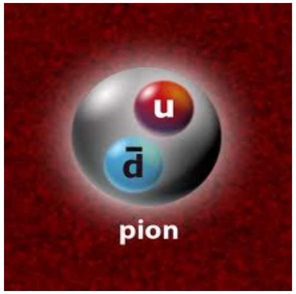
Meson = 2 quarks



Proton

Neutron

Quark composition of a proton and a neutron (diagrams from *Wikipedia*)



Up to the 50's a few hadrons were known ...

Baryons = 3 quarks

Nowadays!!!

Mesons = 2 quarks

QCD HADRON PARTICLE ZOO

Baryon Summary Table

This short table gives the name, the quantum numbers (where known), and the status of baryons in the Review. Only the baryons with 3- or 4-star status are included in the main Baryon Summary Table. Due to insufficient data or uncertain interpretation, the other entries in the short table are not established as baryons. The names with masses are of baryons that decay strongly. For N, Δ, and Ξ resonances, the partial wave is indicated by the symbol $L_{2I,2J}$, where L is the orbital angular momentum (S, P, D, ...), I is the isospin, and J is the total angular momentum. For Λ and Σ resonances, the symbol is $L_{I,2J}$.

p	P_{11}	****	$\Delta(1232)$	P_{33}	****	Λ	P_{01}	****	Σ^+	P_{11}	****	Ξ^0	P_{11}	****
n	P_{11}	****	$\Delta(1600)$	P_{33}	***	$\Lambda(1405)$	S_{01}	****	Σ^0	P_{11}	****	Ξ^-	P_{11}	****
$N(1440)$	P_{11}	****	$\Delta(1620)$	S_{31}	****	$\Lambda(1520)$	D_{03}	****	Σ^-	P_{11}	****	$\Xi(1530)$	P_{13}	****
$N(1520)$	D_{13}	****	$\Delta(1700)$	D_{33}	****	$\Lambda(1600)$	P_{01}	***	$\Sigma(1385)$	P_{13}	****	$\Xi(1620)$		*
$N(1535)$	S_{11}	****	$\Delta(1750)$	P_{31}	*	$\Lambda(1670)$	S_{01}	****	$\Sigma(1480)$		*	$\Xi(1690)$		***
$N(1650)$	S_{11}	****	$\Delta(1900)$	S_{31}	**	$\Lambda(1690)$	D_{03}	****	$\Sigma(1560)$		**	$\Xi(1820)$	D_{13}	***
$N(1675)$	D_{15}	****	$\Delta(1905)$	F_{35}	****	$\Lambda(1800)$	S_{01}	***	$\Sigma(1580)$	D_{13}	*	$\Xi(1950)$		***
$N(1680)$	F_{15}	****	$\Delta(1910)$	P_{31}	****	$\Lambda(1810)$	P_{01}	***	$\Sigma(1620)$	S_{11}	**	$\Xi(2030)$		***
$N(1700)$	D_{13}	***	$\Delta(1920)$	P_{33}	***	$\Lambda(1820)$	F_{05}	****	$\Sigma(1660)$	P_{11}	***	$\Xi(2120)$		*
$N(1710)$	P_{11}	***	$\Delta(1930)$	D_{35}	***	$\Lambda(1830)$	D_{05}	****	$\Sigma(1670)$	D_{13}	****	$\Xi(2250)$		**
$N(1720)$	P_{13}	****	$\Delta(1940)$	D_{33}	*	$\Lambda(1890)$	P_{03}	****	$\Sigma(1690)$		**	$\Xi(2370)$		**
$N(1900)$	P_{13}	**	$\Delta(1950)$	F_{37}	****	$\Lambda(2000)$		*	$\Sigma(1750)$	S_{11}	***	$\Xi(2500)$		*
$N(1990)$	F_{17}	**	$\Delta(2000)$	F_{35}	**	$\Lambda(2020)$	F_{07}	*	$\Sigma(1770)$	P_{11}	*			
$N(2000)$	F_{15}	**	$\Delta(2150)$	S_{31}	*	$\Lambda(2100)$	G_{07}	****	$\Sigma(1775)$	D_{15}	****	Ω^-		****
$N(2080)$	D_{13}	**	$\Delta(2200)$	G_{37}	*	$\Lambda(2110)$	F_{05}	***	$\Sigma(1840)$	P_{13}	*	$\Omega(2250)^-$		***
$N(2090)$	S_{11}	*	$\Delta(2300)$	H_{39}	**	$\Lambda(2325)$	D_{03}	*	$\Sigma(1880)$	P_{11}	**	$\Omega(2380)^-$		**
$N(2100)$	P_{11}	*	$\Delta(2350)$	D_{35}	*	$\Lambda(2350)$	H_{09}	***	$\Sigma(1915)$	F_{15}	****	$\Omega(2470)^-$		**
$N(2190)$	G_{17}	****	$\Delta(2390)$	F_{37}	*	$\Lambda(2585)$		**	$\Sigma(1940)$	D_{13}	***	[illegible]		[illegible]
$N(2200)$	D_{15}	**	$\Delta(2400)$	G_{39}	**				$\Sigma(2000)$	S_{11}	*	Λ_c^+		****
$N(2220)$	H_{19}	****	$\Delta(2420)$	$H_{3,11}$	****				$\Sigma(2030)$	F_{17}	****	$\Lambda_c(2593)^+$		***
$N(2250)$	G_{19}	****	$\Delta(2750)$	$I_{3,13}$	**				$\Sigma(2070)$	F_{15}	*	$\Lambda_c(2625)^+$		***
$N(2600)$	$I_{1,11}$	***	$\Delta(2950)$	$K_{3,15}$	**				$\Sigma(2080)$	P_{13}	**	$\Lambda_c(2765)^+$		*
$N(2700)$	$K_{1,13}$	**							$\Sigma(2100)$	G_{17}	*	$\Lambda_c(2880)^+$		**
			$\Theta(1540)^+$		*				$\Sigma(2250)$		***	$\Sigma_c(2455)$		****
									$\Sigma(2455)$		**	$\Sigma_c(2520)$		***
									$\Sigma(2620)$		**	$\Sigma_c(2800)$		***
									$\Sigma(3000)$		*	Ξ_c^+		***
									$\Sigma(3170)$		*	Ξ_c^0		***
												$\Xi_c'^+$		***
												$\Xi_c'^0$		***
												$\Xi_c(2645)$		***
												$\Xi_c(2790)$		***
												$\Xi_c(2815)$		***
												Ω_c^0		***
												Ξ_{cc}^+		*
												Λ_b^0		***
												Ξ_b^0, Ξ_b^-		*

**** Existence is certain, and properties are at least fairly well explored.
*** Existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, branching fractions, etc. are not well determined.
** Evidence of existence is only fair.
* Evidence of existence is poor.

Meson Summary Table

See also the table of suggested $q\bar{q}$ quark-model assignments in the Quark Model section.
• Indicates particles that appear in the preceding Meson Summary Table. We do not regard the other entries as being established.
† Indicates that the value of J given is preferred, but needs confirmation.

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_2(1270)$	$0^+(2^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$	• $f_2(1950)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$
• $a_2(1320)$	$1^-(2^{++})$	$f_0(2020)$	$0^+(0^{++})$
• $f_0(1370)$	$0^+(0^{++})$	• $a_4(2040)$	$1^-(4^{++})$
$h_1(1380)$	$?^-(1^{+-})$	• $f_4(2050)$	$0^+(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$	$f_0(2100)$	$0^+(0^{++})$
• $f_1(1420)$	0^+([illegible])	$f_2(2150)$	$0^+(2^{++})$
• $\omega(1420)$	0^-([illegible])	$\rho(2150)$	$1^+(1^{--})$
$f_2(1430)$	$0^+(2^{++})$	$f_0(2200)$	$0^+(0^{++})$
• $a_0(1450)$	$1^-(0^{++})$	$f_J(2220)$	$0^+(2 \text{ or } 4^{++})$
• $\rho(1450)$	$1^+(1^{--})$	$\eta(2225)$	$0^+(0^{-+})$
• $\eta(1475)$	$0^+(0^{-+})$	$\rho_3(2250)$	$1^+(3^{--})$
• $f_0(1500)$	$0^+(0^{++})$	• $f_2(2300)$	$0^+(2^{++})$
$f_1(1510)$	$0^+(1^{++})$	$f_4(2300)$	$0^+(4^{++})$
• $f_2'(1525)$	$0^+(2^{++})$	• $f_2(2340)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
$h_1(1595)$	$0^-(1^{+-})$	$a_6(2450)$	$1^-(6^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$f_6(2510)$	$0^+(6^{++})$
$a_1(1640)$	$1^-(1^{++})$	**OTHER LIGHT**	
$f_2(1640)$	$0^+(2^{++})$	Further States	
• $\eta_2(1645)$	$0^+(2^{-+})$		
• $\omega(1650)$	$0^-(1^{--})$		
• $\omega_3(1670)$	$0^-(3^{--})$		

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
$D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D^*(2640)^\pm$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

	$I^G(J^{PC})$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
B_s^*	$0(1^-)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• $B_c^\pm$	$0(0^-)$

$c\bar{c}$

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $X(3872)$	$0^?(?^{?+})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$Y(3940)$	$?^?(?^{??})$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
$Y(4260)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

$b\bar{b}$

$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
$\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

NON $q\bar{q}$ CANDIDATES

How do we make sense of this???

Quantum Chromodynamics

David J. Gross H. David Politzer Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

6 quarks = fundamental fermions with mass

8 gluons = fundamental massless gauge bosons that mediate the interactions

Fundamental Lagrangian density

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f^{N_f} \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f$$

Gluons

Quark fields

It looks like QED but it has a fundamental difference …
gluons interact to each other!!!

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

Self-interaction!

Direct calculations can only be performed when the coupling is very small …

Under which conditions can this happen?????

Quantum Chromodynamics

Fundamental property of QCD: Asymptotic freedom !!!   Gross, Wilczek, Politzer, 2004

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

Self-interaction!

Dimensional transmutation: Even without any quarks there is an energy scale

$$\Lambda_{QCD} \sim 200\ MeV$$

When typical momentum of the interaction q → infinity

$$g^2(q^2) = \frac{16\pi^2}{b_0 \log(q^2/\Lambda^2_{QCD})}, \qquad b_0 = \frac{11}{3}N_c - \frac{2}{3}N_f,$$

Then, at sufficiently high energies QCD is weakly-coupled !!!

QCD is the true origin of mass in the observable universe !!!

Proton mass = 934 MeV

Proton = 3 quarks

Summing up the bare masses of the quarks in the proton < 30 MeV

~ 97 % of the proton mass has nothing to do with the Higgs field !!!!!

Then, 97% of the matter of the observable universe (4% of total) comes from the interactions between quarks and gluons in QCD !!!

Confinement of quarks and gluons inside hadrons remains as one of the fundamental problems in physics.

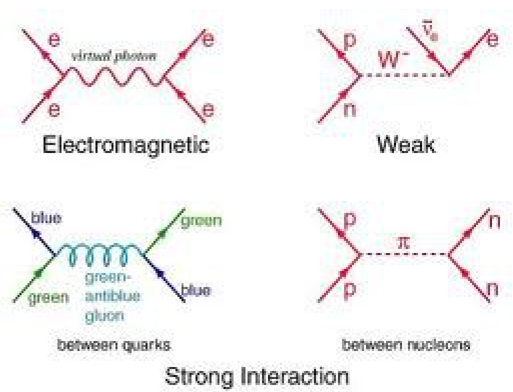
String theory appeared in the early 70's as a theory for mesons. It did not work out because it predicted the existence of a massless spin 2 “meson”.

One man's trash is another man's treasure ...

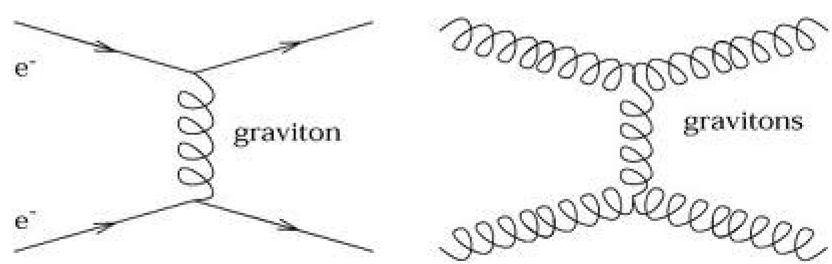
The “old string theory” failed as the theory of strong interactions in the early 70's …

However, at the same time it was realized that the massless spin 2 field that appears in this theory could be understood as the graviton (the quantum of gravitation) ...

In general

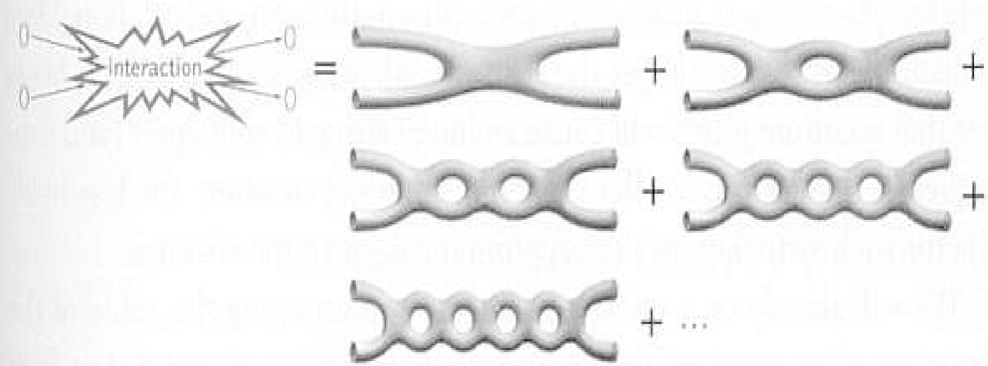


Quantum gravity !!!



String Theory as the theory of everything (1980's) ...

“All the fundamental particles in nature can be understood as different modes of vibration of a single, unified string theory”



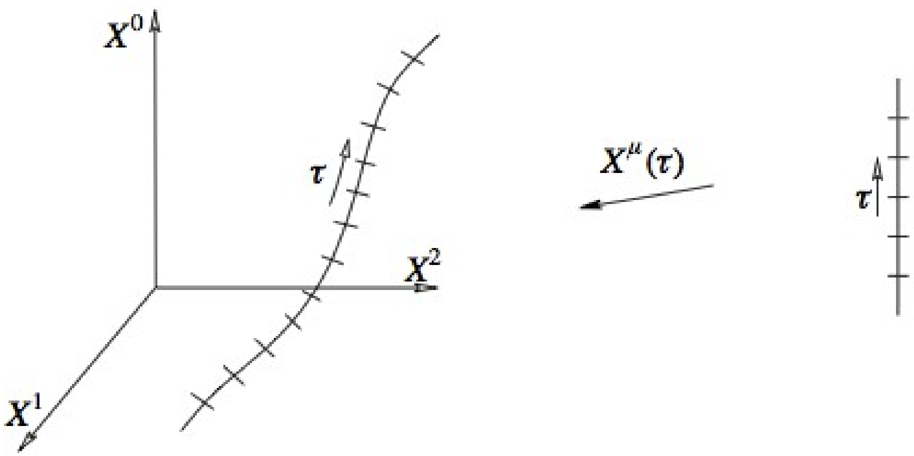
The net influence each incoming string has on the other comes from adding together the influences involving diagrams with ever more loops.

$$\ell_{Planck} \sim 10^{-35} m$$

Let's talk a bit about these things (VERY superficially, though)

See, for instance, C. Johnson's book

Geometrical view of a relativistic particle



$$\frac{d^2x^\lambda}{d\tau^2} = -\Gamma^\lambda_{\mu\nu}(g)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$$

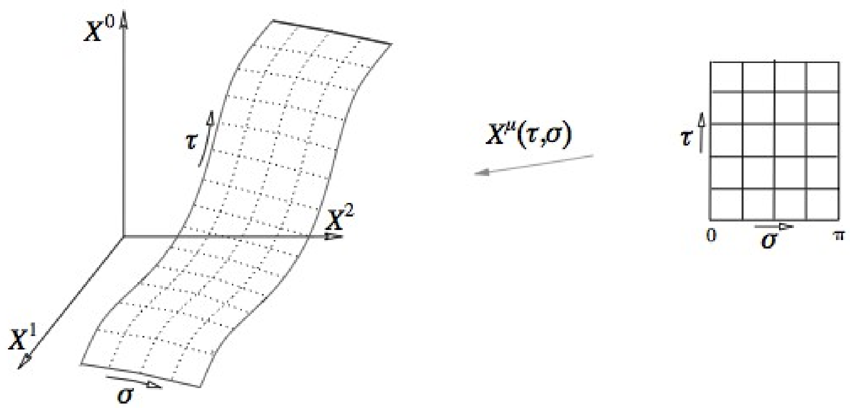
EOM

$$\Gamma^\lambda_{\mu\nu}(g) = \frac{1}{2}g^{\lambda\kappa}\left(\partial_\mu g_{\kappa\nu} + \partial_\nu g_{\kappa\mu} - \partial_\kappa g_{\mu\nu}\right)$$

Action: $$S = -m\int(-g_{\mu\nu}(x)dx^\mu dx^\nu)^{1/2} = -m\int_{\tau_i}^{\tau_f}(-g_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu)^{1/2}d\tau\ ,$$

Length of the particle's worldline in spacetime

String propagating in spacetime



Induced metric on the worldsheet

$$h_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu},$$

Action = Area

we extremise the area of the world-sheet:

$$S = -T \int dA = -T \int d\tau d\sigma \, (-\det h_{ab})^{1/2} \equiv \int d\tau d\sigma \; \mathcal{L}(\dot{X}, X'; \sigma, \tau).$$

$$S = -T \int d\tau d\sigma \left[\left(\frac{\partial X^\mu}{\partial \sigma} \frac{\partial X^\mu}{\partial \tau} \right)^2 - \left(\frac{\partial X^\mu}{\partial \sigma} \right)^2 \left(\frac{\partial X_\mu}{\partial \tau} \right)^2 \right]^{1/2}$$
$$= -T \int d\tau d\sigma \left[(X' \cdot \dot{X})^2 - X'^2 \dot{X}^2 \right]^{1/2},$$

→ Nambu-Goto action

where X' means $\partial X / \partial \sigma$.

We introduce a new field, an independent metric on the worldsheet $\gamma_{ab}(\tau, \sigma)$

$$S = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma (-\gamma)^{1/2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$
$$= -\frac{1}{4\pi\alpha'} \int d^2\sigma \, (-\gamma)^{1/2} \gamma^{ab} h_{ab}.$$

Polyakov action

The theory is invariant under reparametrizations on the worldsheet.

$$S = \frac{1}{4\pi\alpha'} \int_{\mathcal{M}} d^2\sigma \, g^{1/2} g^{ab} \partial_a X^\mu \partial_b X_\mu + \lambda \left\{ \frac{1}{4\pi} \int_{\mathcal{M}} d^2\sigma \, g^{1/2} R + \frac{1}{2\pi} \int_{\partial\mathcal{M}} ds K \right\}$$

Now this is a theory for X and g (like GR coupled to D scalar fields in 2 dimensions).

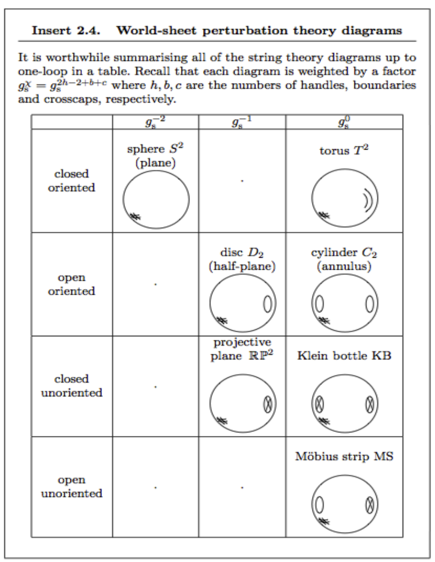
“Statistical mechanics” of surfaces is a hard problem ...

$$\mathcal{Z} = \int \mathcal{D}X\mathcal{D}g\, e^{-S},$$

Euler number

$$\chi = \frac{1}{4\pi}\int_{\mathcal{M}} d^2\sigma\,(-\gamma)^{1/2}R + \frac{1}{2\pi}\int_{\partial\mathcal{M}} ds K,$$

resulting amplitudes will be weighted by a factor $e^{-\lambda\chi}$, where $\chi = 2-2h-b-c$. Here, h, b, c are the numbers of handles, boundaries and crosscaps,



Insert 2.4. World-sheet perturbation theory diagrams

It is worthwhile summarising all of the string theory diagrams up to one-loop in a table. Recall that each diagram is weighted by a factor $g_s^{\chi} = g_s^{2h-2+b+c}$ where h, b, c are the numbers of handles, boundaries and crosscaps, respectively.

	g_s^{-2}	g_s^{-1}	g_s^{0}
closed oriented	sphere S^2 (plane)	·	torus T^2
open oriented	·	disc D_2 (half-plane)	cylinder C_2 (annulus)
closed unoriented	·	projective plane $\mathbb{RP}^2$	Klein bottle KB
open unoriented	·	·	Möbius strip MS

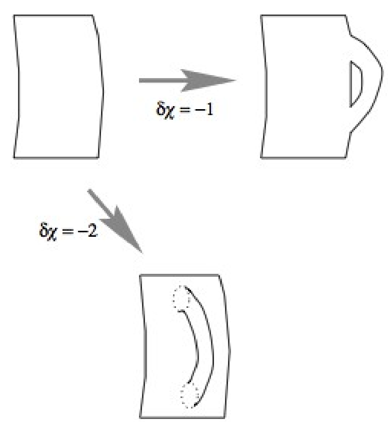


Fig. 2.2. World-sheet topology change due to emission and reabsorption of open and closed strings.

Crosscap = The self-intersection of a one-sided surface

Strings in curved spacetimes (“fancy” non-linear sigma model)

$$S_\sigma = \frac{1}{4\pi\alpha'} \int d^2\sigma \, g^{1/2} \left\{ (g^{ab} G_{\mu\nu}(X) + i\epsilon^{ab} B_{\mu\nu}(X)) \partial_a X^\mu \partial_b X^\nu + \alpha' \Phi R \right\}$$

$B_{\mu\nu}$ Anti-symmetric tensor

$G_{\mu\nu}$ Background metric

Φ Dilaton

Now one has to quantize this respecting the symmetries (Weyl invariance and etc ...)

USP

Weyl invariance requires that the trace of the worldsheet energy-momentum tensor vanishes

$$T^a{}_a = -\frac{1}{2\alpha'}\beta^G_{\mu\nu} g^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{i}{2\alpha'}\beta^B_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\beta^\Phi R$$

$$\beta^G_{\mu\nu} = \alpha'\left(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\Phi - \frac{1}{4}H_{\mu\kappa\sigma}H_\nu{}^{\kappa\sigma}\right) + O(\alpha'^2),$$

$$\beta^B_{\mu\nu} = \alpha'\left(-\frac{1}{2}\nabla^\kappa H_{\kappa\mu\nu} + \nabla^\kappa\Phi H_{\kappa\mu\nu}\right) + O(\alpha'^2),$$

$$\beta^\Phi = \alpha'\left(\frac{D-26}{6\alpha'} - \frac{1}{2}\nabla^2\Phi + \nabla_\kappa\Phi\nabla^\kappa\Phi - \frac{1}{24}H_{\kappa\mu\nu}H^{\kappa\mu\nu}\right) + O(\alpha'^2)$$

$$\boxed{\beta^G_{\mu\nu} = \beta^B_{\mu\nu} = \beta^\Phi = 0}$$

$$H_{\mu\nu\kappa} \equiv \partial_\mu B_{\nu\kappa} + \partial_\nu B_{\kappa\mu} + \partial_\kappa B_{\mu\nu}.$$

The vanishing beta functions are like equations of motion from an action

$$S = \frac{1}{2\kappa_0^2}\int d^DX(-G)^{1/2}e^{-2\Phi}\left[R + 4\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} + O(\alpha')\right].$$

Closed bosonic string

This is why one says that the low energy limit of quantized string theories reduce to higher dimensional GR + stuff …

A nontrivial D-dimensional "spacetime" is created due to the propagation of strings.

There is an infinite number of details I have skipped: critical dimension, spectrum, stability, supersymmetry/superstrings, compactification and etc.

The success and impact of these ideas in high energy particle physics in the 1980's were enormous ...

Famous physicists even said at the time that the fundamental theory of everything was within our grasp !!!

Edward Witten

→ “string theory is 21st century physics which fell into the 20th century ...”

Others were not so enthusiastic in the 80's ...

Richard Feynman

“String theorists don't make predictions, they make excuses.”

Indeed, since string theory is theory of quantum gravity, its natural scale is

$$\ell_{Planck} \sim 10^{-35} m$$

which may never be experimentally accessible.

Mathematical consistency became the major guide for all those studies. After some time this lack of experimental verification started to create some controversy among (non-stringy) physicists.

String theorists did in the end the only thing they could: they kept working hard and generating more and more knowledge!

Things changed radically after Maldacena's breakthrough in 1998.

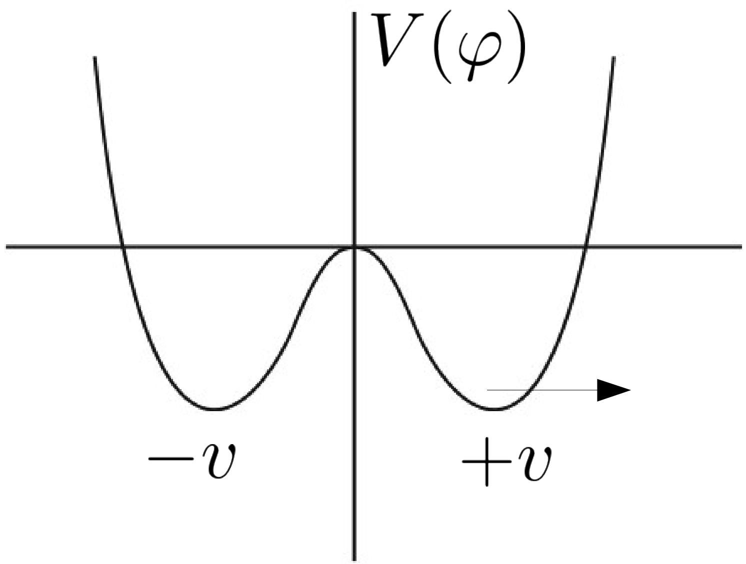
There is more to QFT than just Feynman diagrams …

Anonymous

See A. Zee, QFT in a Nutshell

Solitons and the kink solution of $\lambda\,\phi^4$ in 1+1 dimensions

$$\mathcal{L} = \tfrac{1}{2}(\partial\varphi)^2 - V(\varphi) \qquad V(\varphi) = (\lambda/4)(\varphi^2 - v^2)^2$$

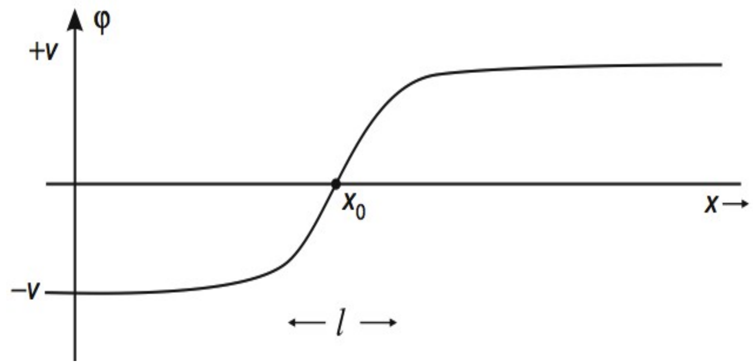


Small oscillations around the minimum produce massive "mesons" particles

$$\mu = (\lambda v^2)^{\frac{1}{2}}$$

The soliton solution $\varphi(x) = v\, tanh\left(\mu \frac{(x - x_0)}{\sqrt{2}}\right)$

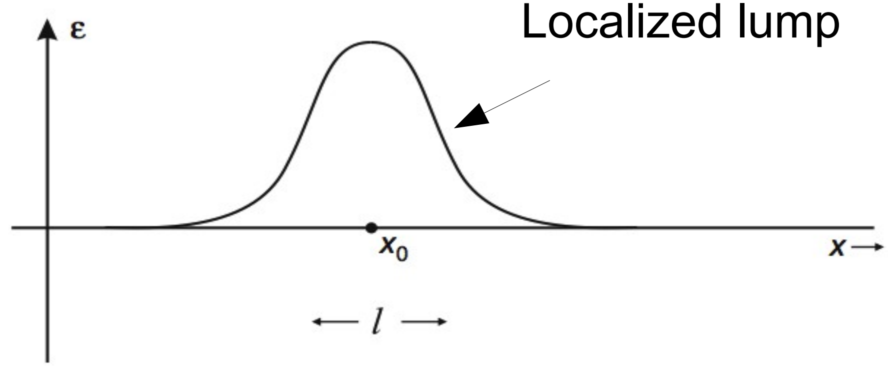
$l \sim 1/\mu$



Energy density $\varepsilon(x) = \frac{1}{2}\left(\frac{d\varphi}{dx}\right)^2 + \frac{\lambda}{4}(\varphi^2 - v^2)^2$

Mass $M = \int dx\, \varepsilon(x)$

Localized lump



Checking the behavior of the soliton's energy

$$M = \int dx \left[\frac{1}{2} \left(\frac{d\varphi}{dx} \right)^2 + \frac{\lambda}{4} \left(\varphi^2 - v^2 \right)^2 \right]$$

$$= \left(\frac{\mu^2}{\lambda} \right) \mu \int dy \left[\frac{1}{2} \left(\frac{df}{dy} \right)^2 + \frac{1}{4} \left(f^2 - 1 \right)^2 \right]$$

$\varphi(x) \to v f(y)$ and $y = \mu x$

$$M = a(\mu^2/\lambda)\mu$$

"a" pure number

The soliton is "heavy" at weak coupling $\lambda \to 0$ and light at strong coupling $\lambda \to \infty$

The "meson" is light at weak coupling and heavy at strong coupling.

At low energies and strong coupling, the soliton becomes the relevant d.o.f.

Conservation of topological charge

$$J^\mu = \frac{1}{2v}\varepsilon^{\mu\nu}\partial_\nu\varphi \quad \longrightarrow \quad \partial_\mu J^\mu = 0$$

Topological charge: $$Q = \int_{-\infty}^{+\infty} dx\, J^0(x) = \frac{1}{2v}[\varphi(+\infty) - \varphi(-\infty)]$$

BPS: $$M \geq \int dx \left(\frac{\lambda}{2}\right)^{\frac{1}{2}} \left|\left(\frac{d\varphi}{dx}\right)(\varphi^2 - v^2)\right| \geq \left(\frac{\lambda}{2}\right)^{\frac{1}{2}} \left|\left[\frac{1}{3}\varphi^3 - v^2\varphi^2\right]_{-\infty}^{+\infty}\right| = \left|\frac{4}{3\sqrt{2}}\mu\left(\frac{\mu^2}{\lambda}\right)Q\right|$$

$$M \geq |Q|$$

The Reissner-Nordstrom Black Hole in 4D

- Black hole has mass M and electric charge Q

$$R_{\mu\nu} - g_{\mu\nu}\frac{R}{2} = 8\pi G_4 T_{\mu\nu}$$

Field tensor

$$T_{\mu\nu} = F_{\mu\rho}F_{\nu}^{\rho} - \frac{g_{\mu\nu}}{4}F_{\alpha\beta}F^{\alpha\beta}$$

Spherical symmetry

$$F_{tr} = E_r = \frac{Q}{r^2}$$

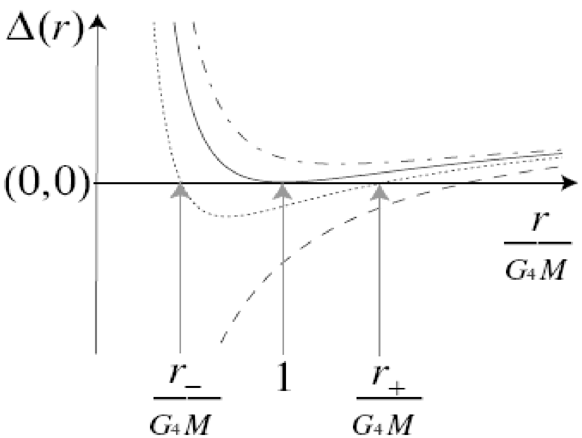
The metric is $$ds^2 = -\Delta dt^2 + \Delta^{-1}dr^2 + r^2 d\Omega_2^2$$

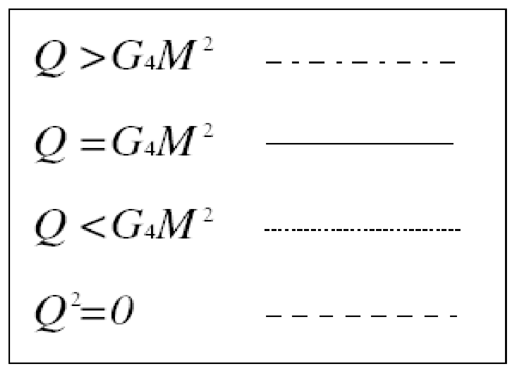
$$\Delta = 1 - \frac{2G_4 M}{r} + \frac{Q^2 G_4}{r^2}$$

- There is still a singularity at r = 0. Note also that $g_{00} \rightarrow 0$ when

$$r = r_{\pm} = MG_4 \pm \sqrt{(MG_4)^2 - Q^2 G_4}$$

$r_{\pm}$ Outer (inner) horizon





$$M \geq |Q|/\sqrt{G_4}$$

To avoid the creation of a naked singularity at r = 0 (cosmic censorship conjecture)

$$\Delta = 1 - \frac{2G_4 M}{r} + \frac{Q^2 G_4}{r^2}$$

This looks like a soliton but in a theory of gravity ...

Extremal Reissner-Nordstrom Black Holes in 4D

$$r_\pm = MG_4 \qquad\qquad |Q| = M\sqrt{G_4}$$

In string theory, the saturation of this bound is often equivalent to the saturation of a BPS bound, which then implies that the theory has some unbroken supersymmetry.

$$ds^2 = -\left(1-\frac{r_0}{r}\right)^2 dt^2 + \left(1-\frac{r_0}{r}\right)^{-2} dr^2 + r^2 d\Omega_2^2$$

$$r_0 = MG_4$$

When $r \sim r_0$ → $AdS_2 \otimes S^2$

Black p-branes

Black branes appear in string theory

They are solutions of type II supergravity in 10 dimensions

$$S^{(p)} = \frac{1}{2G_{10}} \int d^{10}x \sqrt{-g} \left[e^{-2\Phi} \left(\mathcal{R} + 4(\partial\Phi)^2 \right) - \frac{1}{2} |F_{p+2}|^2 \right]$$

If p is odd → Type IIB

If p is even → Type IIA

If p = 3 then $\int_{S^5} F_5 = N_c$ D3-brane charge

Φ Spatially homogeneous

$F_5 = F_5^*$ Self dual

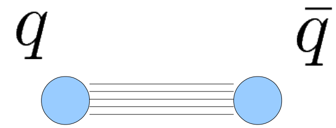
This will be useful later today ...

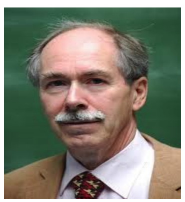
Large N gauge theory and string theory

Large N QCD and string theory

Why should we expect that large N QCD ~ Some string theory ??????

Because of the mesons, of course :D





, can you help us here?

QCD: SU(3) gauge theory with Nf fundamental quarks → SU(Nc)

$g_{YM} = g_{YM}(E)$ No obvious small external parameter close to Λ_{QCD}

Take the limit $N_c \longrightarrow \infty$, expand in powers of $1/N_c$ and check whether

Nc = 3 is a good approximation

Number of gluons ~ N_c^2

What happens in this case?

Number of quarks $N_c N_f \ll N_c^2$

Limit is well defined if $N_c \longrightarrow \infty$

Λ_{QCD} fixed $g_{YM} \longrightarrow 0$

t'Hooft coupling

$\lambda \equiv g_{YM}^2 N_c$ $\rightarrow$ finite

For instance: Beta function

$$\lim_{N_c \to \infty} \frac{d}{d\mu}\lambda = \beta(\lambda) + \mathcal{O}(1/N_c^2)$$

Double line notation:

q $\longrightarrow$

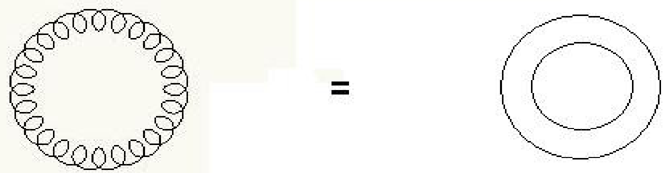
gluon

$\bar{q}$ $\longleftarrow$

Vertex: g_{YM}

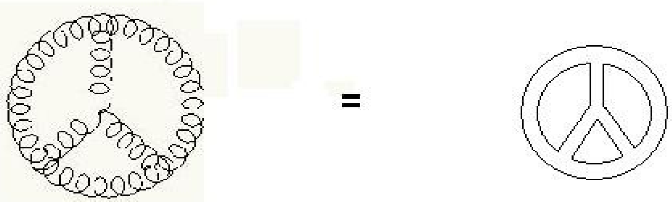
See review by Witten, 1979

Seeing the stringy nature of QCD when $N_c \longrightarrow \infty$

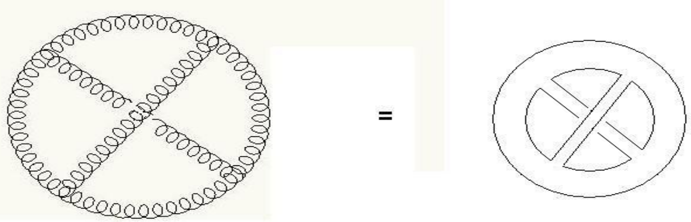


$\sim N_c^2$

Planar diagrams



$\sim g_{YM}^4 N_c^4 = \lambda^2 N_c^2$



$\sim g_{YM}^4 N_c^2 = \lambda^2$

Non-Planar diagrams are suppressed when $N_c \longrightarrow \infty$!!!

Another simplification that occurs when $N_c \longrightarrow \infty$ is factorization!

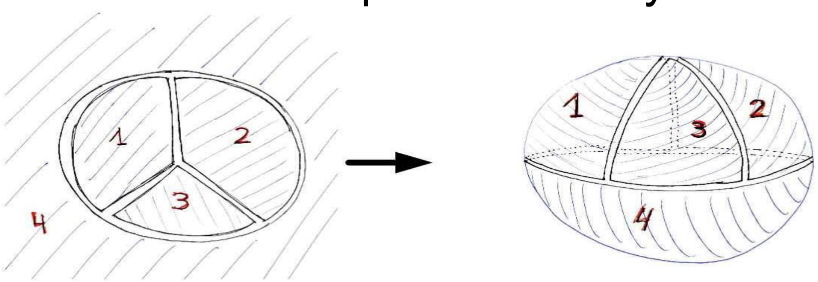
Correlation functions of single trace operators of the form $\mathcal{O}_i = \mathrm{Tr}(\ldots)$ factorize

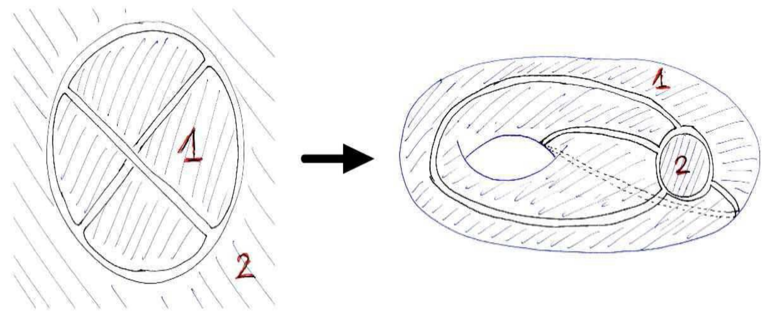
$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\rangle = \langle \mathcal{O}_1(x_1)\rangle\langle \mathcal{O}_2(x_2)\rangle + O\left(\frac{1}{N^2}\right)$$

Note that this does not imply that the gauge theory is free because the 1-point functions are not trivial (you can have a large number of these quantities). For instance, the anomalous dimensions of all operators are controlled by the t'Hooft coupling.

Only planar diagrams survive at $N_c \to \infty$

Planar + point at infinity





In general, for planar diagrams

Euler number

$$\chi = 2 - 2g$$

g = genus

Amplitude of a given process in the gauge theory

$$\mathcal{A} = N^m \left(\mathcal{A}_0(\lambda) + \frac{1}{N^2}\mathcal{A}_1(\lambda) + \frac{1}{N^4}\mathcal{A}_2(\lambda) + \ldots \right) = N^m \sum_g \frac{\mathcal{A}_g(\lambda)}{N^{2g}}$$

Amplitude of a given process in the large N gauge theory

$$\mathcal{A} = N^m \left(\mathcal{A}_0(\lambda) + \frac{1}{N^2}\mathcal{A}_1(\lambda) + \frac{1}{N^4}\mathcal{A}_2(\lambda) + \ldots \right) = N^m \sum_g \frac{\mathcal{A}_g(\lambda)}{N^{2g}}$$

Hmm ... This looks like the loop expansion for a closed string theory

$$\mathcal{A} = g_s^n \left(\mathcal{A}_0(\alpha') + g_s^2 \mathcal{A}_1(\alpha') + g_s^4 \mathcal{A}_2(\alpha') + \ldots \right) = g_s^n \sum_g g_s^{2g} \mathcal{A}_g(\alpha')$$

String tension ~ $1/\alpha'$ if $g_s \sim 1/N_c$

This led several people to think that:

Large Nc gauge theory in 4d should be equivalent to a weakly coupled, classical string theory/gravity in 4d !!!

THIS IS WRONG !!

Remember Witten-Weinberg's theorem?

So, what kind of string theory comes about then?

Bosonic string in D=3+1 dimensions?

Not really ... and this is why:

Classical Polyakov action:

$$S \sim \int d^2\sigma \sqrt{g}\, g^{ab} \partial_a X_\mu \partial^b X^\mu$$

$$\mu = 0, \ldots, D-1$$

$$g_{ab} \longrightarrow e^{\phi}\, g'_{ab}$$ Weyl symmetry

Faddev-Popov ghosts: b,c

$$e^{-S_{eff}} = \int DX\, D(bc) e^{-S[X,g]-S[b,c,g]}$$

Anomaly

$$S_{eff}(g) - S_{eff}(g') = \frac{26-D}{48\pi} \int d^2\sigma \, \frac{1}{2}(\nabla\phi)^2 + R^2\phi + \mu^2 e^{\phi}$$

Liouville action

$$S_{eff}(g) - S_{eff}(g') = \frac{26-D}{48\pi}\int \frac{1}{2}(\nabla\phi)^2 + R^2\phi + \mu^2 e^{\phi}$$

So, integrating over ϕ is like adding a new coordinate, or a new dimension !!!

In general, it is not known how to quantize the Liouville action in D = 4.

Lesson: In order to find the correct string theory description, one has to introduce "at least" one extra dimension where the strings can move. Does it sound familiar?

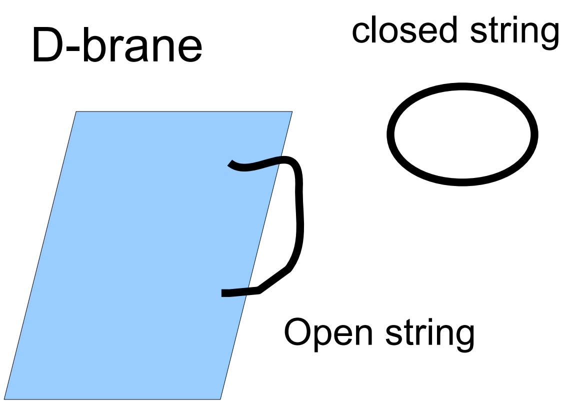
Yes !!!!

We will see in the following how holographic was first discovered in the context of the duality between type IIB string theory in AdS5 x S5 and N=4 SYM.

But, first, let me introduce you to the "key player" of the game ...

D-branes

Solitonic solutions of string theory that carry energy-momentum, mass and charge. Their “tension” is T~1/g so they only contribute to the dynamics at very large coupling.



They are necessary for the consistency of string theory. The end points of open strings are always attached to D-branes.

In general, a Dp-brane is an extended p-dimensional object. For instance, as a D3-brane moves through spacetime it creates a 4-dimensional world volume.

10 dimensional spacetime

$$T_{D3} \sim \frac{1}{g_s l_p^4}$$

$$M_{planck} \sim 10^{19}\mathrm{GeV}$$

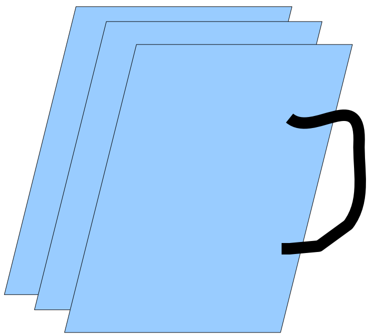
Or 21 micrograms

Planck length

$$l_p \sim 10^{-33}\mathrm{cm}$$

How to construct a SU(Nc) non-Abelian gauge theory out of D3-branes

Type IIB



Stack of
Nc D3-branes

The endpoints of the string can be in any of the D-branes

This gives N_c^2 possibilities => adjoint representation

The endpoints live in a 4-dimensional space. The low energy degrees of freedom will be exactly those in N=4 SYM.

What about the gravitons? How do D-branes curve spacetime?

Range of influence: $L^4 \sim G_{10}\, N_c\, T_{D3}$

$$G_{10} \sim g_s^2\, l_p^8$$

$$T_{D3} \sim 1/g_s l_p^4$$

$$\frac{L^4}{l_p^4} = g_s\, N_c$$

Gravitons decouple at low energies

$$\lim_{E\to 0} G_{10} E^8 = 0$$

Therefore

However, $g_s = g_{open}^2 = g_{YM}^2$

$$\frac{L^4}{l_p^4} = g_s N_c = \lambda$$

If $\lambda \ll 1$, D3-branes are just defects in spacetime (boundary for open strings, which are fluctuations of the D3-branes)

Also, in this case the low-energy effective SYM theory is weakly coupled.

What if $\lambda \gg 1$?

Spacetime is then strongly modified close to the D3-branes (far from them we recover flat space).

However, in this case classical supergravity should be applicable. It has been shown that the geometry close to the branes becomes

BPS: $M = Q$

$AdS_5 \otimes S_5$

L = Radius of AdS and 5-sphere

Extremal solution

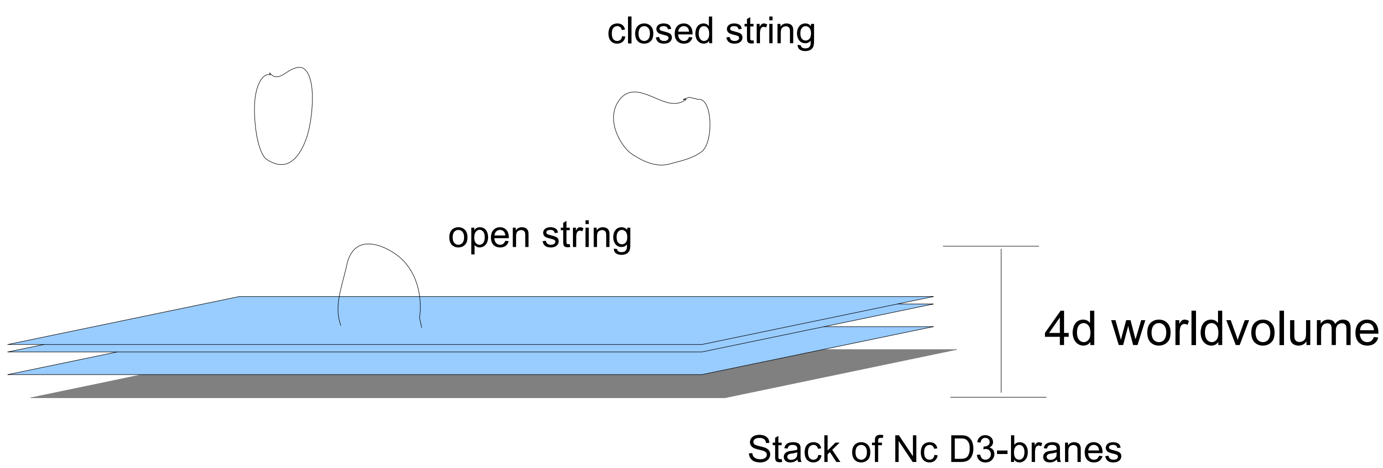
Holography and the decoupling limit

Type IIB string theory

$$G_{\mu\nu},\ \Phi,\ B_{\mu\nu},\ C,\ C_{\mu\nu}, C_{\mu\nu\lambda\rho},\ \ldots$$

$\mathcal{N} = 2$ 32 supercharges

The slide is a 10d spacetime



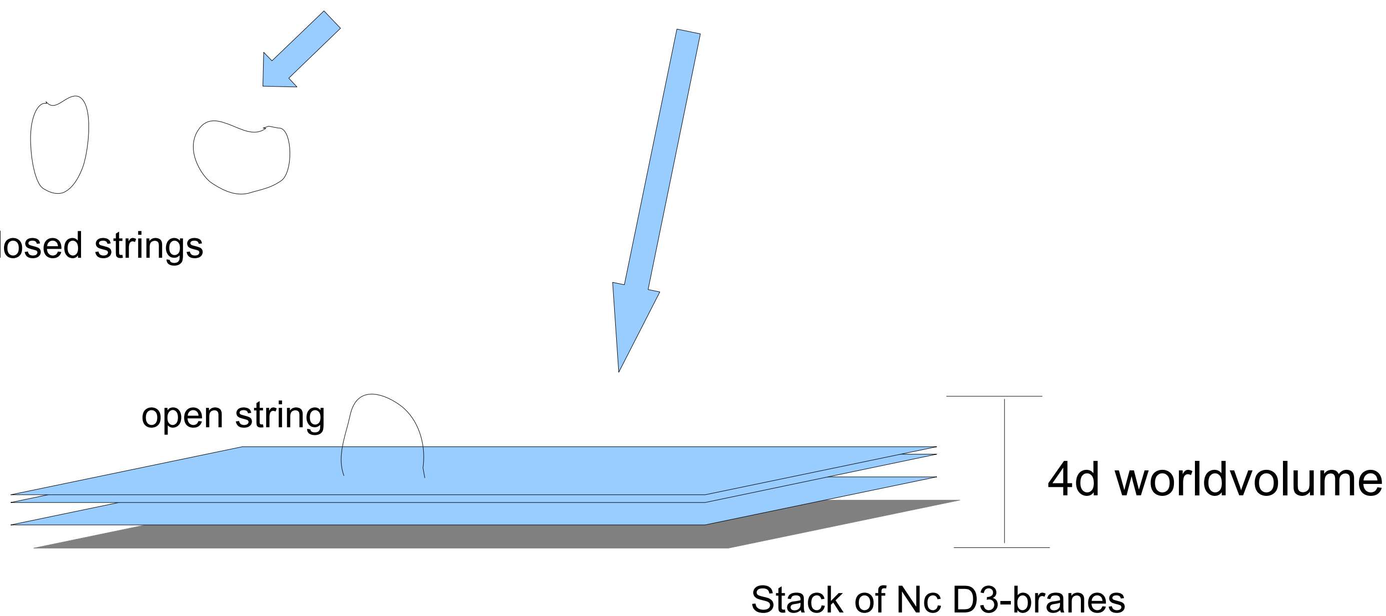
Nc → infinity to suppress loop corrections in the string theory

Low energies $E \ll 1/\ell_s$

First description

Effective action for massless modes

$$S = S_{Bulk} + S_{brane} + S_{int}$$



Low energies $E \ll 1/\ell_s$

First description

Expansion around flat space $G = \eta + \kappa\, h$

$\ell_s \longrightarrow 0$ $\kappa \longrightarrow 0$ Gravitational coupling

$$S_{Bulk} \sim \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G}\, \mathcal{R} \sim \int d^{10}x\, (\partial h)^2 + \kappa(\partial h)^2 h + \ldots$$

Free gravity

$S_{int} \longrightarrow 0$ No interactions between branes and bulk

S_{brane} Non-Abelian Dirac-Born-Infeld → $\mathcal{N} = 4$ SYM in 4d + higher derivative corrections

with Nc → infinity

$\mathcal{N} = 4$ SU(Nc) Supersymmetric Yang-Mills

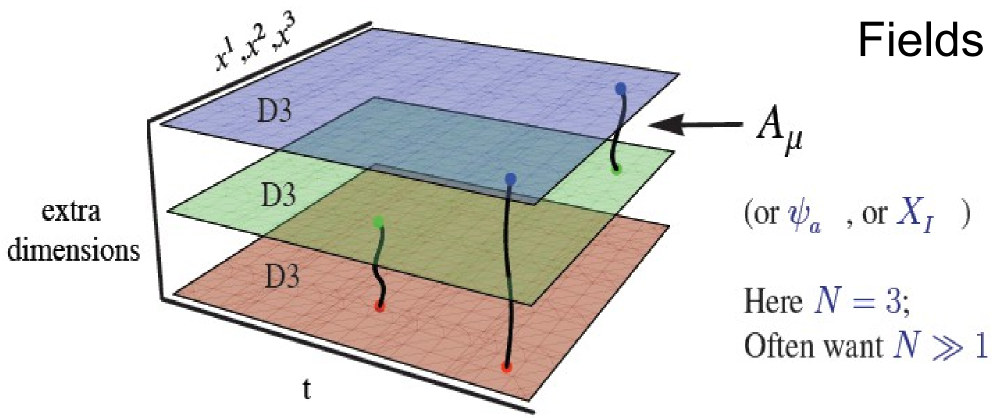


Figure from S. Gubser, QM09

Fields in the adjoint representation of SU(Nc)

- 16 + 16 supercharges
- SU(4) R-symmetry
- Global SO(6) symmetry.

$$\beta(\lambda) = 0$$

Exactly conformal

$$\mathcal{L} = \frac{1}{g^2} Tr \left[F^2 + (D\phi)^2 + \bar{\psi} \not{D} \psi + \sum_{I,J} \left(\phi^I \phi^J\right)^2 + \bar{\psi} \Gamma^I \phi^I \psi \right]$$

+ theta term

Low energies $E \ll 1/\ell_s$

Second description

D3-branes are sources for $C_{\mu\nu\lambda\rho}$

Supergravity solution for the 10d metric

$$ds^2 = f^{-1/2}(-dt^2 + d\vec{x}^2) + f^{1/2}(du^2 + u^2 d\Omega_5^2)$$

$$L^4/\ell_s^4 \sim g_s N_c$$

$$f = 1 + \frac{L^4}{u^4}$$

Redshift $E_\infty = f^{-1/4} E_u \sim u\, E_u / L$

The closer you get to the "throat" u = 0 , the lower is the energy at infinity!!!!

Low energies $E \ll 1/\ell_s$

Observer at infinity sees 2 types of low energy $\omega \longrightarrow 0$ excitations:

1) Decoupled bulk massless particles with very long wavelengths (free gravity)

Low energy graviton cross section $\sigma_{abs} \sim \omega^3 L^8 \rightarrow 0$

2) Near u → 0 excitations $f \sim L^4/u^4$

$$ds^2 = \frac{u^2}{L^2}(-dt^2 + d\vec{x}^2) + L^2 \frac{du^2}{u^2} + L^2 d\Omega_5^2$$

⟹ Type IIB in $AdS_5 \otimes S_5$

Type IIB supergravity in AdS5 x S5

10d low energy action

$$S_{sugra} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-G}\left[\mathcal{R} - \frac{(\partial\phi)^2}{2} - \frac{1}{4.5!}F_5^2 \ldots\right]$$

$$\int_{S_5} F_5 = N_c$$

D3-brane flux

Compactify over S_5

$$S = -\frac{1}{2\kappa_5^2}\int d^5x\,\sqrt{-G}\left[\mathcal{R} + \frac{12}{L^2} + \ldots\right]$$

$\kappa_5^2 \sim 1/N_c^2$

This is the basic starting point in most of the gravity calculations

Maldacena's conjecture, 1998

$\mathcal{N} = 4$ SYM is equivalent to type IIB string theory on $AdS_5 \otimes S_5$

Realization of the holographic principle!!!!

Witten, 1998;
Gubser, Klebanov, Polyakov, 1998

Duality dictionary

$$Z_{string/gravity}\left[\Phi(x,u)\Big|_{u\to\infty} = J(x)\right] = \langle e^{\int d^4x\, \mathcal{O}(x) J(x)} \rangle_{QFT}$$

Sum of all spacetimes asymptotically
AdS5 x S5 !!!!!

Ex: Bulk $\quad$ Boundary

$g_{\mu\nu} \longleftrightarrow T^{\mu\nu}$

Dynamical gravity $\quad$ Translational invariant gauge theory

- Lorentz invariance
- Match conformal dimension
- Use conserved quantities
- **Only gauge invariant observables are used.**

Ubiquitousness of gravity !!!

Simple top-down model for deconfinement

N=4 SYM : Supersymmetric conformal plasma, always in the deconfined phase

General idea (simple model) to describe deconfinement via the gauge/string duality

1998

- Use Nc D4-branes → 5d spacetime, $SU(N_c)$ theory with gluons, adjoint fermions, and scalars.

- Compactify one of the spatial dimensions on a circle with radius Ω.

- Fermions acquire a mass of order $M \sim 1/\Omega$. Scalars should also acquire a mass (though nobody knows how to compute that at strong coupling).

- Gluons remain massless.

At energies $E \ll M$ The effective theory that describes the D4-brane system reduces to 4d pure glue + KK contamination !!

- "True" pure glue has asymptotic freedom (Λ_{QCD}) , confinement, and a 1st order deconfinement phase transition at large N.

- For the gravitational description, we want to study the limit where $\Lambda_{QCD} \ll M$

so that the contribution from additional fields at scale M is (hopefully) negligible.

The ten dimensional metric is

$$ds^2 = \left(\frac{r}{L}\right)^{3/2} \left(-f dt^2 + d\vec{x}^2 + dy^2\right) + \left(\frac{L}{r}\right)^{3/2} \frac{dr^2}{f} + L^{3/2} r^{1/2} d\Omega^2$$

6d space

4d sphere

$f = 1 - r_0^3/r^3$ Periodicity (compactification) → y → y + Ω

Boundary when r → ∞

- Note that the spacetime does not factorize anymore (S^4 not constant)

It suffices here to use the 6d metric

$$ds_1^2 = e^{2\Phi(r)}\left(-f dt^2 + d\vec{x}^2 + dy^2\right) + e^{-2\Phi(r)}\frac{dr^2}{f}$$

Dilaton $$e^{\Phi(r)} = \left(\frac{r}{L}\right)^{3/4}$$

Gives a running coupling const. in the gauge theory (problems in the UV can be fixed, though)

Horizon at

$$r_0 = \left(\frac{4\pi}{3}\right)^2 L^3 T^2$$

However, there is another metric that is also a solution of the EOM

$$ds_2^2 = e^{2\Phi(r)}\left(-dt^2 + d\vec{x}^2 + f\,dy^2\right) + e^{-2\Phi(r)}\frac{dr^2}{f}$$

that is also a solution of the supergravity equations !

Note that here there is no black hole in this case !!!

We still have to avoid the conical singularity in the y coordinate by imposing

Regularity at r_0 ⟶ $r_0 = \left(\frac{4\pi}{3}\right)^2 \frac{L^3}{\Omega^2}$

The Hawking-Page phase transition

The (Euclidean) partition function $$Z_{string} \simeq e^{-F_1 V/T} + e^{-F_2 V/T}$$

$$S_{Euclidean} = F/T$$

At a given T = Tc one solution overcomes the other → deconfinement phase transition!

Entropy from the solution 1 $\sim N_c^2$

Entropy from the solution 2 $\sim N_c^0$

Exact glueball spectral function :

$$\chi(k) = 2\, Im\, i \int d^4x\, e^{-i\,kx} \theta(t) \langle [Tr\, F^2(x), Tr\, F^2(0)] \rangle$$

Solution 1 (with black brane):

Hydrodynamic modes and very broad peaks $T > T_c$

Solution 2 (no black brane):

$T < T_c$ Very sharp peaks - mass gap!

However, there is more than meets the eye ... in fact, the confinement/ deconfinement phase transition in asymptotically free theories cannot be really described using classical gravity ...

- Remember, classical gravity approx. is ok when $N_c \to \infty \quad \lambda \gg 1$

Corrections to the pure Yang-Mills from KK tower are suppressed by powers of $\Lambda_{QCD}/M \ll 1$

But, in general from 1-loop $\Lambda_{QCD} \sim M \exp\left(-\frac{c^2}{g^2_{YM}(M)N_c}\right)$

Thus, the decoupling limit $\Lambda_{QCD} \ll M \Rightarrow \lambda \ll 1$

Therefore, one has to go way beyond classical gravity in order to describe the transition in a way consistent with asymptotic freedom !!!

Summary

- Large Nc gauge theories in 4d should be dual to some string theory.

- The string theory should be defined, however, in a larger number of space dimensions.

- The exact equivalence between N=4 SYM and type IIB string theory in AdS is the most studied (and clear) example where holography is known to work.

- It is possible to devise a top down construction involving Nc D4 branes that displays some of the properties expected for the deconfinement transition in large N gauge theories.

- Asymptotically free gauge theories cannot be studied holographically with the methods known today.

LECTURE V: Effective Holographic Theory for Large N QCD

OUTLINE OF THIS LECTURE

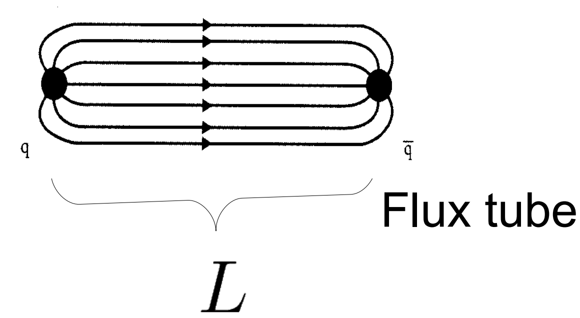
1- Deconfinement in SU(N) gauge theories

2- Holographic effective models for pure glue at large N

3- Summary

Deconfinement in SU(N) Gauge Theories

Simple argument for deconfinement (Polyakov, 1978)



Linearly confining theory $E_{Q\bar{Q}} \sim \sigma L$

Density of states (long string) $\sim \exp c\, L$

Prob. $\sim e^{c\, L - \sigma L / T}$

Deconfinement

$$T \to T_c = \sigma / c$$

Arbitrarily large strings
can be thermally produced !!!

- This shows that any linearly confining theory must deconfine at some finite T.
- No reference to the microscopic theory (the argument is pretty general).
- However, this simple idea is incomplete because it predicts a 2nd order transition

$$\lim_{T \to T_c} \sigma_{eff}(T) = \sigma - cT \to 0$$

Let's use the fact that we know what the microscopic degrees of freedom are ...

$$Z \sim \sum_{glueballs} e^{-E/T} + \sum_{glue} e^{-E/T} = e^{-\mathcal{F}_{glueballs}/T} + e^{-\mathcal{F}_{glue}/T}$$

Entropy: Glueballs $\sim N_c^0$

Entropy: Glue $\sim N_c^2$

$\Rightarrow$ T_c

Latent heat

$$\mathcal{L}_h / T_c^4 \sim N_c^2$$

Strong 1st order at large Nc

- In a 1st order transition the string tension jumps at the transition.

Comparing the vacuum energy density and the free energy in the deconfined phase

$$\lim_{N_c \to \infty} N_c^2 \sigma^2 \sim N_c^2 T_c^4 \quad \Longrightarrow \quad \lim_{N_c \to \infty} T_c/\sqrt{\sigma} \sim \mathcal{O}(1)$$

What is the order parameter of the deconfinement transition?

Polyakov loop
(fundamental rep.)

$$\ell(\vec{x}) = \frac{1}{N_c} \mathrm{Tr}\, P\, e^{i \int_0^{1/T} \hat{A}_0(\vec{x},\tau) d\tau}$$

Heavy quark free energy

$$e^{-\mathcal{F}_{Q\bar{Q}}(|\vec{x}|,T)/T} \equiv \langle \ell(\vec{x}) \ell^*(0) \rangle$$

Note that due to the cluster decomposition principle

$$e^{-F_Q/T} \equiv |\langle \ell(T) \rangle| = \lim_{|\vec{x}| \to \infty} e^{-\mathcal{F}_{Q\bar{Q}}(|\vec{x}|,T)/2T}$$

Polyakov loop as the order parameter of the deconfining transition

In this case confinement (area law) then gives

$$T \leq T_c \rightarrow |\langle \ell(T) \rangle| = 0$$

$$T \geq T_c \rightarrow |\langle \ell(T) \rangle| \neq 0$$

$|\langle \ell(T) \rangle|$ is the order parameter for the transition   Breaking of Z_N

(without dynamical fermions)

What is observed on the lattice?

About the order of the transition

- Nc = 2 is 2nd order
- Nc = 3 is weak 1st order
- Nc > 3 is strong 1st order

$$\frac{T_c}{\sqrt{\sigma}} = 0.5970(38) + \mathcal{O}(1/N_c^2)$$

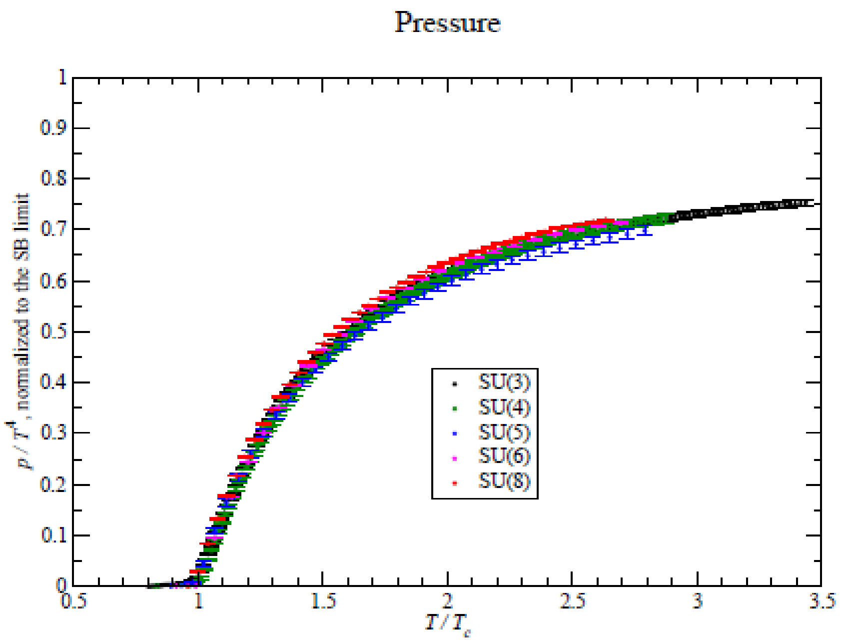
$$T_c \simeq 260\mathrm{MeV}$$

Latent heat

$$\frac{\mathcal{L}_h^{1/4}}{N_c^{1/2} T_c} = 0.766(40) + \mathcal{O}(1/N_c^2)$$

Lucini, Teper, Wenger, hep-lat/0502003

What is observed on the lattice?



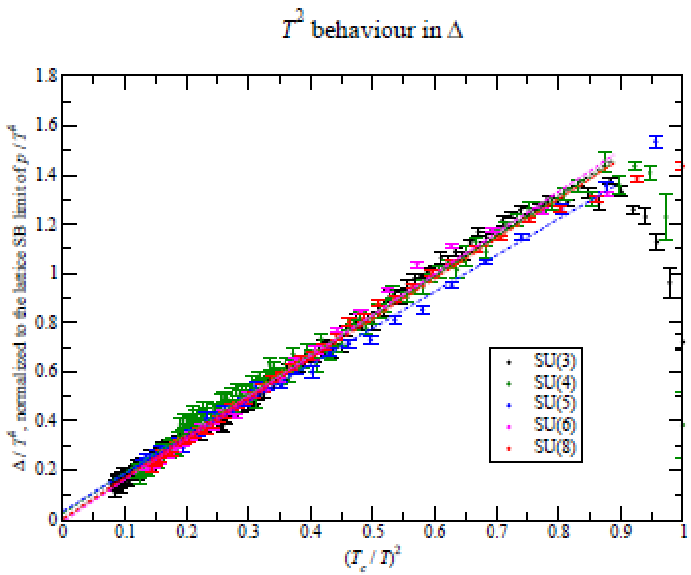
No significant difference between Nc = 3 and Nc = 8

M. Panero, arXiv:0907.3719 [hep-lat]

What is observed on the lattice?

Trace anomaly

$$\Delta = \varepsilon - 3p$$



M. Panero, arXiv:0907.3719 [hep-lat]

T^2 dependence near T_c ???

Meisinger, Miller, Ogilvie, 2001
Pisarski, 2006
Megias, Arriola, Salcedo, 2006

Evidence for a non-perturbative "fuzzy" bag???

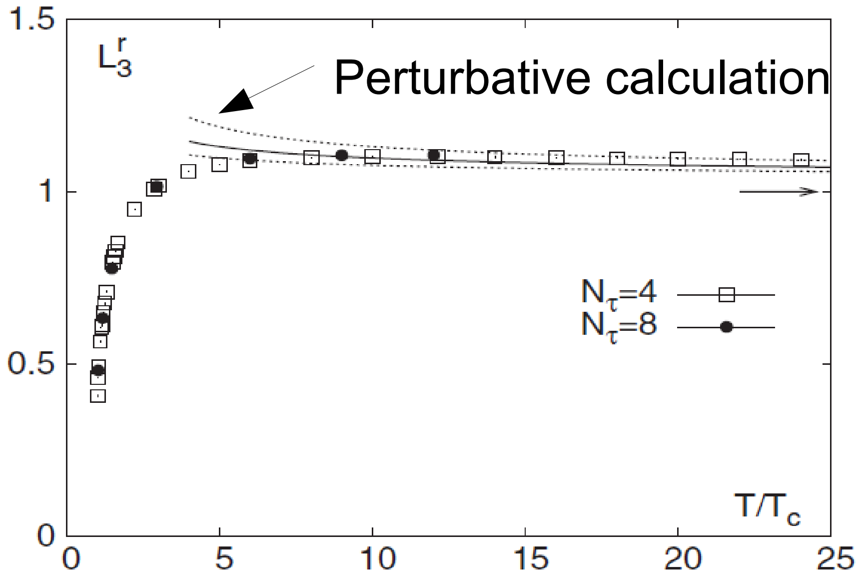
$$p(T) = f_{pert} T^4 - B_{fuzzy} T^2 - B_{MIT}$$

What is observed on the lattice?

Renormalized Polyakov loop in the fundamental representation

Quenched

$SU(3)$



Gupta, Huebner, Kaczmarek, PRD 77, 034503 (2008).

Also in this case Nc=3, 4 and etc give similar results (Panero et al. 2012)

- Thermodynamics does not seem to be strongly sensitive to Nc.

- Non-perturbative contribution to the pressure near Tc ("fuzzy" bag).

- Can holography provide some new insights into the non-perturbative QCD physics at large Nc near Tc ?

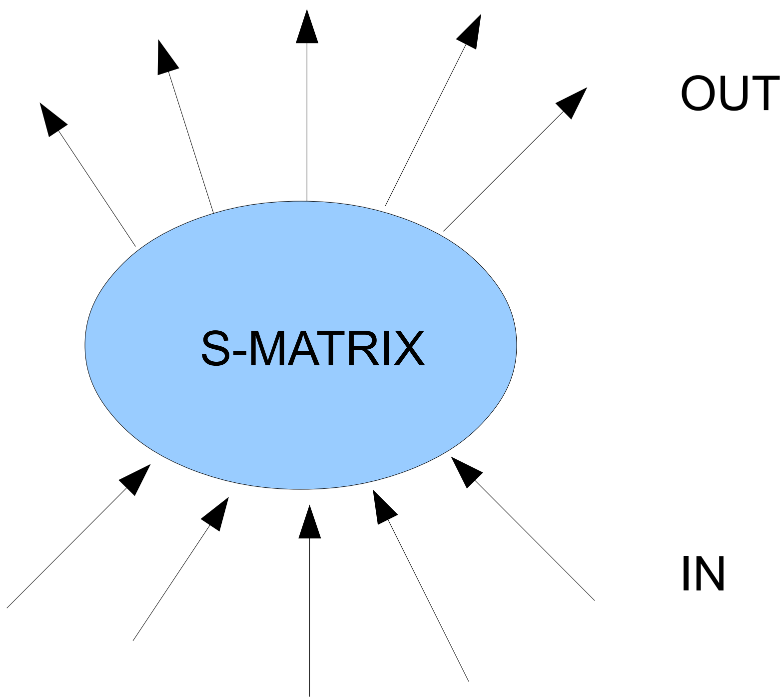
Do we have the branes to understand hot glue???

- I will show you in the next slides that it is possible to construct holographic dual models that can describe the equilibrium properties of SU(Nc) Yang-Mills theories as determined by lattice QCD.

Argument due to Witten

Holography and Boundary Reconstruction

Consider a unitary and relativistic S-matrix in 4D Minkowski spacetime



Actually, consider a family of S-matrices depending on a parameter

$$c$$

Such that when

$$\lim_{c \to 0} S(c) = 1$$

Holography and Boundary Reconstruction

Particle physics is largely based on the following “theorem”:

“Such an S-matrix can always be obtained by perturbation theory using some effective Lagrangian that is free when $c \to 0$ ”

For example:

$$\mathcal{L} = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 + c\phi^3 + \ldots \right]$$

$m^2, \ldots$ = constants

Holography and Boundary Reconstruction

This "theorem" (discussed by Witten), if true, establishes that we can reconstruct a 4-dimensional field theory from boundary data, i.e., the S-matrix, that only depends on 3-dimensional variables – the on-shell momenta $p_\mu p^\mu = m^2$

This "theorem" helps us define low energy effective theory: a theory of massless particles that becomes non-interacting when the energy goes to zero.

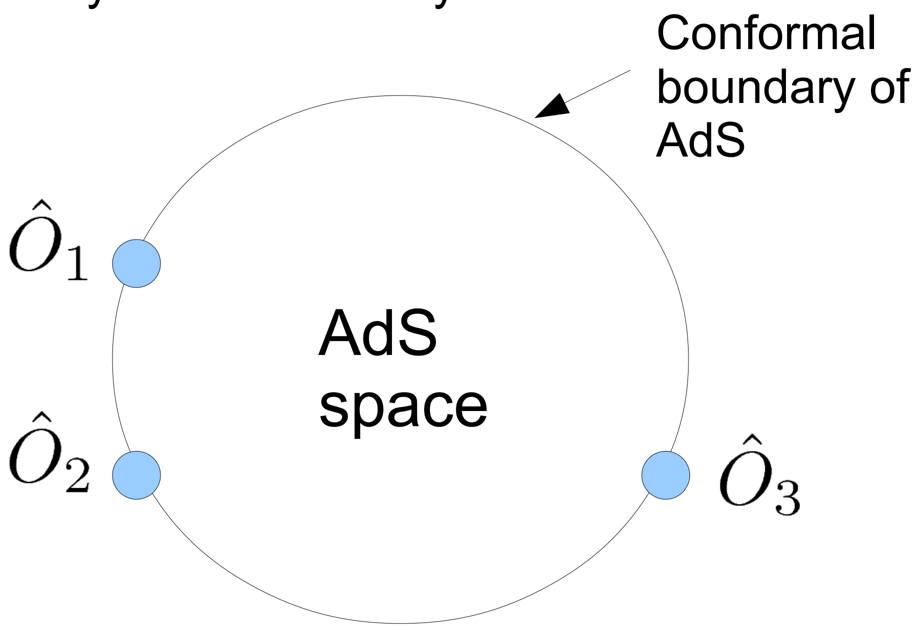
Also, we see that the validity of effective field theory is a special case of this "theorem".

$$\mathcal{L} = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 + \frac{c^4}{4!}(\partial\phi)^4 + \ldots \right]$$

Holography and Boundary Reconstruction

Now, what happens in an AdS space?

Instead of an S-matrix we have a
conformal field theory at the boundary



Operator insertions at the boundary

Holography and Boundary Reconstruction

The analogue of S-matrix → 1 is to say that the correlation functions factorize

$$\langle \hat{O}_1(x_1)\hat{O}_2(x_2)\hat{O}_3(x_3)\hat{O}_4(x_4)\rangle =$$

$$\langle \hat{O}_1(x_1)\hat{O}_2(x_2)\rangle\langle \hat{O}_3(x_3)\hat{O}_4(x_4)\rangle + permutations$$

This occurs in gauge theories when $N_c \rightarrow \infty$

Ex:

$$O_1 = \frac{1}{N} Tr\, F^2$$

Holography and Boundary Reconstruction

In this case, the analogue of what we did before is to consider a family of conformal field theories that depend on a parameter

$c \sim 1/N_c$ such that when $c \to 0$ the correlation functions factorize.

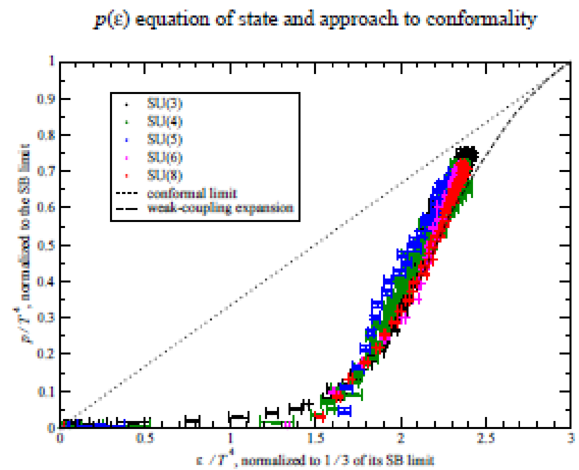
In this case, the reconstruction "theorem" asserts that there should be a local AdS action in the bulk (it must include gravity since the boundary field theory possesses a conserved energy-momentum tensor) from which the boundary correlation functions can be deduced by the usual recipe

$$\mathcal{L} = \int_{AdS} d^5x\sqrt{-G}\,\{N_c^2\mathcal{R} + \ldots\}$$

Holography is a concrete realization of these ideas!!!!

Holographic Models of Pure Glue at Large Nc

Lattice shows that conformal invariance is badly violated near Tc



M. Panero, arXiv:0907.3719 [hep-lat]

From the duality dictionary

$$\phi \sim Tr\, F_{\mu\nu}^2$$

Bulk

In the gauge theory at the boundary

We should look for gravity duals with a dynamical scalar field !!!

Gubser et al. 2008
Kiritsis et al, 2008
Noronha, 2009

Minimal extension of the good and old gravity setup (bottom-up approach)

$$S = -\frac{1}{2\kappa_5^2}\int d^5x\,\sqrt{-G}\left[\mathcal{R} - \frac{(\partial\phi)^2}{2} - V(\phi)\right]$$

Dilaton potential

$$V(\phi)$$

Nontrivial fields in the 5d bulk: $G_{\mu\nu},\ \phi$

Dual to a relevant deformation of a 4d CFT $\mathcal{L}_{CFT} + \Lambda_\phi^{4-\Delta}\mathcal{O}_\phi$

Here Λ_ϕ is the energy scale of the deformation and Δ is the dimension of $\mathcal{O}_\phi$ in the boundary, which is dual to ϕ in the bulk.

General assumptions:

Gubser et al. 2008
Noronha, 2009.

- Relevant deformation (important in the IR) $\Delta < 4$

- Spacetime is asymptotically AdS_5 with radius R

$$\lim_{\phi\to 0} V(\phi) = -\frac{12}{R^2} + \frac{1}{2R^2}\Delta(\Delta - 4)\phi^2 + \mathcal{O}(\phi^4)$$

- Breitenlohner-Freedman bound $1 < \Delta < 4$ $m_\phi^2 < 0$

- Gauge theory is conformal in the UV $E \gg \Lambda_\phi$ (not asymptotically free)

The theory has a nontrivial UV fixed point.

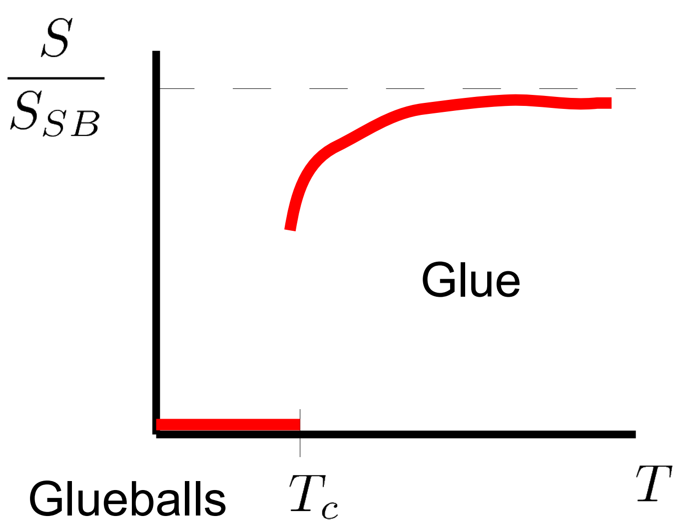
Pure glue at large Nc → Strong 1st order transition

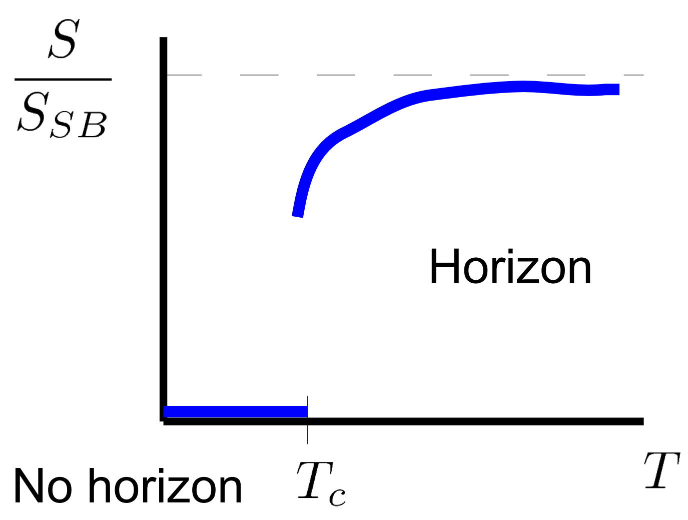
Gauge theory at finite T

Gravity dual with a black brane

T, entropy = T, entropy

$$S = \frac{A}{2k_5^2}$$





Deconfinement is equivalent to the formation of a horizon in the gravity dual

Hawking-Page transition (1983)

Witten, 1998

$$Z_{sugra} \sim e^{-I_c} + e^{-I_d}$$

Basically, there are two solutions of the supergravity equations (two different metric configurations):

$$I_c = \mathcal{F}_c(T)/T$$ No horizon

$$I_d = \mathcal{F}_d(T)/T$$ With Horizon

Equal pressure

At T = Tc $$p_c(T_c) = p_d(T_c)$$ $\rightarrow$ Phase transition

This constraints a lot our choice for $V(\phi)$

Confinement can also be understood via the area law

Area law

Heavy quark potential at T = 0 $$\lim_{L\to\infty} \mathcal{V}_{Q\bar{Q}} \sim \sigma\, L$$

Occurs when

$$\lim_{large\,\phi} V(\phi) \sim -\phi^z\, e^{\sqrt{\frac{2}{3}}\phi} \qquad\qquad z \geq 0$$

Kiritsis et al, 2008

Such type of potentials (with z=0) naturally appear in non-critical 5d string theory.

Examples with linear glueball spectrum $M^2 \sim n$ also known (z = 1/2).

Kiritsis et al, 2008

Correlator of Polyakov Loops

$$Z_{string} = \langle \ell(\vec{x})\ell^*(0)\rangle = e^{-\mathcal{F}_{Q\bar{Q}}(|\vec{x}|,T)/T}$$

where

$$Z_{string} \sim \sum_{\mathcal{D}} e^{-S_{NG}(\mathcal{D})}$$

$\mathcal{C}$ Path in 4d

$\mathcal{D}$ Worldsheet in the bulk

$$\mathcal{C} = \partial\mathcal{D}$$

Heavy quark free energy given by

$$S_{NG}(\mathcal{D}) = \frac{1}{2\pi\alpha'}\int_{\mathcal{D}} d^2\sigma\sqrt{\det h_s^{ab}}$$

Nambu-Goto action for the string in the bulk

$$h_s^{ab} = G_s^{\mu\nu}\partial^a X^\mu \partial^b X^\nu$$

Induced metric on the worldsheet

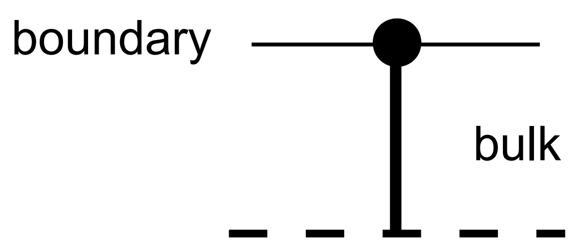
General results: Dual description of the Heavy Quark Free Energy

Assuming

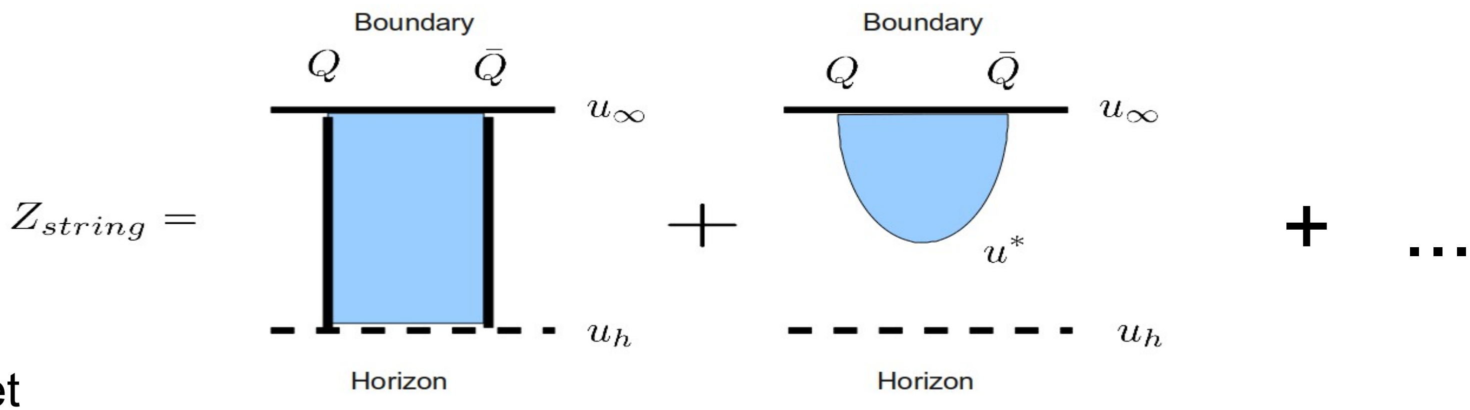
$N_c \gg 1$

$R^4/\alpha' \gg 1$

Infinitely massive heavy quark ~ fundamental string in the bulk



Maldacena, 1998



Sum over worldsheet configurations

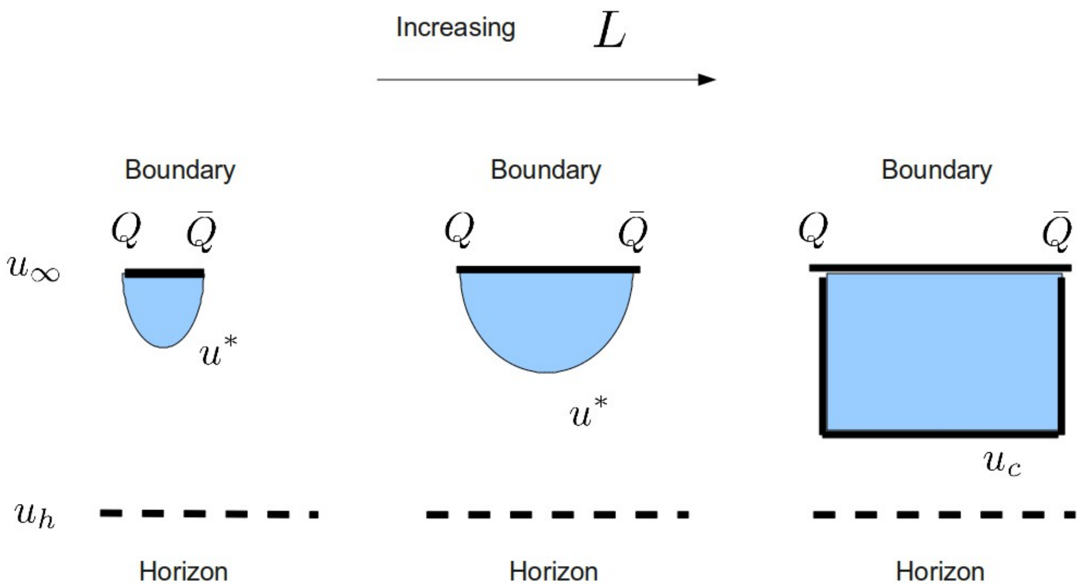
Disconnected

Connected

Confinement and the area law

An area law for the Wilson loop can be obtained if the backreaction of the dilaton on the background geometry is such that the bottom of the string cannot go below a certain critical depth in the bulk.

Below Tc

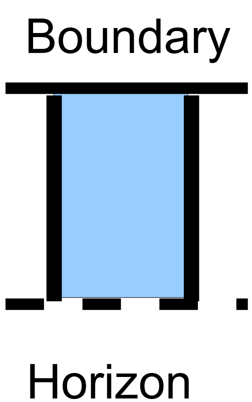


$$\mathcal{F}_{Q\bar{Q}} \to \sigma\, L$$

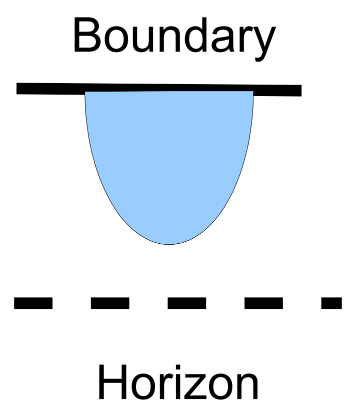
Remarks about the Nc dependence of the loop correlator

Yaffe et al, 2006

The endpoint of a string can attach to the Nc D3-branes in N_c^2 different ways.



$$\sim \mathcal{O}(N_c^2)$$



$$\sim \mathcal{O}(1)$$

Therefore, in the $|\vec{x}| \longrightarrow \infty$ limit $\langle \ell(\vec{x})\ell^*(0)\rangle$ is described by two different classes of worldsheets depending on T :

- T < Tc $\rightarrow$ use the connected U-shaped configuration $\sim \mathcal{O}(1)$

- T > Tc $\rightarrow$ cluster decomposition implies that the full correlator equals the disconnected piece $\sim \mathcal{O}(N_c^2)$

Thus, in these models confinement implies that $|\langle \ell(T)\rangle|$ jumps at the transition, which must be of 1st order.

JN, arXiv:0910.1261 [hep-th]

In general, the single heavy quark free energy is $|\langle \ell(T) \rangle| = e^{-F_Q/T}$

In the deconfined phase the following equation holds for **GOOD** $V(\phi)$

$$\frac{dF_Q}{dT} = \frac{2}{\alpha'} \frac{e^{\sqrt{\frac{2}{3}}\phi(u_h)}}{V(u_h)} \frac{1}{c_s^2} \, .$$

→ Speed of sound

- Note that $dF_Q/dT < 0$ in thermodynamic equil. since $V(\phi) < 0$

- This quantity diverges when $c_s \longrightarrow 0$ (phase transition) .

- The jump in the loop at Tc (aka Gross-Witten point) is related to how quickly the speed of sound vanishes at Tc.

Noronha, 2009

Can these models describe lattice results for both the thermodynamics and the Polyakov loop near Tc?

Yes!!!!

Noronha, 2009

Using for example,

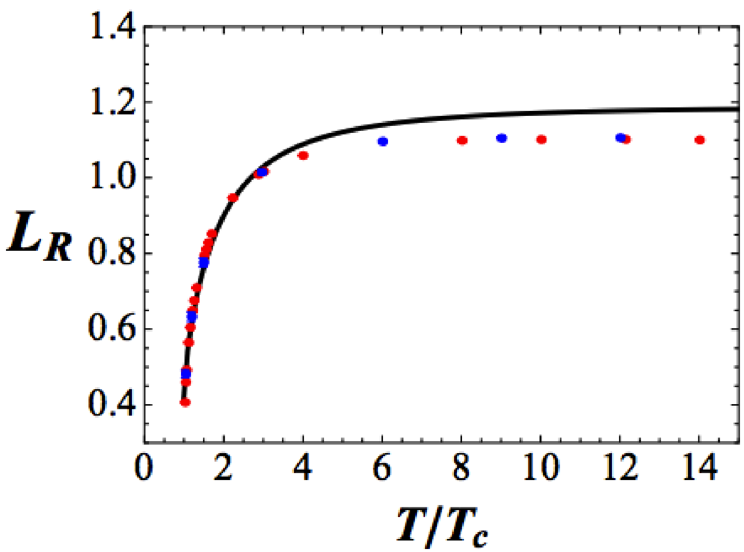
$$V(\phi) = \frac{-12\,\cosh\gamma\phi + b_1\phi^2}{R^2},$$

γ Related to the speed of sound at low T

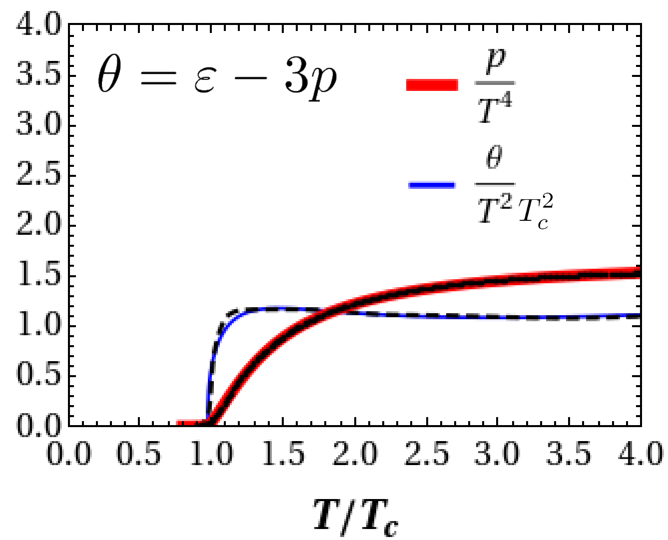
b_1 Gives the scaling dimension Δ

$$V(\phi) = \frac{-12\cosh\sqrt{1/2}\phi + 1.95\phi^2}{R^2}$$

Noronha, 2009



SU(3) data from Kaczmarek et al, 2002



SU(3) data from Boyd et al., 1996.

$$R^2/\alpha' \sim \mathcal{O}(1)$$

Not as large as one hoped ...

Note that the gravity transition is continuous but the agreement is excellent above Tc

Summary

- The gauge/string duality can now provide not only qualitative but also quantitative descriptions of the deconfinement transition in QCD at large Nc.

- Depending on the dilaton potential, a variety of phase transitions at finite T can be obtained. In fact, one can have crossovers, 1st and 2nd order transitions.

- $|\langle \ell \rangle|$ has the expected behavior as a function of T if T is not too large. Indeed, since the gauge theory is conformal at large T we have

$$\lim_{T\to\infty} \frac{F_Q}{T} = -\frac{R^2}{2\alpha'}$$

Gauge theory is not asymptotically free !!!

- Equilibrium quantities (such as the entropy density and the Polyakov loop) have T dependence near Tc that resembles quenched QCD.

- Confinement via Wilson's area law can be obtained.

- One can perform calculations within these confinement gravity duals in Minkowski space (in this case “gravity rules” :).

$$Im\, V_{Q\bar{Q}}(T \to \infty) \sim -\frac{R^2}{\alpha'} T (LT)^4$$

Related to the thermal width of $Q\bar{Q}$ bound states

$$Im\, V_{Q\bar{Q}}(T \leq T_c) = 0$$

JN and A. Dumitru, PRL 103, 152304 (2009).
S. Finazzo, JN, to appear.

Final comments (about the lectures)

- Strongly coupled systems with many d.o.f. in d-dimensions can be described by theories of classical gravity defined in higher dimensions.

- We now can at least start to understand what a theory of quantum gravity in 4 dimensions may be like by studying weakly-coupled 3d QFTs.

- The holographic duality has now become a well studied tool to understand strongly coupled QFTs with many d.o.f.

- According to holography, any strongly interacting quantum many-body system at finite density and temperature with sufficiently many d.o.f /volume is predicted to behave at low energies as a perfect fluid.

SOME OPEN QUESTIONS

- Can holography be rigorously derived from first principles?

- Given a generic QFT, what is the holographic dual?

- Will holographic predictions ever be checked by experiments?

- Is it possible to find the holographic dual of QCD?

- Can holography explain non-perturbative, non-relativistic many-body phenomena (such as high Tc superconductivity)?

- Will holography play a role in the understanding of beyond standard model phenomena?

- Is gravity really an emergent concept? What would be the implications of this idea to cosmology? Do we live in a ``hologram”???

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[7] O.~Aharony, S.~S.~Gubser, J.~M.~Maldacena, H.~Ooguri and Y.~Oz, ``Large N field theories, string theory and gravity,'' Phys.\ Rept.\ {\bf 323}, 183 (2000) [hep-th/9905111]

String Theory Pocket Dictionary

- Axion: GB associated with spontaneously broken PQ symmetry. It is closely related to the graviton and the dilaton in perturbative string theory.

- black p-brane: a p-dimensional extended object with an event horizon. A space that is translational invariant in p-directions and has a black hole in the remaining directions.

- Born-Infeld action: Generalization of the gauge field action that is non-polinomial in the gauge field strength. It appears as a low-energy description of the gauge fields on D-branes.

- Central charge: an operator (which may be a constant) that appears on the right hand side of a Lie algebra and commutes with all operators in the algebra.

String Theory Pocket Dictionary

- D-brane: In type I, IIA and IIB superstring theory, a dynamical object in which open strings can end. The term is a contraction of Dirichlet brane. The coordinates of the attached string obey Dirichlet boundary conditions in the directions transverse to the brane and Neumann boundary conditions in the directions tangent to the D-brane. A Dp-brane has even p for type IIA and odd p for type IIB. The Dp-brane is a source for the (p+1)-form R-R gauge field. The low energy excitations of D-branes are described by supersymmetric gauge theory, which is non-Abelian for coincident branes.

- Heterotic string: Supersymmetric string with different constraint algebras acting on left and right-moving fields.

- Montonen-Olive duality: the weak-strong duality of d=4 N=4 SYM.

- Neveu-Schwarz boundary condition: Fermion fields are anti-periodic on the world sheet.

String Theory Pocket Dictionary

- NS-NS states: In type I and II string theories, these are the closed string states whose left and right-moving modes are bosonic. These include the graviton and the dilaton, and in the type II case there is a 2-form potential.

- Oriented string theory: a string theory in which the world-sheet has a definite orientation.

- p-brane: a p-dimensionally extended object.

- R symmetry: A symmetry that does not commute with the supercharges.

- Ramond boundary condition: the condition that a fermionic field be periodic on the world-sheet.

- RNS superstring: The formulation of type I and II superstrings that has superconf. invariance but not explicit spacetime supersymmetry.

String Theory Pocket Dictionary

- Supergravity: the union between general relativity and supersymmetry. Here supersymmetry is a local symmetry.

- Supersymmetry: A symmetry whose charge transforms like a spinor, which relates the masses and couplings of bosons and fermions.

- Type I superstring: The theory of open and closed unoriented superstrings, which is consistent only for the gauge group SO(32).

- Type IIA superstring: The theory of closed oriented superstrings. Left and right-hand modes have different chiralities.

- Type IIB superstring: The theory of closed oriented superstrings. Left and right-hand modes have the same chirality.

EXTRA SLIDES

The Gauge/Gravity Duality ~ Holography (1998)

Juan Maldacena

String theories defined in a 10 dimensional spacetime are equivalent (or dual) to 4 dimensional gauge theories that describe the interactions among particles that are similar to the quarks and gluons of QCD.

Strongly coupled gauge theories ~ Higher Dimensional Gravity

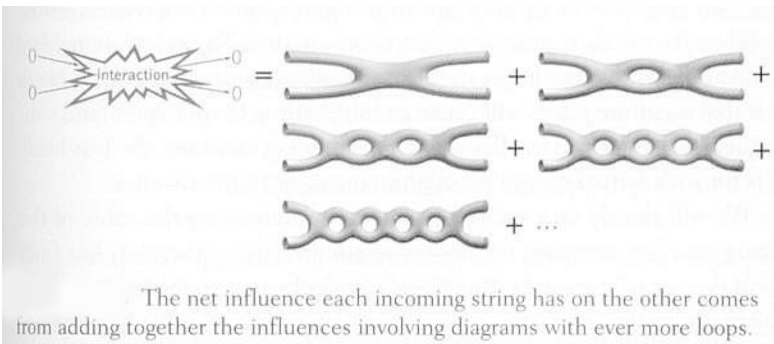
Confinement, hadronic spectrum, etc ~ geometry

Transport properties of gauge theories ~ Properties of black holes

Why is this useful for non-string theorists?????

This duality maps weakly coupled gravity (which is well known) to
strongly coupled gauge theory (which we didn't know how to deal with) !!!!

In 10 dimensions



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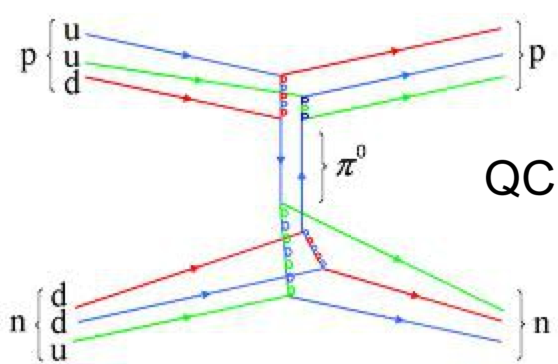
In 4 dimensions



QCD*

Strong coupling → **Weak coupling**

Strong coupling → **Weak coupling**

A brief reminder about black holes

The Schwarzchild solution in D= 3+1

$$S = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\,R \quad \Rightarrow \quad R_{\mu\nu} - g_{\mu\nu}\frac{R}{2} = 0$$

$$R_{\mu\nu} = 0$$ D > 2

Most general solution with spherical symmetry and mass M

$$ds^2 = -\left(1 - \frac{r_0}{r}\right)dt^2 + \left(1 - \frac{r_0}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

$$r_0 = 2G_4 M$$ Event horizon radius

- Note that as $M \rightarrow 0$ we recover Minkowski space.

- Geometry is not singular at horizon – true singularity at $r \rightarrow 0$

$$R_{\mu\nu\alpha\lambda} R^{\mu\nu\alpha\lambda} = 12 \frac{r_0^2}{r^6}$$

- Only real singularity at $r = 0$. Singularity is not “naked” because of the horizon.

A useful analogy: Suppose you want to perform the integral of a real function

$$I = \int_{-\infty}^{\infty} f(x)dx$$

We learn early on as undergrads that one can perform such integrals by adding another dimension → we go to the complex plane where the analytical structure of the function (its poles and etc) are manifest and then we use a suitable contour to compute the value of the integral.

$$I = \int_{-\infty}^{\infty} f(x)dx \quad \Rightarrow \quad I(C) = \oint_C dz f(z) = 2\pi i \sum Res$$

Holography is similar, in a way, to this idea.

On one hand, you have a 4 dimensional conformal gauge theory that is strongly coupled and sits at a UV fixed point of the renormalization group flow.

As you vary the energy with which you probe the different processes in the gauge theory, you move down in the renormalization group flow.

The gauge/gravity duality explains, in a geometrical manner using a theory of gravity in a higher dimensional curved spacetime, what happens in the strongly coupled gauge theory as you vary the energy scale.

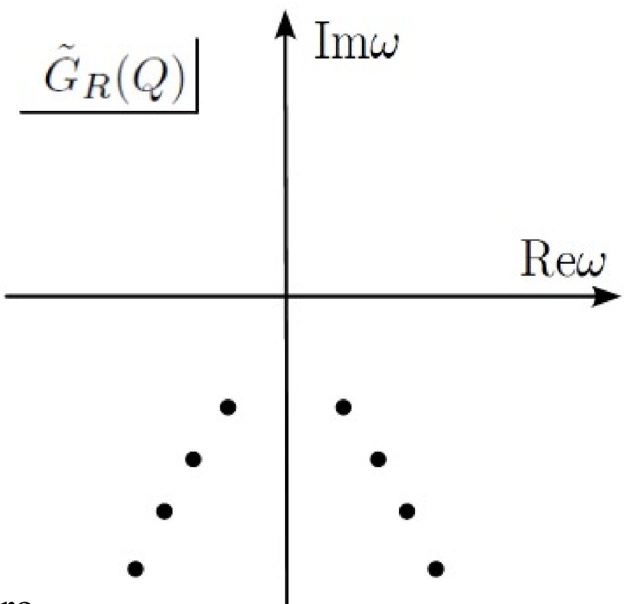
In a few years, these techniques will be taught in regular quantum field lectures to students.

What about the relaxation time coefficient τ_π in strongly-coupled fluids ?

JN, Denicol, 2011

It was shown that for any strongly-coupled conformal plasma with gravity dual

AdS/CFT = **TYPE 2**



At zero wavelength

Poles of $\tilde{G}_R^{xyxy}$ are always

symmetrical !!!!!

Radically different than weakly-coupled gases obtained via the Boltzmann equation !!

Strongly coupled fluids do not approach their Navier-Stokes limit via relaxation equations !!!

The transient dynamics of relativistic conformal fluids is surprisingly sensitive to the nature of the fixed point:

Near a trivial fixed point: **TYPE 1** (this must be the case for high T QCD):

At order $\mathcal{O}(\mathrm{Re}^{-2}, \mathrm{Re}^{-1}\mathrm{Kn}, \mathrm{Kn}^2)$ the EOM for the conformal fluid are

$$\tau_1 D\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + e_1\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\,\alpha} + e_2\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\,\alpha} + e_3\pi_\alpha^{\langle\mu}\Omega^{\nu\rangle\,\alpha} - \sum_{i=1}^{5} 2\eta\, c_i\, O_i^{\mu\nu},$$

$$D\pi^{\langle\mu\nu\rangle} = \dot{\pi}^{\langle\mu\nu\rangle} + 4\pi^{\mu\nu}\theta\,/3$$

10 independent transport coefficients

USP

The transient dynamics of relativistic conformal fluids is surprisingly sensitive to the nature of the fixed point:

Near a trivial fixed point: **TYPE 2** (this is the case of strongly coupled N=4 SYM):

$$\chi_2 D^2 \pi^{\langle\mu\nu\rangle} + \chi_1 D\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + e_1 \pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\,\alpha} + e_2 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\,\alpha} + e_3 \pi_\alpha^{\langle\mu}\Omega^{\nu\rangle\,\alpha}$$
$$-\sum_{i=1}^{5} 2\eta\, c_i\, O_i^{\mu\nu} + \sum_{i=1}^{5} f_i\, \pi_\rho^{\langle\mu} O_i^{\nu\rangle\,\rho} - \xi\, D^{\langle\mu} D_\lambda \pi^{\nu\rangle\,\lambda}.$$

17 independent transport coefficients

$$D^2\pi^{\langle\mu\nu\rangle} \longrightarrow \ddot{\pi}^{\langle\mu\nu\rangle} - 2u_\rho \dot{\pi}^{\rho\langle\mu}\dot{u}^{\nu\rangle} + \frac{20}{9}\pi^{\mu\nu}\theta^2 + 3\theta\,\dot{\pi}^{\langle\mu\nu\rangle} + \frac{4}{3}\pi^{\mu\nu}\dot{\theta}\,.$$

Remarks about the heavy quark free energy

In the deconfined phase the U-shaped configuration is not always the extremum of the action at all temperatures. In fact,

For $\mathcal{N} = 4$ SYM this other configurations must appear when LT > 0.28

Yaffe et al, 2006

- In these Einstein-dilaton models the same phenomenon appears above Tc.

- However, below Tc the U-shaped configuration is the single extremum in the supergravity approximation.

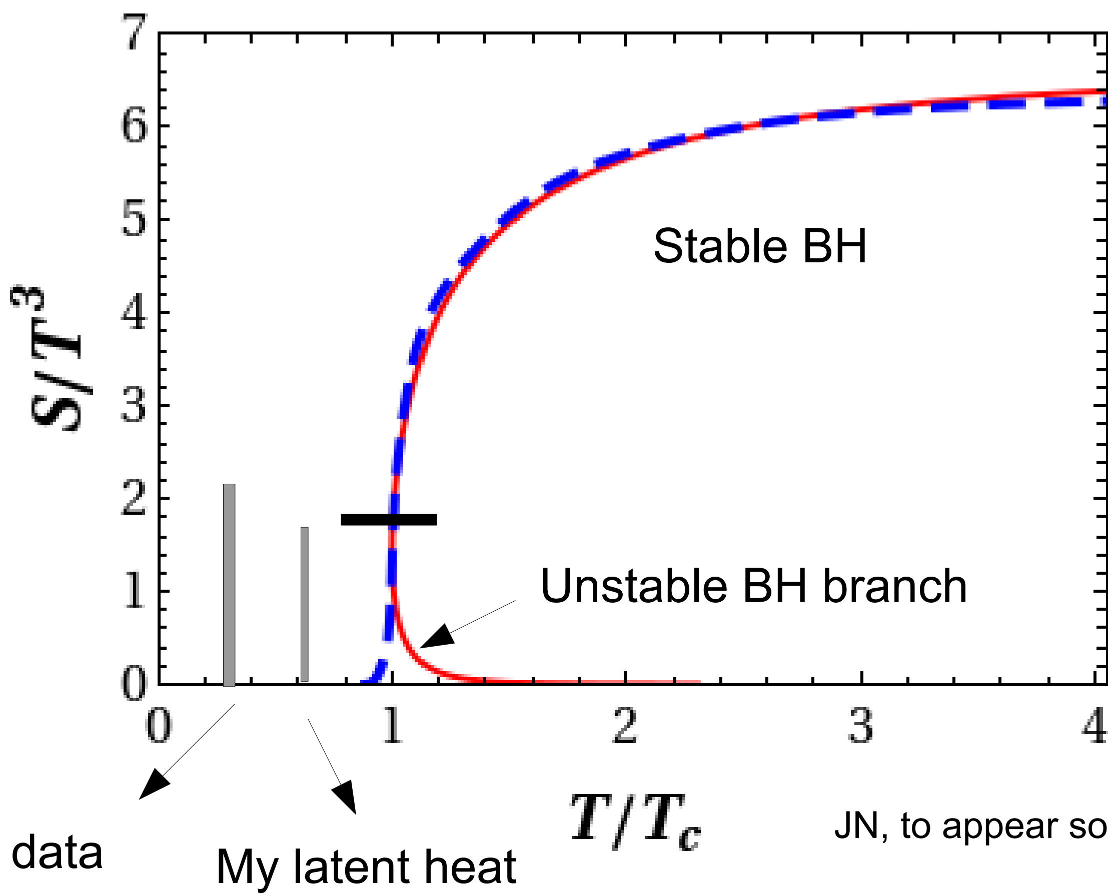
JN, arXiv:0910.1261 [hep-th]

$$V(\phi) = \frac{-12\ \left(1+17\phi^2+5\phi^4\right)^{1/4} \exp \sqrt{2/3\phi} + 18\phi^2}{R^2}$$

SU(3) data from Boyd et al., 1996.

Confinement!!!

1st order



JN, to appear soon.

But what is the QFT and the corresponding string theory?

QFT: Usually, a local field theory that is scale invariant is also conformal invariant (in more than 1+1 dimensions this is tough to prove …)

Scale invariance must have something to do with the spacetime metric and the QFT's energy-momentum tensor. In fact, changing the coordinates

$$x^\mu \to x^\mu + \zeta^\mu(x) \quad \Rightarrow \quad \delta S = \int T^{\mu\nu} \delta g_{\mu\nu}$$

$$\delta g_{\mu\nu} = \nabla_\mu \zeta_\nu + \nabla_\nu \zeta_\mu$$ Scale transf. $$\Rightarrow \quad \delta g_{\mu\nu} = 2 g_{\mu\nu}\, \delta C$$

Conformal transformations

Action is invariant if $T^\mu_\mu = 0$. In this case, $\delta g_{\mu\nu} = f(x)\, g_{\mu\nu}$

In D=3+1, the symmetry group is $SO(2,4)$

This is like the Poincare group + inversion $\vec{x} \to -\vec{x}/x^2$

We saw today that one cannot really know, using only thermodynamic quantities, whether the plasma is strongly or weakly-coupled ...

What about transport properties? Let's talk about the shear viscosity η

Weakly-coupled limit of $\mathcal{N} = 4$ SYM: $N_c \to \infty$ $\lambda \ll 1$

Huot et al., 2007

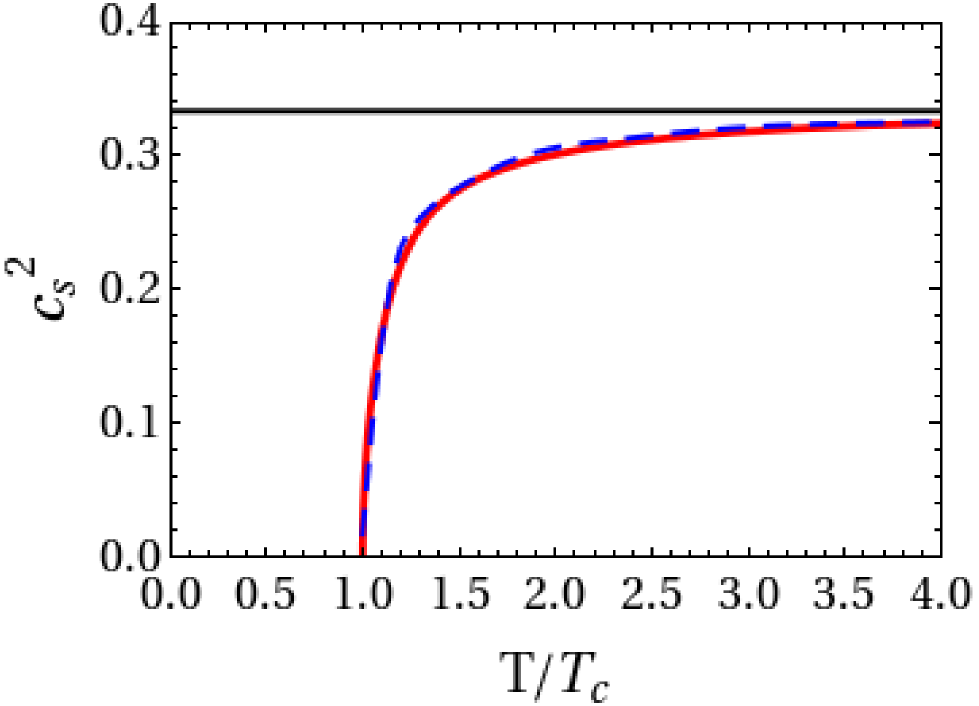
Leading log kinetic theory result for N=4 SYM plasma

$$\frac{\eta}{s} = \frac{6}{\lambda^2 \ln\left(2/\sqrt{\lambda}\right)}$$

Kinetic theory

*I feel uneasy to talk about the Boltzmann equation in a theory that is exactly conformal in the vacuum but anyways …

$$V(\phi) = \frac{-12\ \left(1 + 17\phi^2 + 5\phi^4\right)^{1/4} \exp\sqrt{2/3\phi} + 18\phi^2}{R^2}$$



SU(3) data from Boyd et al., 1996.

Black Hole Temperature

- Black holes emit blackbody radiation at a certain temperature, which is determined by requiring that the Euclidean metric is regular at the horizon.

Starting from Schwarzchild

$$ds^2 = -\left(1 - \frac{r_0}{r}\right) dt^2 + \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

Near the horizon $\tau = it$

Define $r = r_0\left(1 + \rho^2\right)$ and expand Euclidean version around $\rho \longrightarrow 0$

$$ds^2 = 4r_0^2 \left[d\rho^2 + \rho^2 \left(\frac{d\tau}{2r_0}\right)^2 + \frac{1}{4} d\Omega_2^2 \right]$$

Period $\beta = 1/T$

$$1/T = 4\pi r_0 = 8\pi M G_4$$

Bekenstein-Hawking Entropy

Assume 1st law of thermodynamics $dM = TdS$

Using $T = \dfrac{1}{8\pi G_4 M}$ and $S \to 0$ when $M \to 0$ $S = 4\pi M^2 G_4$

Horizon area $A = 4\pi r_0^2 = 16\pi \left(MG_4\right)^2$ then

$$\boxed{S = \frac{A}{4G_4}}$$

This result is only valid for Einstein-like gravity actions (maximum 2 derivatives in the action) even in higher dimension spacetime.

Basic Laws of Black Hole Thermodynamics

We may now state the basic laws of black-hole thermodynamics:

- *The zeroth law* of black-hole thermodynamics states that the surface gravity (temperature) of stationary black holes is constant over the horizon.
- *The first law* states the balance of energy during physical processes

  $$dM = TdS + \mu dQ.$$

  In general there might be more "work" terms, associated with other conserved local charges (angular momentum for example).
- *The second law* states that in all processes the BH entropy is nondecreasing. When one involves also matter outside of black holes the second law is replaced by:
- *The generalized second law* states that the sum of the BH entropy and the entropy of the rest of the world is never decreasing.
- *The third law* states that an infinite number of steps are required to bring the temperature of a black hole to zero.

The BH entropy seems to suggest that a black hole with given M and Q has a large number of degenerate microstates Ω so that

$$S = \log \Omega.$$

The classical theory of gravitation gives no information on the nature of such putative microscopic degrees of freedom.

Some info about Anti-de Sitter spacetime

See Kiritsis' book

Anti-de Sitter space in p+2 dimensions can be defined by the hyperboloid in (p+3) space with metric

$$ds^2 = -dX_0^2 - dX_{p+2}^2 + \sum_{i=1}^{p+1} dX_i^2$$

and the condition

$$X_0^2 + X_{p+2}^2 - \sum_{i=1}^{p+1} X_i^2 = L^2$$

The space has an SO(2,p+1) isometry and it is homogenous and isotropic

The constraint above can be resolved using the coordinates

$$X_0 = L\cosh\rho\cos\tau\,, \quad X_0 = L\cosh\rho\sin\tau\,,$$

$$X_i = L\ \sinh\rho\ \Omega_i \quad \left(i = 1, \ldots, p+1, \sum_{i=1}^{p+1}\Omega_i^2 = 1\right).$$

$\Rightarrow$

$$ds^2 = L^2\left(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_p^2\right)$$

Some info about Anti-de Sitter spacetime

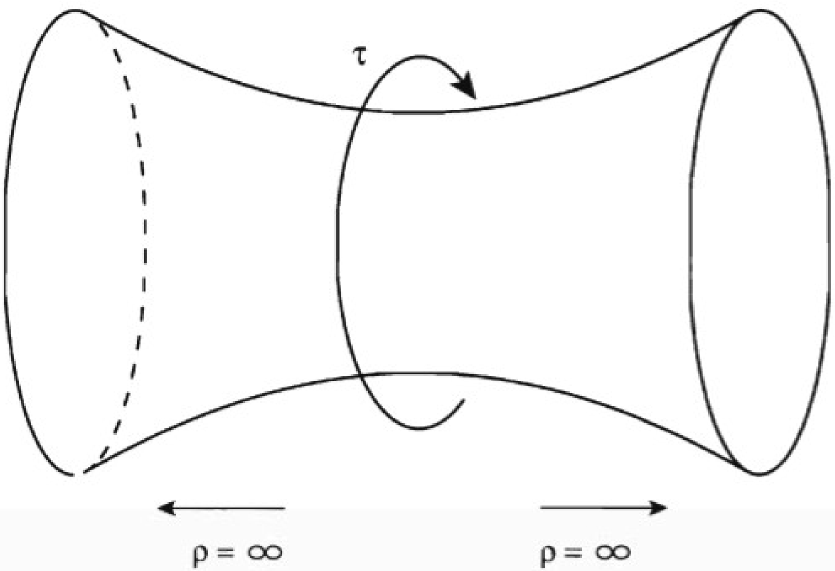


Figure K.1 The space AdS_{p+2} can be constructed as a hyperboloid in $R^{2,p+1}$. It has closed timelike curves in the coordinate τ. To obtain a causal space we must go to the universal cover of the time coordinate.

The isometry group $SO(2, p+1)$ has $SO(2) \times SO(p+1)$ are the maximal compact subgroup. $SO(2)$ acts by translating the τ coordinate. $SO(p+1)$ is the transitive symmetry of S^p.

Some info about Anti-de Sitter spacetime

By taking $\rho \in (0, \infty)$ and $\tau \in [0, 2\pi)$ we cover the hyperboloid once. Therefore, ρ, τ, Ω_i are global coordinates of AdS. When we consider the neighborhood of $\rho \simeq 0$ the metric can be approximated by

$$ds^2 \simeq L^2 \left(-d\tau^2 + d\rho^2 + \rho^2 \, d\Omega_p^2 \right) \tag{K.5}$$

and this makes explicit the fact that AdS_{p+2} has topology $S^1 \times \mathbb{R}^{p+1}$. The S^1 is time-like, and indicates that AdS_{p+2} thus defined, has closed timelike curves (see figure K.1). We may obtain a causal space-time by taking the universal cover of the S^1 coordinate. This practically means that we will take $-\infty < \tau < +\infty$. On this cover, there are no closed time-like curves. Here and in most places in the literature AdS_{p+2} stands for this universal cover.

Some info about Anti-de Sitter spacetime

An important property of AdS is its causal structure. To that end, it is convenient to bring the end points of the ρ coordinate to finite values by introducing a new coordinate θ as

$$\tan\theta = \sinh\rho, \quad \theta \in \left[0, \frac{\pi}{2}\right). \tag{K.6}$$

Then, the metric (K.4) takes the form

$$ds^2 = \frac{L^2}{\cos^2\theta}\left(-d\tau^2 + d\theta^2 + \sin^2\theta\, d\Omega_p^2\right). \tag{K.7}$$

Since the causal structure of a space-time does not change by conformal transformations we multiply the AdS metric by $\cos^2\theta/L^2$ to obtain

$$d\tilde{s}^2 = -d\tau^2 + d\theta^2 + \sin^2\theta\, d\Omega_p^2. \tag{K.8}$$

This is the metric of the Einstein static universe, with one difference: the θ coordinate takes values in $\left[0, \frac{\pi}{2}\right)$ rather than the full range $[0, \pi)$. The equator $\theta = \pi/2$ is a boundary of the space with the topology of S^p. This is shown in figure K.2 in the case of AdS_3. In the case of AdS_2 the boundary consists of two points and the coordinate θ takes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

In general if a space-time is conformal to a space-time that is isomorphic to half of the Einstein static universe then this space-time is called *asymptotically AdS*.

Some info about Anti-de Sitter spacetime

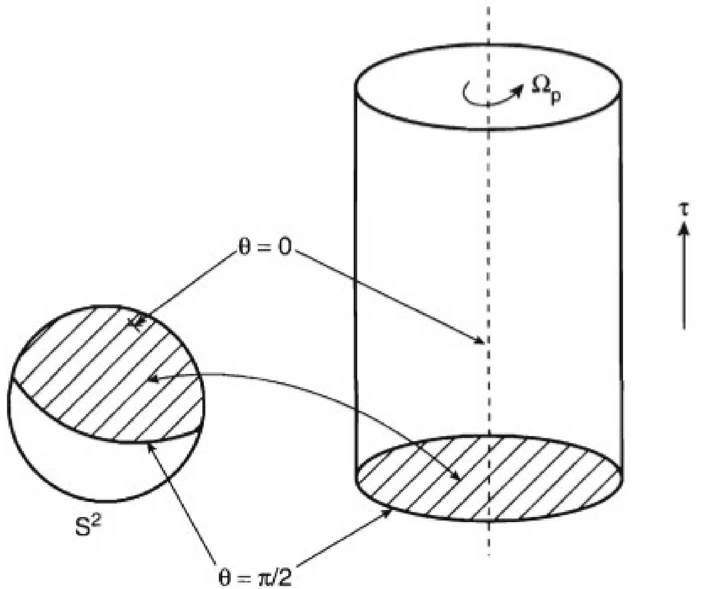


Figure K.2 The space AdS_3 can be conformally mapped into half of the Einstein static universe $\mathbb{R} \times S^2$.

Some info about Anti-de Sitter spacetime

Poincare coordinates:

$$X_0 = \frac{u}{2}\left[1 + \frac{1}{u^2}(L^2 + \vec{x}^2 - t^2)\right], \quad X_i = \frac{L\,x^i}{u},$$

$$X_{p+1} = \frac{u}{2}\left[1 - \frac{1}{u^2}(L^2 - \vec{x}^2 + t^2)\right], \quad X_{p+2} = \frac{L\,t}{u}.$$

The metric in such coordinates is

$$ds^2 = \frac{L^2}{u^2}\left[du^2 - dt^2 + d\vec{x}^2\right].$$

The boundary is at $u = 0$.

Euclidean AdS

AdS has a global time coordinate τ and therefore the continuation to Euclidean signature is straightforward. This amounts to $\tau \to i\tau_E$ which indicates that Euclidean AdS can be defined as the hyperboloid

$$X_0^2 - X_E^2 - \vec{X}^2 = L^2 \tag{K.13}$$

in $\mathbb{R}^{1,p+2}$

$$ds_E^2 = -dX_0^2 + dX_E^2 + d\vec{X}^2. \tag{K.14}$$

We can also obtain the same space by rotating the Poincaré t coordinate as $t \to i\, t$. The metric of Euclidean AdS_{p+2} can be written as

$$\begin{aligned} ds_E^2 &= L^2 \left(\cos^2\rho\, d\tau_E^2 + d\rho^2 + \sin^2\rho\, d\Omega_p^2\right) \\ &= L^2 \left[\frac{dr^2}{r^2} + r^2(dt_E^2 + d\vec{x}^2)\right]. \end{aligned} \tag{K.15}$$

We can obtain the Poincaré metric by transforming $r \to 1/u$

$$ds_E^2 = \frac{L^2}{u^2}\left[du^2 + dx_1^2 + \cdots + dx_{p+1}^2\right]. \tag{K.16}$$

Euclidean AdS_{p+2} is topologically a $(p+2)$-dimensional disk. In the Poincaré coordinates $r = \infty$ is almost all of the boundary. It is topologically an S^{p+1} with a point removed. The full boundary S^{p+1} is recovered by adding the point $r = 0$. This is equivalent to adding the point at infinity, $\vec{x} = \infty$.

The Supergravity Approximation

- When $g_s \ll 1$ and the AdS radius $L^2/\ell_s^2 \gg 1$ we should recover a theory of supergravity (low energy approximation of superstring theory)

- Action is local and composed of several massless fields (at low energies)

$$\frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-G}\, e^{-2\Phi} \left(\mathcal{R} + 4(\partial\Phi)^2 + \dots\right)$$

- $G_{10} \sim g_s^2\, \ell_s^8$ where $g_s \sim 1/N_c \ll 1$ is the string coupling.

- In this limit, $\lambda = R^4/\ell_s^4 \gg 1, \;\; N_c \to \infty \;\; \text{and} \;\; Z_{string} \sim e^{i\, S_{sugra}}$

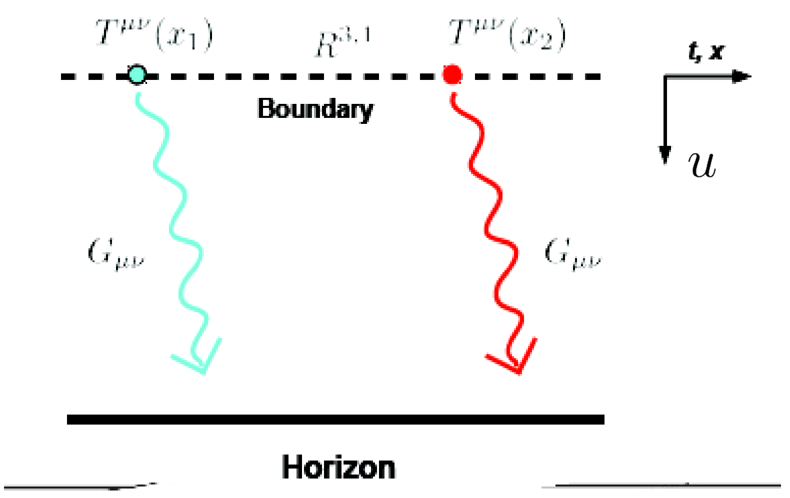
T'Hooft coupling in
the gauge theory

The supergravity approximation: Summary

- Gauge theories em D=4 with Nc >>1 at strong coupling should be dual to a local weakly-coupled effective theory of gravity em D >= 5.

- Why is this useful for QGP physics? We saw that the transport coefficients of relativistic fluids are obtained via the retarded energy-momentum correlator

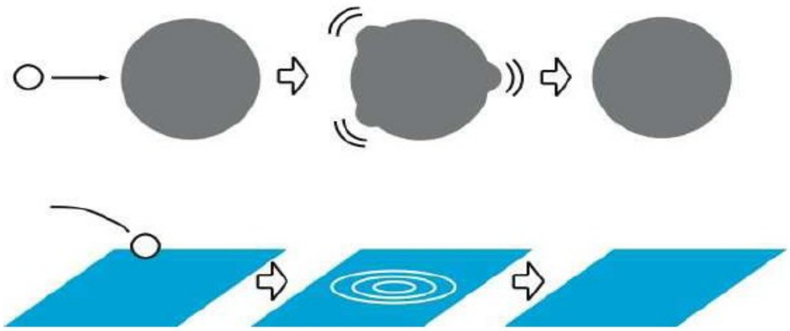
Graviton scattering in the bulk !!!!



Computing the Shear Viscosity from Supergravity

It is more or less like this ...

Black brane defined in 10 dimensions



QCD-like theory in 4 dimensions in a thermal bath

The black brane event horizon will go back to equilibrium, which corresponds
To near equilibrium fluid dynamical behavior in the 4-dimensional plasma

The dissipation of these waves are controlled by the same parameter in both descriptions

η Shear viscosity

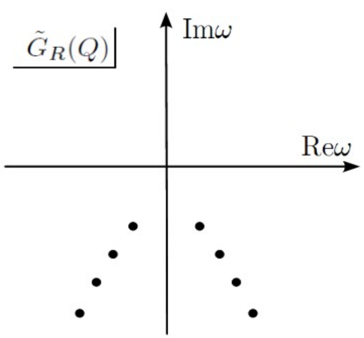
While the specific value of the poles of the correlator can vary according to the theory used, the fact that the lowest modes are symmetrical is universal for any conformal plasma

JN, G. Denicol, 2011

Why???

AdS/CFT predicts that the retarded correlator of T^{xy} is identical to the glueball correlator $\mathrm{Tr}F^2$

Both correlators are obtained by solving the equation for a massless scalar field coupled to gravity in the bulk



$$\chi_2\partial_t^2\delta\Pi^{xy}+\chi_1\partial_t\delta\Pi^{xy}+\delta\Pi^{xy}=\eta\partial_t h_{xy}+\mathcal{O}\left(\partial_t^2 h_{xy},\partial_z^2 h_{xy}\right)$$

N=4 SYM => $\chi_1 \sim 1.27/(4\pi T)$ $\chi_2 \sim 0.93/(4\pi T)^2$

Gradient Expansion: Hydrodynamics based on the **Taylor expansion** of the Green's function around the origin

Linear theory

$$\tilde{J}\left(Q\right) = \tilde{G}_R\left(Q\right)\tilde{F}\left(Q\right)$$

Taylor expansion

$$\tilde{G}_R(Q) \sim \tilde{G}_R(0,\mathbf{0}) + \partial_\omega \tilde{G}_R(0,\mathbf{0})\,\omega + \ldots,$$

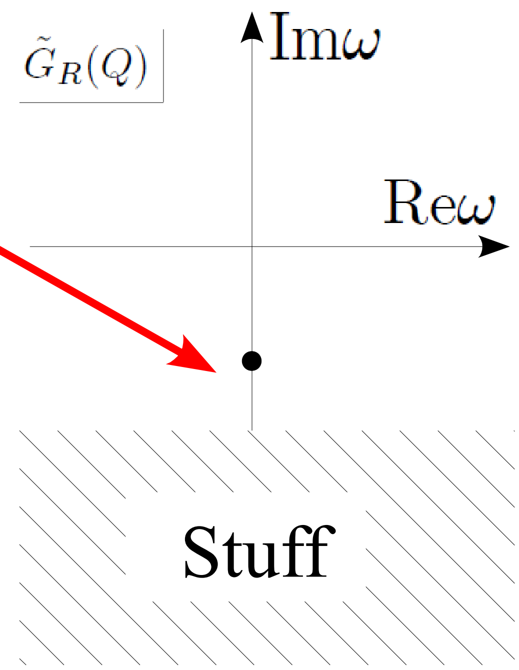
This is the gradient expansion !

$$J\left(X\right) = CF\left(X\right) + C_1\partial_t F\left(X\right) + \ldots,$$

$$C = \tilde{G}_R\left(0,\mathbf{0}\right), \quad C_1 = \partial_\omega \tilde{G}_R\left(Q\right)\Big|_{\omega,\mathbf{k}=0}$$

Limitations of the Gradient expansion

$$\tilde{G}_R\left(Q\right) = \frac{f\left(Q\right)}{\omega - \omega_0\left(\mathbf{k}\right)} + \ldots$$



If the Green's function has poles the **radius of convergence** of the gradient expansion is **limited**

At long times (small frequencies), only the poles **nearest** to the origin are important.

Systematic Approach: Inclusion of one pole

G. Denicol, JN, H. Niemi, D. Rischke, PRD 2011

Linear theory

$$\tilde{J}(Q) = \tilde{G}_R(Q)\,\tilde{F}(Q)$$

Laurent expansion

$$\tilde{G}_R(Q) = \frac{f(\omega_0,\mathbf{k})}{\omega-\omega_0(\mathbf{k})} + \left[\tilde{G}_R(0,\mathbf{k}) + \frac{f(\omega_0,\mathbf{k})}{\omega_0(\mathbf{k})}\right] + \ldots$$

Relaxation equation !

$$\tau_R \partial_t J + J = DF + \ldots$$

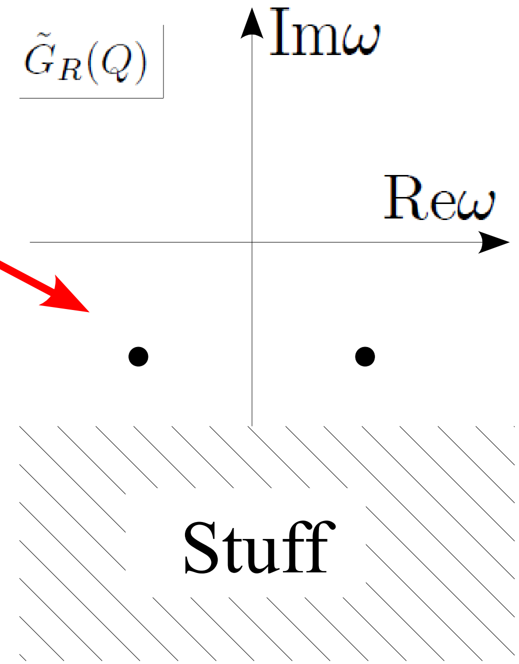
$$\tau_R = \frac{1}{i\omega_0(\mathbf{0})}, \quad D = \tilde{G}_R(0,\mathbf{0})$$

Causality requires the presence of at least one pole !!!

Limitations of the Gradient expansion

Two Symmetric Poles:

$$\tilde{G}_R(Q) = \frac{f_1(Q)}{\omega - \omega_1(\mathbf{k})} + \frac{f_2(Q)}{\omega - \omega_2(\mathbf{k})} + \dots$$



At long times (small frequencies), **both** poles contribute **equally** to the dynamics.

Clearly, both poles must be taken into account in this case !!!!

Two Symmetric Poles

Linear theory

$$\tilde{J}(Q) = \tilde{G}_R(Q)\,\tilde{F}(Q)$$

Laurent expansion

$$\tilde{G}_R(Q) = \frac{f_1(\omega_1,\mathbf{k})}{\omega-\omega_1(\mathbf{k})} + \frac{f_2(\omega_2,\mathbf{k})}{\omega-\omega_2(\mathbf{k})} + \left[\tilde{G}_R(0,\mathbf{k}) + \frac{f_1(\omega_1,\mathbf{k})}{\omega_1(\mathbf{k})} + \frac{f_2(\omega_2,\mathbf{k})}{\omega_2(\mathbf{k})}\right] + \ldots$$

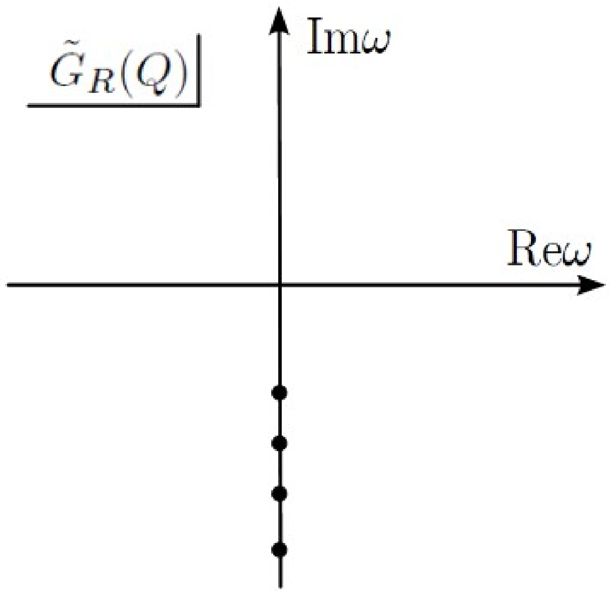
Not a relaxation equation !

$$\chi_2\partial_t^2 J + \chi_1\partial_t J + J = DF + \ldots$$

$$\chi_2 = \frac{-1}{\omega_1(\mathbf{0})\,\omega_2(\mathbf{0})},\quad \chi_1 = \frac{\omega_1(\mathbf{0})+\omega_2(\mathbf{0})}{i\omega_1(\mathbf{0})\,\omega_2(\mathbf{0})},\quad D = \tilde{G}_R(0,\mathbf{0})$$

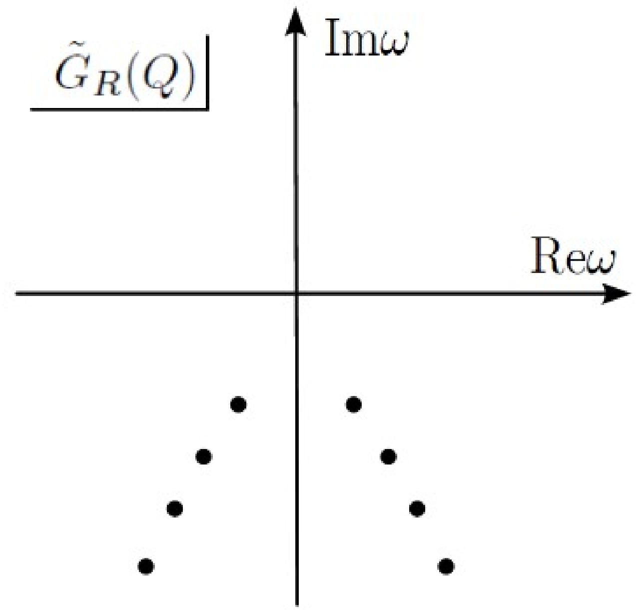
Importance of the analytical structure of the retarded Green's function

TYPE 1



If the pole closest to the origin is **purely imaginary**, the long-time dynamics is given by **relaxation type** equations

TYPE 2



If nearest poles have **real parts**, long time dynamics is given by a **second order** differential equation (oscillations)

When is the relaxation time going to be **IMPORTANT**?

Assuming we can get the qualitative behavior of hydro flow from the Bjorken solution

Typical timescale for time variation of flow: $$\theta(\tau) \equiv \nabla_\mu u^\mu = 1/\tau$$

At the beginning of hydro expansion

Thus, in the beginning $$\tau_\pi \, \theta(\tau_0) \sim 1$$

$$\tau \sim \tau_0 \sim 0.5\, fm/c$$

But this quantity can only **INCREASE** with propertime in a realistic system that decouples at **FREEZE-OUT** (non-conformality here is fundamental)!!!

- It can be shown that such relaxation-type theories appear directly from the Boltzmann equation for relativistic dilute gases.

Israel and Stewart, 1978
Denicol, Koide, Rischke, 2010

- Thus, in the weak coupling limit of QCD at finite temperature $T \gg \Lambda_{QCD}$

$$g_s(T) \ll 1$$

quarks and gluons should form a weakly interacting gas described by those Israel-Stewart-like equations.

- Different than Navier-Stokes equations, the relaxation equations can be solved numerically and they have been fundamental in the determination of the properties of the quark-gluon plasma formed in heavy ion collisions.

- These equations are used to show that QGP is the hottest nearly perfect fluid ever created in the lab ...

Consider an **arbitrary** linear theory for the dissipative current

$$J(X) = \int d^4x' G_R(X - X') F(X')$$

In Fourier space,

$$Q = (\omega, \mathbf{k})$$

$$\tilde{J}(Q) = \tilde{G}_R(Q) \tilde{F}(Q)$$

$$\tilde{A}(Q) = \int d^4X \exp(iQ \cdot X) A(X) \ ,$$

$$A(X) = \int \frac{d^4Q}{(2\pi)^4} \exp(-iQ \cdot X) \tilde{A}(Q) \ .$$

How far are we from finding the exact dual theory of pure glue with Nc = 3??

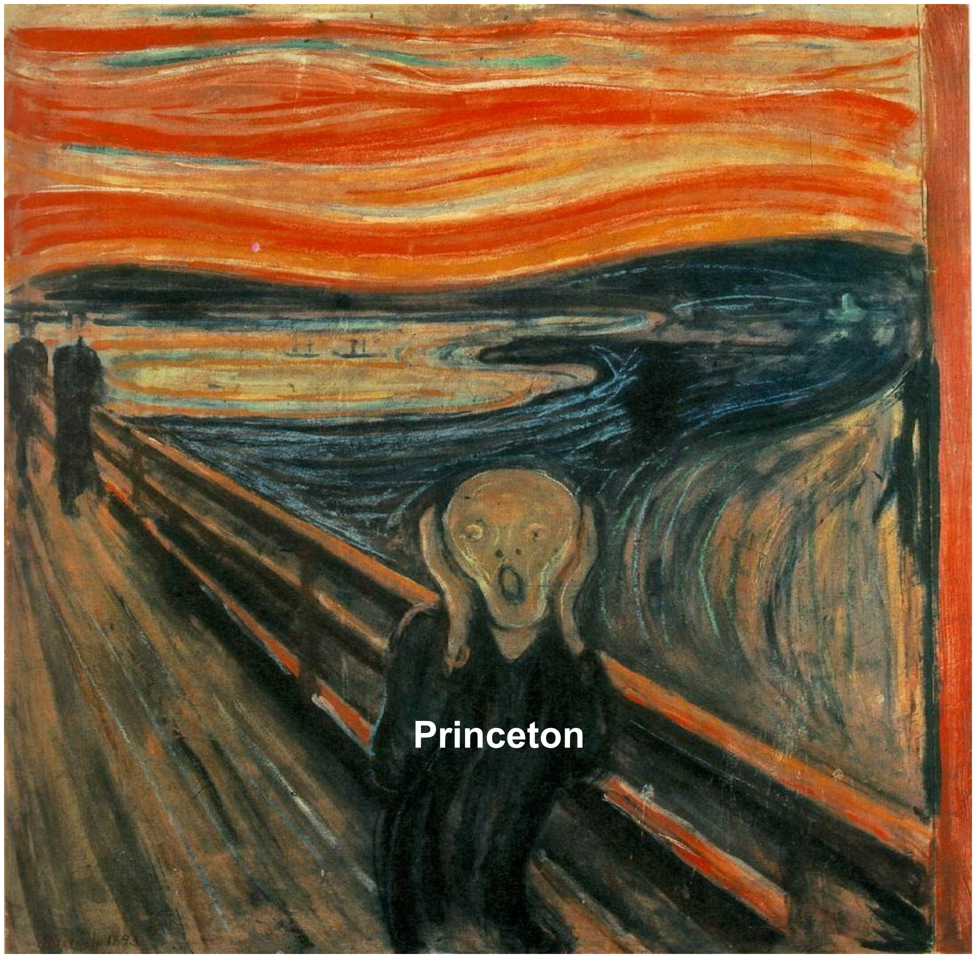
- Lattice says that, for pure glue, Nc = 3 = infinity is a good approximation.
- This means that the planar limit can be used → enormous simplification!!!!

The devil lies in the (ultraviolet) details ...

- Pure glue has a trivial UV fixed point (known since 1973).
- In this talk you will see that, quite generally, SUGRA-like theories

CANNOT BE ASYMPTOTICALLY FREE

Asymptotic freedom may truly require a non-perturbative description of string theory near a curvature singularity!!!!



$$\frac{R^2}{\alpha'} \sim 1$$

Infinite number of corrections!

New ideas are necessary ...

But what can be done at the moment?

Lattice is going to be our guide ...

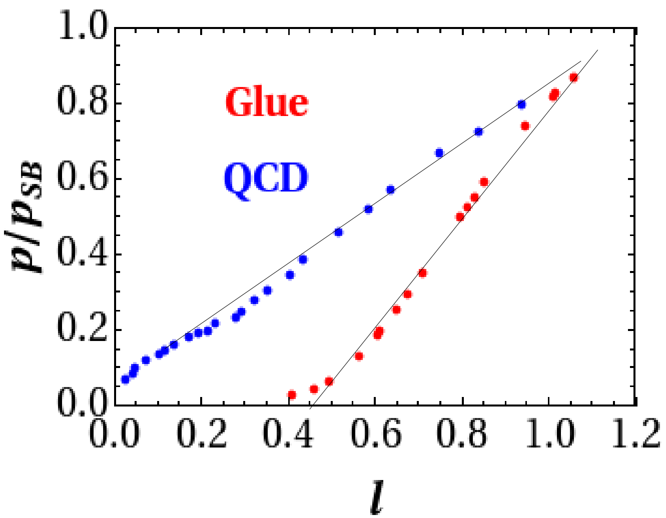
Lattice data suggests that in the temperature region relevant for heavy ion experiments (1 - 3 Tc), QCD at finite T (and zero chemical potential)

- sQGP far from being conformal (sizable trace anomaly)

- Dominated by power-like terms that are beyond the reach of perturbation theory (this could make a big difference in qhat calculations …)

- Thermodynamics looks like pure glue (more on that later).

- Even though the Polyakov loop is not a real order parameter in QCD, the current lattice data suggests that it may be possible to find an effective theory where



Note that $p/p_{SB} \sim \ell$

for glue, glue + quarks (even in the hadronic phase) !!!

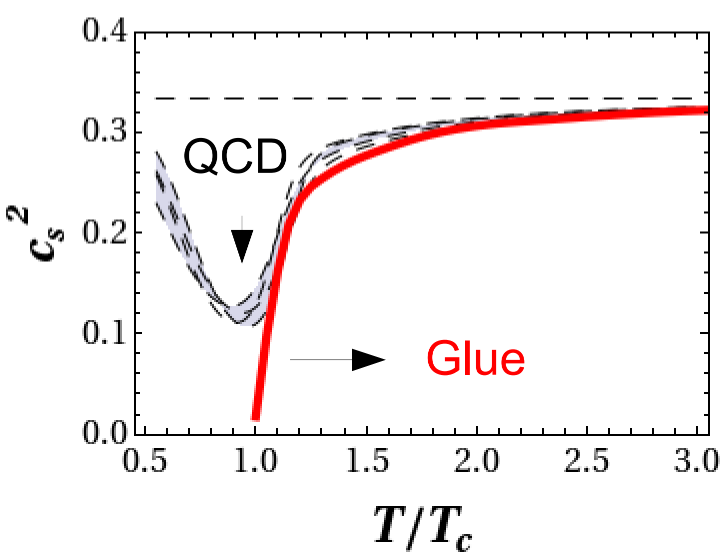
What is this effective theory ???

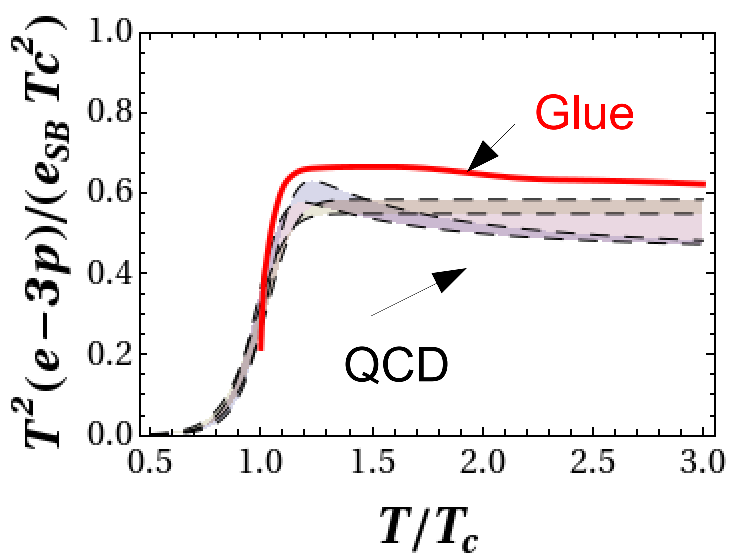
What is observed on the lattice?

QCD with (almost) physical quark masses

Strange quark mass ms = physical
2 light flavors with 1/10 strange mass

Lattice size: $32^3 \times 8$ Data from A. Bazavov et al , 2009.





QCD is a crossover

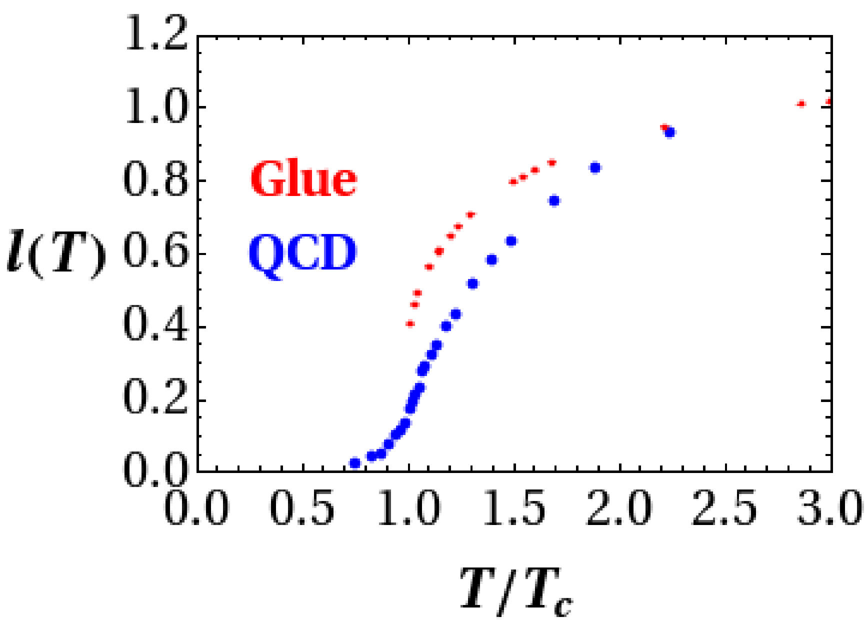
(I defined a Tc = 185 MeV
from the dip in the speed of sound)

Nonperturbative power-like terms
also present in QCD

What is observed on the lattice?

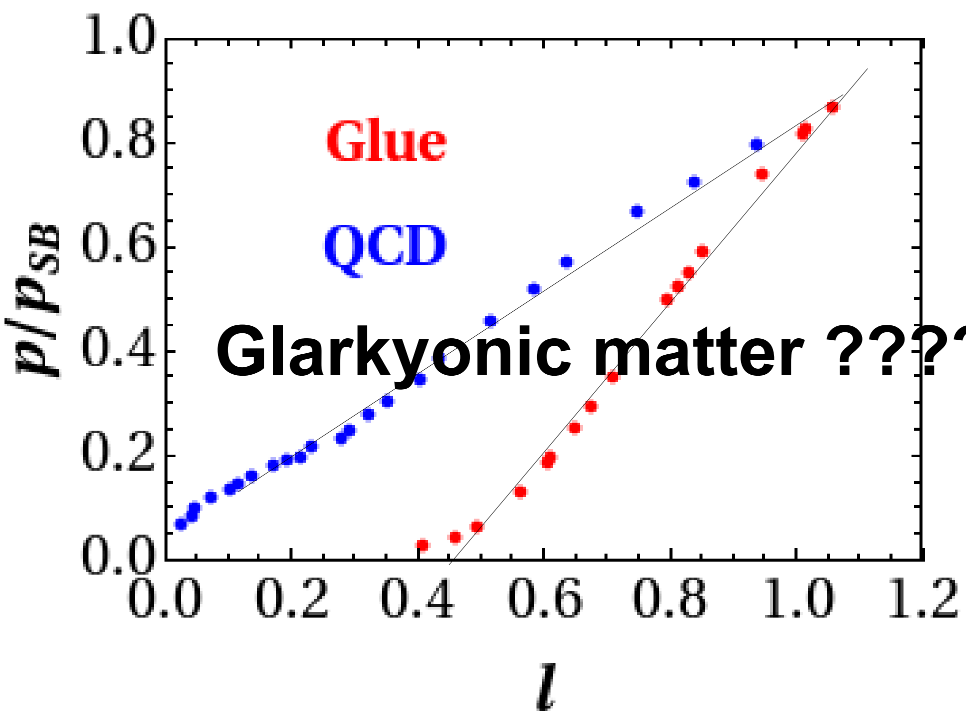
Renormalized Polyakov loop in the fundamental representation

Data from A. Bazavov et al (MILC Collaboration), 2009.



What is observed on the lattice?

While both the pressure and the loop vary significantly near Tc, note that



Glarkyonic matter ????

Whatever causes this linear behavior for the glue is also there in QCD

Nf = 0 to Nf =3 → change in the slope!

Glue

Quarks

$$p/p_{SB} \sim \ell$$

An important property of these theories is that the thermodynamic quantities exhibit a power-like expansion in terms of

$$\left(\frac{\Lambda_\phi}{T}\right)^{2(4-\Delta)}$$

Cherman, Nellore 2009
Hohler, Stephanov, 2009

Thus, the power-like behavior seen on the data should be captured by these models for a convenient choice of parameters.

Here I will require that there is linear confinement below Tc.

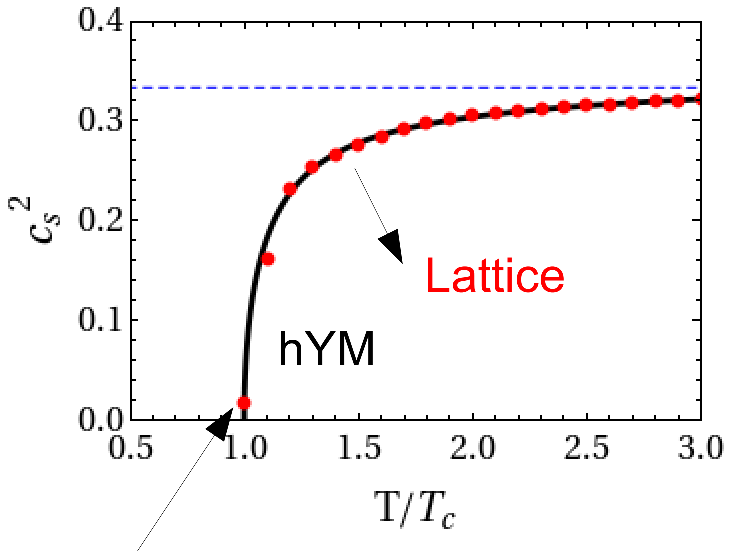
A choice for the scalar potential that describes the lattice data is

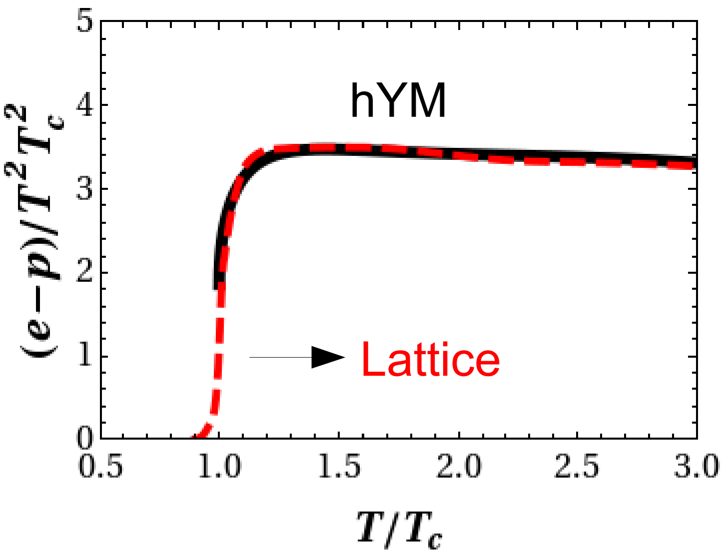
$$V(\phi) = -12(1+a\phi^2)^{1/4} \cosh\sqrt{\frac{2}{3}}\phi + b_2\phi^2 + b_4\phi^4 + b_6\phi^6$$

$a = 1$ $\quad$ $b_4 = 0.4$

$b_2 = 5$ $\quad$ $b_6 = 0.0098$ $\quad$ $\Delta = 2$

This choice gives an amazing match to the lattice data …





1st order transition

What about the Polyakov loop ????

It is more intuitive to think about this the following way

Imagine that the quarks are infinitely heavy and also infinitely far apart (this is well defined above Tc).

For a single(infinitely massive) quark one could sketch what the partition function is

$$Z \sim \frac{\int \mathcal{D}A\mathcal{D}x \, e^{-S_{glue}(A)+\int dt\left(\frac{M\dot{x}^2}{2}+A\dot{x}\right)}}{\int \mathcal{D}A \, e^{-S_{glue}(A)}} \sim \langle \mathrm{Tr}\, e^{\int_C A} \rangle \sim \langle \ell \rangle$$

$\langle \ell \rangle$ somehow measures what happens to the gluon medium once this heavy quark probe is included

This idea can be made more precise …

McLerran, Svetitsky, 1981

$$\mathbf{Q} \qquad \bar{\mathbf{Q}} \quad = \langle \ell(r)\ell^*(0)\rangle \equiv e^{-F_{Q\bar{Q}}(r,T)/T}$$

Difference between free energy densities

$$F_{Q\bar{Q}}(r,T) \equiv \mathcal{F}(r,T) - \mathcal{F}_{glue}(T)$$

glue+Q pure glue

$$|\langle \ell(T)\rangle| = \lim_{r\to\infty} e^{-F_{Q\bar{Q}}(r,T)/2T} \equiv e^{-F_Q(T)/T}$$

What is $F_Q(T)$??? In general, it is NOT a true free energy !!!

Noronha, 2010

When Nc is large

$$\mathcal{F}_{glue}(T) = N_c^2 F_2^g(T) + F_0^g(T) + \mathcal{O}(1/N_c^2)$$

$$\mathcal{F}(r,T) = N_c^2 F_2(T) + F_0(r,T) + \mathcal{O}(1/N_c^2)$$

Glue + probe quark

$$\lambda_{YM} = g_{YM}^2 N_c$$

$$\lim_{N_c \gg 1} F_Q(T) = (F_0(r \to \infty, T) - F_0^g(T))/2$$

The difference between free energies is not necessarily a free energy

High T properties of the Polyakov Loop

Let's define

$$U_Q(T) \equiv F_Q(T) - T\frac{dF_Q}{dT}$$

One can show that

$$\frac{d\ell(T)}{dT} = U_Q(T)\,\frac{\ell(T)}{T^2}$$

Immediately above Tc one should have $U_Q(T \sim T_c) > 0$

Let's for now assume that $F_Q(T)$ is a true thermodynamic free energy density

Then, in equilibrium one should have a positive specific heat, i.e.,

$$\frac{dU_Q}{dT} > 0 \qquad T > T_c$$

Since $U_Q(T \sim T_c) > 0$ is always true, in this case one obtains

$$\frac{d\ell(T > T_c)}{dT} \geq 0$$

If $F_Q(T)$ is a true free energy

then the Polyakov loop is a monotonic function of T.

Is this true for pure glue with Nc=3 ?

NO!!!!

In this region T ~ 10 Tc

$$\frac{d\ell}{dT} < 0$$

$F_Q(T)$ is not a free energy in pure glue.

Data from Gupta, Huebner, Kaczmarek, 2008.

This should always be true in an asymptotically free theory

Noronha, 2010

At very high temperatures HTL predicts that

$$\ell \sim 1 + \lambda_{YM}(m_D/T) + \ldots \qquad m_D \sim \sqrt{\lambda_{YM}}T$$

Debye mass

Asymptotic freedom → Loop approaches 1 from above.

$$\frac{d\ell}{dT} < 0 \qquad \text{because} \qquad \frac{d\lambda_{YM}}{dT} < 0$$

Thus, in any confining theory that is also asymptotically free there must be a value of temperature where

Noronha, 2010

$$\frac{d\ell(T^*)}{dT} = 0$$

I would expect same in QCD although the exact value of T* may be regularization dependent.

What happens in the classes of gravity duals described earlier?

Those theories do not have a trivial UV fixed point (QCD or pure glue do).

Is $F_Q(T)$ a true free energy in this case? Is the loop a monotonic function of T?

In supergravity the loop should be a monotonic function of T because

Noronha, 2010

$F_0^g(T) = 0$ in this approximation.

Then, $$\lim_{N_c \gg 1} F_Q(T) = F_0(r \to \infty, T)/2$$

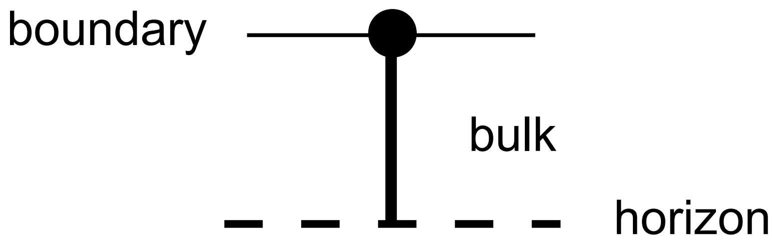
which implies that $F_Q(T)$ is a free energy and hence

$$\frac{d\ell(T > T_c)}{dT} \geq 0$$

Can this be used to constraint the dual theory of pure glue?

Asymptotic freedom cannot be described by supergravity !!!!!!

Infinitely massive heavy quark ~ fundamental string in the bulk



Maldacena, 1998
Rey et al. 1998
Brandhuber et al 1998

Assuming

$R^2/\alpha' \gg 1$

$$|\langle \ell(T) \rangle| \sim e^{-S_{NG}(\mathcal{D})}$$

$$S_{NG}(\mathcal{D}) = \frac{1}{2\pi\alpha'} \int_{\mathcal{D}} d^2\sigma \, q(\phi) \sqrt{\det h^{ab}}$$

Nambu-Goto action for the string in the bulk

$$h^{ab} = G^{\mu\nu} \partial^a X^\mu \partial^b X^\nu$$

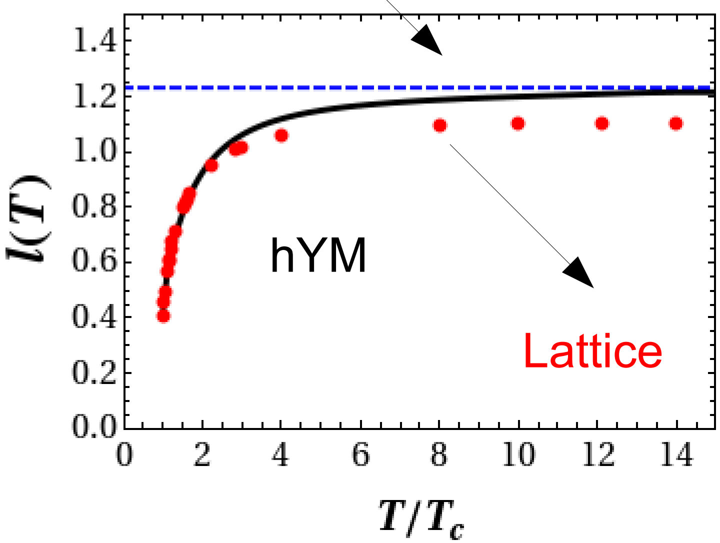
Induced metric on the worldsheet

$q(\phi)$ coupling between string and the scalar field

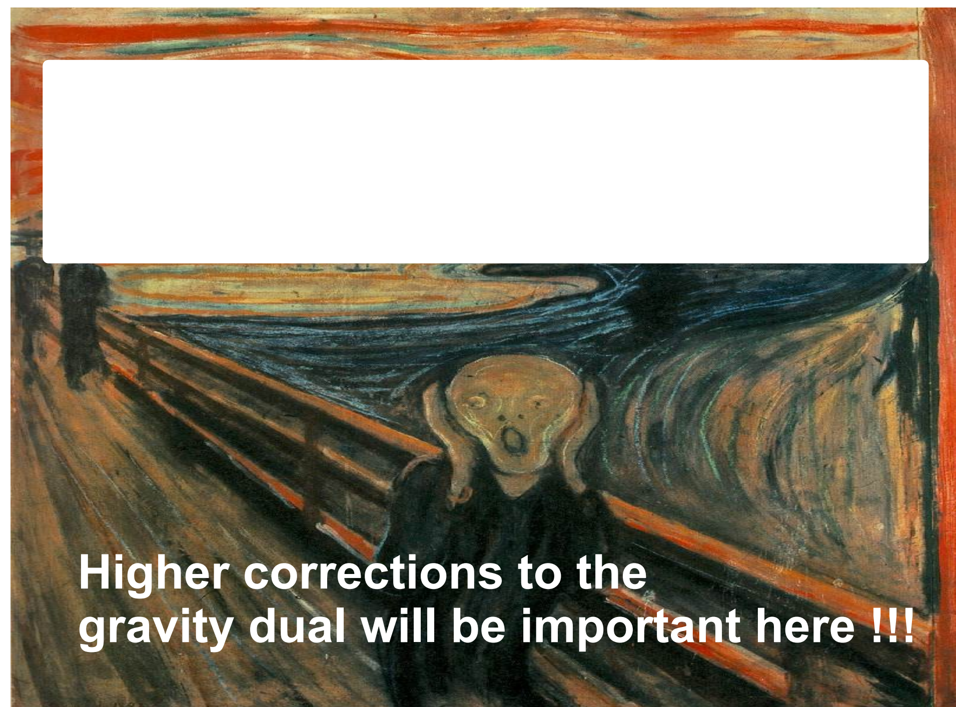
Let's assume a dilaton-like coupling function $q(\phi) = e^{\sqrt{2/3}\phi}$

Kiritsis et al, 2008

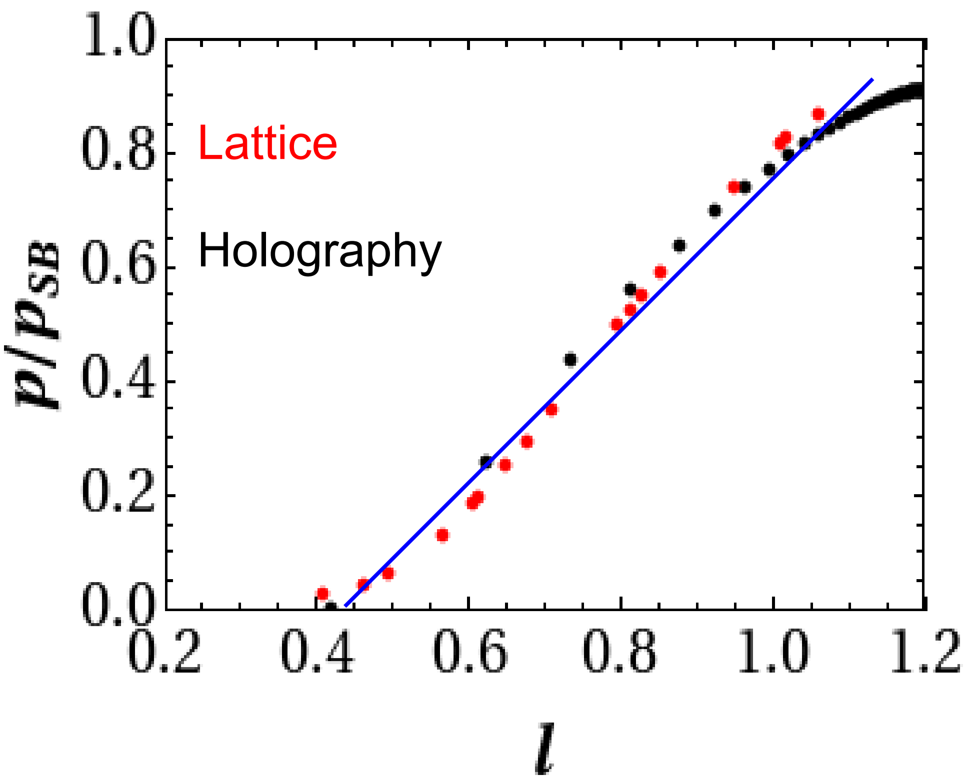
Value at T/Tc >> 1



Good agreement below 4Tc …



In this case the characteristic linear relation between P and the loop is reproduced



- The Polyakov loop

But F_Q is not a free energy in QCD (although it becomes a free energy in gravity duals in the supergravity approximation). The fact that this is not a free energy in QCD directly affects, for instance, the determination of the binding energy of heavy mesons in the plasma (do we understand how these heavy states melt in the plasma ???).

- A good agreement between the holographic calculation for the Polyakov loop and the lattice data near Tc only happens if $R^2/\alpha' \sim 1$

- Would that lead to problems in other observables that have been compared to RHIC data (such as heavy quark energy loss)?