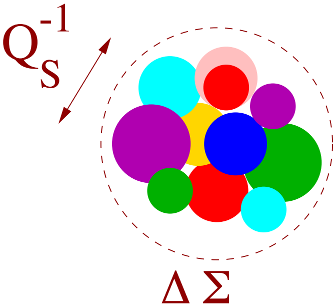
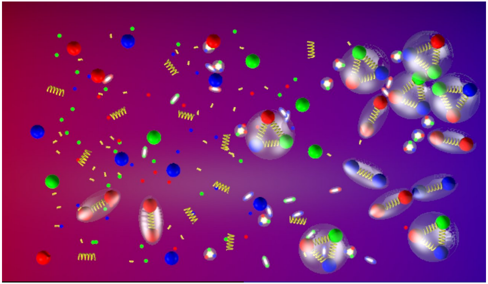
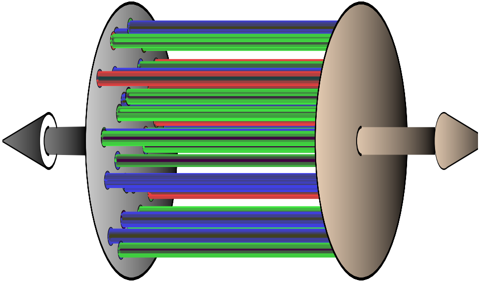
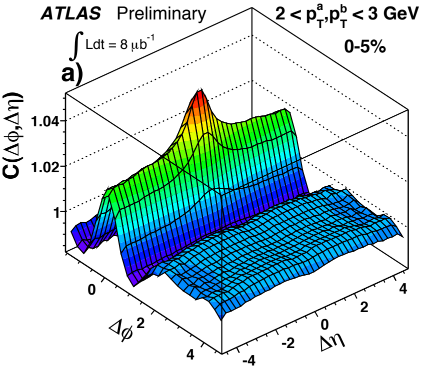
From Colour Glass Condensate to Quark–Gluon Plasma

Edmond Iancu
Institut de Physique Théorique de Saclay



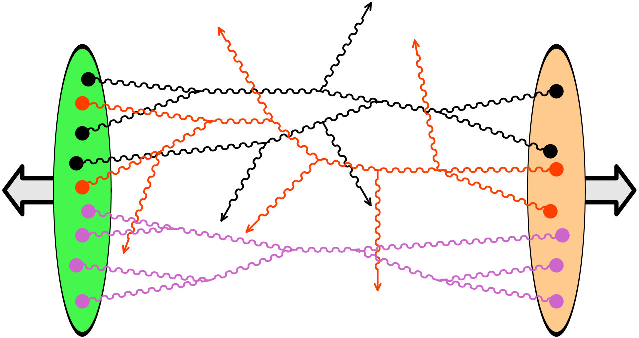






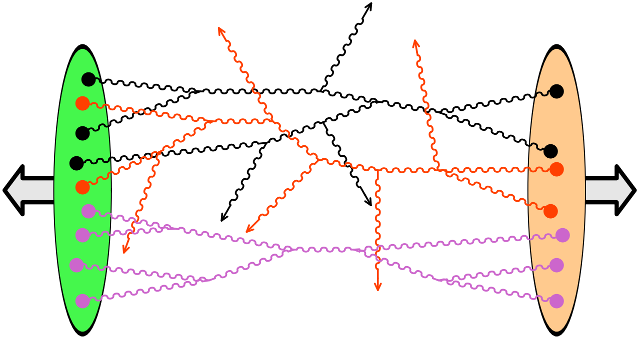
ATLAS Preliminary
2 < p$_T^a$,p$_T^b$ < 3 GeV
∫Ldt = 8 μb^{-1}
0-5%
a)
C(Δφ,Δη)
1.04
1.02
1
0
Δφ 2
4
4
2
0
-2
-4
Δη

Nucleus–nucleus collisions



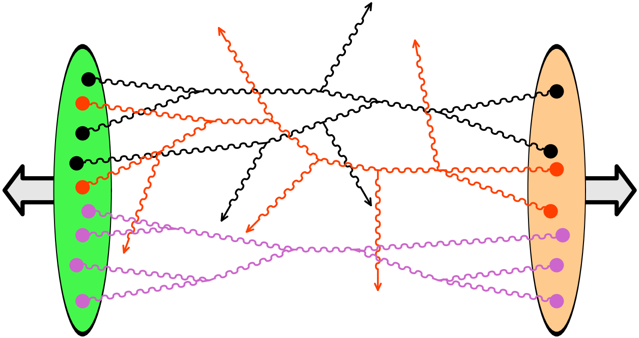
- How to compute particle production in AA collisions ?
- Very complicated : non–linear effects enter at all stages !
  - in both incoming wavefunctions: gluon saturation
  - in the scattering process : multiple interactions
  - in the partonic medium created by the scattering: final–state interactions

Nucleus–nucleus collisions



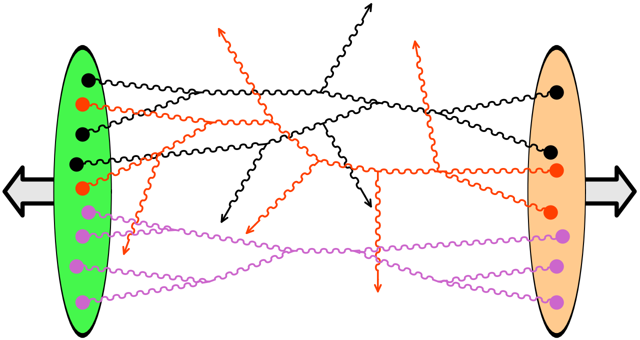
- How to compute particle production in AA collisions ?
- Very complicated : non–linear effects enter at all stages !
  - treat each of the incoming nucleus as a CGC
  - in the scattering process : multiple interactions
  - in the partonic medium created by the scattering: final–state interactions

Nucleus–nucleus collisions



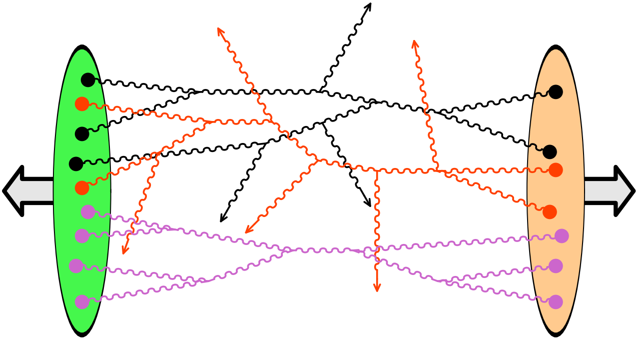
- How to compute particle production in AA collisions ?
- Very complicated : non–linear effects enter at all stages !
  - treat each of the incoming nucleus as a CGC
  - exactly solve the classical Yang–Mills equations with 2 sources
  - in the partonic medium created by the scattering: final–state interactions

Nucleus–nucleus collisions



- How to compute particle production in AA collisions ?
- Very complicated : non–linear effects enter at all stages !
  - treat each of the incoming nucleus as a CGC
  - exactly solve the classical Yang–Mills equations with 2 sources
  - use the above solution as an initial condition for the subsequent evolution of this partonic matter (e.g. for hydrodynamics)

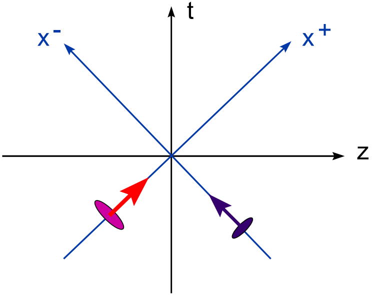
Nucleus–nucleus collisions



- The Color Glass Ccondensate is the right effective theory to describe the initial conditions for heavy ion collisions

Light–cone variables

- At high energy it is convenient to use light–cone variables



$$x^{\pm} = \frac{1}{\sqrt{2}}(t \pm z)$$

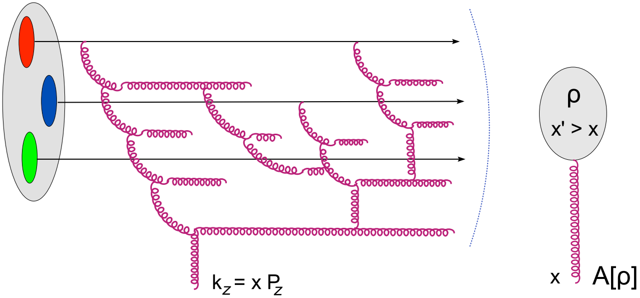
$$p^{\pm} = \frac{1}{\sqrt{2}}(p_0 \pm p_z)$$

$$p^{\mu} = (p^+, p^-, \boldsymbol{p}_\perp)$$

$$p \cdot x = p^+ x^- + p^- x^+ - \boldsymbol{p}_\perp \cdot \boldsymbol{x}_\perp$$

- Ultrarelativistic right mover :
  - $z \simeq t \implies x^- \simeq 0$ (Lorentz contraction) & $x^+ \simeq \sqrt{2}t$ (LC time)
  - $p_z \simeq p_0 \equiv E \implies p^\mu \simeq (p^+, 0, \mathbf{0}_\perp)$ with $p^+ = \sqrt{2}E$
- Left mover: the roles of x^+ and x^- (or p^+ and p^-) get interchanged

The CGC effective theory

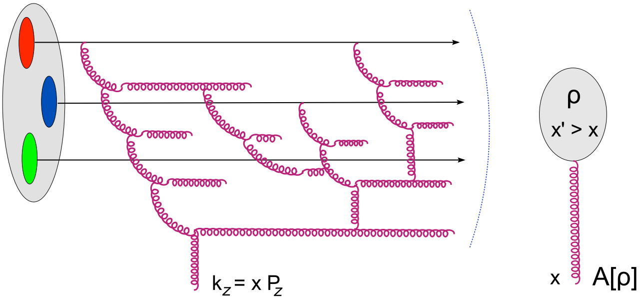


- An effective theory for the small–x gluons in the nuclear wavefunction
  - classical color fields A_a^μ radiated by randomly distributed color charges representing the 'fast' partons with $x' \gg x$
  - obtained by solving the classical Yang–Mills equations

$$D_\nu^{ab} F_b^{\nu\mu}(x) = J_a^\mu(x) \simeq \delta^{\mu +}\delta(x^-)\rho^a(\boldsymbol{x}_\perp)$$

  - large occupation numbers $n \sim 1/\alpha_s \Longleftrightarrow$ strong fields $A_a^i \sim 1/g$

The CGC effective theory



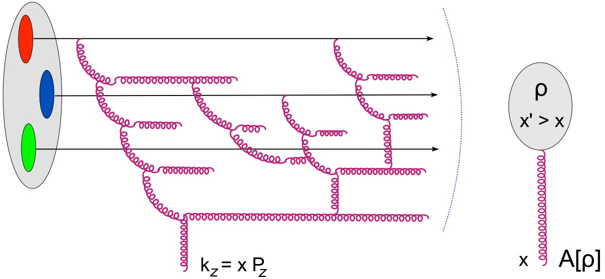
- $W_Y[\rho]$: functional probability distribution for the color charges
  - a kind of Master 'unintegrated gluon distribution'
  - information about all the n–point gluon correlations with $n \geq 2$

$$\langle \rho^a(\boldsymbol{x})\rho^b(\boldsymbol{y})\dots\rangle_Y = \int [\mathcal{D}\rho]\; W_Y[\rho]\; \rho^a(\boldsymbol{x})\rho^b(\boldsymbol{y})\dots$$

  - for uncorrelated color charges: a Gaussian in ρ (MV model)
  - obtained by integrating out the 'fast' gluons in layers of $Y = \ln 1/x$

Balitsky–JIMWLK equation

(Jalilian-Marian, Iancu, McLerran Weigert, Leonidov, Kovner; 1997–2000)



- JIMWLK : Functional evolution equation for $W_Y[\rho]$

$$\frac{\partial}{\partial Y} W_Y[\rho] \;=\; H\, W_Y[\rho] \qquad H = \alpha_s \frac{\delta}{\delta\rho}\,\chi[\rho]\,\frac{\delta}{\delta\rho}$$

  - initial condition: randomly distributed valence quarks (MV model)
  - equivalent to an infinite hierarchy of non–linear equations *(Balitsky, 96)*
  - exact numerical solutions available (2D–lattice)
  - recently extended to next-to-leading-logarithimic accuracy: $\alpha_s(\alpha_s Y)^n$

The CGC factorization for AA

- Numerically solve classical YM equations with 2 sources (2D lattice)

$$D_\nu F^{\nu\mu}(x) = \delta^{\mu+}\rho_1(x) + \delta^{\mu-}\rho_2(x)$$

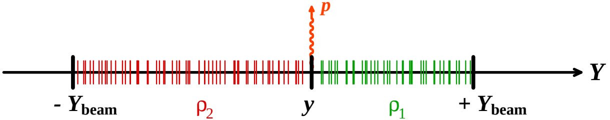
- Decompose the solution $A_a^\mu(x)$ in Fourier modes

  ▷ gluon spectrum 'event-by-event' (for given configurations of ρ_1 and ρ_2)

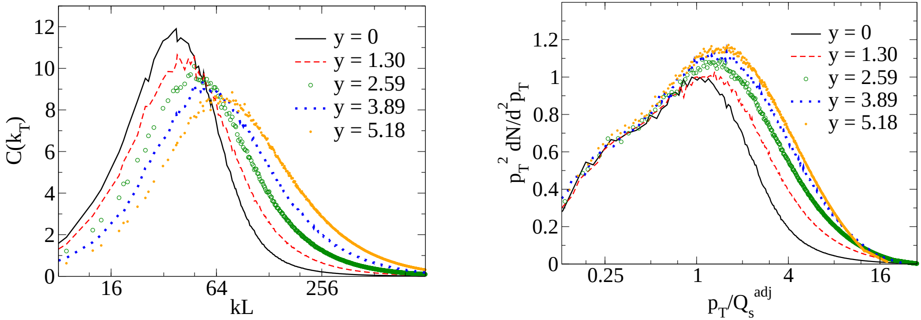
- Average over ρ_1 and ρ_2 using the CGC distributions of the 2 nuclei:

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}Y\,\mathrm{d}^2p_\perp} \right\rangle = \int [\mathcal{D}\rho_1 \mathcal{D}\rho_2]\, W_{Y_{\rm beam}-Y}[\rho_1]\, W_{Y_{\rm beam}+Y}[\rho_2]\, \frac{\mathrm{d}N}{\mathrm{d}Y\,\mathrm{d}^2p_\perp}\bigg|_{\rm class}$$

  ▷ JIMWLK evolution from $Y_{\rm beam}$ up to the rapidity Y of the produced gluon



Gluon spectrum from classical Yang–Mills



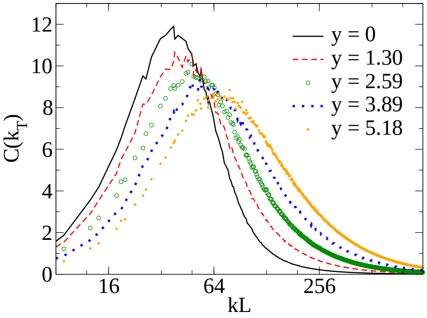
▷ *Numerical solutions to JIMWLK & CYM eqs. by T. Lappi (2011)*

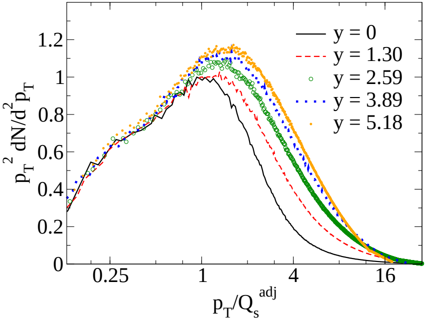
▷ *Left: unintegrated gluon distribution for different values of* $Y = \ln(1/x)$

▷ *Right: spectrum of gluons produced in* AA *for different energies* ($y \propto \ln E$)

- Particle production at high energy can be computed from QCD ☺

Gluon spectrum from classical Yang–Mills





▷ *Numerical solutions to JIMWLK & CYM eqs. by T. Lappi (2011)*

▷ *Left: unintegrated gluon distribution for different values of* $Y = \ln(1/x)$

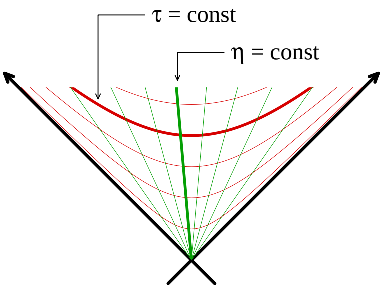
▷ *Right: spectrum of gluons produced in* AA *for different energies* ($y \propto \ln E$)

- Particle production at high energy can be computed from QCD ☺
- Hadron spectra can be modified by final state interactions ...
- ... but gross features and special correlations will survive !

Boost invariance & longitudinal expansion

- The classical field is invariant under a boost along the collision axis

  ▷ depends upon the proper time τ but not upon the space–time rapidity η_s



$$\tau \equiv \sqrt{t^2 - z^2} = \sqrt{2x^+x^-}$$

$$\eta_s \equiv \frac{1}{2}\ln\frac{t+z}{t-z} = \frac{1}{2}\ln\frac{x^+}{x^-}$$

- Under a boost with velocity v_0 :

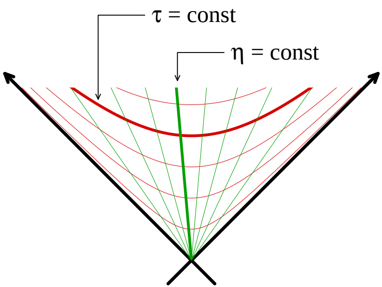
  τ is invariant

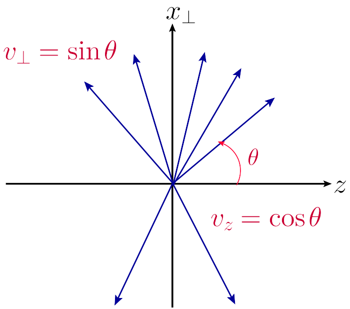
  $\eta_s \longrightarrow \eta_s + \beta$ with $\tanh\beta = v_0$

Boost invariance & longitudinal expansion

- The classical field is invariant under a boost along the collision axis

  ▷ depends upon the proper time τ but not upon the space–time rapidity η_s





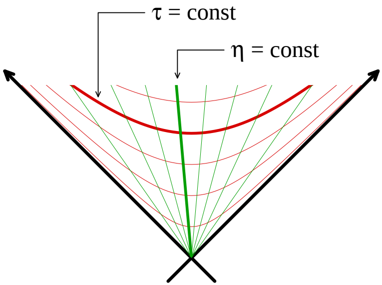
- Particle distribution $\mathrm{d}N/\mathrm{d}\eta$ is independent of η

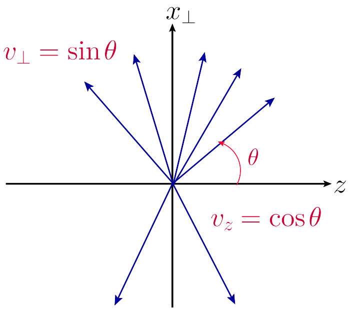
  ▷ particles move away from the interaction point at the speed of light

$$z \simeq v_z t \implies \eta_s \simeq \frac{1}{2}\ln\frac{1+v_z}{1-v_z} = -\ln\tan\frac{\theta}{2} = \eta$$

Boost invariance & longitudinal expansion

- The classical field is invariant under a boost along the collision axis
  - ▷ depends upon the proper time τ but not upon the space–time rapidity η_s

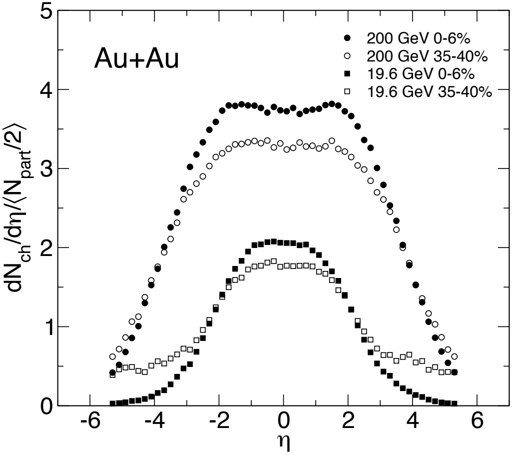




- Free streaming leading to longitudinal expansion *(Bjorken, 1983)*
  - ▷ particles separate from each other in the z direction
  - ▷ radial expansion remains negligible until $\tau \sim R_A$

Multiplicity : rapidity dependence

- RHIC (PHOBOS) data for $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$ as a function of η

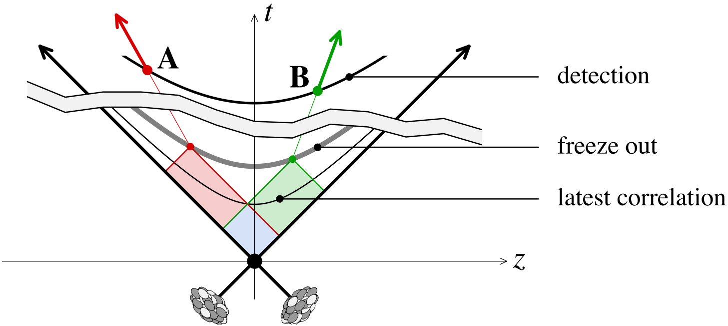


▷ flat in η around midrapidity : 'Feynman plateau'

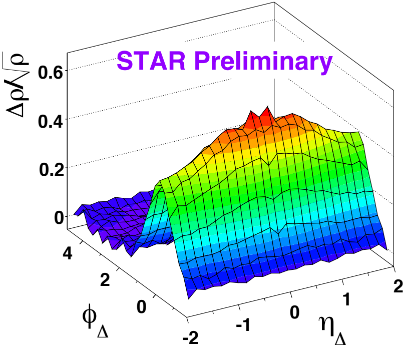
▷ for produced particles, $|\eta| \leq \eta_{\mathrm{beam}}$

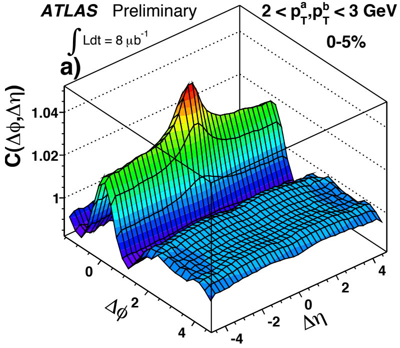
Long–range rapidity correlations probe early times

- Boost invariance leads to long–range correlations in rapidity
- Such correlations can be measured in the final state and traced back to the early stages
- Indeed, long–range correlations in rapidity are necessarily generated at early stages, where particles propagating along very different angles were still in causal contact with each other



- A natural explanation for the 'ridge' :
  - di–hadron correlations long–ranged in $\Delta\eta$ & narrow in $\Delta\phi$
  - abundantly observed in AA collisions at RHIC and the LHC

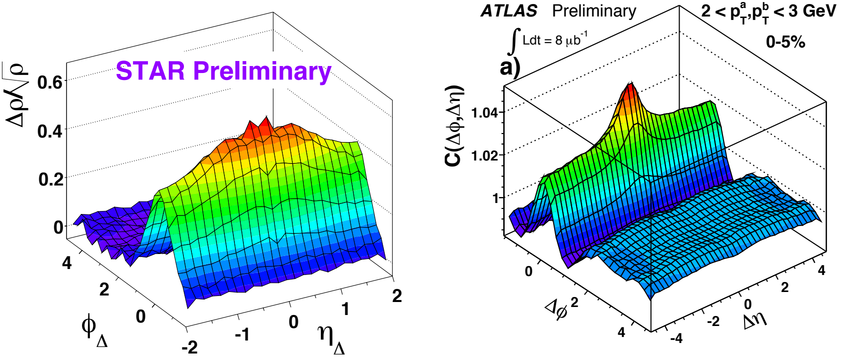




$$\mathcal{C}(\Delta\phi,\Delta\eta) \equiv \frac{\mathrm{d}N_{\rm pair}}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2} - \frac{\mathrm{d}N}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1}\,\frac{\mathrm{d}N}{\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2}$$

The Ridge in AA

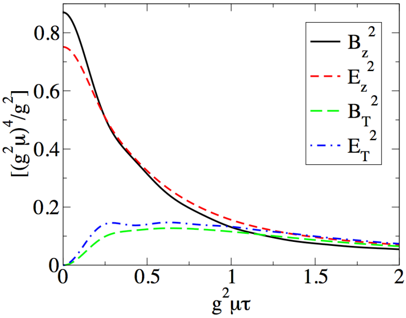
- A natural explanation for the 'ridge' :
  - long–range correlations in $\Delta\eta$: boost invariance at early times
  - collimation in $\Delta\phi$ can be explained by radial flow

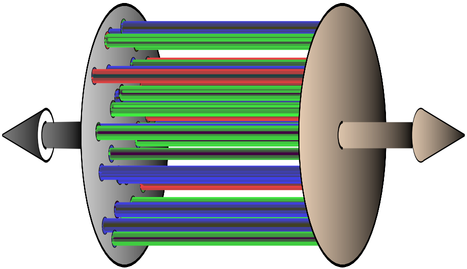


$$\mathcal{C}(\Delta\phi, \Delta\eta) \equiv \frac{\mathrm{d}N_{\rm pair}}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2} - \frac{\mathrm{d}N}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1}\,\frac{\mathrm{d}N}{\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2}$$

Glasma

- Right after the collision, the chromo-electric and chromo-magnetic fields are purely longitudinal
- Flux tubes which extend between the recessing nuclei
  'glasma' (from 'glass' + 'plasma') *(McLerran and Lappi, 06)*

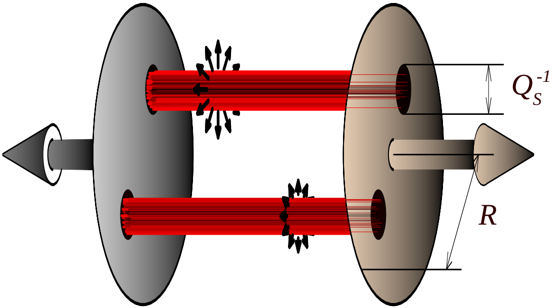




- At time $\tau \sim 1/Q_s$, the transverse fields are regenerated

From flux tubes to particles

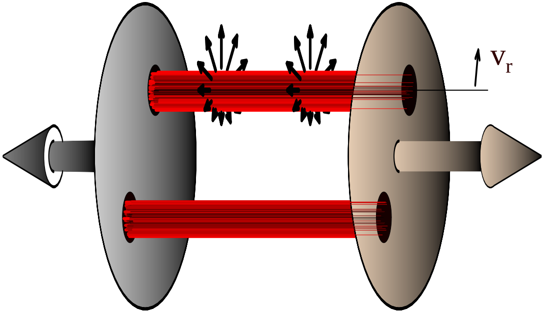
- At time $\tau \sim 1/Q_s$, the glasma flux tubes break into particles (gluons)
- Gluons emitted from the same flux tube are correlated with each other



- correlation length in the transverse plane: $\Delta r_\perp \sim 1/Q_s$
- correlation length in rapidity : $\Delta\eta \sim 1/\alpha_s$
- to start with, this correlation is isotropic in $\Delta\Phi$

From flux tubes to particles

- At time $\tau \sim 1/Q_s$, the glasma flux tubes break into particles (gluons)
- Gluons emitted from the same flux tube are correlated with each other

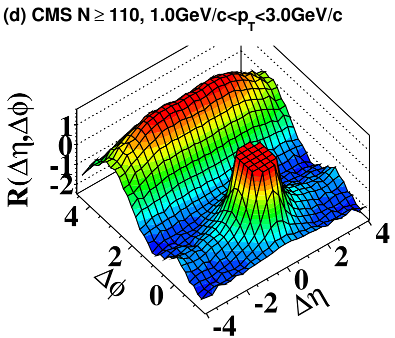


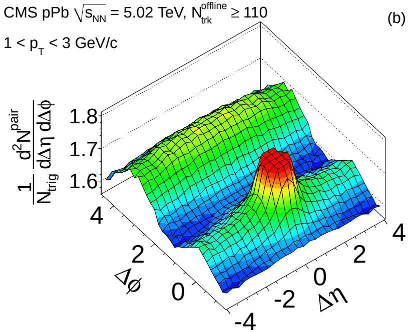
- correlation length in the transverse plane: $\Delta r_\perp \sim 1/Q_s$
- correlation length in rapidity : $\Delta\eta \sim 1/\alpha_s$
- in presence of radial flow, there is a bias leading to collimation in $\Delta\Phi$

  ▷ more particles along the radial velocity $\boldsymbol{v}_r$ than perpendicular to it

The Ridge in pp and pA

- LHC : quite surprisingly, a ridge is also observed in p+p and p+A events with unusually high multiplicity





- What is the origin of the azimuthal collimation ?
- Can flow develop in such small systems (~ 1 fm) ?
- This might reflect the momentum correlations at early times (glasma)

The thermalization puzzle

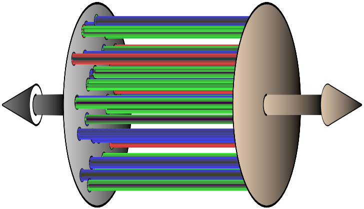
- Strong experimental evidence (RHIC, LHC) in favor of an intermediate phase of quark–gluon plasma in 'local thermal equilibrium'
  - the parton distribution is isotropic in momentum space and slowly varying in space and time; e.g.

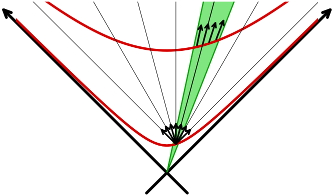
$$n(t, \mathbf{x}, \mathbf{p}) = \frac{1}{\mathrm{e}^{E_p/T} \mp 1} \quad \text{where} \quad T = T(t, \mathbf{x}) \quad \text{is slowly varying}$$

- Strongest evidence in that sense: the great success of nearly ideal hydrodynamics in describing collective phenomena like elliptic flow
  - requires small thermalization time: $\tau_0 \lesssim 1$ fm $\sim 10^{-23}$ secs
- This is very puzzling though
  - the early distribution is highly anisotropic ('glasma flux tubes')
  - to equilibrate, particles need to efficiently exchange 4–momentum
  - difficult to achieve for an expanding, weakly-coupled, system

The thermalization puzzle (2)

- Just after the collision, the partonic matter is highly anisotropic





  - the glasma flux tubes have 'negative longitudinal pressure' : they oppose to expansion (like a string of rubber)

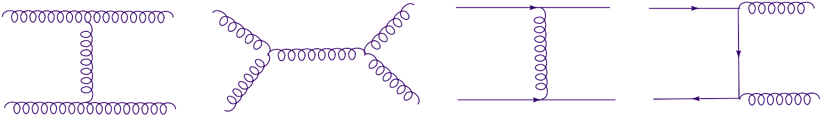
$$T^{\mu\nu}_{\rm eq} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon/3 & 0 & 0 \\ 0 & 0 & \varepsilon/3 & 0 \\ 0 & 0 & 0 & \varepsilon/3 \end{pmatrix} \qquad T^{\mu\nu}_{\rm initial} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & -\varepsilon \end{pmatrix}$$

  - in equilibrium: $P_T = P_L = \varepsilon/3$; in the early glasma: $P_T = \varepsilon = -P_L$

- The original anisotropy can be amplified by the longitudinal expansion

Thermalization in perturbation theory

- Particles can exchange energy and momentum through collisions.
- Weak coupling: the dominant mechanism is $2 \rightarrow 2$ elastic scattering



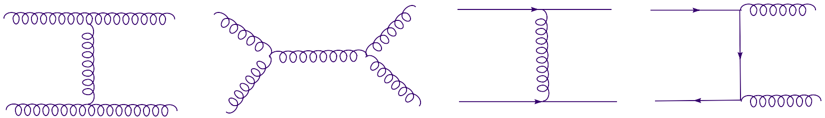
- Cross–section (σ) scales like $|\text{amplitude}|^2$, hence like $g^4 \sim \alpha_s^2$
- Mean free path (ℓ) = average distance between successive collisions

$$\ell \sim \frac{1}{\text{density} \times \sigma} \propto \frac{1}{\alpha_s^2}$$

- Typical equilibration time: $\tau_{\text{eq}} \sim \ell/v \propto 1/\alpha_s^2$
- Weakly coupled systems have large equilibration times ! ☹

Thermalization in perturbation theory

- Particles can exchange energy and momentum through collisions.
- Weak coupling: the dominant mechanism is $2 \to 2$ elastic scattering



- Cross–section (σ) scales like $|\text{amplitude}|^2$, hence like $g^4 \sim \alpha_s^2$
- Mean free path (ℓ) = average distance between successive collisions

$$\ell \sim \frac{1}{\text{density} \times \sigma} \propto \frac{1}{\alpha_s^2}$$

- Typical equilibration time: $\tau_{\rm eq} \sim \ell/v \propto 1/\alpha_s^2$
- A compelling argument in favor of strong coupling and AdS/CFT

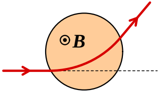
The role of the strong fields

- Heisenberg's uncertainty principle requires

$$\text{mean free path } \ell \gtrsim \text{ de Broglie wavelength } \lambda \sim \frac{1}{p}$$

- In general, weakly interacting systems have $\ell \gg \lambda$
  - weakly coupled QGP, temperature T : $\lambda \sim 1/T$ while $\ell \sim 1/[\alpha_s^2 T]$
- However, the situation can change for a particle interacting with a strong electric, or magnetic, field, as in the glasma
  - domain of size Q_s^{-1} where the (chromo) magnetic field is $|\boldsymbol{B}| \sim Q_s^2/g$

Lorentz force : $\dfrac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = g\boldsymbol{v} \times \boldsymbol{B} \implies \dot{\theta} \sim \dfrac{gB}{p} \sim Q_s$



  - time spent in the domain $\tau \sim Q_s^{-1} \implies \Delta\theta \sim \mathcal{O}(1)$
- Mean free path $\ell \sim Q_s^{-1} \sim 1/p$: as low as permitted by Heisenberg

The role of the strong fields

- Heisenberg's uncertainty principle requires

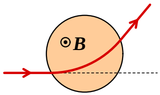
$$\text{mean free path } \ell \gtrsim \text{ de Broglie wavelength } \lambda \sim \frac{1}{p}$$

- In general, weakly interacting systems have $\ell \gg \lambda$
  - weakly coupled QGP, temperature T : $\lambda \sim 1/T$ while $\ell \sim 1/[\alpha_s^2 T]$

- However, the situation can change for a particle interacting with a strong electric, or magnetic, field, as in the glasma
  - domain of size Q_s^{-1} where the (chromo) magnetic field is $|\boldsymbol{B}| \sim Q_s^2/g$

Lorentz force : $\dfrac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = g\boldsymbol{v} \times \boldsymbol{B} \implies \dot{\theta} \sim \dfrac{gB}{p} \sim Q_s$

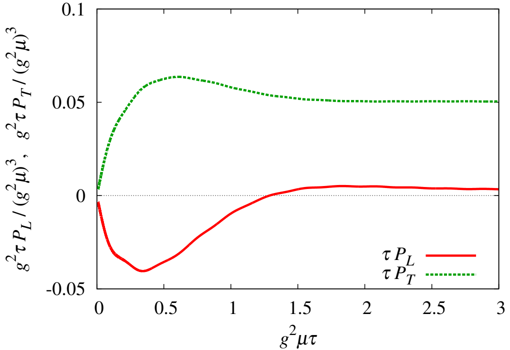
  - time spent in the domain $\tau \sim Q_s^{-1} \implies \Delta\theta \sim \mathcal{O}(1)$



- Short mean free path $\implies$ rapid thermalization !

(Epelbaum and Gelis, 2013)

- Numerical solution to classical Yang–Mills eq. confirms the anisotropy



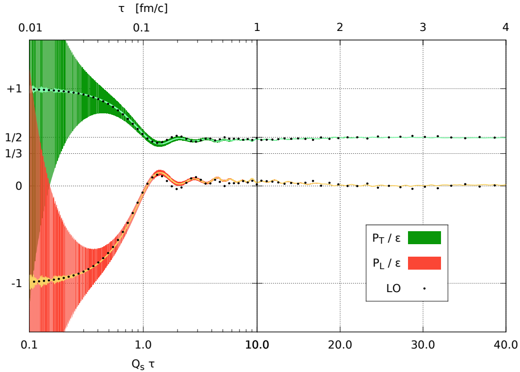
- the saturation momentum $Q_s = g^2\mu$ sets the scale
- $\tau\varepsilon = \tau(2P_T + P_L) \approx$ const. (longitudinal expansion)
- τP_L starts by being negative, then it becomes positive, but it remains much smaller than τP_T

Thermalization at weak coupling & strong fields

(Epelbaum and Gelis, 2013)

- However, this (boost–invariant) classical solution is unstable under (rapidity–dependent) quantum fluctuations.
- The fluctuations can be added to the initial conditions

$\alpha_s = 8\,10^{-4}$ $(g = 0.1)$



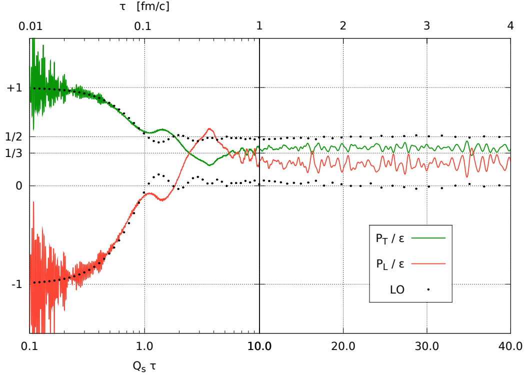
- for very small $g = 0.1$, the solution preserves boost invariance, as at LO

Thermalization at weak coupling & strong fields

(Epelbaum and Gelis, 2013)

- However, this (boost–invariant) classical solution is unstable under (rapidity–dependent) quantum fluctuations.
- The fluctuations can be added to the initial conditions

$\alpha_s = 2\,10^{-2}\ (g = 0.5)$

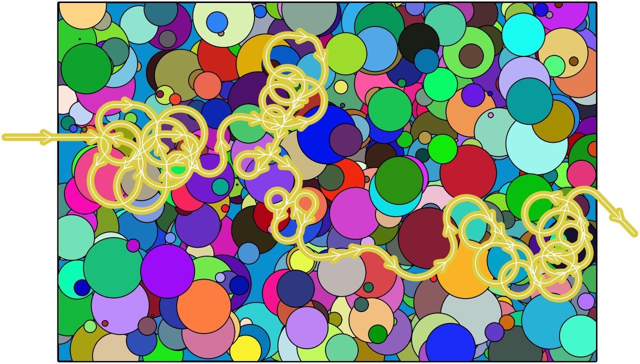


- for $g \gtrsim 0.5$, it approaches isotropy: $P_L/P_T \simeq 0.7$ ☺

Small η/s at weak coupling but strong fields

$$\frac{\text{viscosity}}{\text{entropy density}} \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}} \gtrsim \hbar$$

- Infinitely strong coupling (AdS/CFT) : $\eta/s = 1/4\pi$

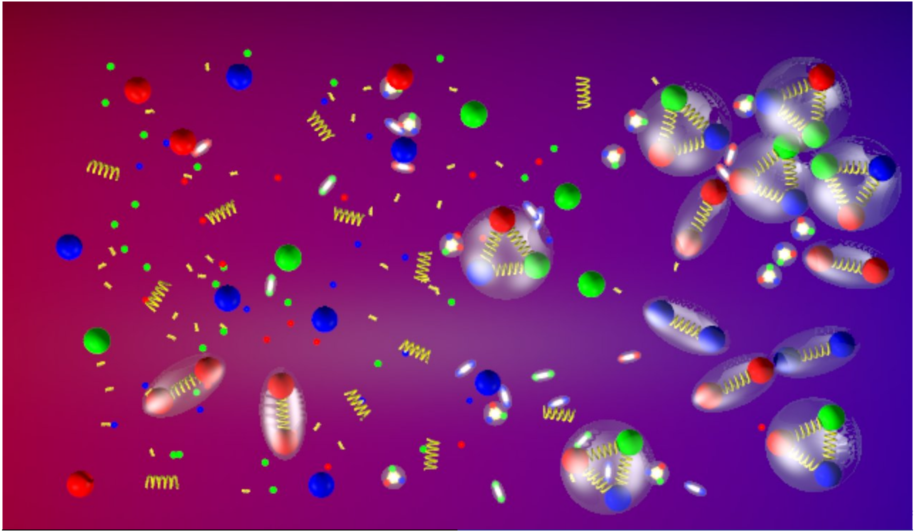


$$\frac{\eta}{s} \sim \frac{\ell}{\lambda} \sim \mathcal{O}(1)$$

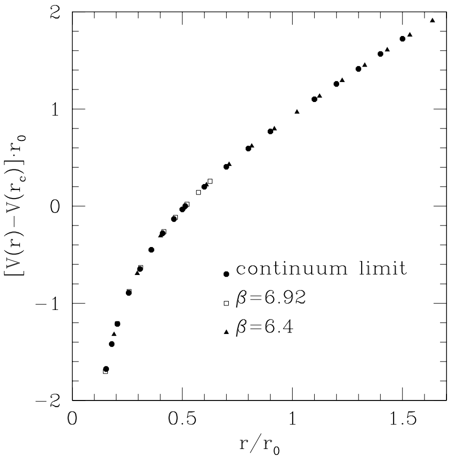
(in units of $\hbar$)

- Glasma : strong classical Yang–Mills fields at weak coupling

Quark–Gluon Plasma



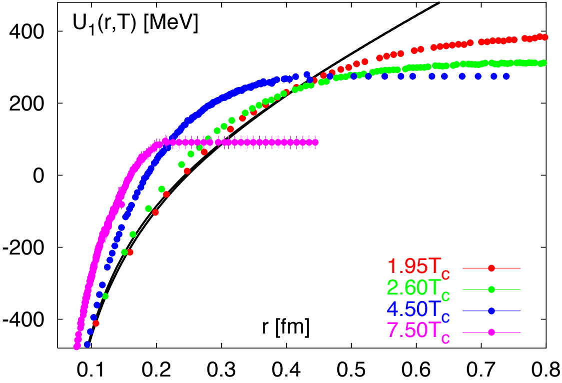
Confinement



- The quark–antiquark potential increases linearly with the distance.
- Quarks (and gluons) are confined into colorless hadrons

Quark–antiquark potential at finite T

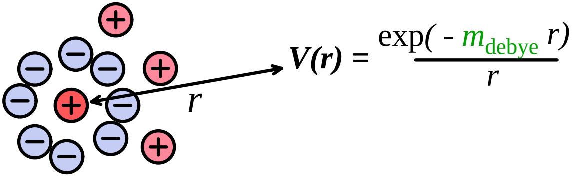
- With increasing the temperature T, the potential flattens out at shorter and shorter distances



- This leads to a 'phase transition' at some 'critical temperature' T_c : from Hadron Gas to a Quark–Gluon Plasma (QGP)

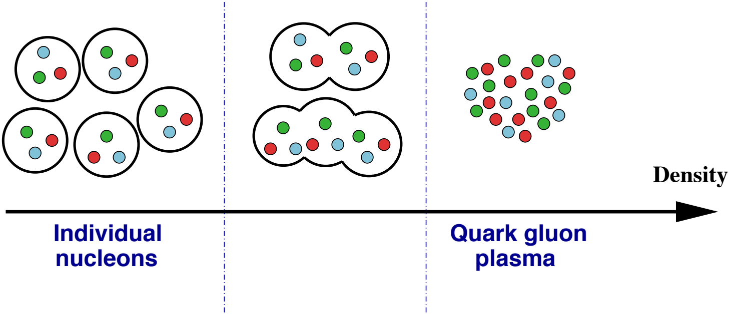
Debye screening

- QGP : a system of quarks and gluons which got free of confinement
- How is that possible ???



- In a dense medium, color charges are screened by their neighbors
- The interaction potential decreases exponentially beyond the Debye radius $R_{\rm Debye} = 1/m_{\rm Debye}$
- Hadrons whose sizes are larger than R_{Debye} cannot bind anymore

Deconfinement phase transition



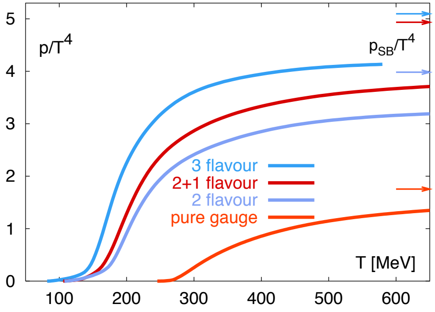
- When the nucleon density increases, they overlap, enabling quarks and gluons to hop freely from a nucleon to its neighbors

  ▷ R_{Debye} becomes smaller than the typical hadron radius $R_h \sim 1$ fm

- The hadrons melt into quarks and gluons

Quark–Gluon Plasma

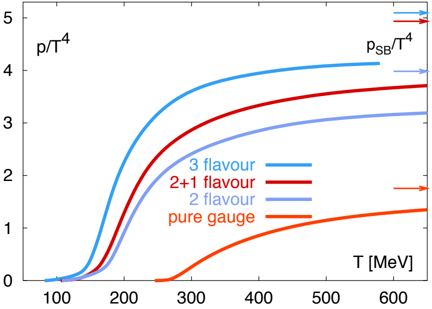
- Lattice calculations of the pressure in QCD at finite T



- Rapid increase of the pressure
  - around $T \simeq 270$ MeV with gluons only ('pure gauge')
  - around $T \simeq 150$ to 180 MeV with light quarks

Quark–Gluon Plasma

- Lattice calculations of the pressure in QCD at finite T

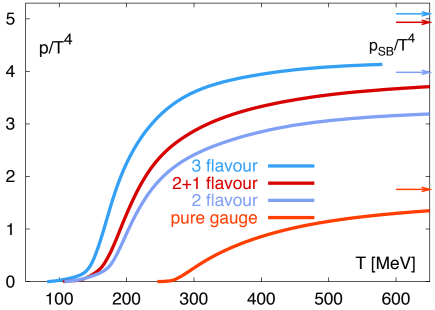


$$\varepsilon_\pi = d_\pi \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{E_p}{e^{E_p/T} - 1}$$

$$d_\pi = 3, \quad E_p = \sqrt{p^2 + m_\pi^2}$$

- The expected rise in the number of active degrees of freedom due to the liberation of quarks and gluons
  - at $T < T_c$: 3 light mesons $(\pi^0, \pi^\pm)$
  - at $T > T_c$: 52 d.o.f. (gluons: $8 \times 2 = 16$; quarks: $3 \times 3 \times 2 \times 2 = 36$)

- Lattice calculations of the pressure in QCD at finite T

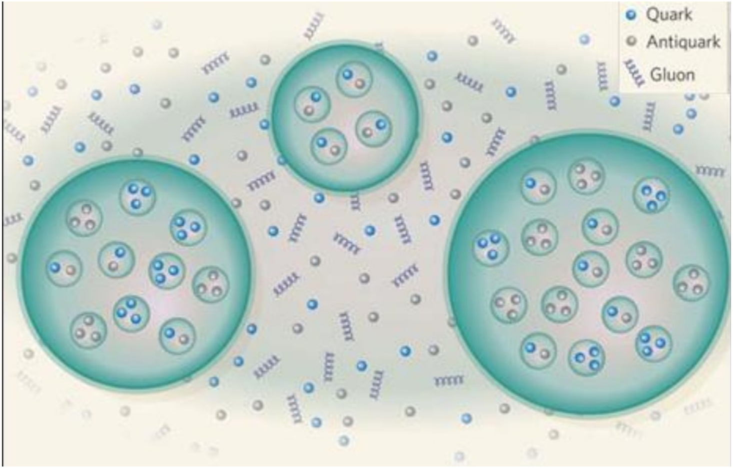


$$\varepsilon_g = d_g \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p}{e^{p/T} - 1}$$

$$\varepsilon_q = d_q \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{E_p}{e^{E_p/T} + 1}$$

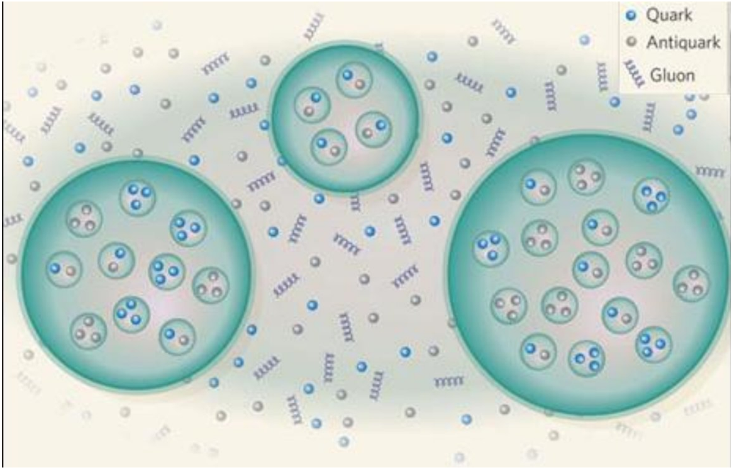
- The expected rise in the number of active degrees of freedom due to the liberation of quarks and gluons
  - at $T < T_c$: 3 light mesons $(\pi^0, \pi^\pm)$
  - at $T > T_c$: 52 d.o.f. (gluons: $8 \times 2 = 16$; quarks: $3 \times 3 \times 2 \times 2 = 36$)

Possible first–order scenario with critical bubbles



- If the transition was first–order, it would go through a mixed phase containing a mixture of hadronic and QGP phases

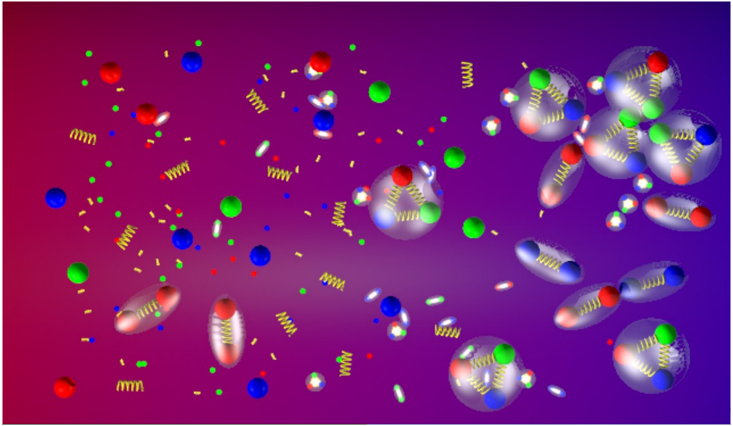
Possible first–order scenario with critical bubbles



- This would be the case if the 3 'active' quarks (u, s, d) were either massless or infinitely massive ('pure gauge')

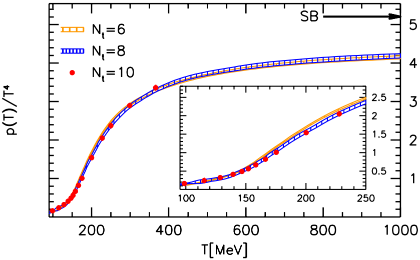
A cross–over

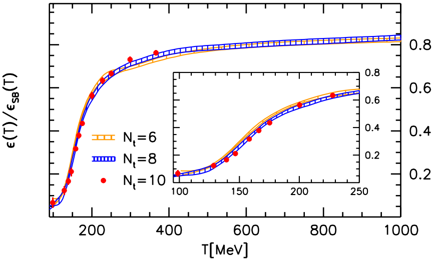
- This is not the case for the physical quark masses (2 light + 1 massive)



- The actual scenario is a 'cross–over' (no discontinuity)
  the Wuppertal–Budapest lattice group, Nature, 443 (2006) 675)

QCD thermodynamics: lattice





- With increasing temperature, the coupling $g(T)$ decreases, so the exact result approaches towards the Stefan–Boltzmann limit

$$P_{SB} = \frac{\pi^2}{90}\left\{2(N_c^2-1)+\frac{7}{2}N_cN_f\right\}T^4$$

- For $T \gtrsim 2.5T_c$, $P(T) - P_{SB}(T)$ is about 20%

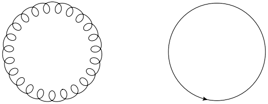
  ... is this small or large ?

- Can one understand this difference in perturbation theory ?

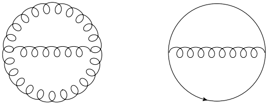
Perturbation theory for the pressure

$$P = \frac{T}{V} \ln \mathcal{Z}\,, \qquad \mathcal{Z} \equiv \sum_n \mathrm{e}^{-\beta E_n} = \mathrm{Tr}\ \mathrm{e}^{-\beta H}$$ (partition function)

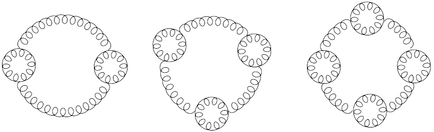
- Zero order $(g \to 0)$: one-loop graphs



- Order $g^2 \sim \alpha_s$: two-loop graphs

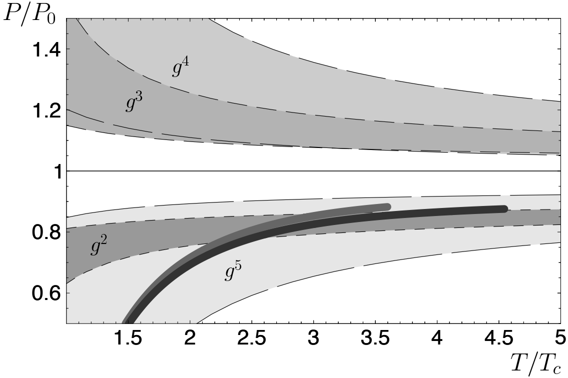


- Order $g^3 \sim \alpha_s^{3/2}$: ring diagrams

 ...

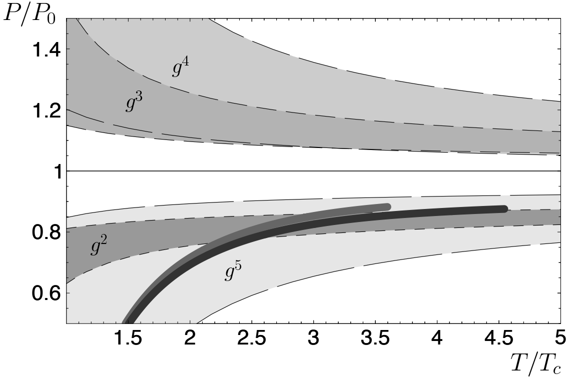
  - Infinitely many diagrams formally starting at $\mathcal{O}(g^4)$ but which contribute already at $\mathcal{O}(g^3)$: 'plasmon effect' (see below)

Perturbation theory shows no convergence

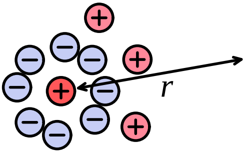


- By itself, the $\mathcal{O}(g^2)$ seems to do a pretty good job. However...
- Successive perturbative approximations — $\mathcal{O}(g^2)$, $\mathcal{O}(g^3)$, $\mathcal{O}(g^4)$, $\mathcal{O}(g^5)$ — jump up and down, without any sign of convergence.
- Increasingly larger renormalization scale uncertainties $(\mu \to 4\mu)$

Perturbation theory shows no convergence

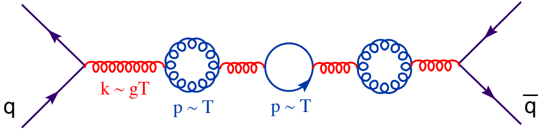


- Is this a non-perturbative effect inherent to QCD ?
  An indication of strong coupling ?
- A similar problem appears for any field theory at finite temperature, including weakly coupled QED, or scalar ϕ^4 theory !
- At finite T, perturbation theory gets complicated by medium effects



$$V(r) = \frac{\exp(-m_{\text{debye}}\, r)}{r}$$

- Thermal effect associated with dressing the propagator: $m_{\text{Debye}} \sim gT$

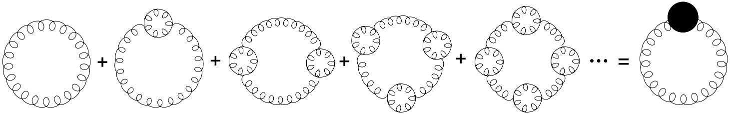


- The electric gluon acquires a mass which is 'non–perturbative' at 'soft' momenta $k \sim gT$:

$$G_{00}(k) = \underbrace{\frac{1}{k^2 + m_D^2}}_{\text{fine !}} = \underbrace{\frac{1}{k^2}\left[1 - \frac{m_D^2}{k^2} + \left(\frac{m_D^2}{k^2}\right)^2 \cdots\right]}_{\text{not fine !}}$$

Ring diagrams

- The sum of the ring diagrams reconstructs the dressed propagator :



- The Bose-Einstein thermal distribution is divergent as $k \to 0$

$$n_B(k) = \frac{1}{e^{k/T} - 1} \simeq \frac{T}{k} \sim \frac{1}{g} \qquad \text{when} \quad k \sim gT$$

▷ large occupation numbers for the soft thermal gluons

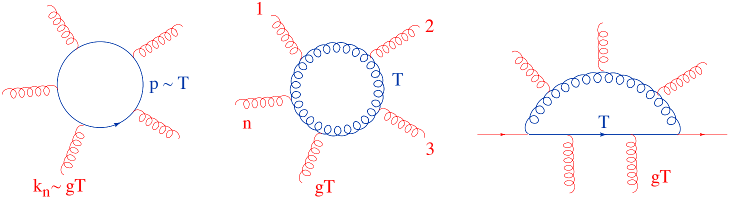
- This divergence is cut off by Debye screening at $k \sim gT$, but this results in an enhancement $\sim 1/g$
- The resummation of $m_D \Longrightarrow$ odd powers in g in perturbation theory
- An expansion in powers of g and not $\alpha_s \Longrightarrow$ lack of convergence !

$$\alpha_s = g^2/4\pi = 0.2 \div 0.3 \implies g \simeq 1.5 \div 2$$

Hard Thermal Loops

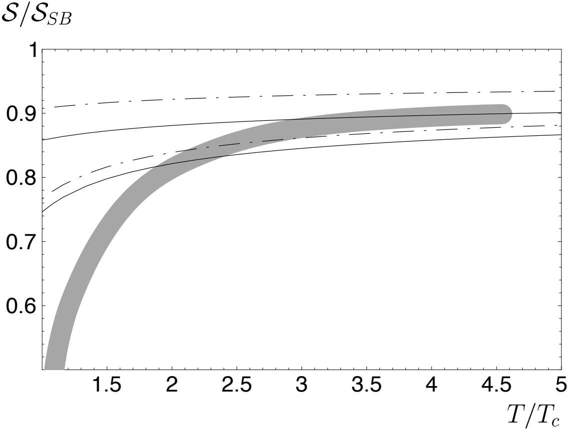
- In a gauge theory, gauge symmetry requires (via Ward identities) the generalization of the Debye mass to generic n–point amplitudes:

  'Hard Thermal Loops' *(Braaten and Pisarski, 1990; Blaizot, E. I., 1992)*



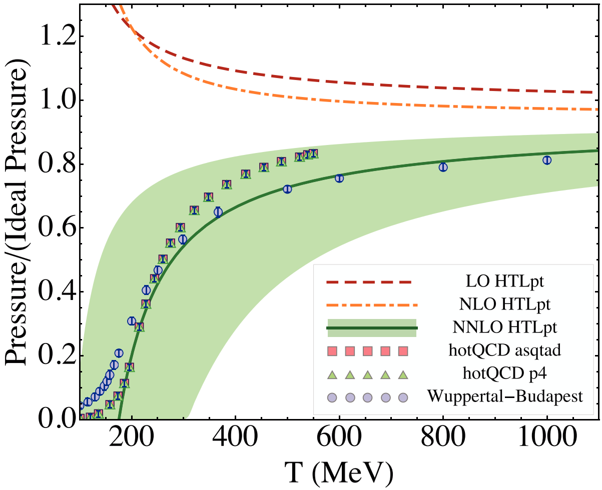
- HTL's : one loop diagrams with internal momenta $p \sim \mathcal{O}(T)$ ('hard') and external momenta $k_i \sim \mathcal{O}(gT)$ ('soft')
- Physical interpretation: collective phenomena in the QGP
- Genuinely leading order effects that must be resummed to all orders, via reorganizations of the perturbative expansion

HTL–resummed entropy



- '2-particle-irreducible' resummation (HTL–dressed propagators) *(J.-P. Blaizot, A. Rebhan, E. I., 2000)*
- Physical picture: weakly coupled quasiparticles.
- Good agreement with the lattice data (Bielefeld) for $T \gtrsim 2.5T_c$.

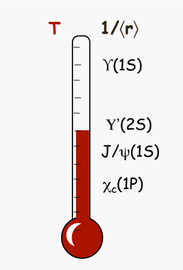
HTL–resummed pressure



- HTL–resummed perturbation theory to 3 loop order *(Andersen, Leganger, Strickland, Nan Su, 2011)*
- When properly organized, perturbation theory is remarkably successful

Quarkonia melting

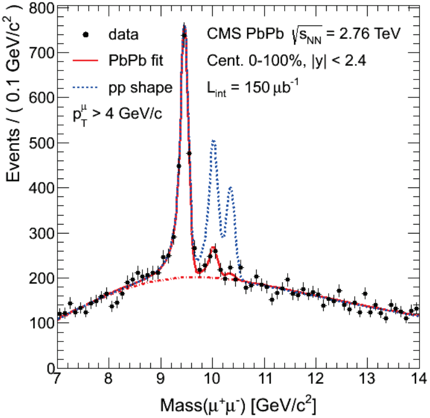
- Recall : a hadron whose size is larger than the Debye radius $R_D = 1/m_D$ cannot survive in the plasma
- Quarkonia : bound states of heavy quarks (charm c or bottom b)
  - ▷ small size $R \sim 1/m_Q \Longrightarrow$ can survive up to higher temperatures
- Two families (including excited states) :
  - ▷ $c\bar{c}$ (charmonium, $m_c = 1.3$ GeV) : $J/\psi(1S)$, $\psi(2S)$, $\chi_c(1P)$
  - ▷ $b\bar{b}$ (bottomium, $m_b = 4.2$ GeV) : $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$
- Sequential suppression :
  - ▷ excited states are larger and melt before the low energy ones
  - ▷ bottomium family melts after the charmonium one
- Quarkonia melting acts as a thermometer !



- The Υ family is better suited since less subjected to ambiguities

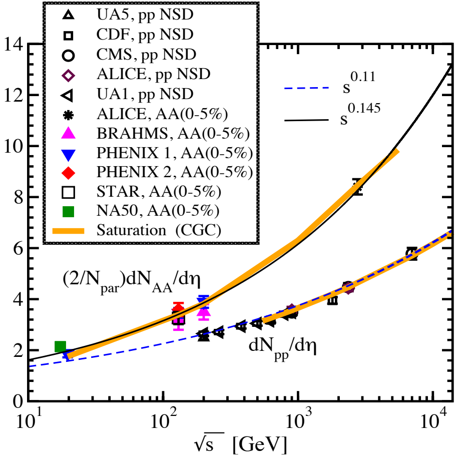
  ▷ no recombination since less $b\bar{b}$ pairs than $c\bar{c}$

  (at LHC : $\sim 100\, c\bar{c}$ pairs in central Pb–Pb collisions $\Longrightarrow$ recombination)

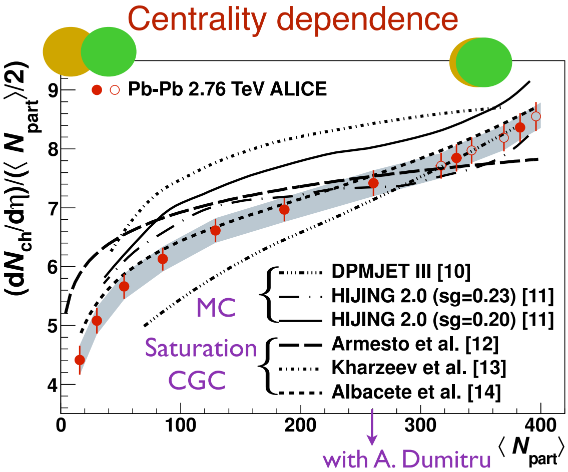


- Very clean successive suppression pattern for the Υ's

- Particle multiplicity $\mathrm{d}N/\mathrm{d}\eta \propto Q_s^2(A) \sim s^{\lambda_s/2}$

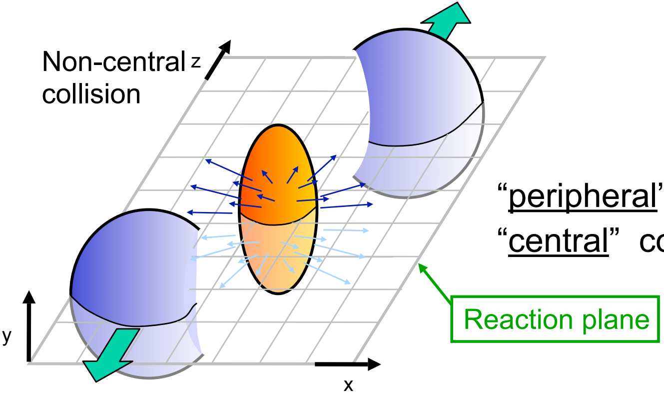


- Slight difference between energy growth in pp and AA
  (see Levin, Rezaeian, '11)



- Excellent fit by the CGC approach
- All the models include some form of saturation
  - ▷ HIJING : energy dependent low–p_T cutoff

The geometry of a HIC



"peripheral" collision ($b \sim b_{max}$)

"central" collision ($b \sim 0$)

Number of participants (N_{part}): number of incoming nucleons (participants) in the overlap region