From Colour Glass Condensate to Quark–Gluon Plasma

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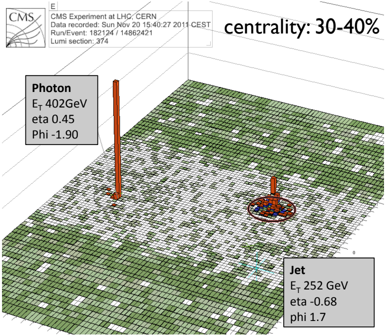
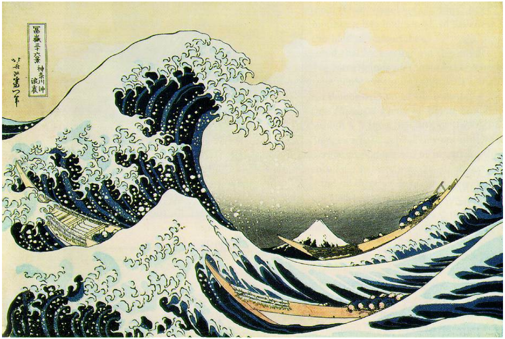
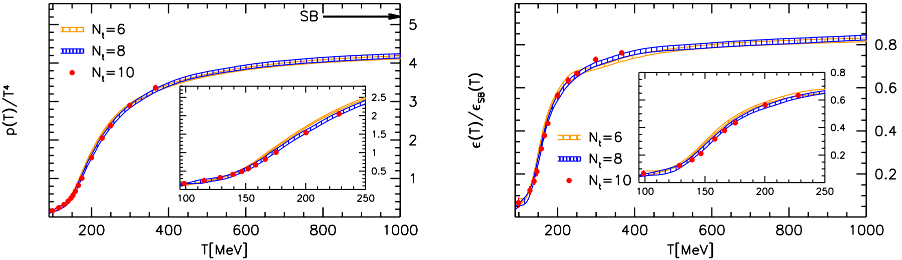




QCD thermodynamics: lattice



- With increasing temperature, the coupling $g(T)$ decreases, so the exact result approaches towards the Stefan–Boltzmann limit

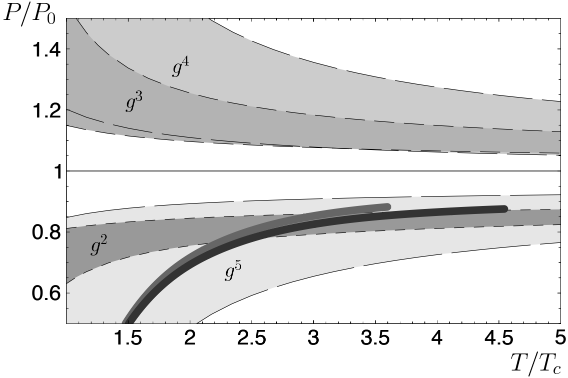
$$P_{SB} = \frac{\pi^2}{90}\left\{2(N_c^2-1) + \frac{7}{2}N_c N_f\right\}T^4$$

- For $T \gtrsim 2.5T_c$, $P(T) - P_{SB}(T)$ is about 20%

  ... is this small or large ?

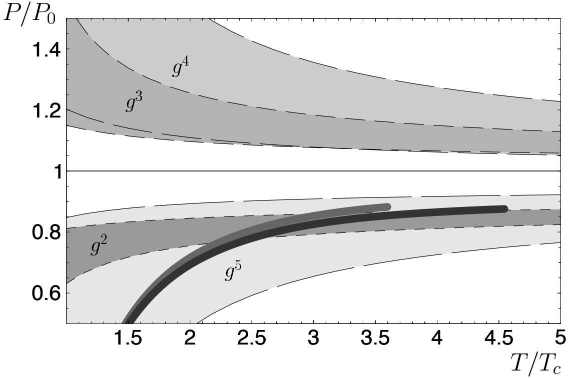
- Can one understand this difference in perturbation theory ?

Perturbation theory shows no convergence

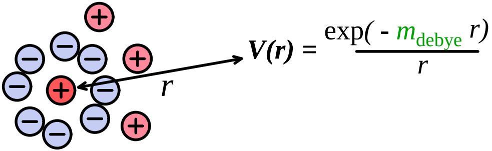


- By itself, the $\mathcal{O}(g^2)$ seems to do a pretty good job. However...
- Successive perturbative approximations — $\mathcal{O}(g^2)$, $\mathcal{O}(g^3)$, $\mathcal{O}(g^4)$, $\mathcal{O}(g^5)$ — jump up and down, without any sign of convergence.
- Increasingly larger renormalization scale uncertainties $(\mu \to 4\mu)$

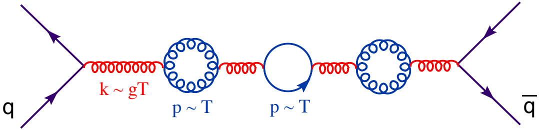
Perturbation theory shows no convergence



- Is this a non-perturbative effect inherent to QCD ?
  A signal of strong coupling ?
- A similar problem appears for any field theory at finite temperature, including QED and scalar ϕ^4 theory with small coupling
- At finite T, perturbation theory gets complicated by medium effects



- Thermal effect associated with dressing the propagator: $m_{\rm Debye} \sim gT$



- The Debye mass is not perturbative at soft momenta $k \sim gT$:

$$G_{00}(k) = \underbrace{\frac{1}{k^2 + m_{\rm D}^2}}_{\text{fine !}} = \underbrace{\frac{1}{k^2}\left[1 - \frac{m_{\rm D}^2}{k^2} + \left(\frac{m_{\rm D}^2}{k^2}\right)^2 \cdots\right]}_{\text{not fine !}}$$

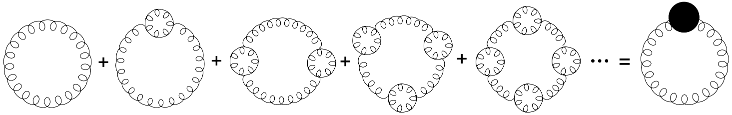
Ring diagrams

- The Bose-Einstein thermal distribution is divergent as $k \to 0$

$$n_B(k) = \frac{1}{e^{k/T} - 1} \simeq \frac{T}{k} \sim \frac{1}{g} \qquad \text{when} \quad k \sim gT$$

▷ large occupation numbers for the soft thermal gluons

- This divergence is cut off by Debye screening at $k \sim gT$, but this results in an enhancement $\sim 1/g$
- E.g.: the sum of the ring diagrams yields a term $\sim g^3 \sim \alpha_s^{3/2}$

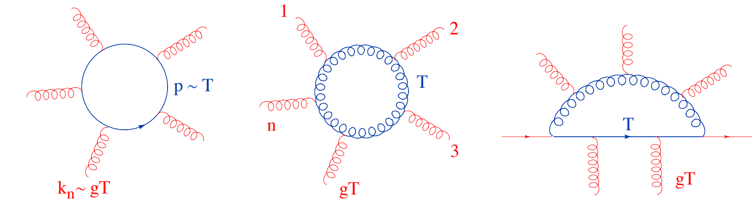


- The resummation of $m_D \Longrightarrow$ odd powers in g in perturbation theory
- An expansion in powers of g and not $\alpha_s \Longrightarrow$ lack of convergence !

$$\alpha_s = g^2/4\pi = 0.2 \div 0.3 \;\Longrightarrow\; g \simeq 1.5 \div 2$$

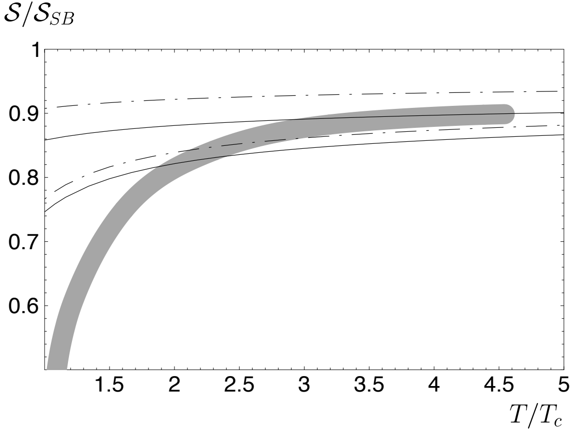
Hard Thermal Loops

- In a gauge theory, gauge symmetry requires (via Ward identities) the generalization of the Debye mass to generic n–point amplitudes:
  'Hard Thermal Loops' *(Braaten and Pisarski, 1990; Blaizot, E. I., 1992)*



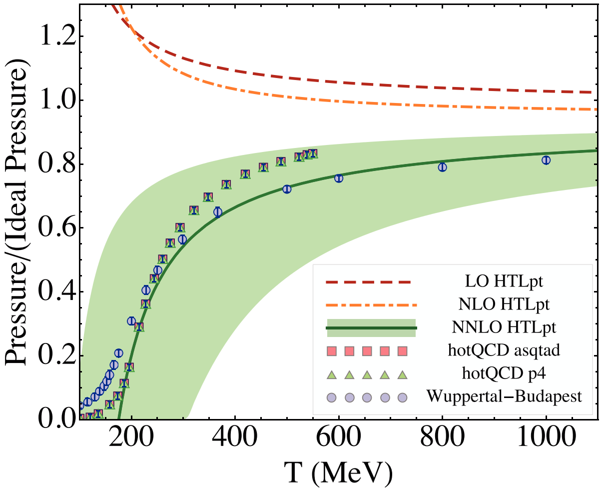
- HTL's : one loop diagrams with internal momenta $p \sim \mathcal{O}(T)$ ('hard') and external momenta $k_i \sim \mathcal{O}(gT)$ ('soft')
- Physical interpretation: collective phenomena in the QGP
- Genuinely leading order effects that must be resummed to all orders, via reorganizations of the perturbative expansion

HTL–resummed entropy



- '2-particle-irreducible' resummation (HTL–dressed propagators) *(J.-P. Blaizot, A. Rebhan, E. I., 2000)*
- Physical picture: weakly coupled quasiparticles.
- Good agreement with the lattice data (Bielefeld) for $T \gtrsim 2.5T_c$.

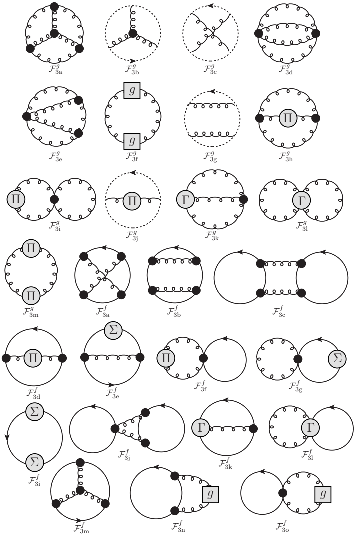
HTL–resummed pressure



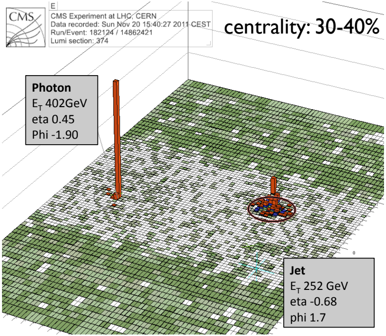
- HTL–resummed perturbation theory to 3 loop order
  (Andersen, Leganger, Strickland, Nan Su, 2011)
- When properly organized, perturbation theory is remarkably successful

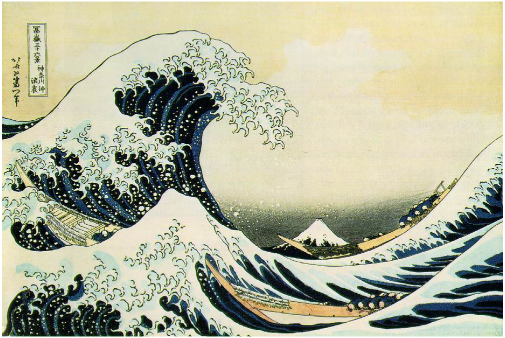
HTL–resummed pressure at 3 loop order

- Not an easy job though ! ☺



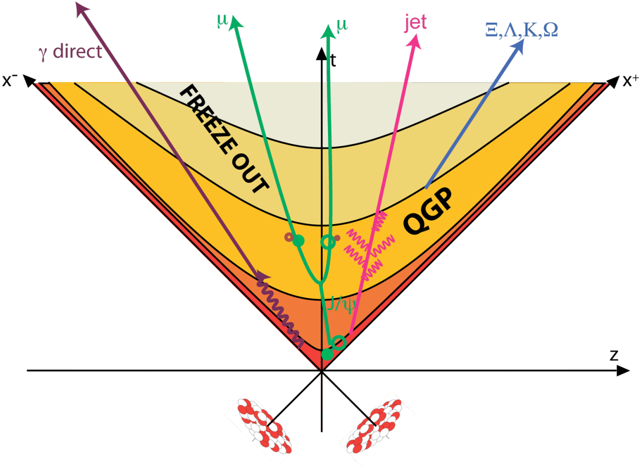
Di-jet asymmetry & wave turbulence





Hard probes

- How the probe the ephemeral QGP phase at intermediate stages ?

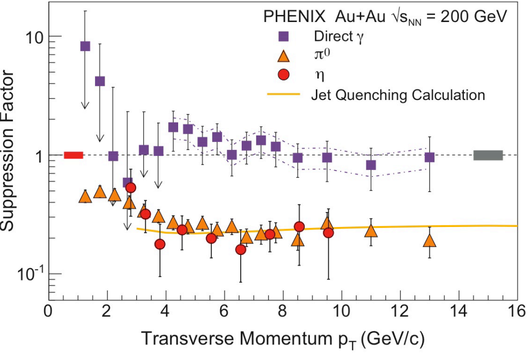


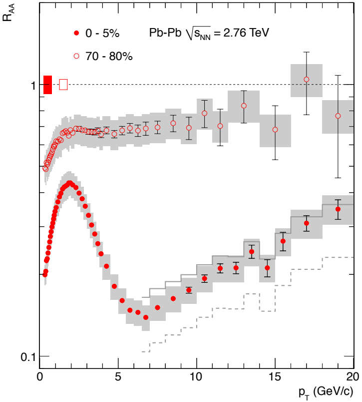
- Hard $(E \gg T)$ partons, photons, leptons created at early times
- Interact with the surrounding medium on their way to the detectors

Nuclear modification factor

- Use p+p collisions as a benchmark for particle production

$$R_{\mathrm{A+A}} \equiv \frac{1}{A^2}\,\frac{\mathrm{d}N_{\mathrm{A+A}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}{\mathrm{d}N_{\mathrm{p+p}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}$$





- No suppression for photons, small suppression in peripheral collisions

Nuclear modification factor

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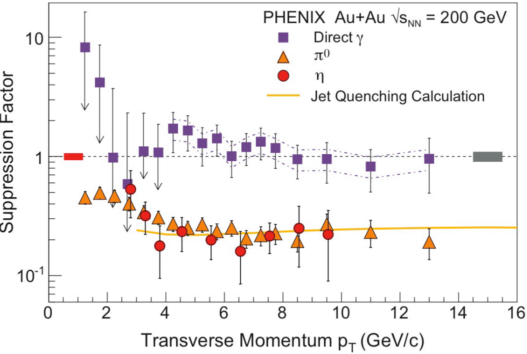
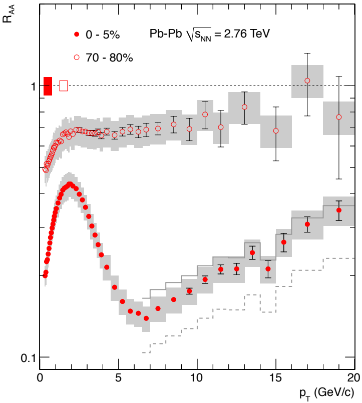


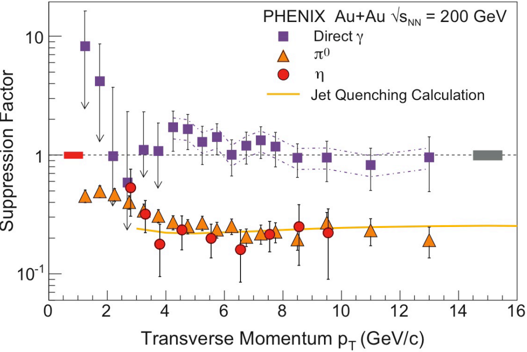


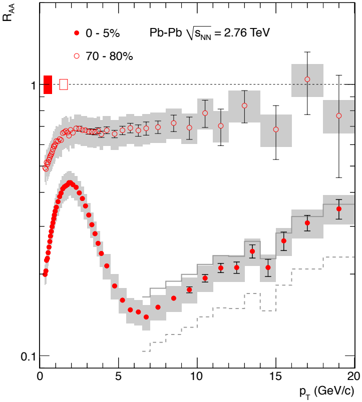
- Strong suppression $(R_{AA} \lesssim 0.2)$ in central collisions

Nuclear modification factor

- Use p+p collisions as a benchmark for particle production

$$R_{\mathrm{A+A}} \equiv \frac{1}{A^2}\,\frac{\mathrm{d}N_{\mathrm{A+A}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}{\mathrm{d}N_{\mathrm{p+p}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}$$

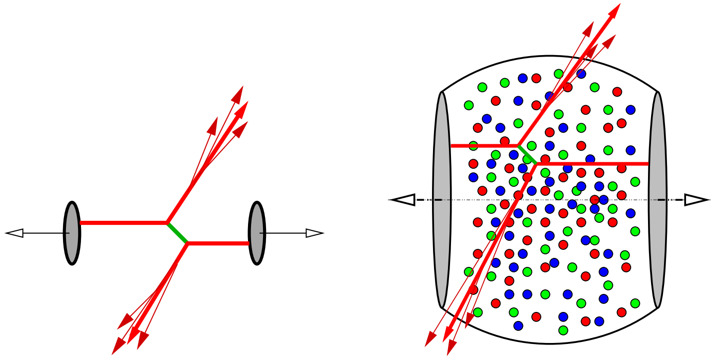




- Large energy loss via interactions in the medium

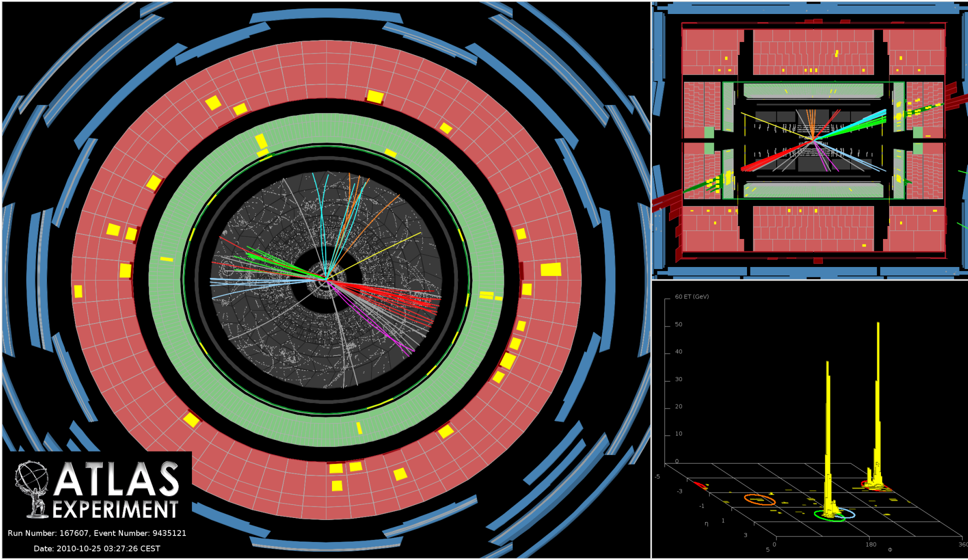
Jet quenching

- Hard partons are typically created in pairs which propagate back–to–back in the transverse plane



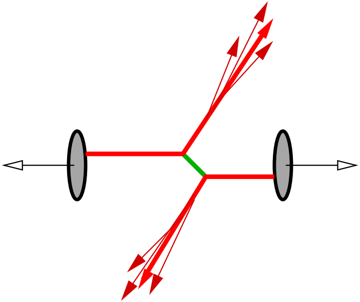
- 'Jet': 'leading particle' + 'products of fragmentation'
- AA collisions : jet propagation and fragmentation can be modified by the surrounding medium: 'jet quenching'

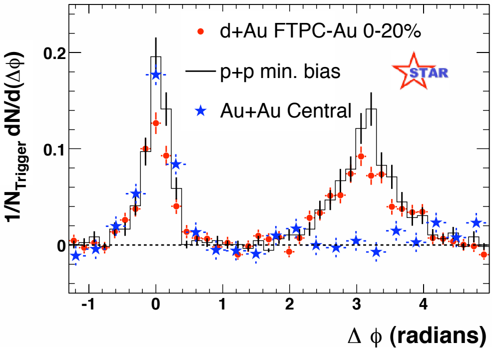
Di–jets in p+p collisions at the LHC



Di–hadron azimuthal correlations

- Distribution of pairs of particles w.r.t. the relative angle $\Delta\Phi$ in the transverse plane

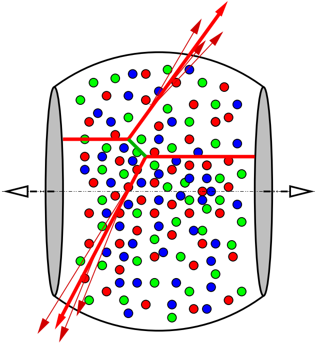


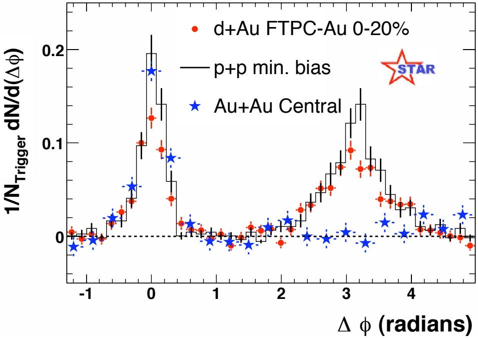


- Di–hadron azimuthal correlations at RHIC:
  - p+p or d+Au : a peak at $\Delta\Phi = \pi$ $\quad (\boldsymbol{p}_1 + \boldsymbol{p}_2 \simeq 0)$

Di–hadron azimuthal correlations

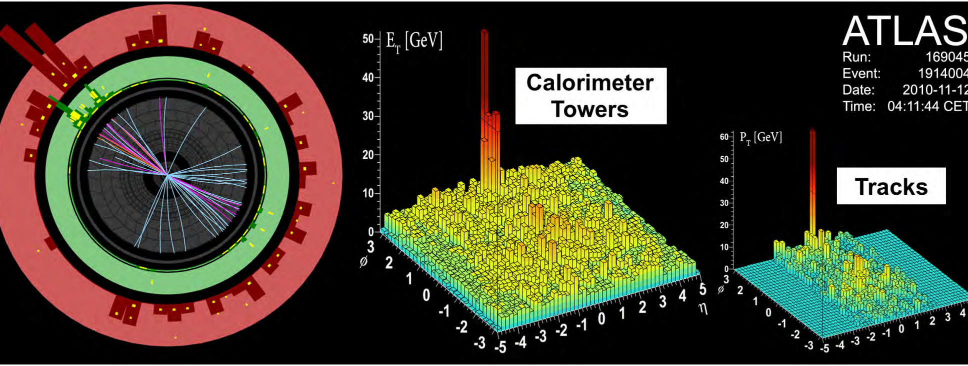
- Distribution of pairs of particles w.r.t. the relative angle $\Delta\Phi$ in the transverse plane





- Di–hadron azimuthal correlations at RHIC:
  - Au+Au : the away jet has disappeared !
- Collisions in the medium lead to transverse momentum broadening

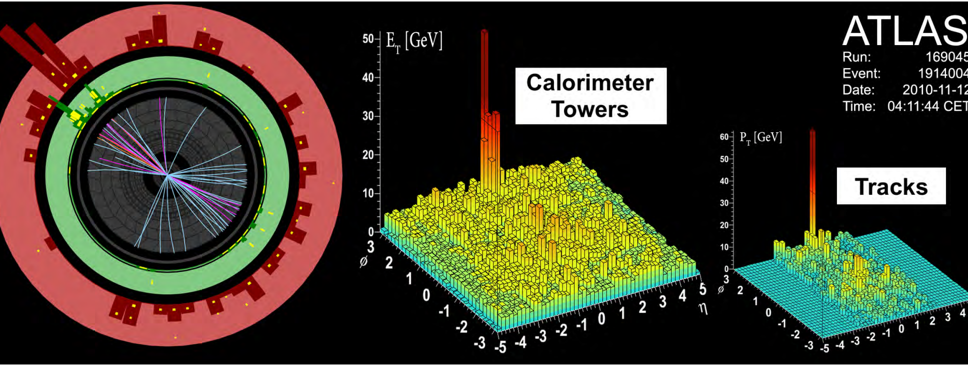
Di–jet asymmetry *(ATLAS)*



Question: How many jets do you see in the event ?

- none
- one
- it depends
- could you please remind me the definition of a jet ?

Di–jet asymmetry *(ATLAS)*



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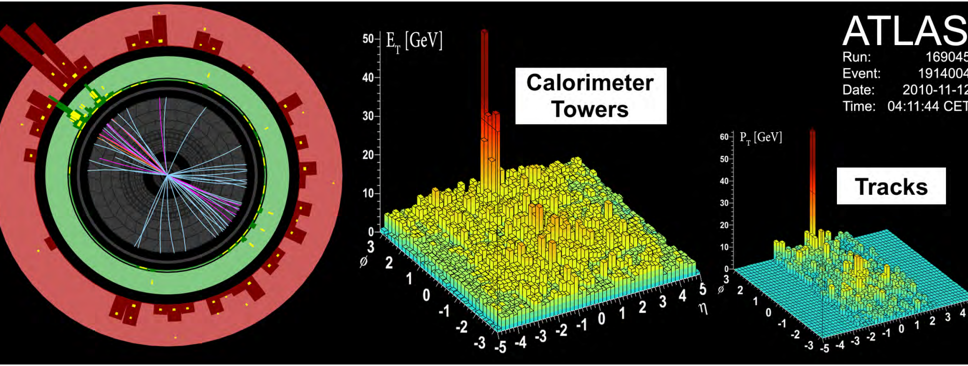
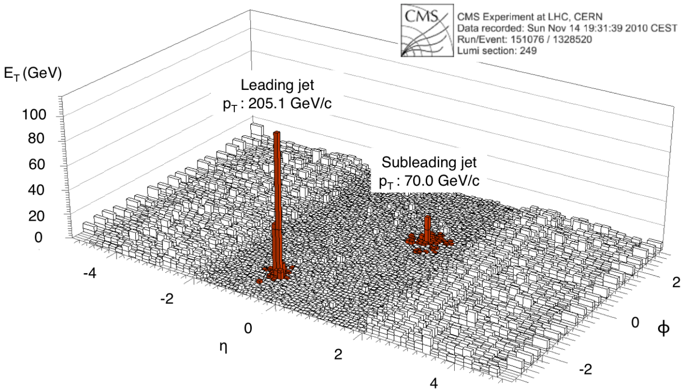


- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

Di–jet asymmetry *(CMS)*

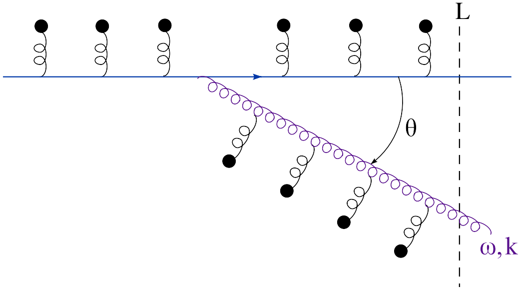


- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Detailed studies show that the 'missing energy' is carried by many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

  ▷ a surprising fragmentation pattern from the standard viewpoint of pQCD

Jet quenching in pQCD

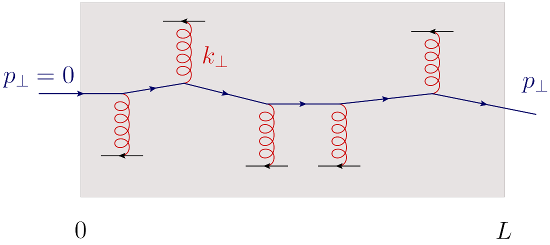
- Can one understand such phenomena from first principles ?
- In perturbative QCD, they all find a common denominator: incoherent multiple scattering off the medium constituents



  - random kicks provide transverse momentum broadening
  - medium induced radiation leading to large energy loss
  - large emission angles, especially for the softest emitted quanta
  - coherence phenomena leading to enhanced jet fragmentation

Transverse momentum broadening (1)

- An energetic quark (or gluon) acquires a transverse momentum $p_\perp$ via collisions in the medium, after propagating over a distance L



- Weakly coupled medium $\Rightarrow$ quasi independent scattering centers

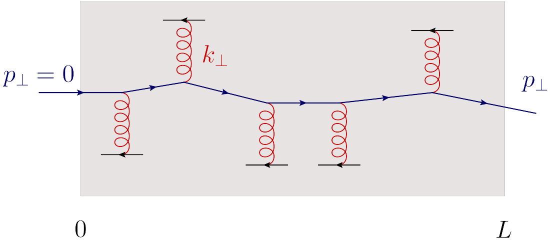
  ▷ successive collisions give random kicks

  ▷ Brownian motion in $p_\perp$: $\langle p_\perp^2 \rangle \simeq \hat{q}\, L$

- $\hat{q}$: the 'jet quenching parameter' (a medium transport coefficient)

  ▷ a fundamental quantity for what follows

Transverse momentum broadening (2)



- QGP at weak coupling : a simple estimate (kinetic theory)

$$\hat{q} \simeq \frac{\mu^2}{\ell}$$

  - average $(\text{momentum})^2$ transfer per scattering μ^2
  - parton mean free path $\ell \sim 1/n\sigma$
  - n : density of medium constituents; σ : elastic cross–section

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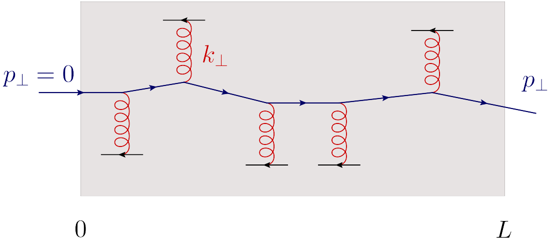

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$$\hat{q} \simeq \frac{\mu^2}{\ell} \sim \alpha_s^2 T^3 \sim 1\ \text{GeV}^2/\text{fm}$$

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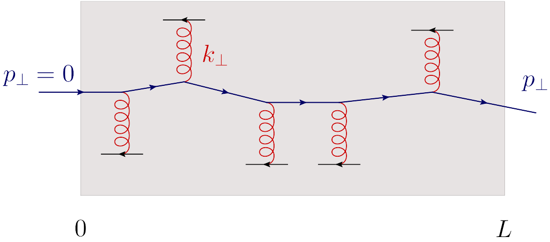

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  - average (momentum)^2 transfer per scattering $\mu^2 \sim T^2$
  - parton mean free path $\ell \sim 1/n\sigma \sim 1/\alpha_s^2 T$

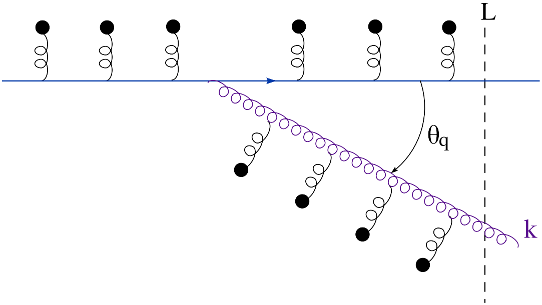
- The softer the particle, the stronger its deviation :

$$\theta(\omega) \simeq \frac{p_\perp}{\omega} \simeq \frac{\sqrt{\hat{q}L}}{\omega}$$

Medium–induced gluon radiation

Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov (BDMPS-Z) ∼ 1995

- Collisions with the plasma constituents provide acceleration (transverse momentum kicks) and thus allow for additional radiation



- Gluon missions can occur anywhere inside the medium (with size L)

  ... but they are not instantaneous : formation time τ_f

- The formation time determines the rate for emission

The formation time

- By the uncertainty principle, it takes some time to emit a gluon !

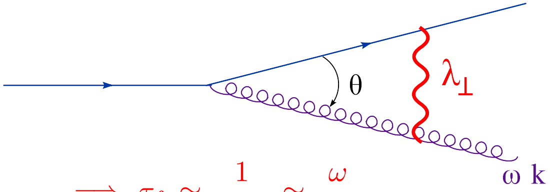
  ▷ the gluon must lose quantum coherence with respect to its source

- Gluon with energy ω and transverse momentum $k_\perp$:

  ▷ the quark–gluon transverse separation $b_\perp$ at the formation time τ_f must be larger than the gluon transverse wavelength $\lambda_\perp$

$b_\perp \simeq \theta \tau_f \gtrsim \lambda_\perp \simeq 1/k_\perp$

$k_\perp \simeq \omega \theta$



$$\Longrightarrow \tau_f \simeq \frac{1}{\theta k_\perp} \simeq \frac{\omega}{k_\perp^2}$$

- So far, the medium did not play any role

In–medium formation time

- The gluon acquires a (momentum)$^2 \sim \hat{q}$ per unit time ...
- ... and hence a momentum $k_f^2 \simeq \hat{q}\,\tau_f$ during its formation.

$$\tau_f \simeq \frac{\omega}{k_\perp^2} \quad \& \quad k_\perp^2 \simeq \hat{q}\tau_f \quad \Longrightarrow \quad \tau_f \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

- Maximal ω for this mechanism: $\tau_f \le L \Rightarrow \omega \le \omega_c \equiv \hat{q}L^2$
- Minimal emission angle: $\theta_c \equiv \theta(\omega_c) \sim 1/\sqrt{\hat{q}L^3}$
- Some typical values: $L \simeq 5$ fm, $\omega_c \simeq 50$ GeV, $\theta_c \simeq 0.1$
- The relatively soft gluons with $\omega \ll \omega_c$ have
    - small formation times: $\tau_f(\omega) \ll L$
    - large emission angles : $\theta(\omega) \gg \theta_c$

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- The relatively soft gluons with $\omega \ll \omega_c$ have
  - small formation times: $\tau_f(\omega) \ll L$
  - large emission angles : $\theta(\omega) \gg \theta_c$
- Potentially relevant for the di–jet asymmetry

Emission probability

- Spectrum : Bremsstrahlung × average number of emissions

$$\omega \frac{\mathrm{d}P}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega_c = \hat{q} L^2)$$

  - emission probability depends upon the energy ω of the emitted gluon
    LPM effect *(from Landau, Pomeranchuk, Migdal, within QED)*

- Energy loss by the 'leading particle' via a single emission :

$$\Delta E = \int^{\omega_c} \mathrm{d}\omega \, \omega \frac{\mathrm{d}P}{\mathrm{d}\omega} \sim \alpha_s \omega_c$$

  - integral dominated by its upper limit $\omega = \omega_c$ (hard emission)
  - rare event : probability of $\mathcal{O}(\alpha_s)$
  - small emission angle $\theta_c \Rightarrow$ the energy remains inside the jet
  - important for the nuclear modification factor R_{AA}

Soft emissions at large angles

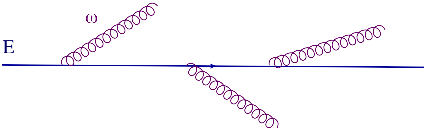
- Spectrum : Bremsstrahlung × average number of emissions

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega_c = \hat{q}L^2)$$

- Relatively soft emissions with $\omega \ll \omega_c$:
  - small formation times : $\tau_f \ll L$
  - quasi–deterministic : probability of $\mathcal{O}(1)$ for $\omega \lesssim \alpha_s^2 \omega_c$
  - a relatively smaller contribution to the energy loss : $\Delta E_s \sim \alpha_s^2 \omega_c$
  - ... but this can be lost at very large angles : $\theta \gtrsim \theta_c/\alpha_s^2 \sim 0.5$
- Control the energy loss at large angles $\Longrightarrow$ di–jet asymmetry ☺
- When probability of $\mathcal{O}(1) \Longrightarrow$ multiple branchings become important

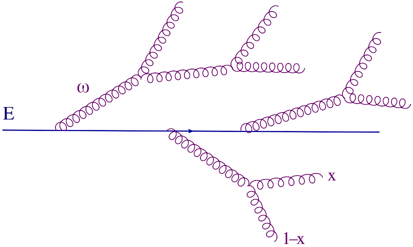
  Casalderrey-Solana, E. I. '11; Blaizot, Dominguez, E.I., Mehtar-Tani '12

- Multiple 'primary' emissions with $\omega \lesssim \alpha_s^2 \omega_c$ by the leading particle



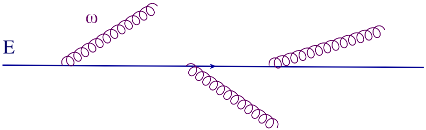
$$\omega \frac{\mathrm{d}P}{\mathrm{d}\omega} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

- Each primary gluon develops its own gluon cascade



Multiple branchings

- Multiple 'primary' emissions with $\omega \lesssim \alpha_s^2 \omega_c$ by the leading particle



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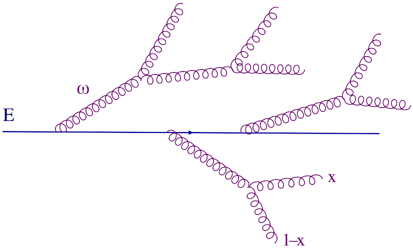

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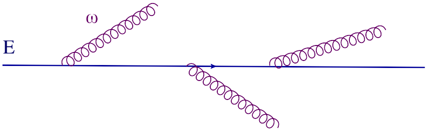

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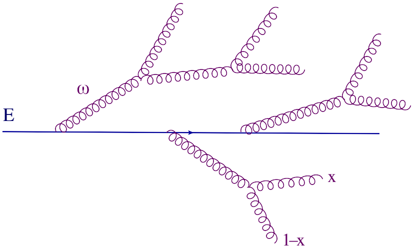
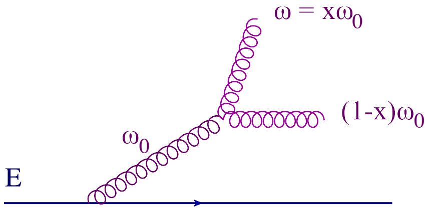


- Their subsequent branchings are quasi–democratic
  - the daughter gluons carry comparable energy fractions: $x \sim 1/2$

Quasi–democratic branchings

- Non–trivial ! Not true for bremsstrahlung in the vacuum !
- Bremsstrahlung in the vacuum : splittings are strongly asymmetric



$$\mathrm{d}P \sim \alpha_s \frac{\mathrm{d}\omega}{\omega} \sim \alpha_s \frac{\mathrm{d}x}{x}$$

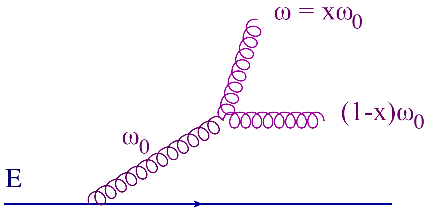
$$\Delta P \sim \alpha_s \int \frac{\mathrm{d}x}{x} \sim \alpha_s \ln \frac{1}{x}$$

- probability of $\mathcal{O}(1)$ when $\alpha_s \ln(1/x) \sim 1 \Longrightarrow$ favors $x \ll 1$
- argument independent of the parent energy ω_0

  ▷ all that matters is the splitting fraction x
- 'soft singularity' $(x \to 0)$ of bremsstrahlung

Quasi–democratic branchings

- In–medium radiation : a consequence of the LPM effect



$$\mathrm{d}P \sim \alpha_s \frac{\mathrm{d}\omega}{\omega}\sqrt{\frac{\omega_c}{\omega}}$$

$$\sim \alpha_s \frac{\mathrm{d}x}{x}\sqrt{\frac{\omega_c}{x\omega_0}}$$

- the rate also depends upon the parent gluon energy ω_0
- probability of $\mathcal{O}(1)$ when $\omega_0 \sim \alpha_s^2\omega_c$ for any value of x
- the phase space favors generic values of x: 'quasi–democratic'

Quasi–democratic branchings

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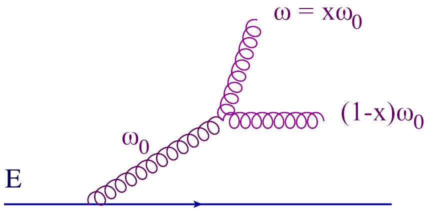

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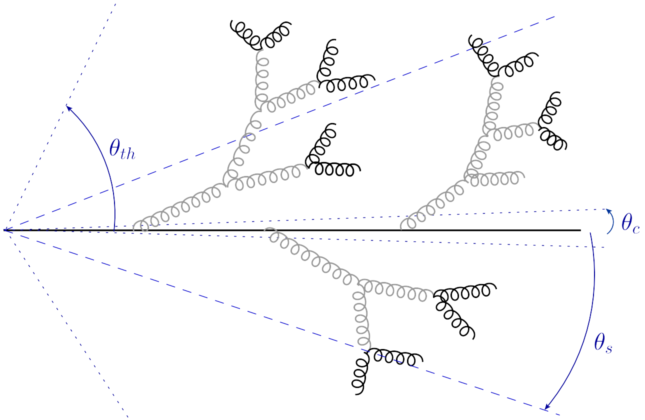
  - the rate also depends upon the parent gluon energy ω_0
  - probability of $\mathcal{O}(1)$ when $\omega_0 \sim \alpha_s^2\omega_c$ for any value of x
  - the phase space favors generic values of x: 'quasi–democratic'
- A similar scenario at strong coupling *(Y. Hatta, E.I., Al Mueller '08)*
- ... but no other known example in a weakly coupled gauge theory

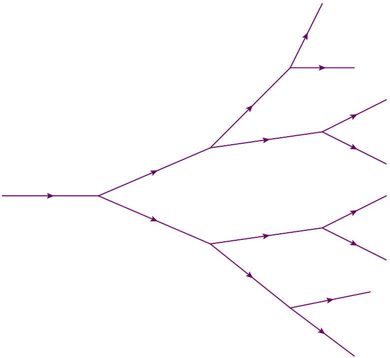
A typical gluon cascade



- The leading particle emits mostly soft gluons: $\omega \lesssim \alpha^2 \omega_c$
- These primary gluons rapidly split into even softer ones.
- The primary gluons propagate along typical angles $\theta_s \simeq \theta_c/\alpha^2 \sim 0.5$
- The final gluons $(\omega \sim T)$ make even larger angles $\theta_{\rm th} > \theta_s \gtrsim 1$

Wave turbulence

- Democratic branchings lead to turbulent energy flow
  (Richardson, '24; Kolmogorov, '41; Zakharov, '92)
- The energy flows from large x to small x without accumulating at any intermediate value of x



- the cascade stops when $\omega \sim T$
- gluons with $\omega \sim T$ 'thermalize'
  (lose their energy towards the medium)
- since very soft, such gluons propagate at very large angles w.r.t. jet axis

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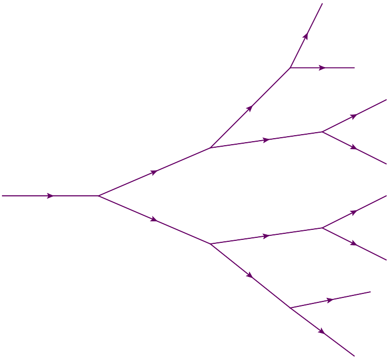
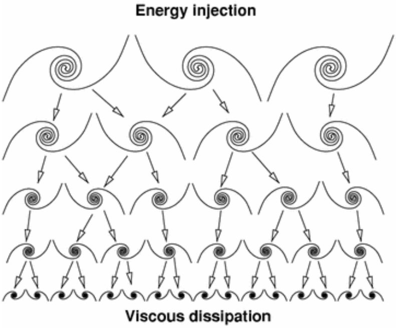
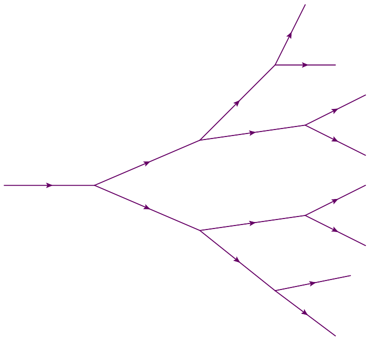


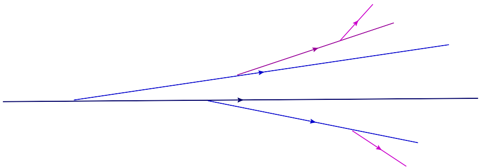


- The prototype: Richardson cascade for breaking-up vortices

Compare to DGLAP cascade (jet in the vacuum)



in–medium cascade

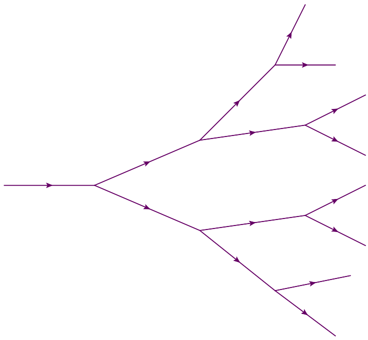


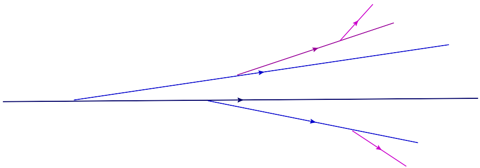
DGLAP cascade

$\tau = \ln Q^2$ ('virtuality')

- The asymmetric splittings amplify the number of gluons at small x
- Yet, the energy remains in the few partons with larger values of x
- In the DGLAP cascade, the energy remains at small angles

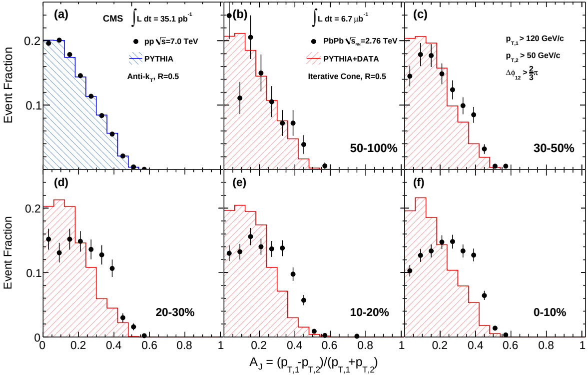
Compare to DGLAP cascade (jet in the vacuum)





- The asymmetric splittings amplify the number of gluons at small x
- Yet, the energy remains in the few partons with larger values of x
- Di-jet asymmetry demonstrates a turbulent cascade

  J.-P. Blaizot, Iancu, Y. Mehtar-Tani (2013); Fister, Iancu (2014)

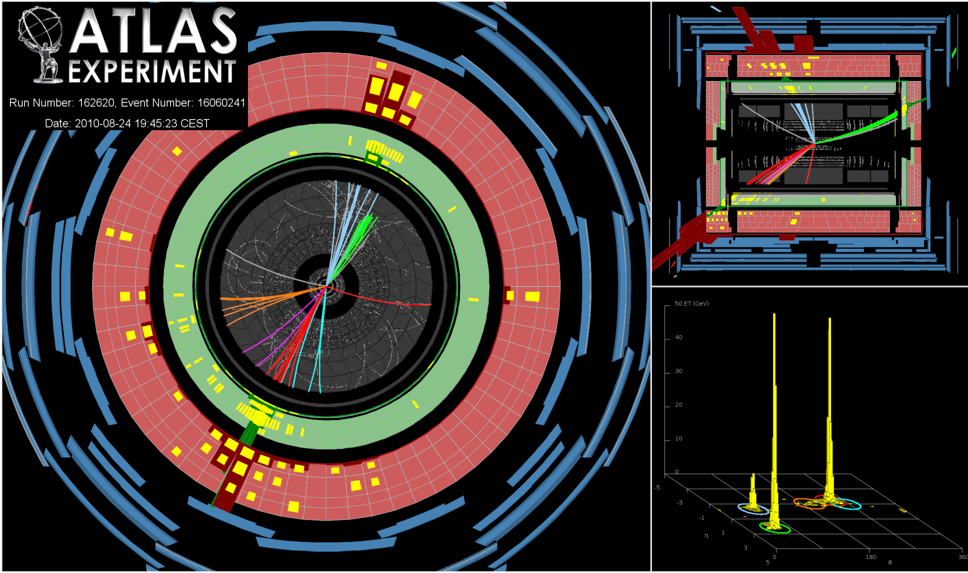


- Event fraction as a function of the di–jet energy imbalance in p+p (a) and Pb+Pb (b–f) collisions for different bins of centrality

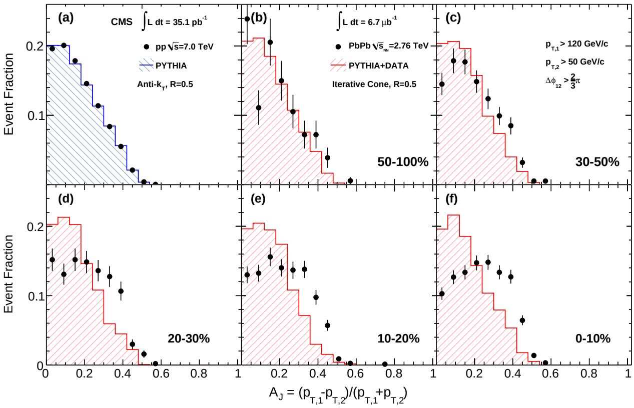
$$A_J = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ jet energies})$$

- N.B. A pronounced asymmetry already in the the p+p collisions !

Tri–jets in p+p collisions at the LHC



Di–jet asymmetry : A_J (CMS)

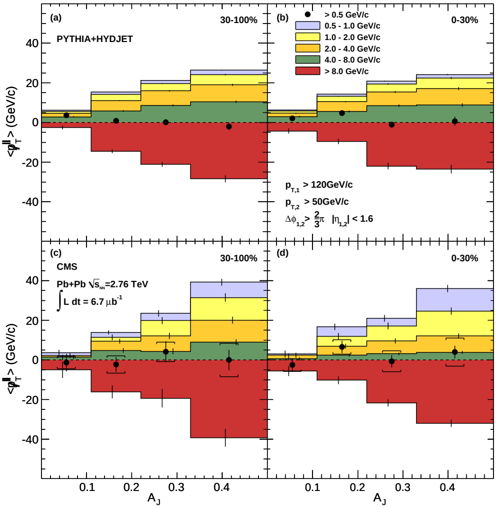


- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Considerably larger than the typical scale in the medium: the 'temperature' $T \sim 1$ GeV (average $p_\perp$)

No missing energy ! *(CMS, arXiv:1102.1957)*

- ... but a pronounced difference in its distribution in bins of $\omega \equiv p_T$

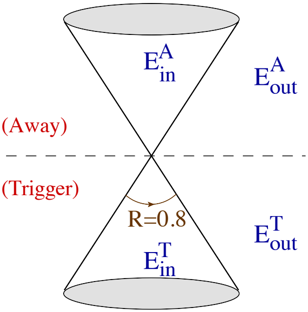
- $p_T^{\parallel}$: projection of a hadron energy along the jet axis
- $p_T^{\parallel} < 0$: same hemisphere as the trigger jet
- $p_T^{\parallel} > 0$: same hemisphere as the away jet
- all hadrons with $p_T > 0.5$ GeV are measured

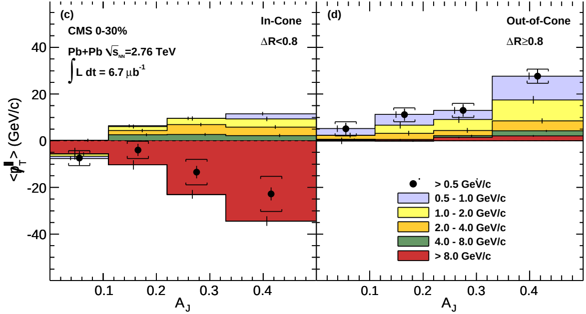


- Pb+Pb: excess of soft hadrons (≤ 2 GeV) in the 'away' hemisphere

These soft hadrons are found at large angles

- The energy imbalance for a jet with a wide opening : $R = 0.8$

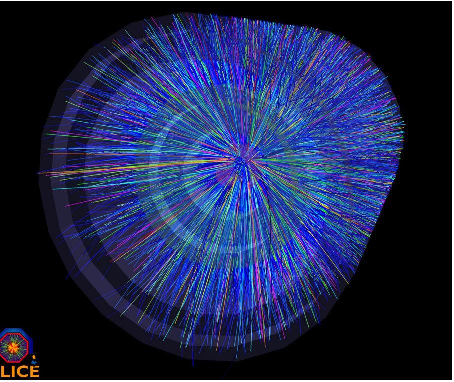


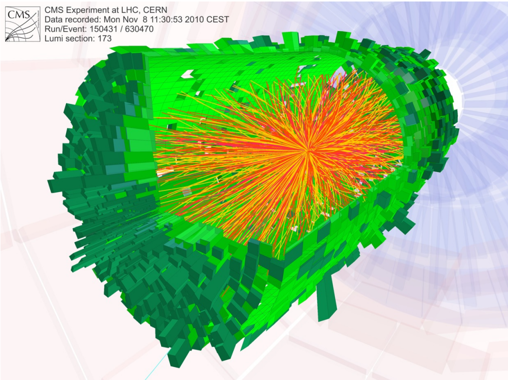


- Di–jet asymmetry : $E_{\rm in}^{\rm T} > E_{\rm in}^{\rm A}$
- No missing energy : $E_{\rm in}^{\rm T} + E_{\rm out}^{\rm T} = E_{\rm in}^{\rm A} + E_{\rm out}^{\rm A}$
- The energy lost at large angles, $E_{\rm out}^{\rm A} - E_{\rm out}^{\rm T}$ ...

... is carried mostly by soft hadrons with $p_T < 2$ GeV

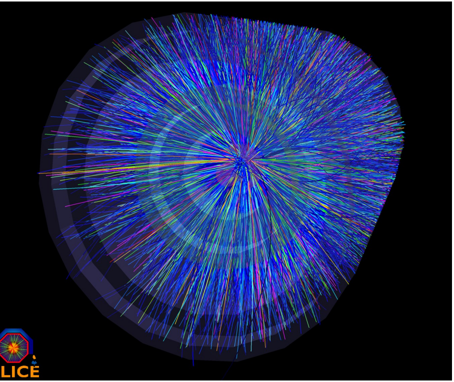
Instead of conclusions

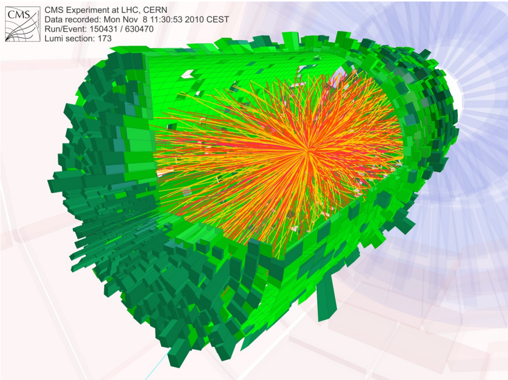




- All that may look hopelessly complicated ... ☹

Instead of conclusions





- All that may look hopelessly complicated ... ☹
- ... but in reality it is JUST PLAIN QCD ☺

THANK YOU !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