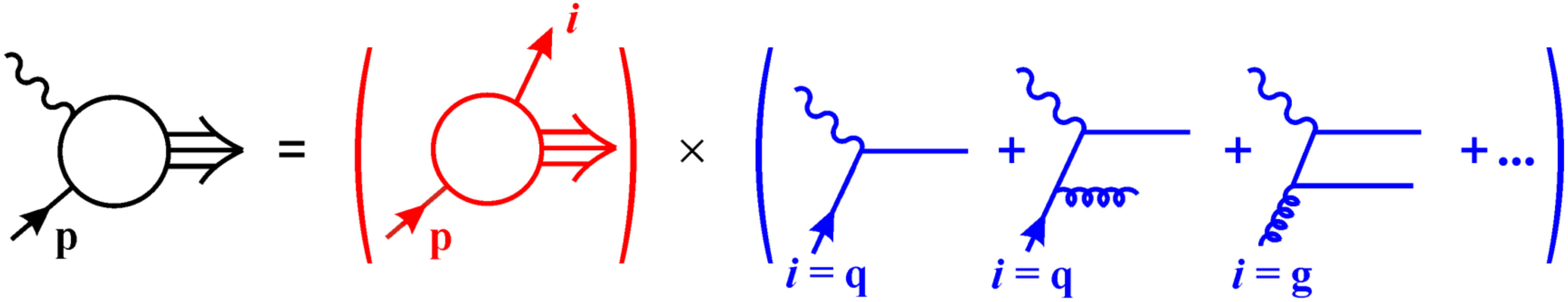
Factorization theorem



$$F_a(x,Q^2) \;=\; \sum_{i=q,\bar{q},g} \int \frac{dy}{y}\; f_i(y,Q^2)\;\; C_{a,i}\left(\frac{x}{y},\alpha_S(Q^2)\right)$$

UNIVERSAL parton densities f_i,
absorb singularities.
Q dep. given in pQCD by DGLAP eqs.

Coeff. fns, $C_{a,i}$ KNOWN
from pQCD
as power series in α_s

	LO	NLO	NNLO...
1	α_s	α_s^2	α_s^3

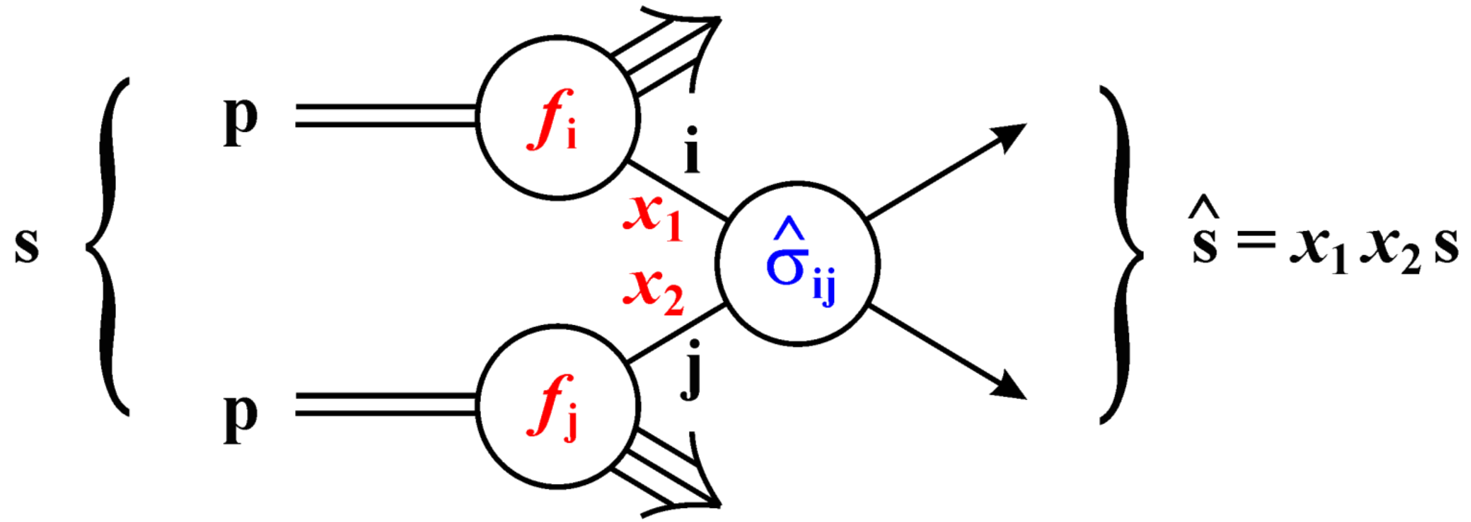
$$\frac{\partial f_i(x,Q^2)}{\partial \log(Q^2)} = \sum_j \alpha_S \int \frac{dy}{y}\, \underbrace{P_{ij}(x/y)}_{P_{ij}^{(0)} + \alpha_S P_{ij}^{(1)} + \alpha_S^2 P_{ij}^{(2)} + \ldots}\; f_j(y,Q^2)$$

sums LL	$\alpha_S^n \log^n Q^2$	NLL	NNLL
	1972-77	1977-80	2004

parametrize $f_i(x,Q_0^2)$
→ know $f_i(x, \text{all } Q^2)$
→ global fit → f_i's

$pp \rightarrow A$

$M=\mu=Q=$
hard scale

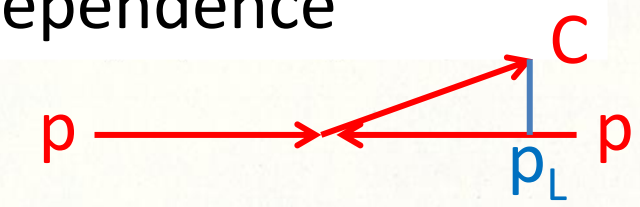


$$\sigma = \sum_{i,j} \int dx_1 dx_2 \; \underbrace{f_i(x_1, Q^2) f_j(x_2, Q^2)}_{\text{universal parton densities}} \; \underbrace{\hat{\sigma}_{ij}(x_1 x_2, \alpha_S(Q^2))}_{\text{calculable}}$$

So LHC and Tevatron data also constrain the PDFs

Process	Subprocess	Partons		x range
$\ell^\pm \{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	fixed target	$x \gtrsim 0.01$
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u		$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$		$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$		$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \to \mu^-(\mu^+) X$	$W^* q \to q'$	$q, \bar{q}$		$0.01 \lesssim x \lesssim 0.5$
$\nu N \to \mu^-\mu^+ X$	$W^* s \to c$	s		$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$		$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	HERA	$10^{-4} \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+ \{d, s\} \to \{u, c\}$	d, s		$x \gtrsim 0.01$
$e^\pm p \to e^\pm c\bar{c} X, e^\pm b\bar{b} X$	$\gamma^* c \to c$, $\gamma^* g \to c\bar{c}$	c, b, g		$10^{-4} \lesssim x \lesssim 0.01$
$e^\pm p \to$ jet+X	$\gamma^* g \to q\bar{q}$	g		$0.01 \lesssim x \lesssim 0.1$
$p\bar{p}, pp \to$ jet+X	$gg, qg, qq \to 2j$	g, q	LHC, Tevatron	$0.005 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm \nu) X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$		$x \gtrsim 0.05$
$pp \to (W^\pm \to \ell^\pm \nu) X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d},$		$x \gtrsim 0.001$
$p\bar{p}(pp) \to (Z \to \ell^+\ell^-) X$	$uu, dd, ..(u\bar{u}, ..) \to Z$	$u, d, ..(g)$		$x \gtrsim 0.001$
$pp \to W^- c,\ W^+ \bar{c}$	$gs \to W^- c$	$s, \bar{s}$		$x \sim 0.01$
$pp \to (\gamma^* \to \ell^+\ell^-) X$	$u\bar{u}, d\bar{d}, .. \to \gamma^*$	$\bar{q}, g$		$x \gtrsim 10^{-5}$
$pp \to b\bar{b} X, \quad t\bar{t} X$	$gg \to b\bar{b}, \quad t\bar{t}$	g		$x \gtrsim 10^{-5}, 10^{-2}$
$pp \to$ exclusive $J/\psi, \Upsilon$	$\gamma^*(gg) \to J/\psi, \Upsilon$	g		$x \gtrsim 10^{-5}, 10^{-4}$
$pp \to \gamma X$	$gq \to \gamma q$, $g\bar{q} \to \gamma \bar{q}$	g		$x \gtrsim 0.005$

Rapidity y is a way to display p_L dependence



$$y = \frac{1}{2}\log\left(\frac{E+p_L}{E-p_L}\right)$$

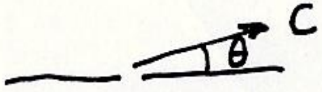
- Depends on frame, but has the advantage of being additive under Lorentz boosts, say of velocity u

$$\left.\begin{array}{l} E \to \gamma(E+up_L) \\ p_L \to \gamma(p_L+uE) \end{array}\right\} \gamma = \frac{1}{\sqrt{1-u^2}} \qquad \begin{array}{l} y \to y + \frac{1}{2}\log\left(\frac{1+u}{1-u}\right) \\ \quad = y + y_{boost} \end{array}$$

- In non-rel. limit $v \ll 1$: $E \to m$, $p \to mv$, $y \to v$

- $$\frac{p_L}{E} = \tanh y \qquad \left\{\begin{array}{l} p_L = m_T \sinh y \\ E = m_T \cosh y \end{array}\right. \qquad \text{with } m_T = \sqrt{m_c^2 + p_T^2}$$

- For massless particles $E \doteq |\vec{p}|$

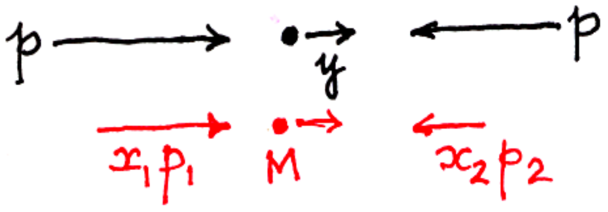


$$y = \frac{1}{2}\log\frac{1+\cos\theta}{1-\cos\theta} = -\log\tan\frac{\theta}{2}$$ pseudorapidity

- $y = \frac{1}{2}\log\left(\frac{E+p_L}{E-p_L}\right) = \frac{1}{2}\ln\left(\frac{(E+p_L)^2}{E^2-p_L^2}\right) = \ln\left(\frac{E+p_L}{m_T}\right)$ $\qquad m_T^2 \equiv m^2+p_T^2$

Kinematics of pp → M

in c.m. frame



$$M^2 = (x_1p_1+x_2p_2)^2$$
$$\simeq x_1x_2\,2p_1\cdot p_2 \simeq x_1x_2 s$$

neglect masses of beam pts.

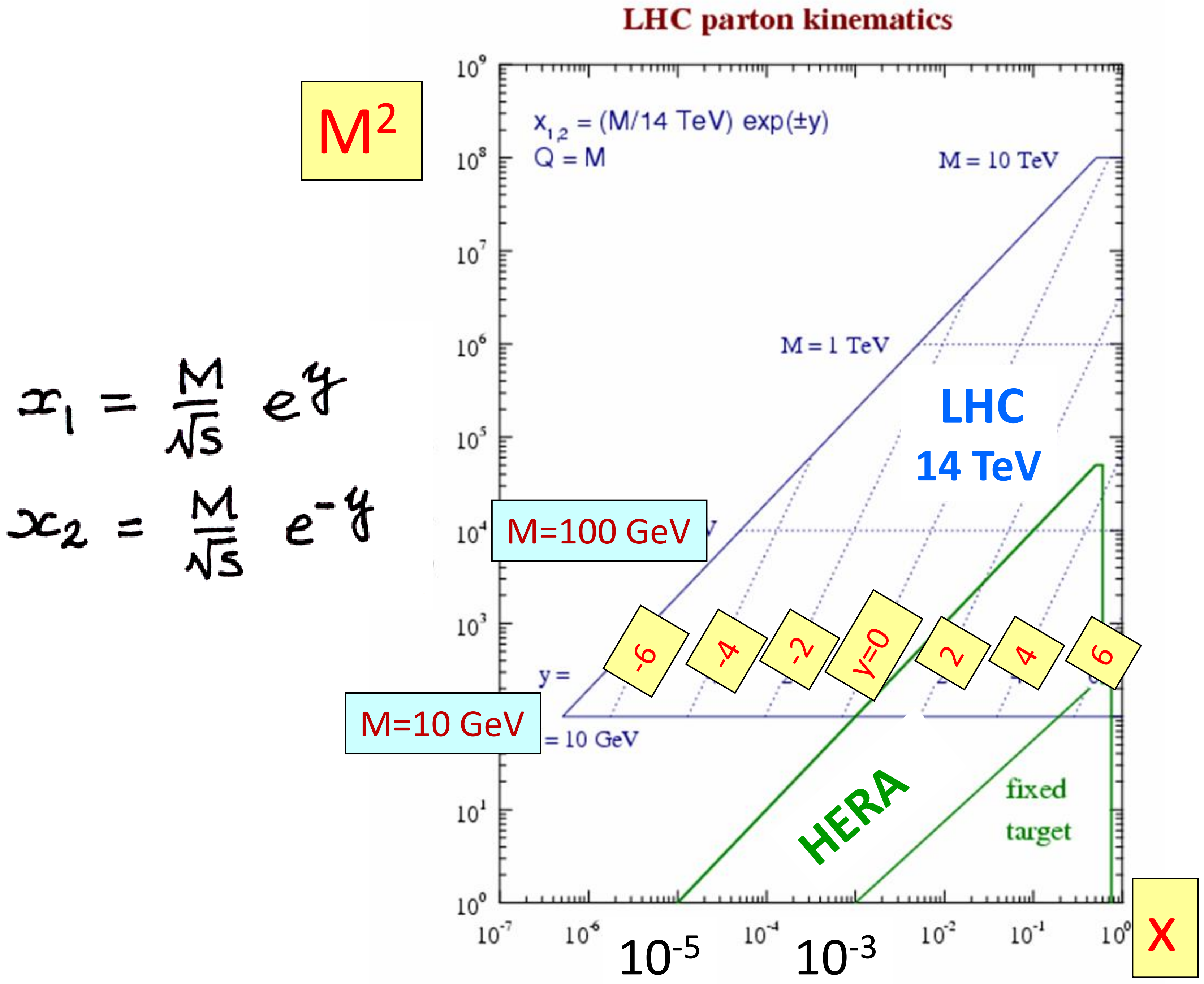
For M: $\begin{cases} E \simeq x_1p + x_2p \\ p_L = x_1p - x_2p \end{cases}$

so $y = \ln\left(\frac{E+p_L}{M}\right) \simeq \ln\frac{2x_1p}{M} = \ln\frac{x_1\sqrt{s}}{M}$

$$s \simeq (p+p)^2 = 4p^2$$

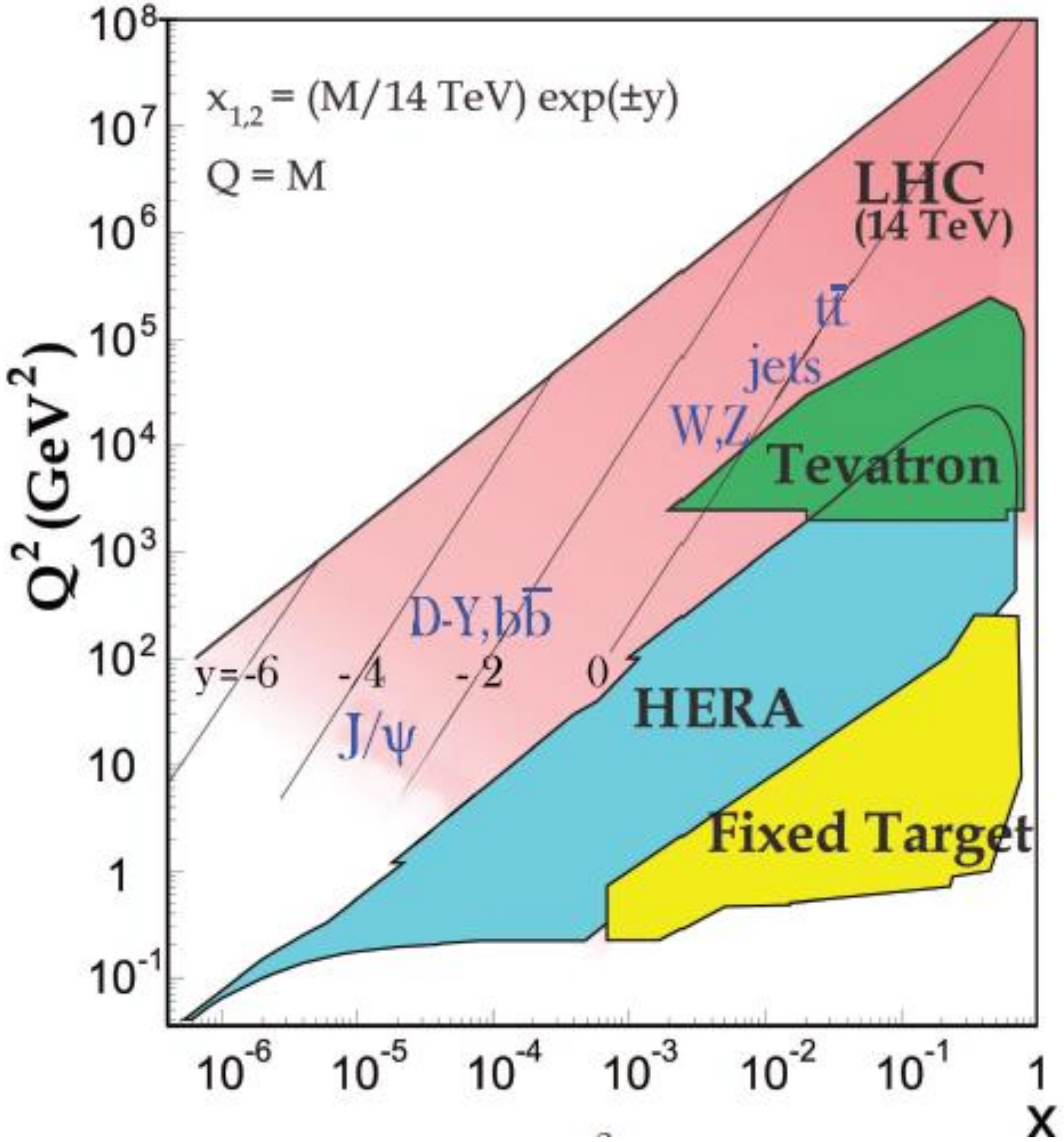
$$E_{proton} \simeq |\vec{p}| \equiv p$$

$$\therefore \boxed{\begin{aligned} x_1 &= \frac{M}{\sqrt{s}}\,e^{y} \\ x_2 &= \frac{M}{\sqrt{s}}\,e^{-y} \end{aligned}}$$



$$x_1 = \frac{M}{\sqrt{s}} e^{y}$$

$$x_2 = \frac{M}{\sqrt{s}} e^{-y}$$



$\ell^\pm \{p,n\} \to \ell^\pm X$

$\ell^\pm n/p \to \ell^\pm X$

$pp \to \mu^+\mu^- X$

$pn/pp \to \mu^+\mu^- X$

$\nu(\bar{\nu}) N \to \mu^-(\mu^+) X$

$\nu N \to \mu^-\mu^+ X$

$\bar{\nu} N \to \mu^+\mu^- X$

Fixed target

$e^\pm p \to e^\pm X$

$e^+ p \to \bar{\nu} X$

$e^\pm p \to e^\pm c\bar{c}X, e^\pm b\bar{b}X$

$e^\pm p \to$ jet$+X$

HERA

$p\bar{p}, pp \to$ jet$+X$

$p\bar{p} \to (W^\pm \to \ell^\pm \nu) X$

$pp \to (W^\pm \to \ell^\pm \nu) X$

$p\bar{p}(pp) \to (Z \to \ell^+\ell^-)X$

$pp \to W^- c,\ \ W^+ \bar{c}$

$pp \to (\gamma^* \to \ell^+\ell^-)X$

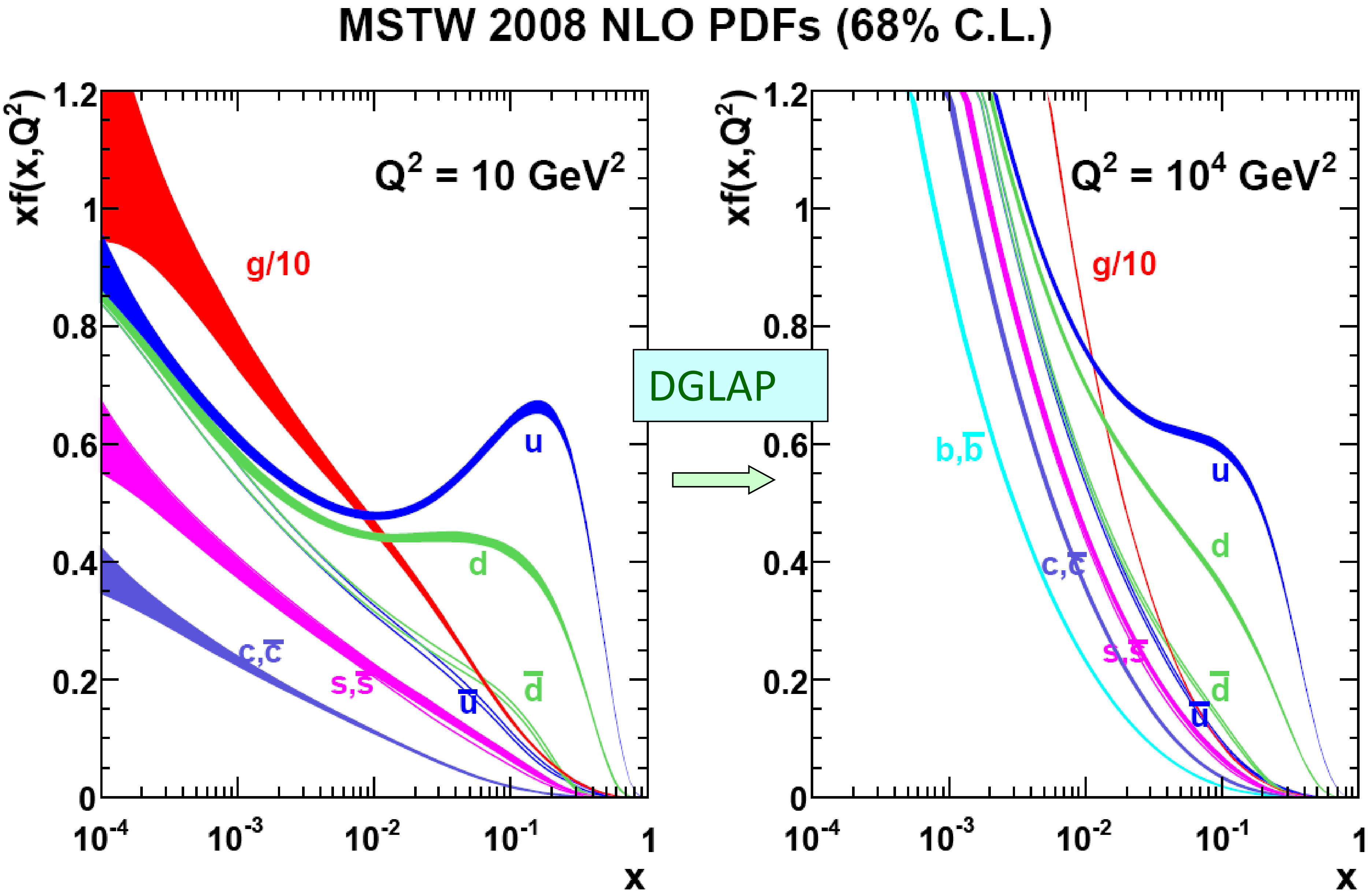
$pp \to b\bar{b}X, \quad t\bar{t}X$

$pp \to$ exclusive $J/\psi, \Upsilon$

$pp \to \gamma X$

LHC

Tevatron

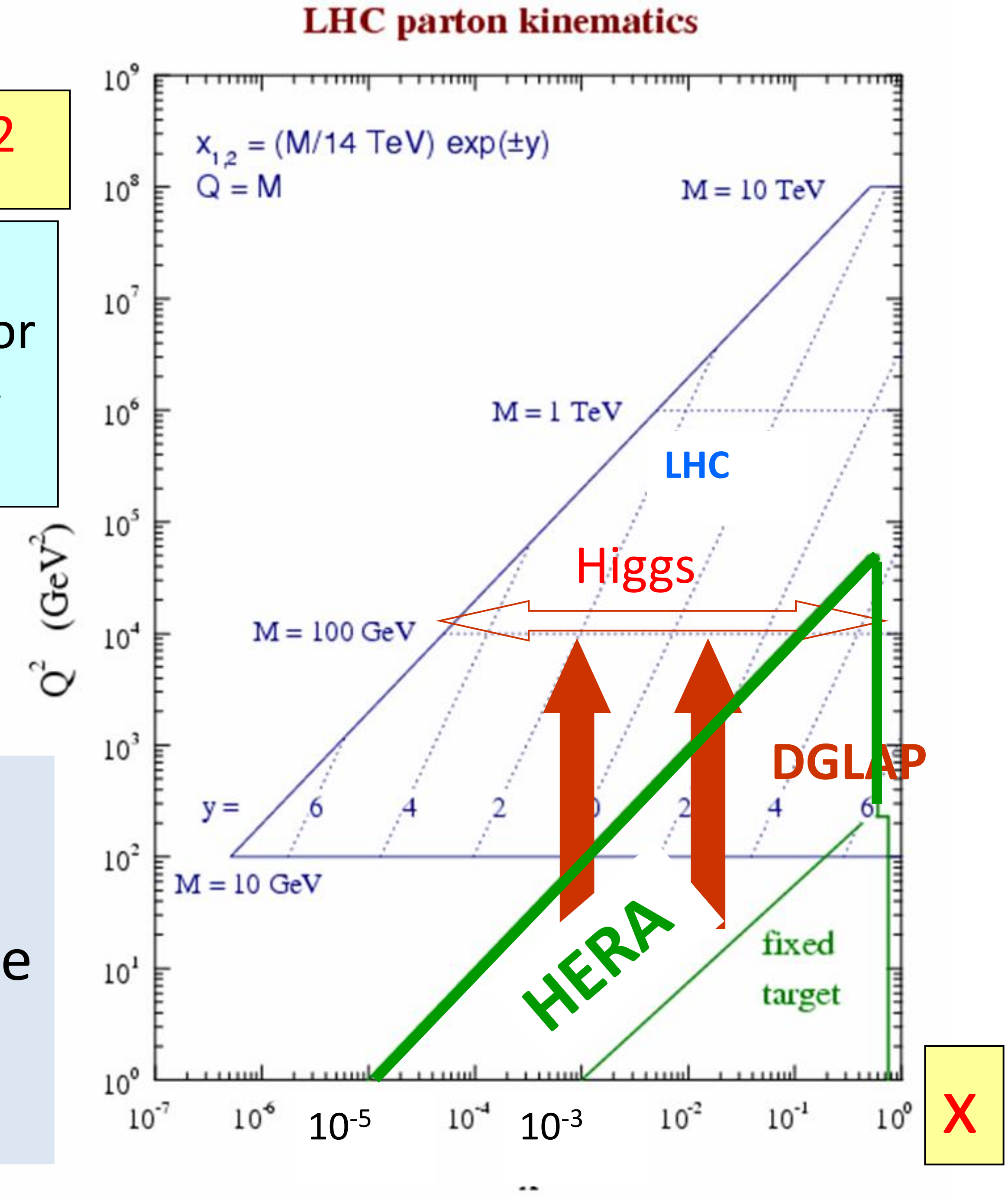


MSTW 2008 NLO PDFs (68% C.L.)

DGLAP

Q^2

DGLAP appears to work down to $x \sim 10^{-3}$ or less, and $Q^2 \sim 2\text{-}5$ GeV^2 --- so PDFs known

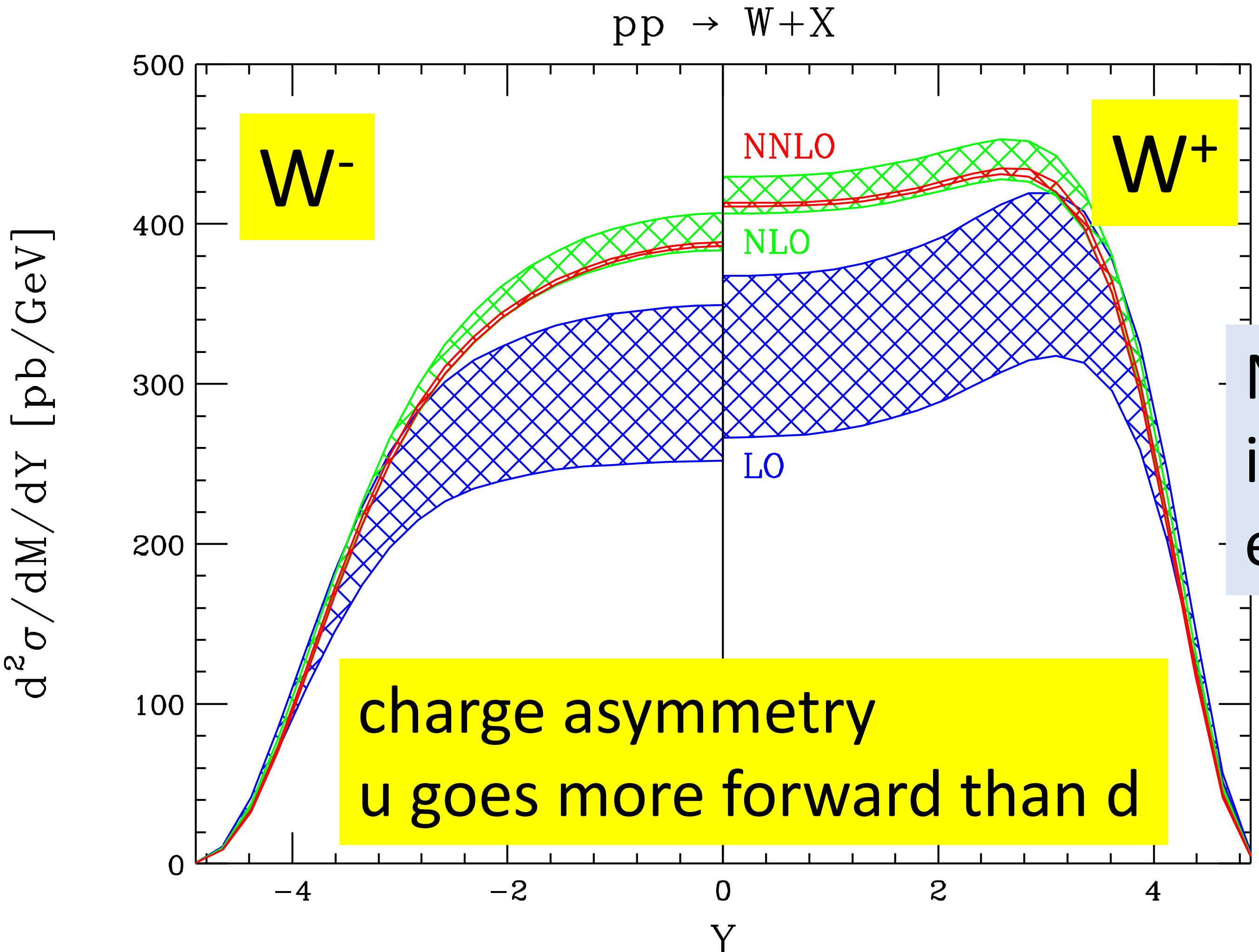


So even before the LHC we could reliably extrapolate PDFs and make predictions

LHC physics: an example -- W production
14 TeV

LO: $d\bar{u} \rightarrow W^{--}$ $u\bar{d} \rightarrow W^{+}$

Anastasiou, Dixon, Melnikov, Petriello

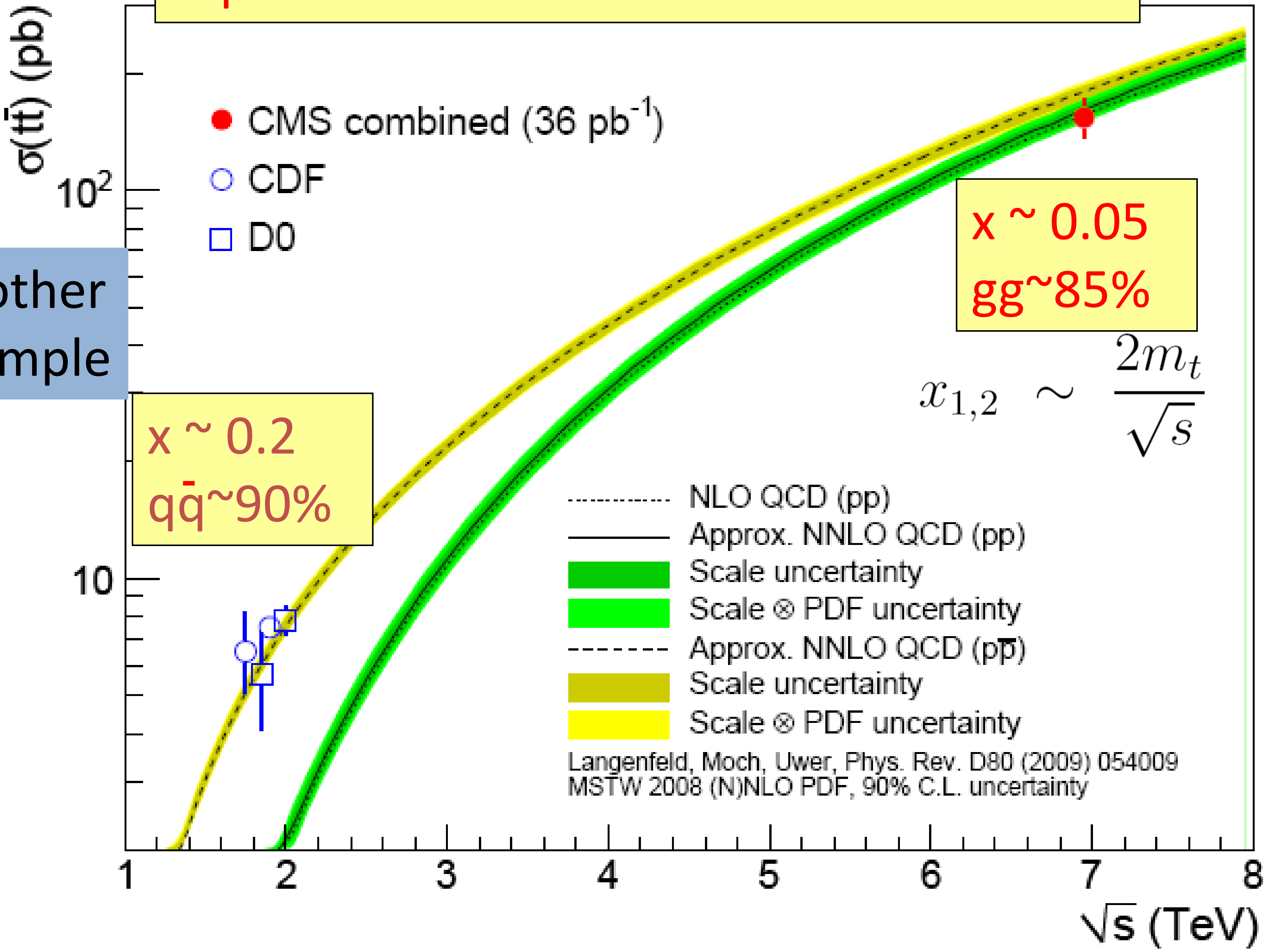


Now confirmed in detail by LHC experiments

charge asymmetry
u goes more forward than d

$t\bar{t}$ production at the LHC and Tevatron

Another example

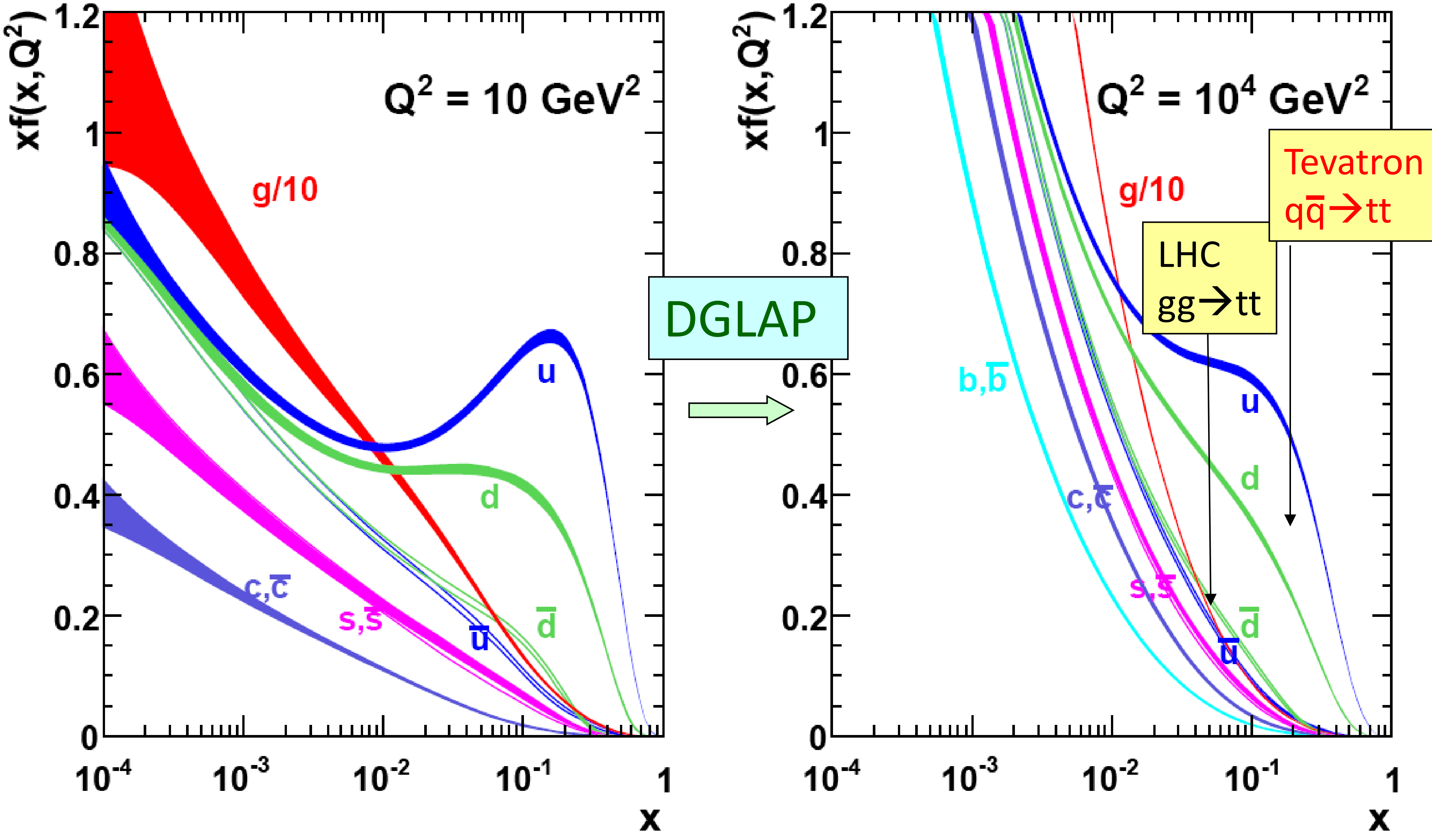


Langenfeld, Moch, Uwer, Phys. Rev. D80 (2009) 054009
MSTW 2008 (N)NLO PDF, 90% C.L. uncertainty

x ~ 0.05
gg~85%

x ~ 0.2
$q\bar{q}$~90%

$$x_{1,2} \sim \frac{2m_t}{\sqrt{s}}$$

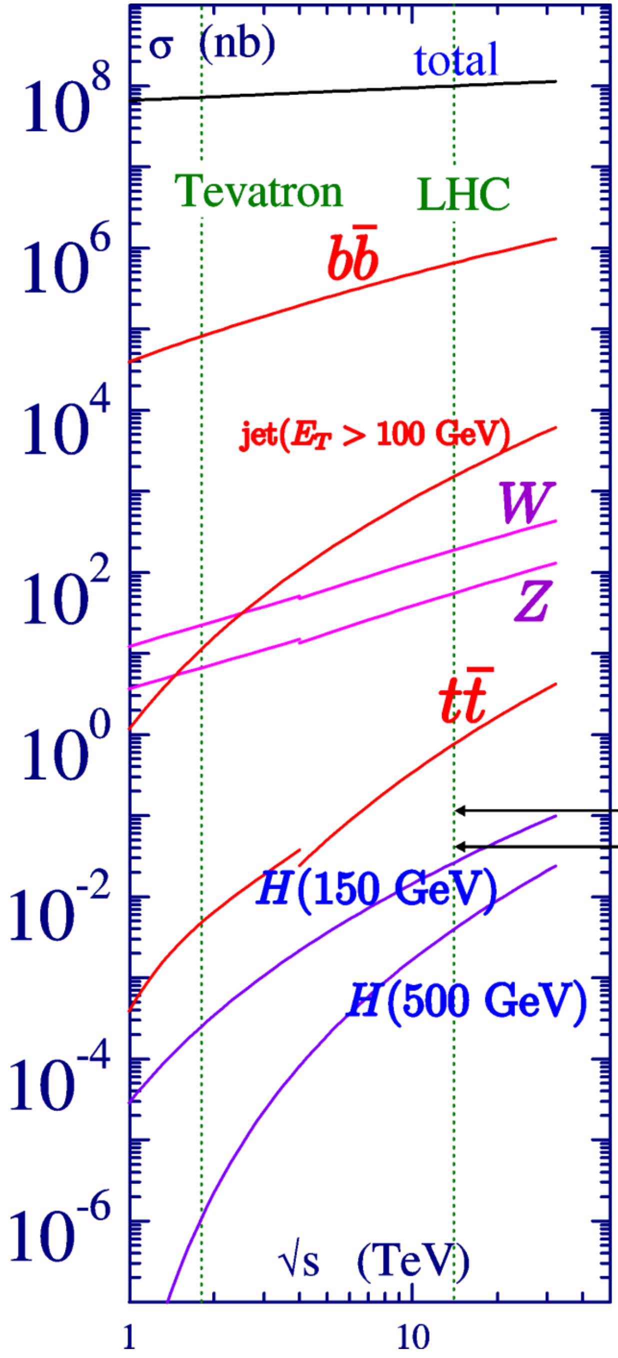


MSTW 2008 NLO PDFs (68% C.L.)

DGLAP

LHC
gg→tt

Tevatron
qq̄→tt



events/sec if $\mathcal{L} = 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

$b\bar{b} \sim 10^6/\mathrm{sec}$ CKM, CP violation

$W \to e\nu \sim 20/\mathrm{s}$

$Z \to ee \sim 2/\mathrm{s}$

M_W, Γ_W luminosity

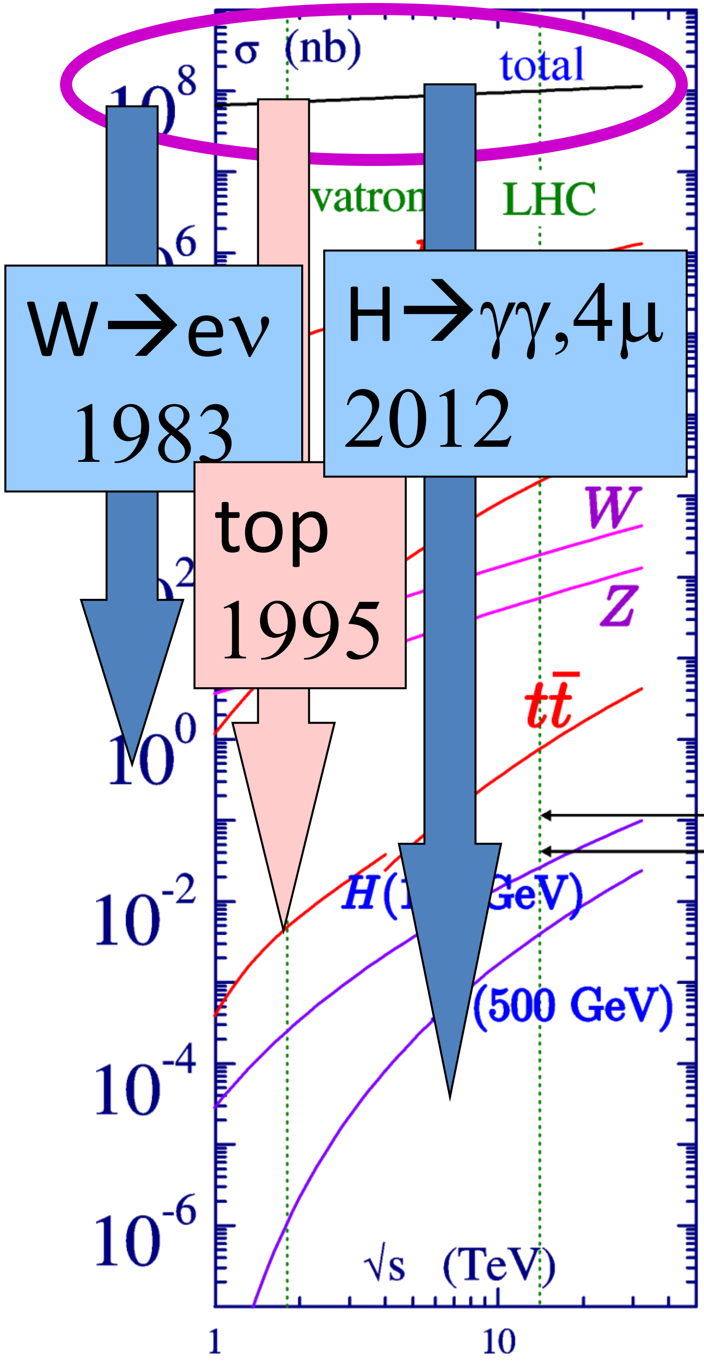
$t\bar{t} \sim 1/\mathrm{s}$ m_t

gluino-pair?
squark-pair?
if m ~ 500 GeV

discovery?

Rates at LHC

events/sec if $\mathcal{L} = 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$



$b\bar{b} \sim 10^6/\mathrm{sec}$ CKM, CP violation

$W \rightarrow e\nu \sim 20/\mathrm{s}$ $\mathrm{M_W}$, Γ_W

$Z \rightarrow ee \sim 2/\mathrm{s}$ luminosity

$t\bar{t} \sim 1/\mathrm{s}$ $\mathrm{m_t}$

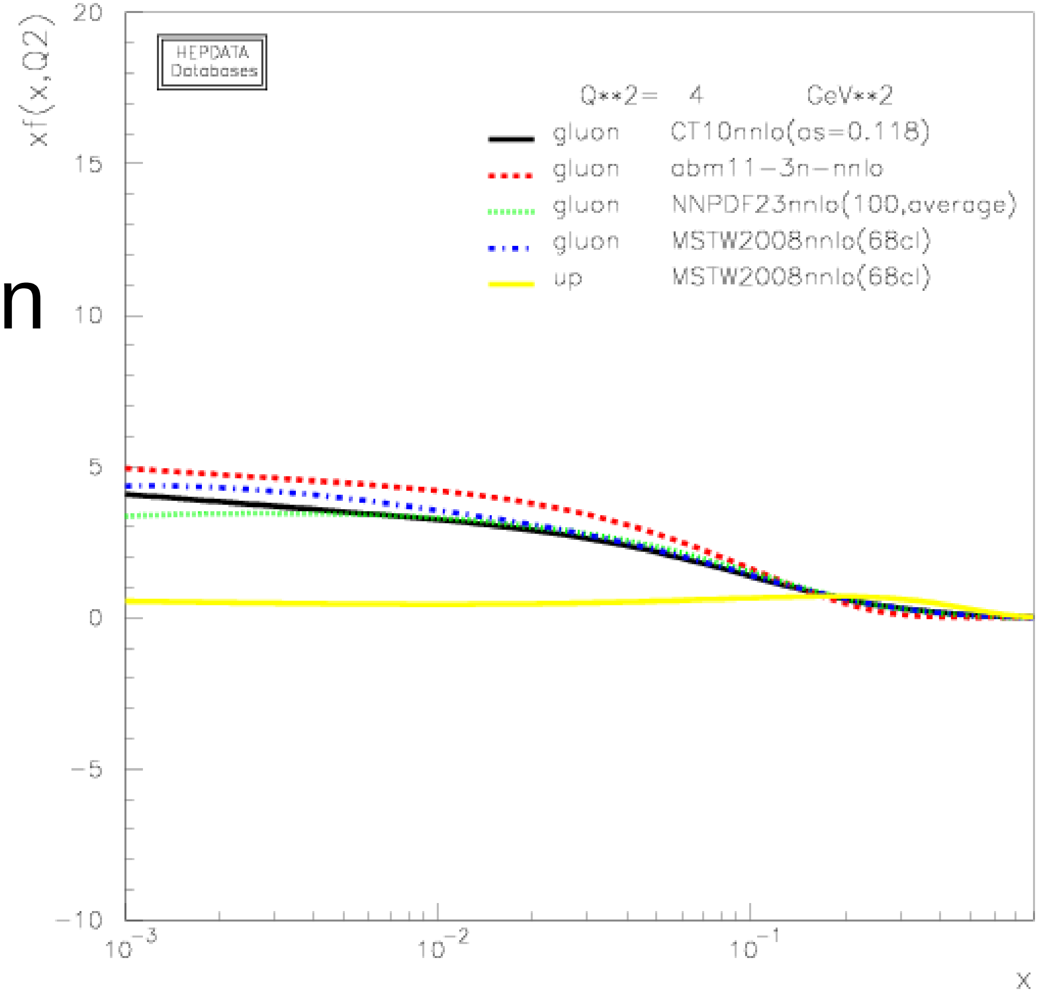
gluino-pair?
squark-pair?
if m ~ 500 GeV

discovery?

or, more exciting, something totally unexpected

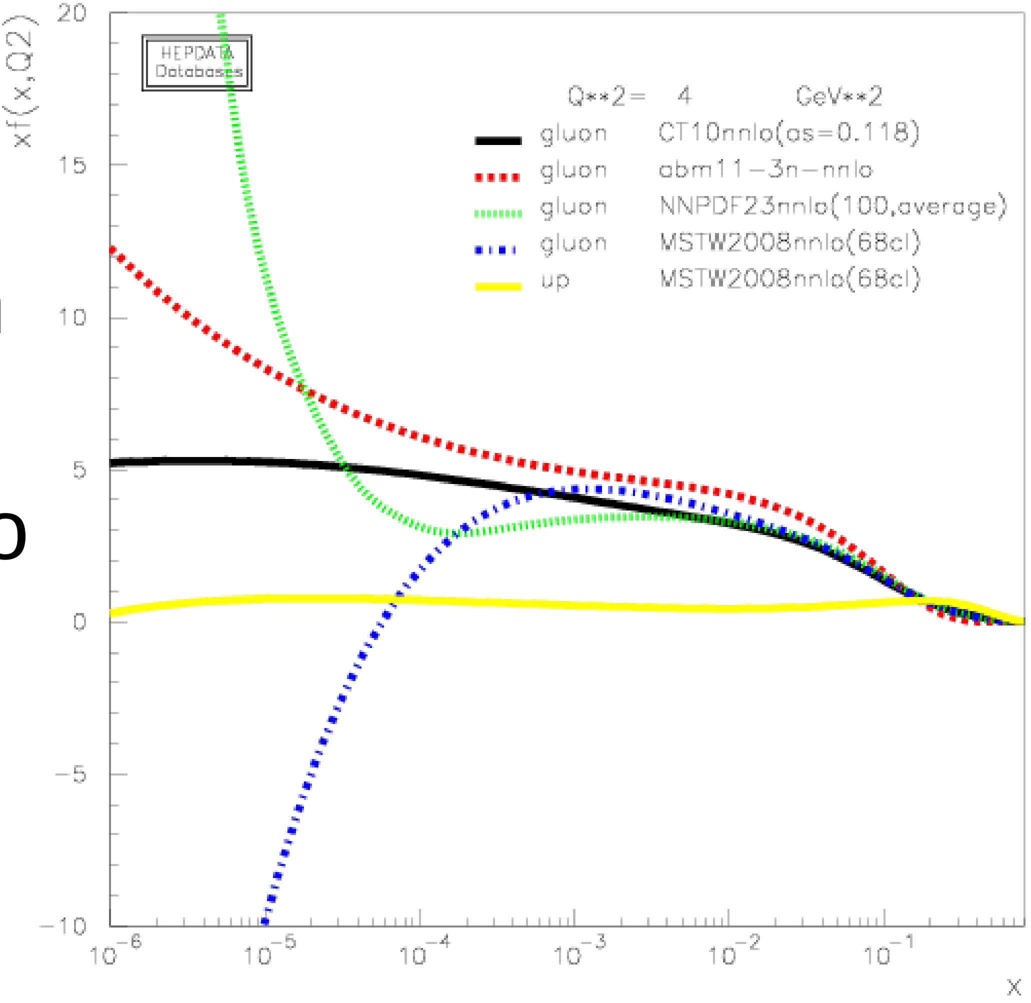
Global analysis (Durham HepData)

- They use the same theory.
- At small *x*, gluon is dominant.

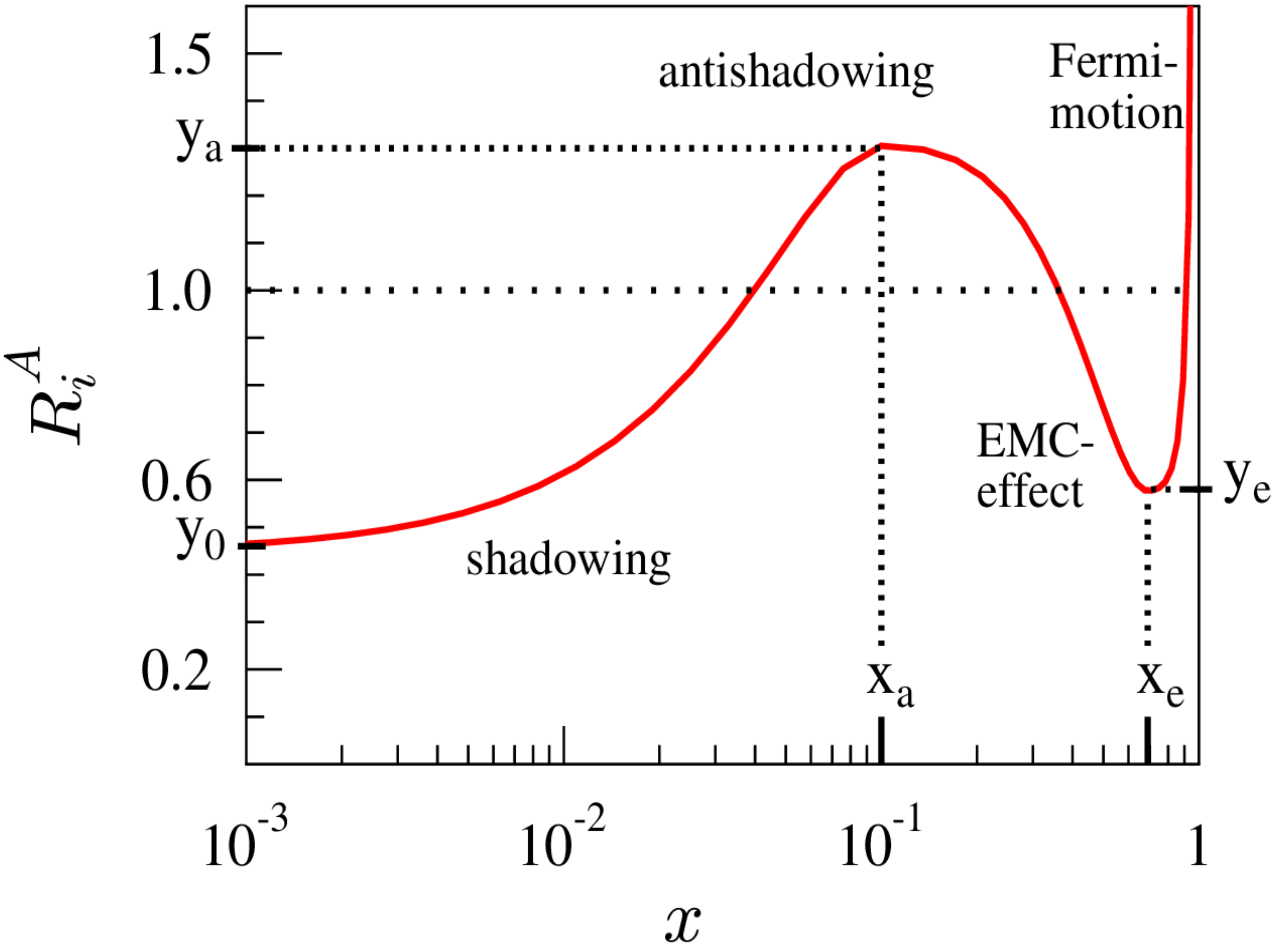


Global analysis (Durham HepData)

- They use the same theory.
- At small *x*, gluon is dominant.
- Very important to the LHC.
- Negative gluon
- LHAPDF

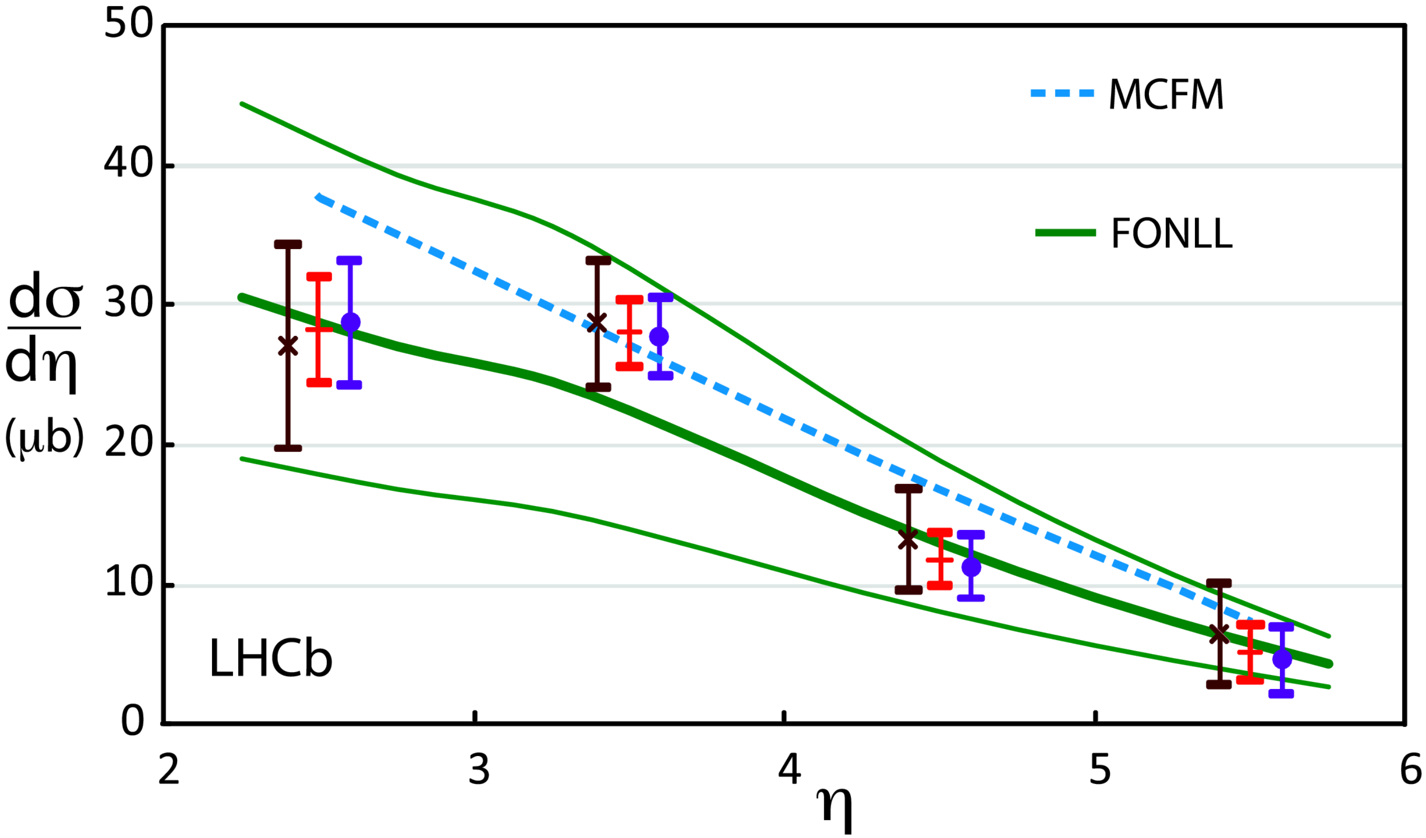


Nuclear partons



Factorization scale dependence

- LHCb – Phys.Lett.B694:209-216,2010



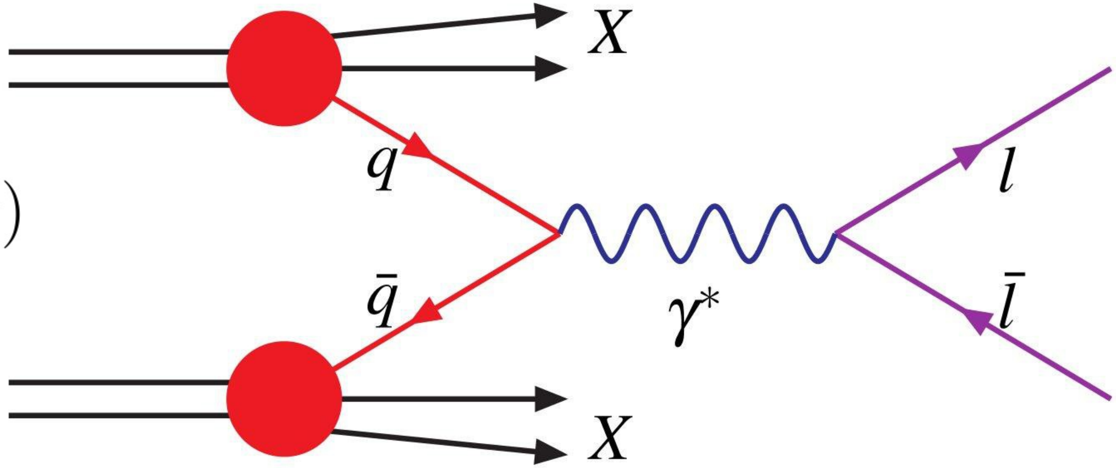
Scale dependence of Drell-Yan production

- LHC: pp collisions -- seen as parton collisions

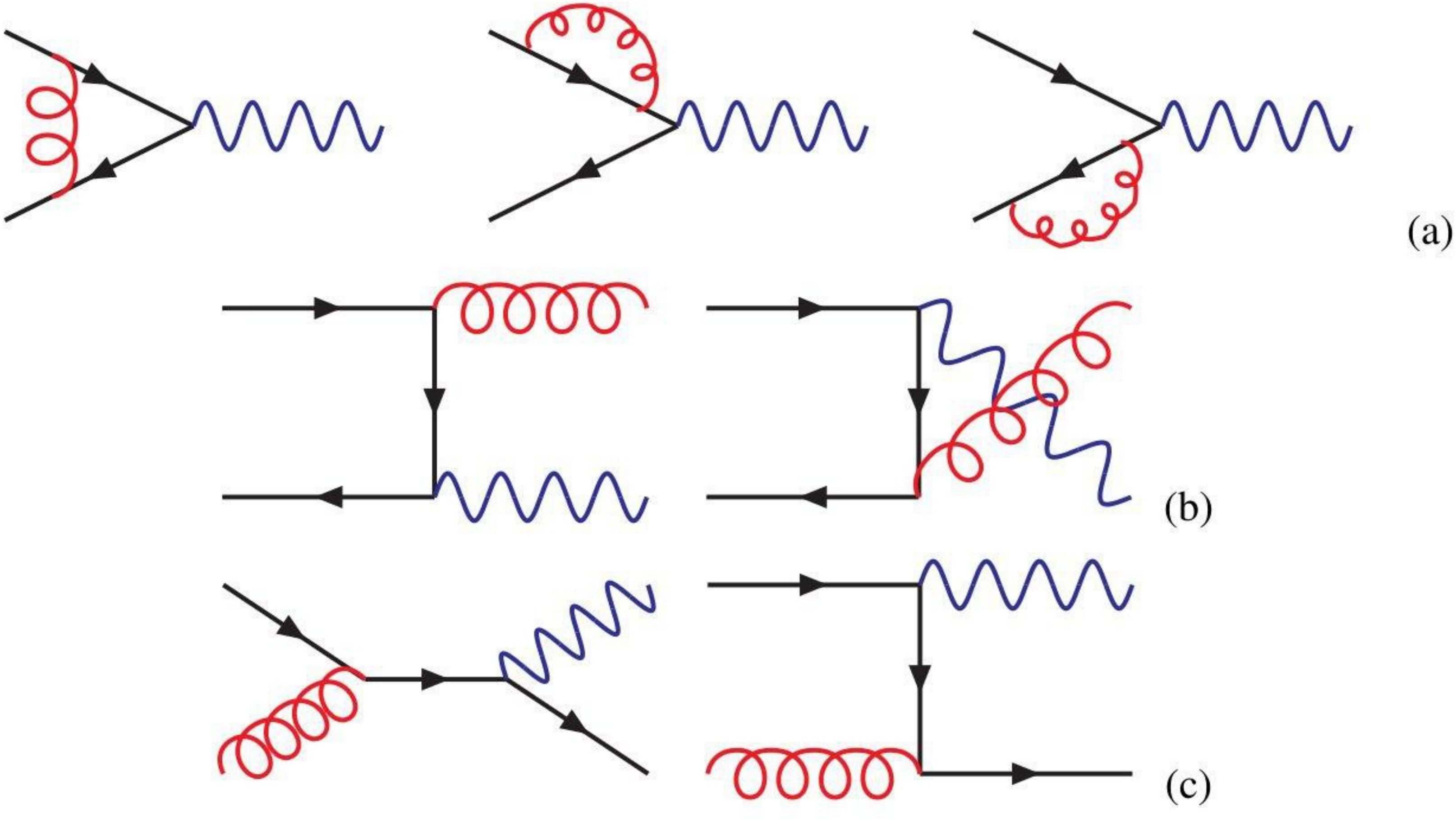
$$d\sigma/d^3p \;=\; \int dx_1 dx_2 \; \mathrm{PDF}(x_1,\mu_F)\; |\mathcal{M}(p;\mu_F,\mu_R)|^2 \; \mathrm{PDF}(x_2,\mu_F)\ ,$$

- x variables

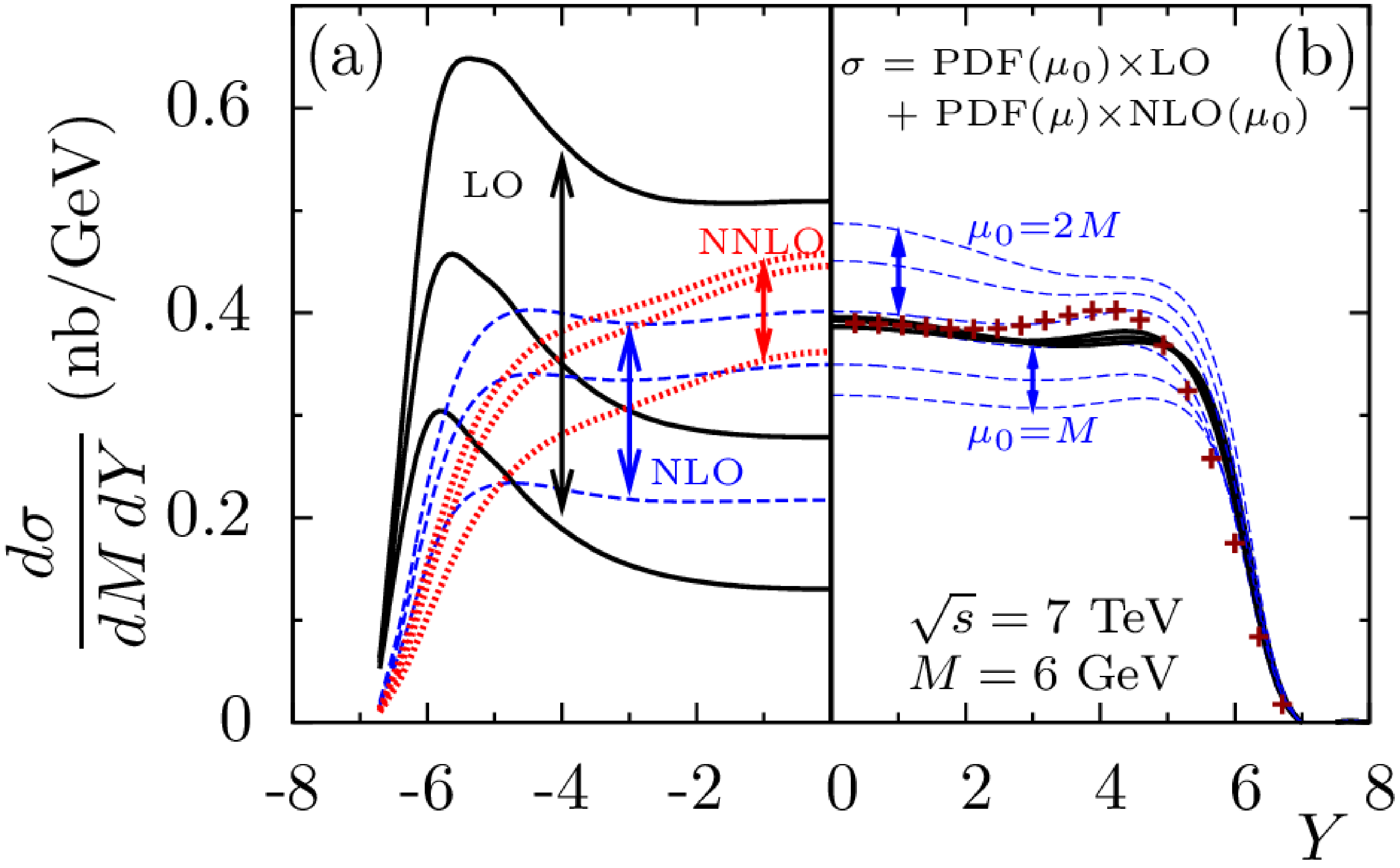
$$x_{1,2} = \frac{m_{\mathrm{hard}}}{\sqrt{s}} \exp(\pm y)$$



NLO diagrams



DY scale dependence



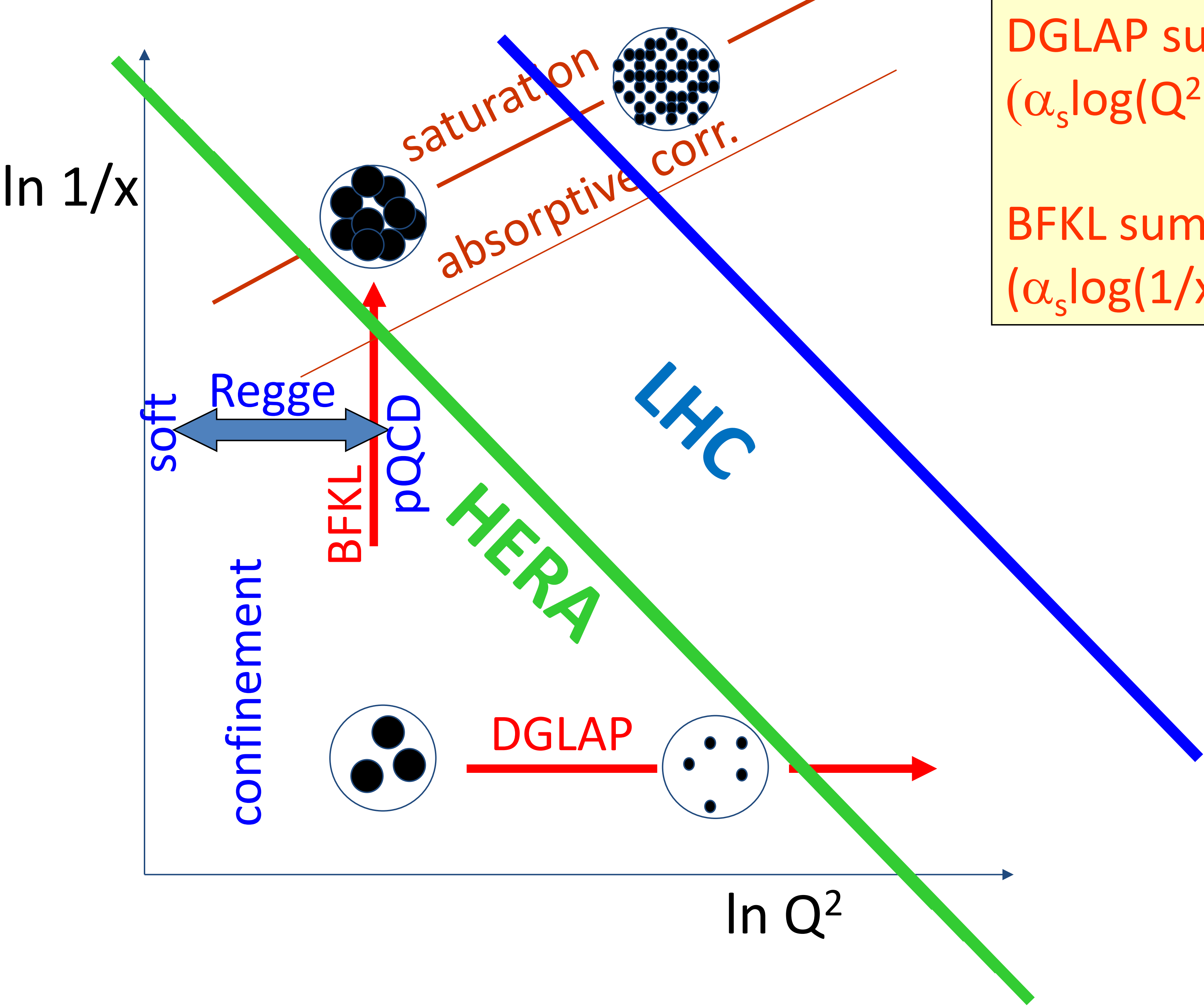
small x

DGLAP sums $(\alpha_s \log Q^2)^n$ terms --- as Q^2 increases probe finer and finer structure of proton

but as x decreases need to sum $(\alpha_s \log(1/x))^n$ terms
→ BFKL

as 1/x increases gluons interact (combine) – absorptive corrections and eventually saturation

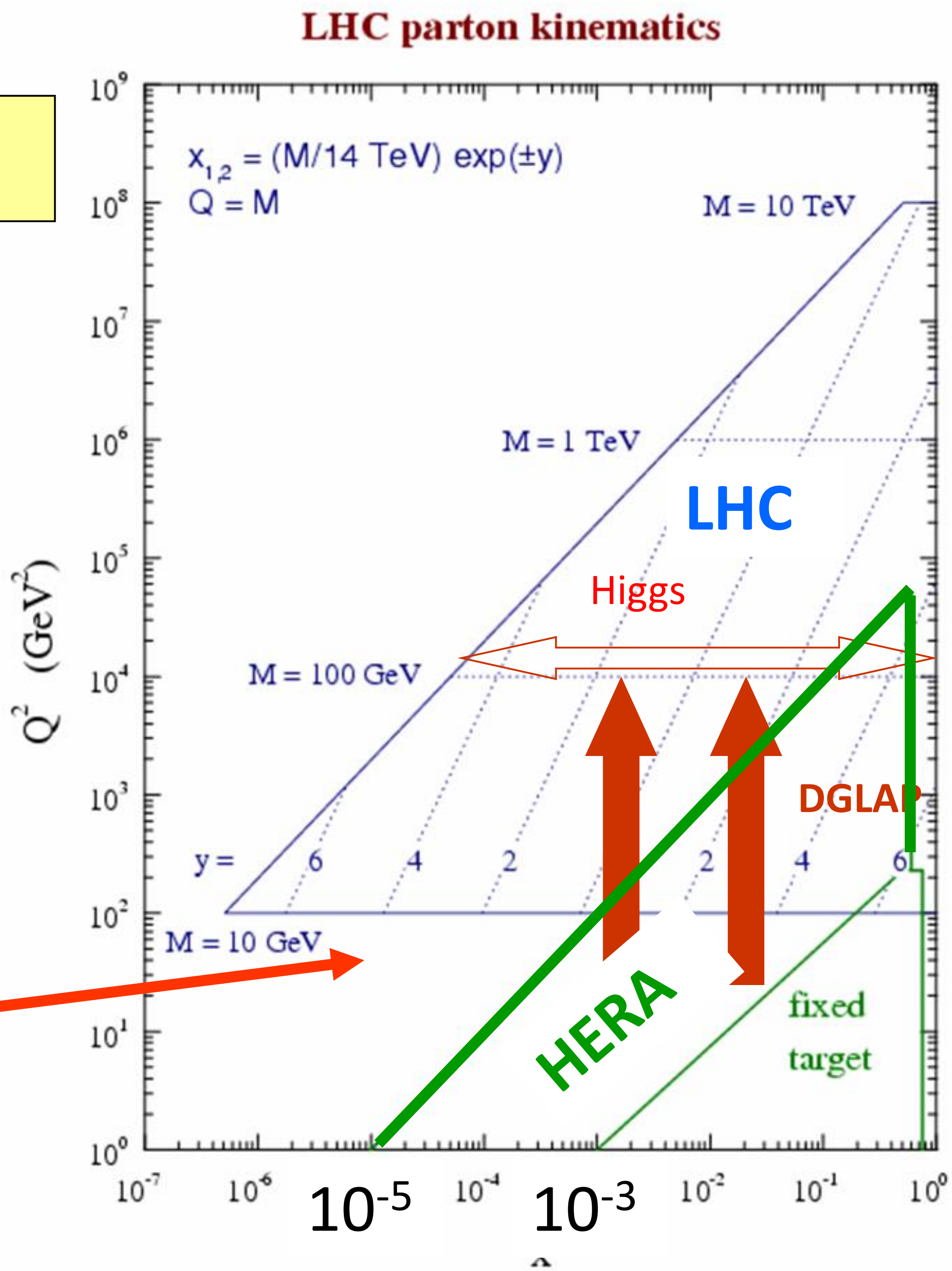
→ sketch



DGLAP sums $(\alpha_s\log(Q^2))^n$

BFKL sums $(\alpha_s\log(1/x))^n$

Q^2



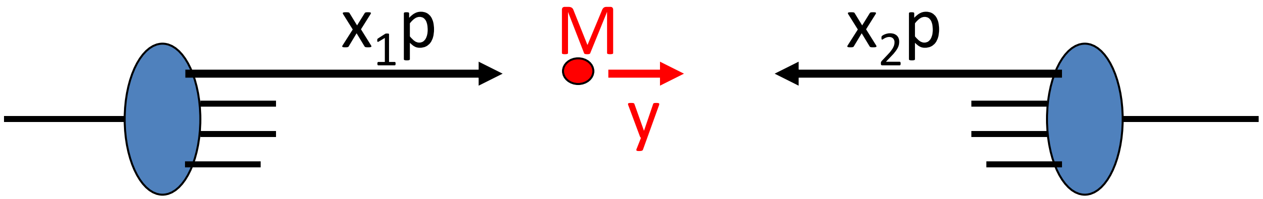
But LHC can probe v.small x

x

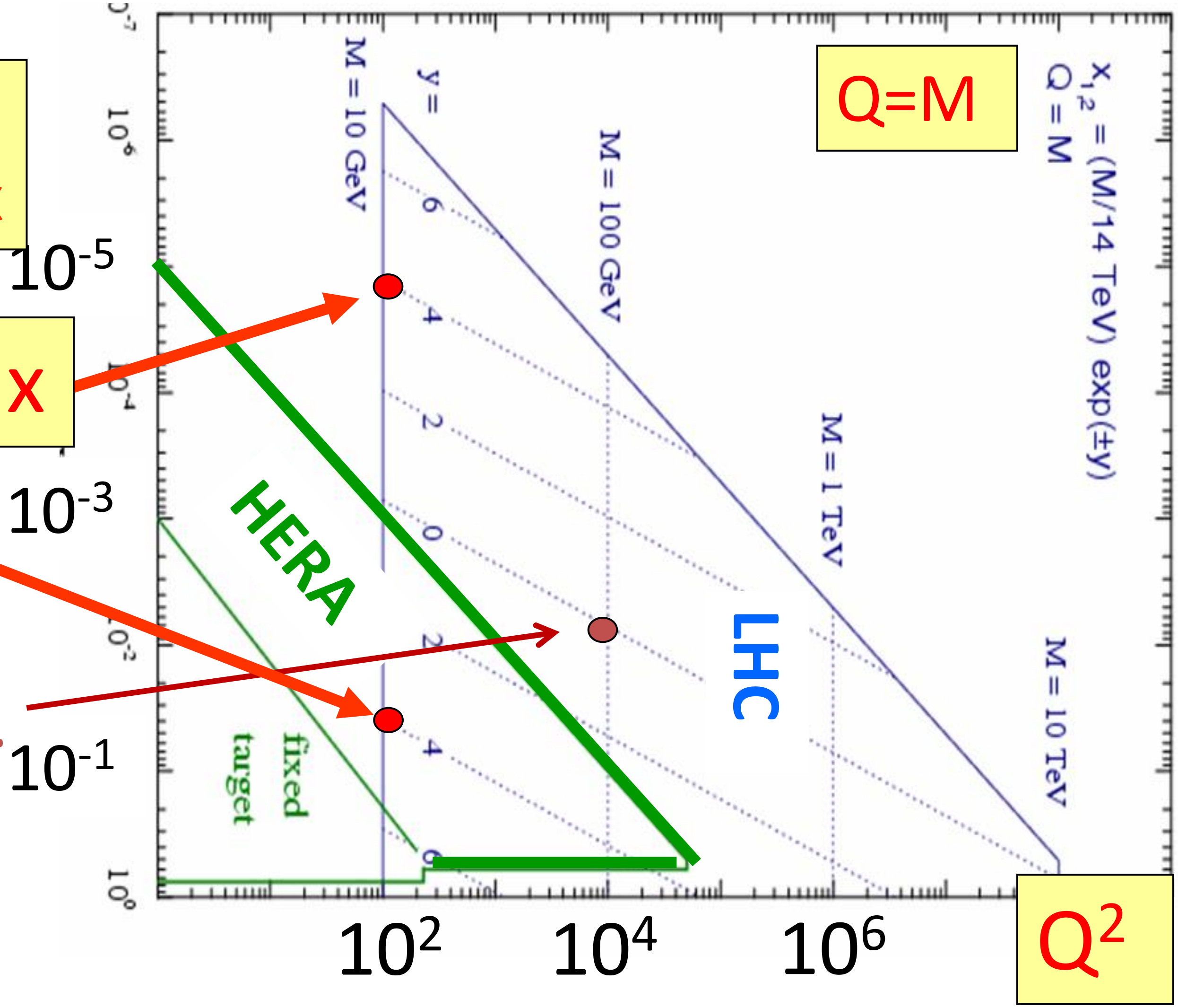
$x_1=(M/E)\ e^{y}$

$x_2=(M/E)\ e^{-y}$

E=14 TeV



But LHC can probe v.small x



LHCb $\mu^+\mu^-$

$|y| \sim 0.4$

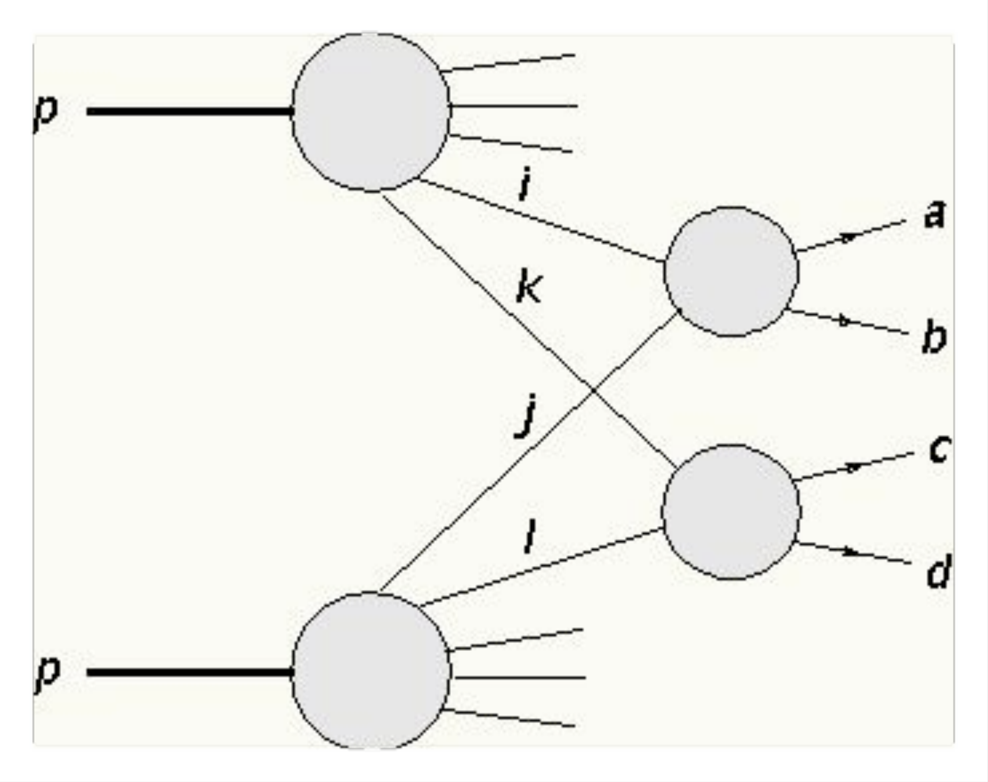
$x \sim 10^{-5}$

central W prod.

$x_{1,2}=0.007$

double parton scattering (DPS)

- Two INDEPENDENT scatterings in ONE proton-proton collision:



- Cross section expressed as a product of two SPS cross sections:

$$\sigma_{DPS} = \frac{\sigma_a \sigma_b}{\sigma_{eff}},$$

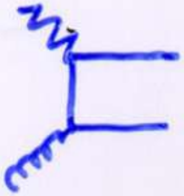
- Motivation?
  - QCD: non-perturbative dynamics, parton distributions, etc.
  - Searches for complex signatures typically rely on fact that new, heavy particles decay "spherically" while QCD backgrounds are correlated.
  - Higgs searches? New Physics searches?

Heavy quarks c (also b)

Important at small x, particularly as Q^2 increases

Two distinct regimes

- $Q^2 \sim m_c^2$:



c is not a parton.
Created in final state
$\gamma^* g \to c\bar{c}$

- $Q^2 \gg m_c^2$: behaves like massless parton

Thus should use a Variable Flavour Number Scheme in which a matching is performed at the heavy quark thresholds

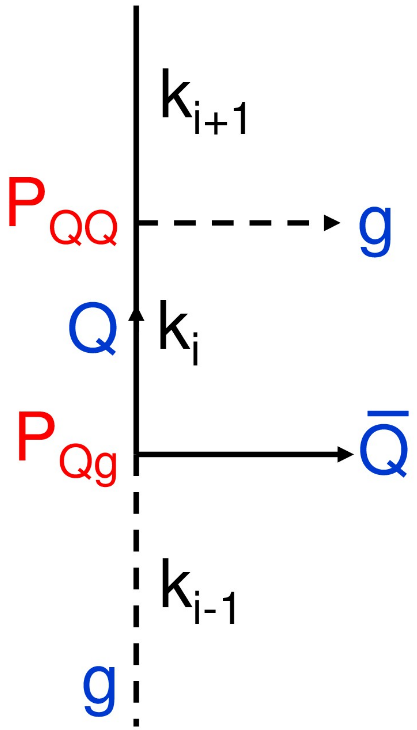
Heavy quarks evolution

Usually quarks are taken as massless.

GM-VFNS (General mass – variable flavour number scheme)

→ quarks as massless for scale bigger than quark mass

→ zero contribution for scale smaller than the quark mass



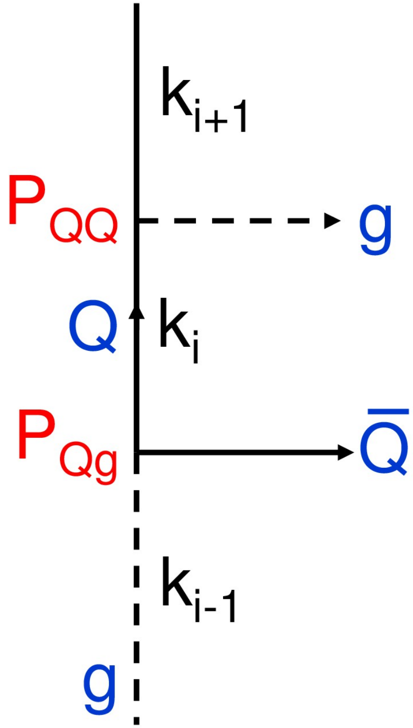
Heavy quarks evolution

GM-VFNS is justified at LO

Produces kinks at quark masses.

We want a smooth behaviour

$$\cdots \int \frac{dk_{i-1}^2}{k_{i-1}^2} \int \frac{dk_i^2 \; k_i^2}{(k_i^2 + m_{\mathtt{Q}}^2)^2} \int \frac{dk_{i+1}^2}{k_{i+1}^2} \cdots$$



g → Q splitting function

$$P_{\mathtt{Q}g}(z, Q^2) = T_R \left([z^2 + (1-z)^2] \frac{Q^2}{m_{\mathtt{Q}}^2 + Q^2} \; + \; \frac{2 m_{\mathtt{Q}}^2 Q^2 z(1-z)}{(Q^2 + m_{\mathtt{Q}}^2)^2} \right) \Theta \left(Q^2 - \frac{z m_{\mathtt{Q}}^2}{1-z} \right)$$

- Color factor, *z*, scale, quark mass
- No divergence!
- Correct high scale limit
- Split flip
- Step function (putting the emitted heavy quark on shell)

Other splitting functions

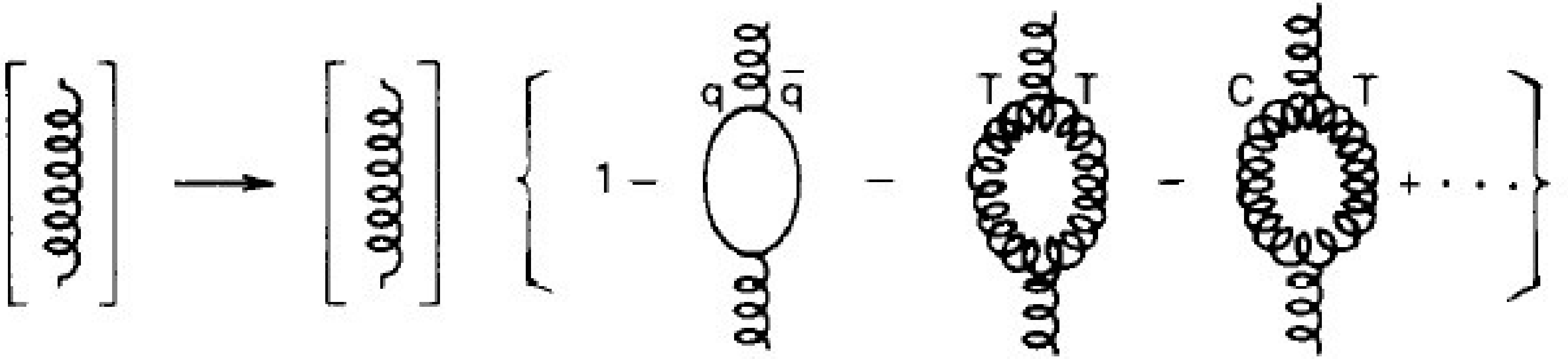
- Momentum conservation
- Intrinsic heavy flavour

$$\delta P_{gg} = -\delta(1-z)\sum_{\mathtt{Q}}\int_0^{z_{\mathtt{Q}}} P_{\mathtt{Q}g}(z',Q^2)dz'$$

$$P_{\mathtt{QQ}}(z,Q^2) = C_F\left(\frac{1+z^2}{1-z}\frac{Q^2}{m_{\mathtt{Q}}^2+Q^2} \;+\; \frac{z(1-3z)}{1-z}\frac{Q^2 m_{\mathtt{Q}}^2}{(Q^2+m_{\mathtt{Q}}^2)^2}\right)$$

$$P_{g\mathtt{Q}}(z,Q^2) = C_F\left(\frac{1+(1-z)^2}{z}\frac{Q^2}{m_{\mathtt{Q}}^2+Q^2} \;+\; \frac{z^2+z-2}{z}\frac{Q^2 m_{\mathtt{Q}}^2}{(Q^2+m_{\mathtt{Q}}^2)^2}\right)\Theta\left(Q^2-\frac{z m_{\mathtt{Q}}^2}{1-z}\right)$$

Running of the coupling



- Conventional approach
- Active number of light (zero mass) flavours

  = 3 for scales between strange and charm masses

  = 4 for scales between charm and bottom masses

  = 5 etc...

Running equation

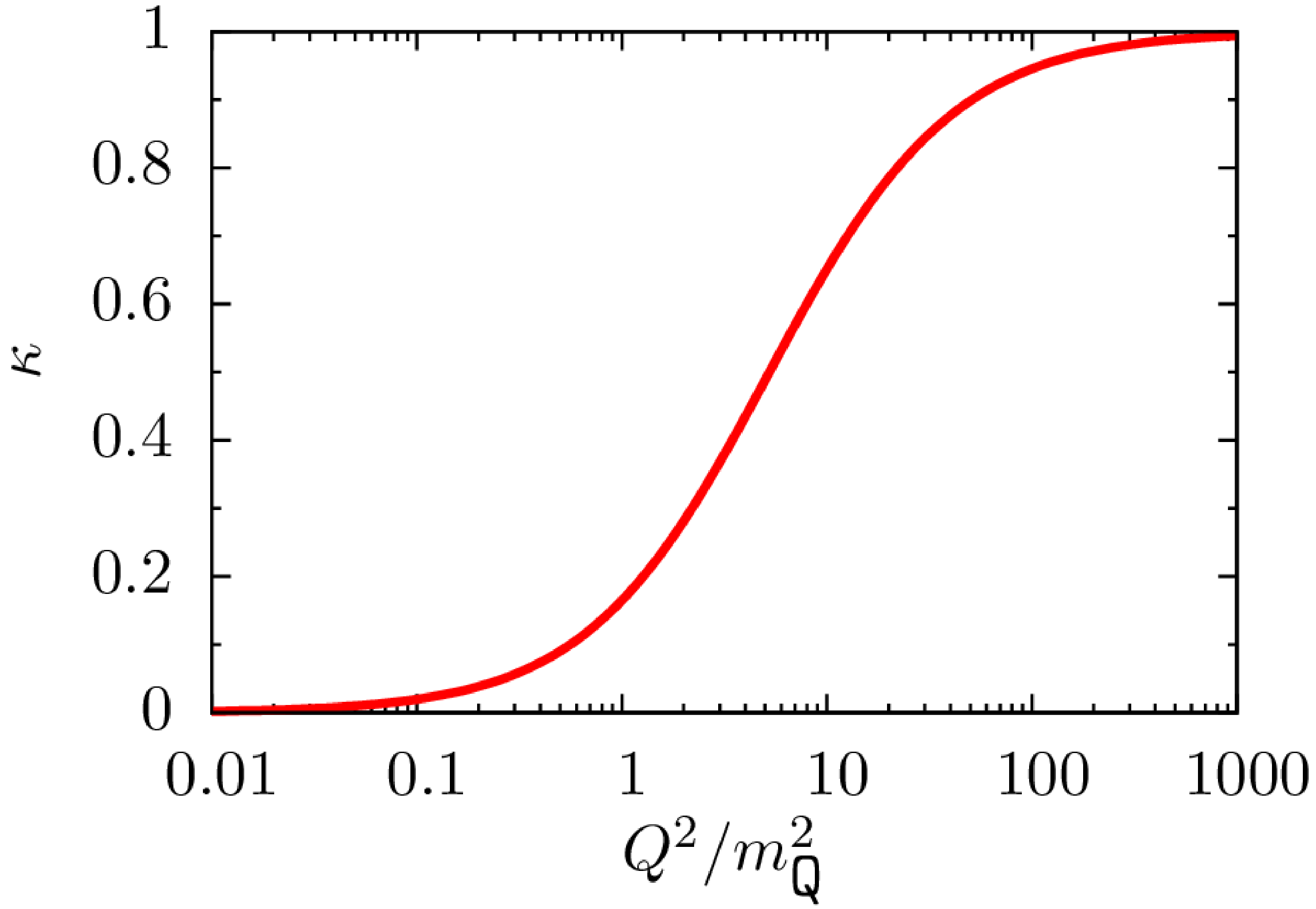
$$\frac{d}{d\ln Q^2}\left(\frac{\alpha_s}{4\pi}\right) \;=\; -\beta_0\left(\frac{\alpha_s}{4\pi}\right)^2 - \beta_1\left(\frac{\alpha_s}{4\pi}\right)^3$$

$$\beta_0(n_f) = 11 - \frac{2}{3}n_f, \qquad \beta_1(n_f) = 102 - \frac{38}{3}n_f$$

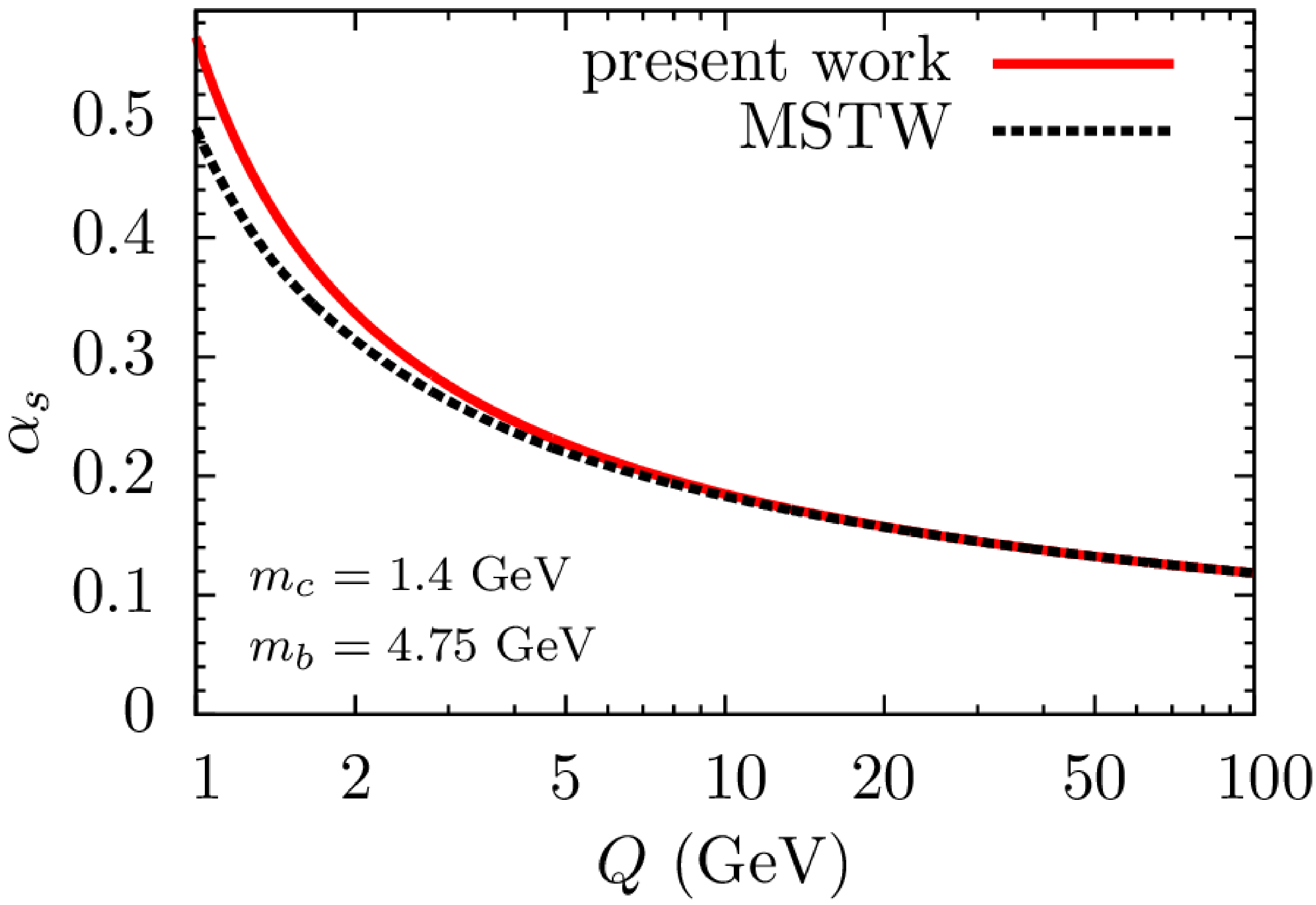
(Our work) Smooth transition: multiply each unit in n_F by: $r = m_Q^2/Q^2$

$$\kappa(r) \;=\; \left[1 - 6r + 12\frac{r^2}{\sqrt{1+4r}}\ln\frac{\sqrt{1+4r}+1}{\sqrt{1+4r}-1}\right]$$

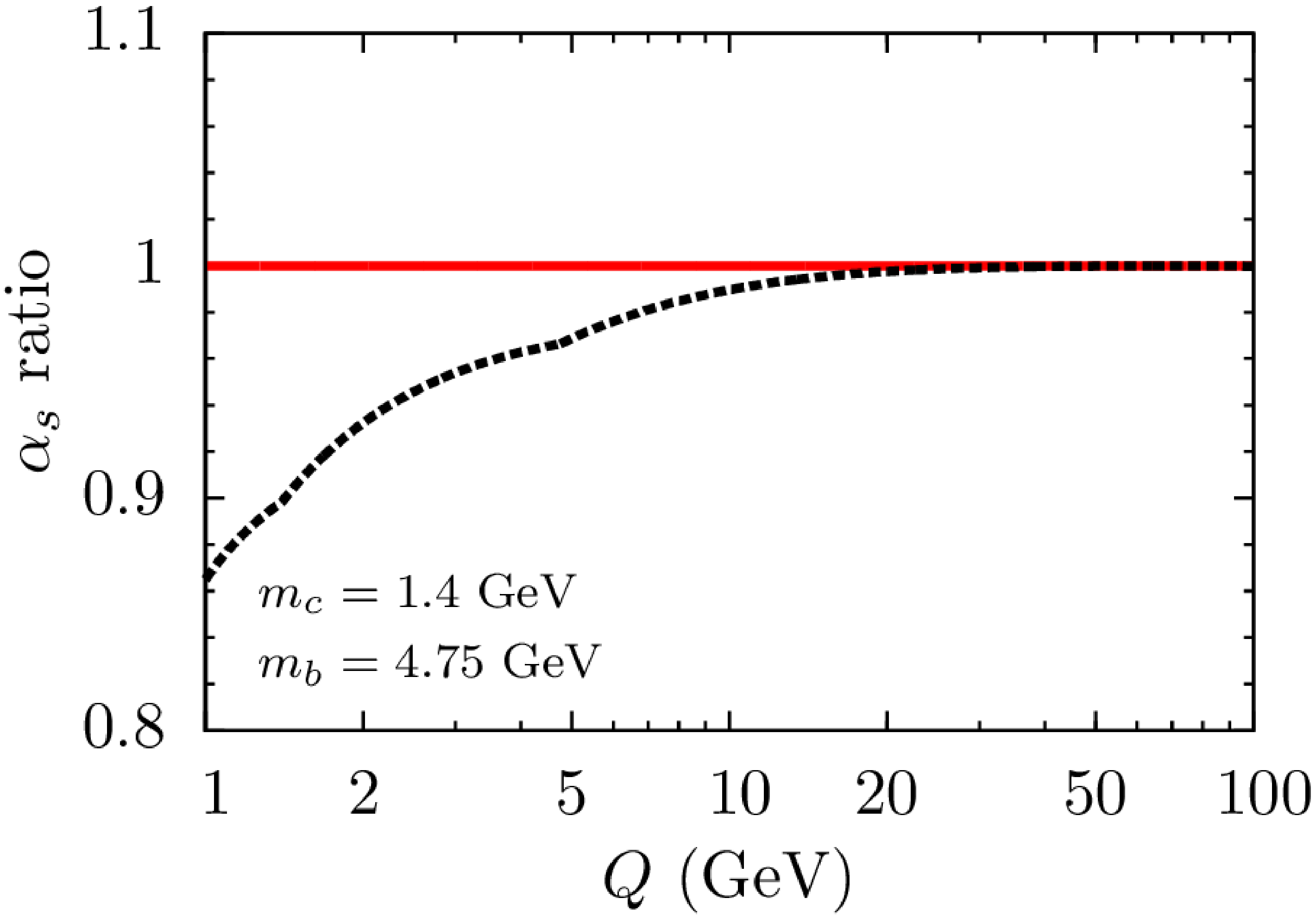
Smooth running



Change in the running



Ratio of change



Muito obrigado!

- Thank the organizers for having me.
- Special thanks to Alan Martin.

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