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BFKL Phenomenology

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In the last lecture we finally derived the BFKL equation

There was bibliography in the last slide of the lecture but we should stress that the method presented was based mainly on two books:

Vincenzo Barone   Enrico Predazzi

High-Energy Particle Diffraction

With 188 Figures

Springer

Quantum Chromodynamics and the Pomeron

J.R.FORSHAW
and
D.A.ROSS

Remember the gluon reggeization

$$A_8(s,t) = A^{(0)}(s,t)\left(1 + \ln\left(\frac{s}{|t|}\right)\epsilon(t) + \frac{1}{2}\ln^2\left(\frac{s}{|t|}\right)\epsilon^2(t) + \ldots\right)$$

An ansatz seems natural: $$A_8(s,t) = A^{(0)}(s,t)\left(\frac{s}{|t|}\right)^{\epsilon(t)}$$

$$D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2}\left(\frac{s}{\mathbf{k}^2}\right)^{\epsilon(q^2)}$$

$$\epsilon(t) = \frac{N_c\alpha_s}{4\pi^2}\int -\mathbf{q}^2\frac{d^2\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

The reggeization of the gluon; Bootstrap equation

$$\alpha_g(t) = 1 + \epsilon(t)$$

Let us pick it up from here...

Again, time to iterate, set the t-channel gluons to reggeized gluons, use the conditions:

$$\mathbf{k}_1^2 \simeq \mathbf{k}_2^2 \simeq \ldots \mathbf{k}_i^2 \simeq \mathbf{k}_{i+1}^2 \ldots \simeq \mathbf{k}_n^2 \simeq \mathbf{k}_{n+1}^2 \gg \mathbf{q}^2 \simeq s_0,$$

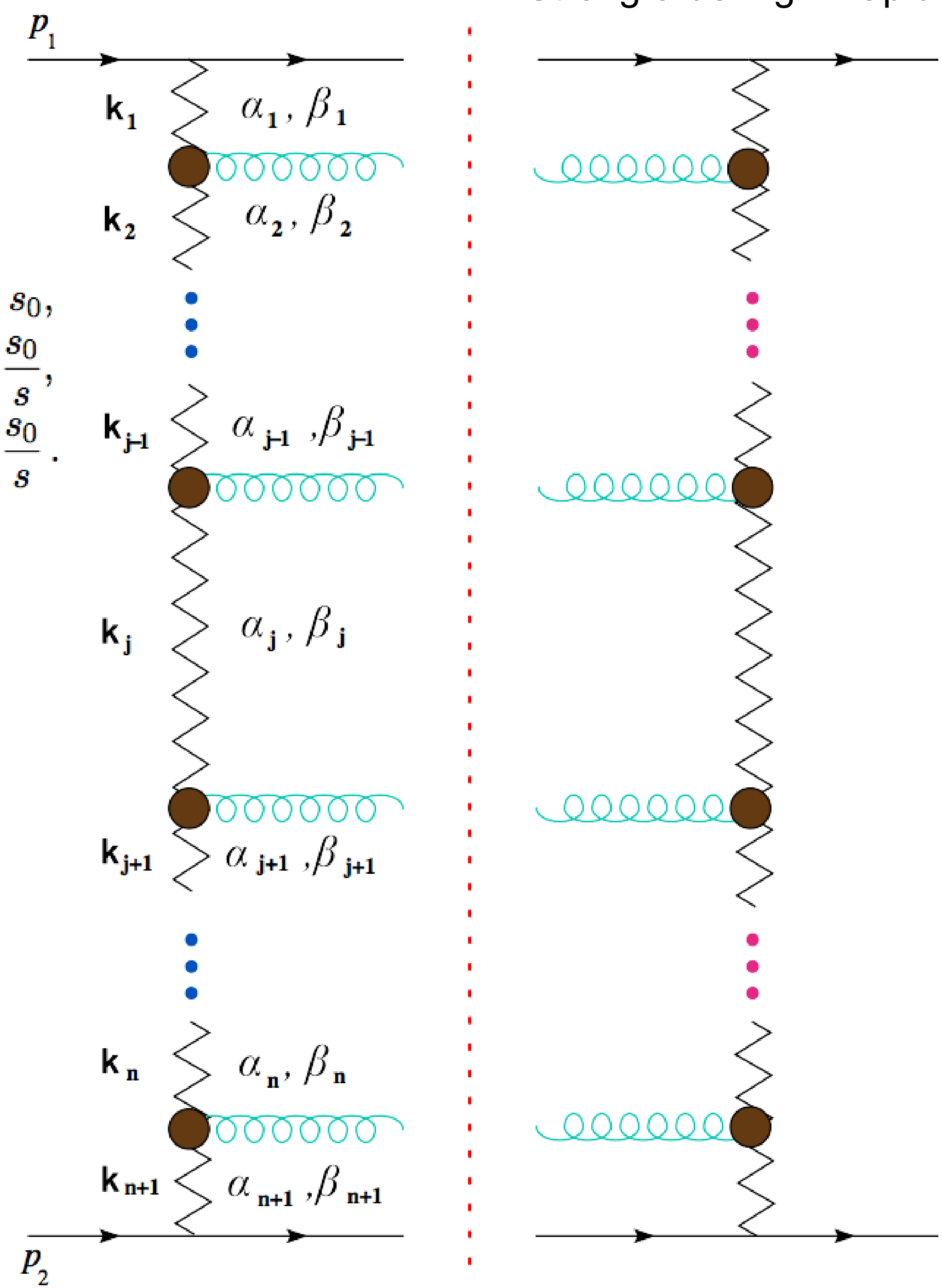
$$1 \gg \alpha_1 \gg \alpha_2 \gg \ldots \alpha_i \gg \alpha_{i+1} \gg \alpha_{n+1} \gg \frac{s_0}{s},$$

$$1 \gg |\beta_{n+1}| \gg |\beta_n| \gg \ldots \gg |\beta_2| \gg |\beta_1| \gg \frac{s_0}{s}.$$

and after the Mellin transform to unfold the nested integrations over phase space, you finally get:

$$\begin{aligned}
\omega f_\omega(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}) &= \delta^2(\mathbf{k}_1 - \mathbf{k}_2) \\
&+ \frac{\bar{\alpha}_s}{2\pi} \int d^2\mathbf{l} \Big\{ \frac{-\mathbf{q}^2}{(\mathbf{l}-\mathbf{q})^2 \mathbf{k}_1^2} f_\omega(\mathbf{l}, \mathbf{k}_2, \mathbf{q}) \\
&+ \frac{1}{(\mathbf{l}-\mathbf{k}_1)^2} \left(f_\omega(\mathbf{l}, \mathbf{k}_2, \mathbf{q}^2) - \frac{\mathbf{k}_1^2 f_\omega(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q})}{\mathbf{l}^2 + (\mathbf{k}_1 - \mathbf{l})^2} \right) \\
&+ \frac{1}{(\mathbf{l}-\mathbf{k}_1)^2} \left(\frac{(\mathbf{k}_1 - \mathbf{q})^2 \mathbf{l}^2 f_\omega(\mathbf{l}, \mathbf{k}_2, \mathbf{q}^2)}{(\mathbf{l}-\mathbf{q})^2 \mathbf{k}_1^2} \right. \\
&\left. - \frac{(\mathbf{k}_1 - \mathbf{q})^2 f_\omega(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}^2)}{(\mathbf{l}-\mathbf{q})^2 (\mathbf{k}_1 - \mathbf{l})^2} \right) \Big\},
\end{aligned}$$

Strong ordering in rapidity



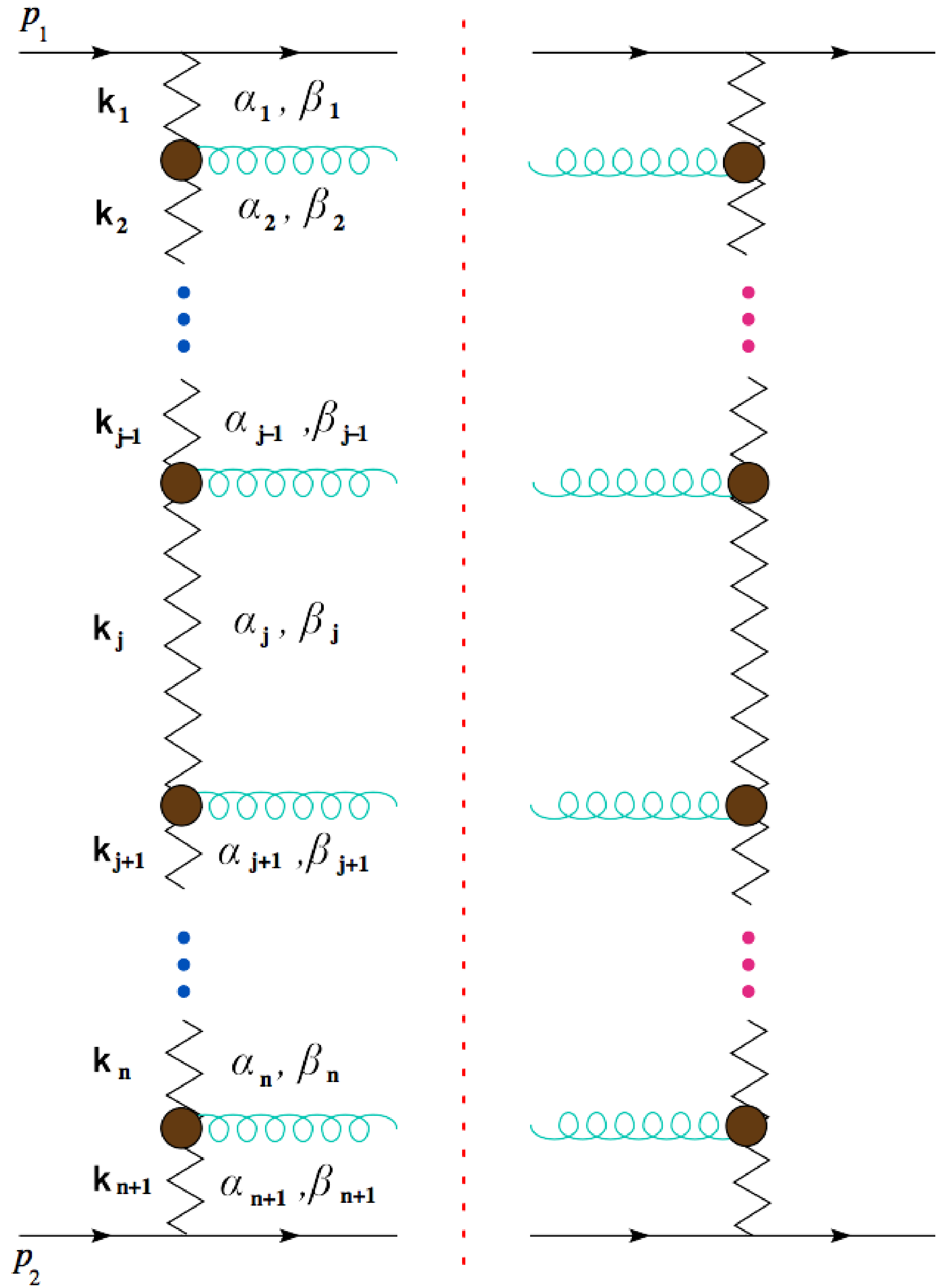
Fixed order VS resummation

Again on the whiteboard...

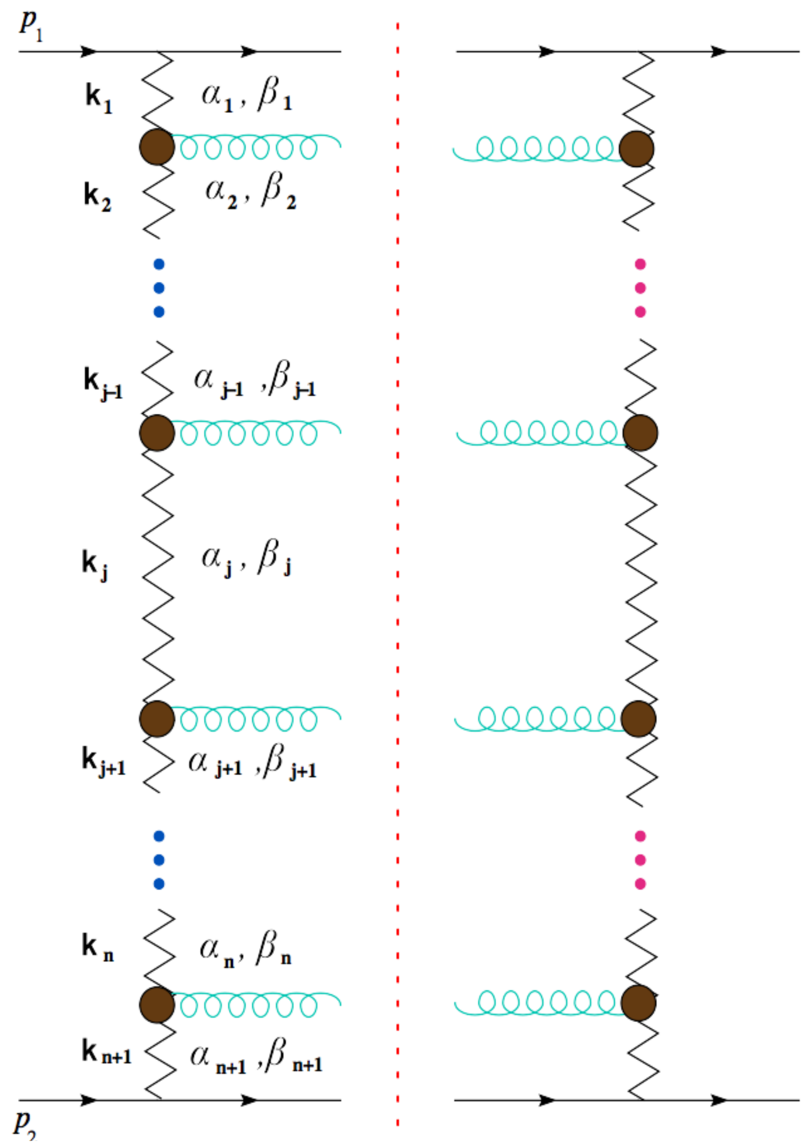
We should keep in mind that we are discussing a calculation in perturbation theory

Let us try to understand the BFKL equation

At this point we were calculating the imaginary part of the amplitude to the right. This kind of diagrams are the so-called ladder diagrams



Let us try to understand the BFKL equation



$$\mathrm{Im}\, A(s,t) = \frac{1}{2}\,(-1)^n\, g_{\rho_1\sigma_1}\cdots g_{\rho_n\sigma_n}$$

$$\times \int \mathrm{d}\Pi_{n+2}\, A^{\rho_1\ldots\rho_n}_{2\to n+2}(k_1,\ldots,k_n)\, A^{\sigma_1\ldots\sigma_n\,\dagger}_{2\to n+2}(k_1-q,\ldots,k_n-q)$$

Let us try to understand the BFKL equation

$$\mathrm{d}\Pi_{n+2} = \frac{s^{n+1}}{2^{n+1}(2\pi)^{3n+2}} \int \prod_{i=1}^{n+1} \mathrm{d}\alpha_i \, \mathrm{d}\beta_i \, \mathrm{d}^2\boldsymbol{k}_i$$
$$\times \delta\left(-\beta_1(1-\alpha_1)s - \boldsymbol{k}_1^2\right) \delta\left(\alpha_{n+1}(1+\beta_{n+1})s - \boldsymbol{k}_{n+1}^2\right)$$
$$\times \prod_{j=1}^{n} \delta\left((\alpha_j - \alpha_{j+1})(\beta_j - \beta_{j+1})s - (\boldsymbol{k}_j - \boldsymbol{k}_{j+1})^2\right) .$$

Remember we had to do something about the (n+2)-body phase space

After integrating over β_i we obtain:

$$\mathrm{d}\Pi_{n+2} = \frac{1}{2^{n+1}(2\pi)^{3n+2}} \prod_{i=1}^{n} \int_{\alpha_{i+1}}^{1} \frac{\mathrm{d}\alpha_i}{\alpha_i} \int_0^1 \mathrm{d}\alpha_{n+1}$$
$$\times \prod_{j=1}^{n+1} \int \mathrm{d}^2\boldsymbol{k}_j \, \delta\left(\alpha_{n+1}s - \boldsymbol{k}^2\right) .$$

Let us try to understand the BFKL equation

$$\operatorname{Im} A(s,t) = \frac{1}{2}\,(-1)^n\, g_{\rho_1\sigma_1}\cdots g_{\rho_n\sigma_n} \times \int \mathrm{d}\Pi_{n+2}\, A^{\rho_1\ldots\rho_n}_{2\to n+2}(k_1,\ldots,k_n)\, A^{\sigma_1\ldots\sigma_n\dagger}_{2\to n+2}(k_1-q,\ldots,k_n-q)$$

$$\operatorname{Im} \mathcal{A}_R(s,t) = \frac{1}{2}\sum_{n=0}^{\infty} 4s^2 g_s^4\,\mathcal{G}_R \int \mathrm{d}\Pi_{n+2}\,\frac{1}{\boldsymbol{k}_1^2(\boldsymbol{k}_1-\boldsymbol{q})^2}\left(\frac{1}{\alpha_1}\right)^{\epsilon(k_1^2)+\epsilon((k_1-q)^2)} \times \prod_{i=1}^{n}\left\{\frac{g_s^2}{\boldsymbol{k}_{i+1}^2(\boldsymbol{k}_{i+1}-\boldsymbol{q})^2}\,(-2\eta_R)\,K(\boldsymbol{k}_i,\boldsymbol{k}_{i+1}) \times \left(\frac{\alpha_i}{\alpha_{i+1}}\right)^{\epsilon(k_{i+1}^2)+\epsilon((k_{i+1}-q)^2)}\right\}$$

Contraction of Lipatov's effective vertices

$$C^{\rho_i}(k_i,k_{i+1})\,C_{\rho_i}(-k_i+q,-k_{i+1}+q) = -2\left[\boldsymbol{q}^2 - \frac{\boldsymbol{k}_i^2\,(\boldsymbol{k}_{i+1}-\boldsymbol{q})^2}{(\boldsymbol{k}_i-\boldsymbol{k}_{i+1})^2} - \frac{\boldsymbol{k}_{i+1}^2\,(\boldsymbol{k}_i-\boldsymbol{q})^2}{(\boldsymbol{k}_i-\boldsymbol{k}_{i+1})^2}\right] \equiv -2\,K(\boldsymbol{k}_i,\boldsymbol{k}_{i+1})$$

Let us try to understand the BFKL equation

Remember also that to unfold the nested integration we took a Mellin transform

$$f_R(\omega,t) = \int_1^\infty \mathrm{d}\left(\frac{s}{|t|}\right) \left(\frac{s}{|t|}\right)^{-\omega-1} \frac{\mathrm{Im}\,\mathcal{A}_R(s,t)}{s}$$

$$\frac{\mathrm{Im}\,\mathcal{A}_R(s,t)}{s} = \frac{1}{2\pi\mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \mathrm{d}\omega \left(\frac{s}{|t|}\right)^{\omega} f_R(\omega,t)$$

Let us try to understand the BFKL equation

$$f_R(\omega, \boldsymbol{q}^2) = (4\pi\alpha_s)^2\, \mathcal{G}_R \sum_{n=0}^{\infty} \prod_{i=1}^{n+1} \frac{\mathrm{d}^2\boldsymbol{k}_i}{(2\pi)^2}$$

$$\times \frac{1}{\boldsymbol{k}_1^2(\boldsymbol{k}_1 - \boldsymbol{q})^2} \frac{1}{\omega - \epsilon(k_1^2) - \epsilon((k_1 - q)^2)}$$

$$\times (-2\alpha_s\eta_R)\, K(\boldsymbol{k}_1, \boldsymbol{k}_2)$$

$$\times \frac{1}{\boldsymbol{k}_2^2(\boldsymbol{k}_2 - \boldsymbol{q})^2} \frac{1}{\omega - \epsilon(k_2^2) - \epsilon((k_2 - q)^2)}$$

$$\vdots$$

$$\times (-2\alpha_s\eta_R)\, K(\boldsymbol{k}_n, \boldsymbol{k}_{n+1})$$

$$\times \frac{1}{\boldsymbol{k}_{n+1}^2(\boldsymbol{k}_{n+1} - \boldsymbol{q})^2} \frac{1}{\omega - \epsilon(k_{n+1}^2) - \epsilon((k_{n+1} - q)^2)}$$

Let us try to understand the BFKL equation

Let us define the following:

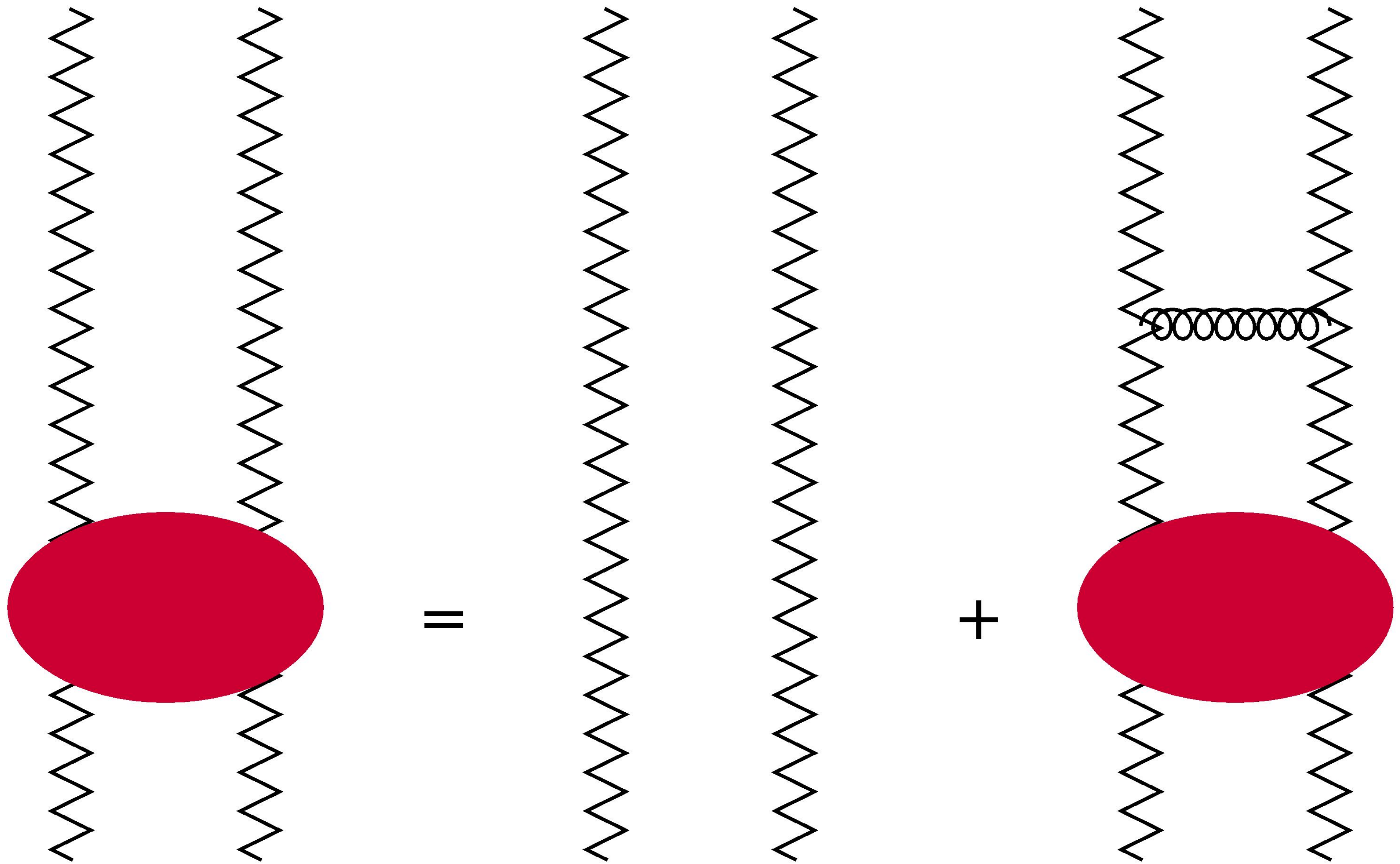
$$f_{\underline{1}}(\omega, \boldsymbol{q}^2) = (8\pi^2\alpha_s)^2 \frac{N_c^2 - 1}{4N_c} \int \frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{k}'}{(2\pi)^2} \frac{F(\omega, \boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})}{\boldsymbol{k}'^2(\boldsymbol{k}-\boldsymbol{q})^2}$$

Then we will have the following integral equation in which we encode the behaviour of $f_1(\omega, \mathbf{q}^2)$:

$$\left[\omega - \epsilon(-\boldsymbol{k}^2) - \epsilon(-(\boldsymbol{k}-\boldsymbol{q})^2)\right] F(\omega, \boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})$$
$$= \delta^2(\boldsymbol{k}-\boldsymbol{k}') - \frac{N_c\alpha_s}{2\pi^2} \int \mathrm{d}^2\boldsymbol{\kappa} \frac{K(\boldsymbol{k}, \boldsymbol{\kappa})}{\boldsymbol{k}^2(\boldsymbol{\kappa}-\boldsymbol{q})^2} F(\omega, \boldsymbol{\kappa}, \boldsymbol{k}', \boldsymbol{q})$$

The subscript R will be from now on 1

Let us try to understand the BFKL equation



The BFKL equation

$$
\begin{aligned}
\omega\, F(\omega, \boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) &= \delta^2(\boldsymbol{k} - \boldsymbol{k}') \\
&+ \frac{N_c \alpha_s}{2\pi^2} \int \mathrm{d}^2\boldsymbol{\kappa} \left\{ \frac{-\boldsymbol{q}^2}{(\boldsymbol{\kappa} - \boldsymbol{q})^2 \boldsymbol{k}^2}\, F(\omega, \boldsymbol{\kappa}, \boldsymbol{k}', \boldsymbol{q}) \right. \\
&+ \frac{1}{(\boldsymbol{\kappa} - \boldsymbol{k})^2} \left[F(\omega, \boldsymbol{\kappa}, \boldsymbol{k}', \boldsymbol{q}) - \frac{\boldsymbol{k}^2\, F(\omega, \boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})}{\boldsymbol{\kappa}^2 + (\boldsymbol{k} - \boldsymbol{\kappa})^2} \right] \\
&+ \frac{1}{(\boldsymbol{\kappa} - \boldsymbol{k})^2} \left[\frac{(\boldsymbol{k} - \boldsymbol{q})^2\, \boldsymbol{\kappa}^2\, F(\omega, \boldsymbol{\kappa}, \boldsymbol{k}', \boldsymbol{q})}{(\boldsymbol{\kappa} - \boldsymbol{q})^2\, \boldsymbol{k}^2} \right. \\
&\left.\left. - \frac{(\boldsymbol{k} - \boldsymbol{q})^2\, F(\omega, \boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})}{(\boldsymbol{\kappa} - \boldsymbol{q})^2 + (\boldsymbol{k} - \boldsymbol{\kappa})^2} \right] \right\}
\end{aligned}
$$

To complete the story...

Suppose now that we know $F(\omega, \boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})$

The we take an inverse Mellin transform to go back to s-space

$$F(s, \boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) = \frac{1}{2\pi \mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \mathrm{d}\omega \left(\frac{s}{|t|} \right)^{\omega} F(\omega, \boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})$$

And to recover the imaginary part of the ladder diagrams all we need to do is:

$$\frac{\operatorname{Im} \mathcal{A}_{\underline{1}}(s,t)}{s} = (8\pi^2 \alpha_s)^2 \frac{N_c^2 - 1}{4N_c} \int \frac{\mathrm{d}^2 \boldsymbol{k}}{(2\pi)^2} \int \frac{\mathrm{d}^2 \boldsymbol{k}'}{(2\pi)^2} \frac{F(s, \boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})}{\boldsymbol{k}'^2 (\boldsymbol{k} - \boldsymbol{q})^2}$$

The BFKL equation for zero momentum transfer, **q**=0

$$\omega F(\omega, \boldsymbol{k}, \boldsymbol{k}') = \delta^2(\boldsymbol{k} - \boldsymbol{k}') + \frac{N_c \alpha_s}{\pi^2} \int \frac{\mathrm{d}^2 \boldsymbol{\kappa}}{(\boldsymbol{k} - \boldsymbol{\kappa})^2} \times \left[F(\omega, \boldsymbol{\kappa}, \boldsymbol{k}') - \frac{\boldsymbol{k}^2}{\boldsymbol{\kappa}^2 + (\boldsymbol{k} - \boldsymbol{\kappa})^2} F(\omega, \boldsymbol{k}, \boldsymbol{k}') \right]$$

Or symbolically:

$$\omega F(\omega, \boldsymbol{k}, \boldsymbol{k}') = \delta^2(\boldsymbol{k} - \boldsymbol{k}') + \int \mathrm{d}^2 \boldsymbol{\kappa} \, \mathcal{K}(\boldsymbol{k}, \boldsymbol{\kappa}) \, F(\omega, \boldsymbol{\kappa}, \boldsymbol{k}')$$

where $$\mathcal{K}(\boldsymbol{k}, \boldsymbol{\kappa}) = 2 \epsilon(-\boldsymbol{k}^2) \, \delta^2(\boldsymbol{k} - \boldsymbol{\kappa}) + \frac{N_c \alpha_s}{\pi^2} \frac{1}{(\boldsymbol{k} - \boldsymbol{\kappa})^2}$$

$$\mathcal{K}_{\mathrm{virt}}(\boldsymbol{k}, \boldsymbol{\kappa}) = 2 \epsilon(-\boldsymbol{k}^2) \, \delta^2(\boldsymbol{k} - \boldsymbol{\kappa})$$

$$\mathcal{K}_{\mathrm{real}}(\boldsymbol{k}, \boldsymbol{\kappa}) = \frac{N_c \alpha_s}{\pi^2} \frac{1}{(\boldsymbol{k} - \boldsymbol{\kappa})^2}$$

SOLVING THE BFKL EQUATION

Solution for zero momentum transfer

Let us write symbolically:

$$\omega F = \mathbb{1} + \mathcal{K} \otimes F$$

By solving the equation we mean finding eigenfunctions such that:

$$\mathcal{K} \otimes \phi_\alpha = \omega_\alpha \phi_\alpha$$

The eigenfunction obey the completeness relation:

$$\sum_\alpha \phi_\alpha(\boldsymbol{k})\, \phi^*_\alpha(\boldsymbol{k}') = \delta^2(\boldsymbol{k} - \boldsymbol{k}')$$

Then the solution to the first equation will be:

$$F(\omega, \boldsymbol{k}, \boldsymbol{k}') = \sum_\alpha \frac{\phi_\alpha(\boldsymbol{k})\, \phi^*_\alpha(\boldsymbol{k}')}{\omega - \omega_\alpha}$$

α denotes a set of indices that can be discrete or continuous and the summation symbol can hide an integration

Solution for zero momentum transfer

Let us write symbolically: $$\omega F = \mathbb{1} + \mathcal{K} \otimes F$$

By solving the equation we mean finding eigenfunctions such that: $$\mathcal{K} \otimes \phi_\alpha = \omega_\alpha \phi_\alpha$$

Actually, if we use polar coordinates $$\boldsymbol{k} \equiv (|\boldsymbol{k}|, \vartheta)$$

the eigenfunctions are: $$\phi_{n\nu}(|\boldsymbol{k}|, \vartheta) = \frac{1}{\pi\sqrt{2}} \left(\boldsymbol{k}^2\right)^{-\frac{1}{2}+\mathrm{i}\nu} \mathrm{e}^{\mathrm{i}n\vartheta}$$

obeying: $$\int \mathrm{d}^2\boldsymbol{k}\, \phi_{n\nu}(\boldsymbol{k})\, \phi_{n'\nu'}(\boldsymbol{k}) = \delta_{nn'}\, \delta(\nu - \nu')$$

whereas the eigenvalues are: $$\omega_n(\nu) = -\frac{2\alpha_s N_c}{\pi} \mathrm{Re} \left[\psi\left(\frac{|n|+1}{2} + \mathrm{i}\nu\right) - \psi(1)\right]$$

Solution for zero momentum transfer

The solution will then be:

$$F(\omega, \boldsymbol{k}, \boldsymbol{k}') = \frac{1}{2\pi^2 \left(\boldsymbol{k}^2 \boldsymbol{k}'^2\right)^{\frac{1}{2}}} \sum_{n=0}^{\infty} \mathrm{e}^{\mathrm{i}n(\vartheta - \vartheta')} \int_{\infty}^{+\infty} \mathrm{d}\nu \, \frac{\mathrm{e}^{\mathrm{i}\nu \, \ln\left(\frac{\boldsymbol{k}^2}{\boldsymbol{k}'^2}\right)}}{\omega - \omega_n(\nu)}$$

Here, n is also called conformal spin, it is connected to the angular information encoded in the gluon Green's function.

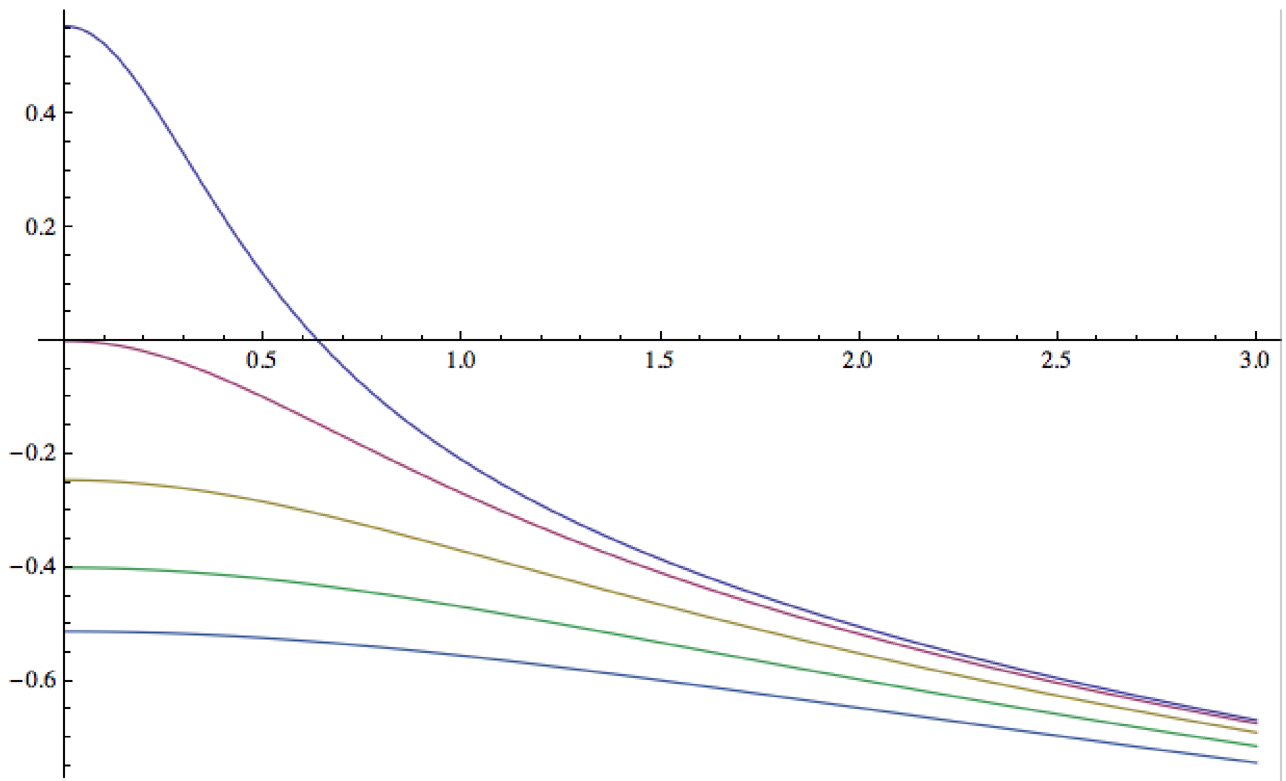
Solution for zero momentum transfer

Hands on... Let us use Mathematica to plot things and draw conclusions

```
omega[n_, v_] := Module[{asBar = 1/5},
   Return[2 asBar (PolyGamma[0, 1] -
     Re[PolyGamma[(Abs[n] + 1)/2 + I v]])]];

Plot[{omega[0,  ], omega[1,  ], omega[2,  ],
  omega[3,  ], omega[4,  ]}, { , 0, 3}]
```

Out[6]=



Solution for zero momentum transfer

$$F(\omega, \boldsymbol{k}, \boldsymbol{k}') = \frac{1}{2\pi^2 \left(\boldsymbol{k}^2 \boldsymbol{k}'^2\right)^{\frac{1}{2}}} \sum_{n=0}^{\infty} \mathrm{e}^{in(\vartheta - \vartheta')} \int_{\infty}^{+\infty} \mathrm{d}\nu \, \frac{\mathrm{e}^{\mathrm{i}\nu \, \ln\left(\frac{\boldsymbol{k}^2}{\boldsymbol{k}'^2}\right)}}{\omega - \omega_n(\nu)}$$

Retain only the n=0 term, this from the analysis before

$$F(\omega, \boldsymbol{k}, \boldsymbol{k}') = \frac{1}{2\pi^2 \left(\boldsymbol{k}^2 \boldsymbol{k}'^2\right)^{\frac{1}{2}}} \int_{-\infty}^{+\infty} \mathrm{d}\nu \, \frac{\mathrm{e}^{\mathrm{i}\,\nu \, \ln\left(\frac{\boldsymbol{k}^2}{\boldsymbol{k}'^2}\right)}}{\omega - \omega_0(\nu)}$$

Expanding around zero where we have the maximum gives:

$$\omega_0(\nu) = \frac{N_c \alpha_s}{\pi} \left(4 \, \ln 2 - 14 \, \zeta(3) \, \nu^2 + \ldots\right)$$

Solution for zero momentum transfer

$$\omega_0(\nu) = \frac{N_c \alpha_s}{\pi} \left(4 \ln 2 - 14 \zeta(3) \nu^2 + \ldots\right)$$

Set: $$\lambda = \frac{N_c \alpha_s}{\pi} 4 \ln 2 , \quad \lambda' = \frac{N_c \alpha_s}{\pi} 28 \zeta(3)$$

Take the inverse Mellin transform

$$F(s, \boldsymbol{k}, \boldsymbol{k}') = \frac{1}{2\pi^2 (\boldsymbol{k}^2 \boldsymbol{k}'^2)^{\frac{1}{2}}} \int_{-\infty}^{+\infty} \mathrm{d}\nu \left(\frac{s}{\boldsymbol{k}^2}\right)^{\omega_0(\nu)} \mathrm{e}^{\mathrm{i}\,\nu\, \ln\left(\frac{\boldsymbol{k}^2}{\boldsymbol{k}'^2}\right)}$$

$$F(s, \boldsymbol{k}, \boldsymbol{k}') = \frac{1}{\sqrt{2\pi^3 \lambda' \boldsymbol{k}^2 \boldsymbol{k}'^2}} \frac{1}{\sqrt{\ln(s/\boldsymbol{k}^2)}} \times \left(\frac{s}{\boldsymbol{k}^2}\right)^{\lambda} \exp\left[-\frac{\ln^2(\boldsymbol{k}^2/\boldsymbol{k}'^2)}{2\lambda' \ln(s/\boldsymbol{k}^2)}\right]$$

Pomeron solution of the BFKL equation

Solution for zero momentum transfer

$$F(s,\boldsymbol{k},\boldsymbol{k}') = \frac{1}{\sqrt{2\pi^3\lambda'\,\boldsymbol{k}^2\boldsymbol{k}'^2}}\,\frac{1}{\sqrt{\ln(s/\boldsymbol{k}^2)}} \times \left(\frac{s}{\boldsymbol{k}^2}\right)^{\lambda} \exp\left[-\frac{\ln^2(\boldsymbol{k}^2/\boldsymbol{k}'^2)}{2\lambda'\,\ln(s/\boldsymbol{k}^2)}\right]$$

$$\alpha_{I\!P}(0) = 1 + \lambda = 1 + \frac{N_c\alpha_s}{\pi}\,4\,\ln 2$$

QCD Pomeron intercept way too large in comparison to the soft Pomeron intercept

Solution for zero momentum transfer

Again in Mathematica:

```
omega[n_, v_] := Module[{asBar = 1/5},
   Return[2 asBar (PolyGamma[0, 1] -
     Re[PolyGamma[(Abs[n] + 1)/2 + I v]])]];
```

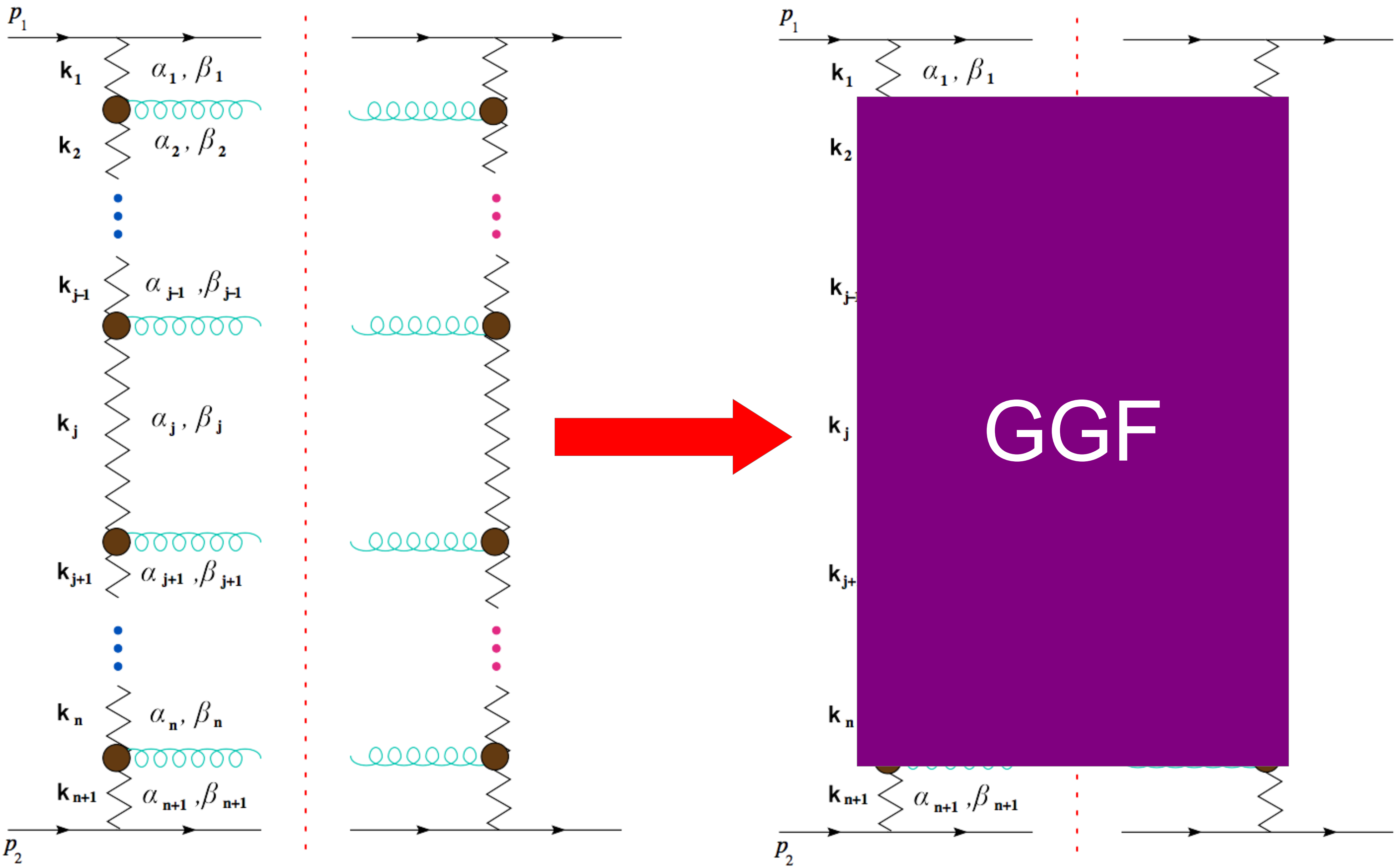
```
analytic[n_, Y_, ka_, kb_, angle_] :=
NIntegrate[Exp[I*n*angle]/(2Pi^2)/ka/kb*2*Exp[omega[n,v]Y]*
Cos[2 Log[(ka/kb)] v], {v, 0, Infinity}, WorkingPrecision -> 20];
```

Now you can calculate the LO gluon Green's function for a given rapidity Y, conformal spin n, and certain momenta of the reggeized gluons.

Note: Many times, in the literature, the leading eigenvalue is denoted as X_0. It is also called sometimes as the LO BFKL kernel!

$$\chi_0(\nu) = -2\,\mathrm{Re}\left\{\psi\left(\frac{1}{2}+i\nu\right)-\psi(1)\right\}$$

The gluon Green's function



To connect with the lectures by Beatriz and Edmond

$$\frac{\partial F(s,\boldsymbol{k},\boldsymbol{k}')}{\partial \ln(s/\boldsymbol{k}^2)} = \frac{1}{2\pi \mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \mathrm{d}\omega \left(\frac{s}{\boldsymbol{k}^2}\right)^{\omega} \omega\, F(\omega,\boldsymbol{k},\boldsymbol{k}')$$

$$\frac{\partial F(s,\boldsymbol{k},\boldsymbol{k}')}{\partial \ln(s/\boldsymbol{k}^2)} = \frac{N_c \alpha_s}{\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{\kappa}}{(\boldsymbol{k}-\boldsymbol{\kappa})^2} \times \left[F(s,\boldsymbol{\kappa},\boldsymbol{k}') - \frac{\boldsymbol{k}^2}{\boldsymbol{\kappa}^2 + (\boldsymbol{k}-\boldsymbol{\kappa})^2} F(s,\boldsymbol{k},\boldsymbol{k}')\right]$$

} Evolution eq. in rapidity

$$f(x,\boldsymbol{k}_\perp^2) \equiv \frac{\partial\left[xg(x,\boldsymbol{k}_\perp^2)\right]}{\partial \ln \boldsymbol{k}_\perp^2}$$

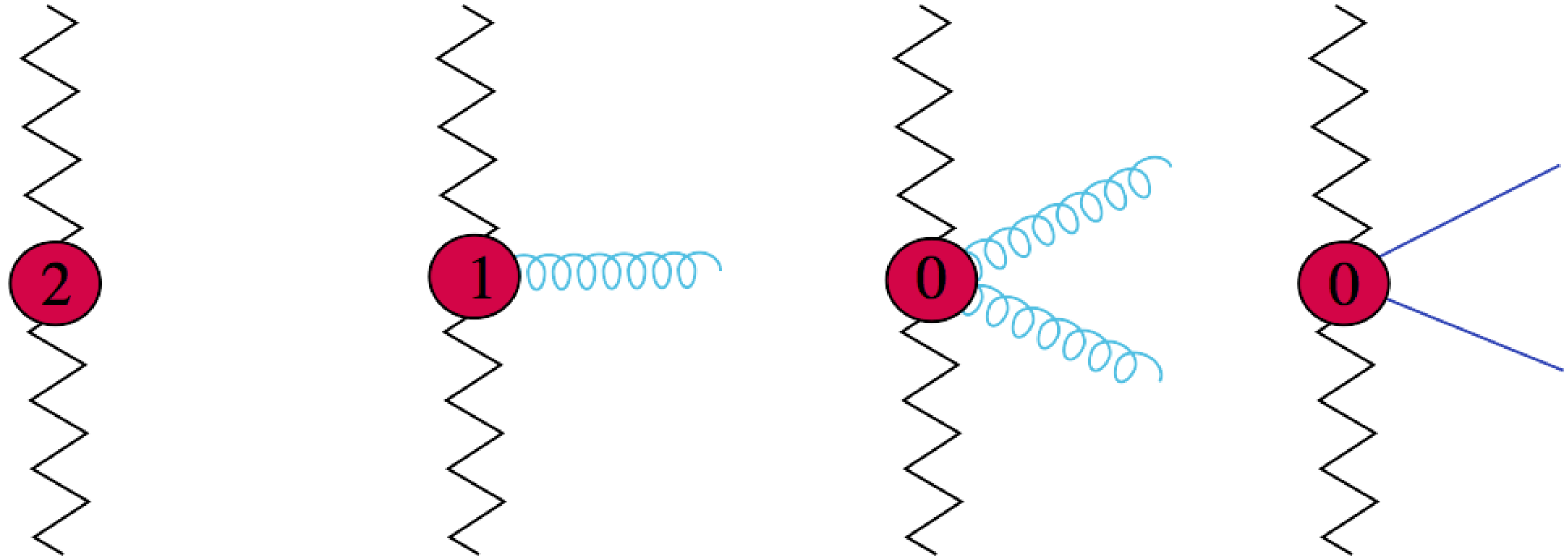
Unintegrated gluon distribution: the probability to find a gluon with longitudinal momentum fraction x and transverse momentum $\boldsymbol{k}$

} DIS

Bibliography II (very incomplete)

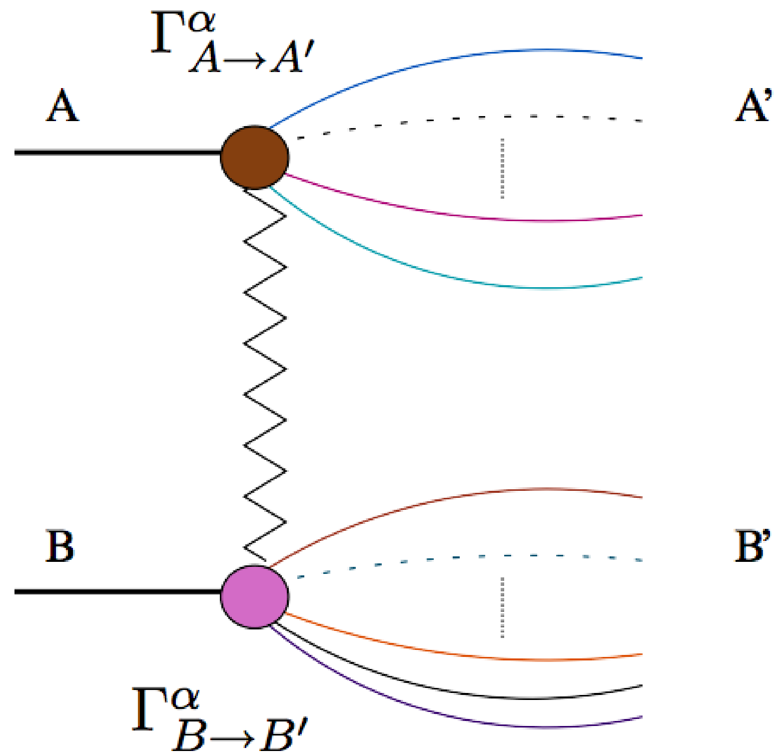
- Forshaw & Ross, “Quantum Chromodynamics and the Pomeron”
- Barone & Predazzi, High Energy Particle Diffraction
- Ioffe, Fadin & Lipatov, “Quantum Chromodynamics: Perturbative and Nonperturbative Aspects”
- Kovchegov & Levin, “Quantum Chromodynamics at High Energy”
- Many many review articles...

NLO BFKL



2–loop trajectory 1–loop g emission pair production

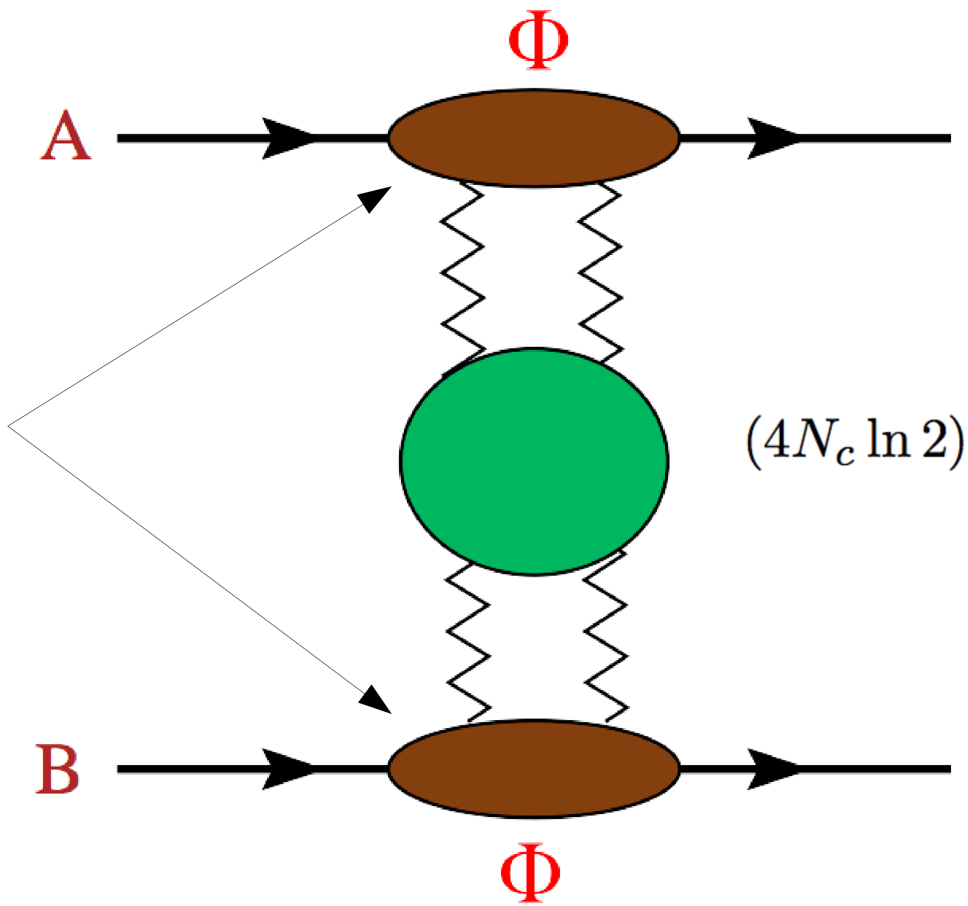
Regge ansatz



$$\mathcal{M}_{AB} = \frac{s}{t}\Gamma^{\alpha}_{A\to A'}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{\alpha}_{B\to B'}$$

A hadronic elastic amplitude

Impact factors... ?



$(4N_c \ln 2)/\pi)\alpha_s \approx 0.5.\ (\alpha_s \sim 0.2)$

$$\mathcal{A}(s,t) = i\, s\, \mathcal{C} \int \frac{d^2\mathbf{k}_1}{(2\pi)^2} \frac{d^2\mathbf{k}_2}{(2\pi)^2} \Phi_A(\mathbf{k}_1,\mathbf{q}) \frac{f(s,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})}{\mathbf{k}_2^2(\mathbf{k}_1-\mathbf{q})^2} \Phi_B(\mathbf{k}_2,\mathbf{q})$$

Impact factors

Impact factors are effective couplings of the BFKL gluon Green's function to the colliding projectiles

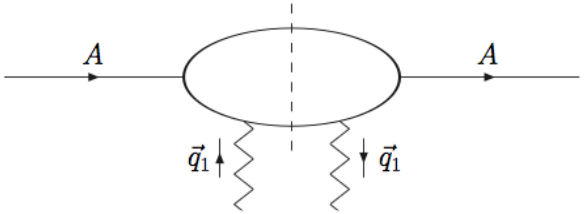
They are process dependent objects

One needs to calculate them at a certain order of the perturbative expansion, preferably the same one as that of the BFKL gluon Green's function.

It is not an easy task to calculate impact factors to NLO.

Impact factors

- **Impact factors** are **process-dependent**;



  only very few have been calculated in the NLA:

  - colliding partons
    [V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
    [M. Ciafaloni and G. Rodrigo (2000)]
  - $\gamma^* \longrightarrow V$, with $V = \rho^0, \omega, \phi$, forward case
    [D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]
  - forward jet production
    [J. Bartels, D. Colferai, G.P. Vacca (2003)]
    [F. Caporale, D.Yu. Ivanov, B. M., A. Papa, A. Perri (2012)]
    (small-cone approximation) [D.Yu. Ivanov, A. Papa (2012)]
  - forward identified hadron production [D.Yu. Ivanov, A. Papa (2012)]
  - $\gamma^* \longrightarrow \gamma^*$ [J. Bartels *et al.* (2001) $\rightarrow$]
    [I. Balitsky, G.A. Chirilli (2011)-(2014)]

Taken from a talk by B. Murdaca