SPONTANEOUSLY BROKEN GAUGE THEORIES

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The well known example of gauge theory with spontaneously broken symmetry is given by the Weinberg-Salam model. The Weinberg-Salam model is the model of electro-weak interaction on the basis of the group $U(2) = SU(2) \times U(1)$. The electron and neutrino are described by the multiplets with respect to the group $SU(2)$. Electron and neutrino with the left polarisation form a doublet

$$L = 1/2(1+\gamma_5)(\nu, e) \tag{1}$$

where ν and e are four component Dirac spinors. Right handed electron is the singlet with respect to the $SU(2)$-group

$$R = 1/2(1-\gamma_5)e \tag{2}$$

That means under the transformations belonging to $SU(2)$

$$L(x) \to L(x) - ig\frac{\tau^a}{2}\xi^a(x)L(x); \quad R(x) \to R(x) \tag{3}$$

Under the transformations $U(1)$

$$L(x) \to L(x) - \frac{ig_1}{2}\eta(x)L(x); \quad R(x) \to -ig_1\eta(x)R(x) \quad (4)$$

The group $U(2)$ is not simple, therefore the constants g and g_1 are independent. The special choice, given above corresponds to the standard form of the electromagnetic current.

The gauge invariant Lagrangian describing the interaction of the fields R and L with the gauge fields corresponding to the groups $SU(2)$ and $U(1)$ is

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \frac{1}{4}G_{\mu\nu}G_{\mu\nu} + i\bar{L}\gamma_\mu(\partial_\mu + ig\frac{\tau^a}{2}A^a_\mu + \frac{ig}{2}B_\mu)L + i\bar{R}\gamma_\mu(\partial_\mu + ig_1 B_\mu)R. \tag{5}$$

The interaction contains the terms

$$\bar{e}\gamma_\mu(1+\gamma_5)\nu_e A^-_\mu \tag{6}$$

It contains charged lepton currents, but if one quantize the fields A^a_μ in the usual way, these fields will have zero mass.

If the fields A_μ^a were massive their propagators would be

$$D_{\mu\nu}^{ab} = \delta^{ab} \frac{g_{\mu\nu} - k_\mu k_\nu m^{-2}}{k^2 - m^2} \quad (7)$$

If $k^2 << m^2$, at low energies we have the interaction $j_\mu^+ j_\mu$. Introduction of the mass term for vector mesons breaks the gauge invariance and makes the theory nonrenormalizable. Gauge invariance plays the essential role in the proof of renormalizability. It provides the equivalence of the unitary gauge (as Coulomb gauge in the Yang-Mills theory) to some explicitely renormalizable gauge (for example the Lorentz gauge $\partial_\mu A_\mu = 0$).

The main achievement of Englert-Brout-Higgs was a disco-

very that the gauge invariance may be saved at the expense of introducing the additional interaction with the scalar field. They considered the Abelian theory. Generalization to the non-Abelian case was done by Kibble, and the renormalizability of the Weinberg-Salam model was proven by G.t'Hooft.

For that purpose one may introduce the complex doublet of scalar fields $\varphi = (\varphi_1, \varphi_2)$ Gauge invariant Lagrangian, which describes the interaction of scalar fields with gauge fields looks as follows

$$L = |\partial_\mu \varphi + ig\frac{\tau^a}{2} A^a_\mu \varphi + \frac{ig_1}{2} B_\mu \varphi|^2 + \frac{m^2}{2}\varphi^+ \varphi - \lambda^2 (\varphi^+ \varphi)^2 \tag{8}$$

Gauge invariant interaction between scalar fields and fermions may be also added

$$\tilde{L} = -G[(\bar{L}\varphi)R + \bar{R}(\varphi^+ L)] \tag{9}$$

The mass term for the scalar field enters with the wrong sign. Hence the state $\varphi = 0$ is unstable, and to develop perturbation theory in the vicinity of a stable extremum, we should perform a shift

$$\varphi \to \varphi' = (\varphi_1, \varphi_2 + \kappa); \quad Im\kappa = 0 \tag{10}$$

After such a shift the mass terms for the fields A_μ, B_μ, L, R and one component of the scalar field φ appears.

After the shift the explicit form of the gauge transformation of the fields φ changes. The field φ also becomes the gauge field: it changes by arbitrary function

$$\varphi \to \varphi - ig\xi^a \frac{\tau^a}{2}\varphi + \frac{i\mu}{2} g(\xi_1 + i\xi_2, \xi_3) \qquad (11)$$

One can choose the functions ξ_1, ξ_2, ξ_3 in such a way to nulify the component φ_1 and imaginary part of φ_2.

In the following we shall consider in details the non-Abelian case, corresponding to the $SU(2)$ gauge field interacting with the scalar doublet. Let us start with the gauge invariant Lagrangian

$$L = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + (D_\mu \varphi)^+ D_\mu \varphi - [-\mu^2 \varphi^+ \varphi + \lambda(\varphi^+ \varphi)^2] \quad (12)$$

As we mentioned before this Lagrangian is unstable and to develop the perturbation theory around the stable minimum one should shift the field φ

$$\varphi \to \varphi' = (\varphi_1, \varphi_2 + \kappa); \quad Im\kappa = 0 \qquad (13)$$

After this shift the theory obviously remains gauge invariant, but the field φ' becomes a gauge field, that means under the gauge transformation it changes by arbitrary three component function.

We can choose this function in such a way, that the first component of φ' disappears, and the second component becomes purely real. In the following we denote $\varphi'_2 = \sigma; \quad Im\sigma = 0$. In this gauge the shifted Lagrangian (12) looks as follows

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{m_1^2}{2}A^a_\mu A^a_\mu + \frac{1}{2}\partial_\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_2^2\sigma^2 +$$
$$\frac{m_1 g}{2}\sigma A^a_\mu A^a_\mu + \frac{g^2}{8}\sigma^2 A^a_\mu A^a_\mu + \frac{g m_2^2}{4m_1}\sigma^3 - \frac{g^2 m_2^2}{32 m_1^2}\sigma^4$$
$$m_1 = \frac{\mu g}{\sqrt{2}}; \quad m_2 = 2\lambda\mu. \tag{14}$$

We can define the canonical momenta P_k^a, P_σ as usual

$$P_k^a = \frac{\delta L}{\delta \dot{A}_k^a} = F_{0k}^a; \quad P_\sigma = \frac{\delta L}{\delta \dot{\sigma}}\tau = \partial_0 \sigma \tag{15}$$

The variable A_0 is not dynamical. Varying the Lagrangian (14) with respect to A_0 we get a constraint

$$(m_1 + \frac{g\sigma}{2})^2 A_0^a = (D_k F_0 k)^a = (D_k P_k)^a \tag{16}$$

This equation does not contain the derivatives with respect to time, and therefore it is not a dynamical equation, but constraint.

The solution of the constraint may be substituted to the Lagrangian (14),removing the degeneracy of this Lagrangian. The constraint (16) allows to express the field A_0 in terms of other canonical variables. In this way we obtain the nondegenerate Lagrangian depending on the variables $A^a_k, P^a_k, \sigma, P_\sigma$. To study the spectrum we shall look at the free Hamiltonian H_0

$$H_0 = \frac{1}{2}P_k^2 - \frac{1}{2m_1^2}(\partial_k P_k)^2 + \frac{1}{4}F^a_{ik}F^a_{ik} + \frac{m_1^2}{2}A^a_k A^a_k + \frac{1}{2}P_\sigma^2 + \frac{1}{2}\partial_k\sigma\partial_k\sigma + \frac{m_2^2}{2}\sigma^2. \quad i,k = 1,2,3. \tag{17}$$

The asymptotic states are described by the harmonic oscillator variables. That means this Hamiltonian should be expressed in terms of holomorfic coordinates $a_i^b, (a_i^b)^*$ and a_σ, a_σ^* It can be done with the help of representation

$$A_l^b(x) = (2\pi)^{-3/2} \sum_{i=1}^{3} \int (\exp\{i\mathbf{kx}\} a_i^b(\mathbf{k}) e_l^i(\mathbf{k}) + \exp\{-i\mathbf{kx}\}(a_i^b)^*(\mathbf{k}) e_l^i(\mathbf{k})) \frac{d^3k}{\sqrt{2\omega}}$$

$$P_l^b(x) = (2\pi)^{-3/2} \sum_{i=1}^{3} \int (\exp\{i\mathbf{kx}\} a_i^b(\mathbf{k}) \tilde{e}_l^i(\mathbf{k}) - \exp\{-i\mathbf{kx}\}(a_i^b)^*(\mathbf{k}) \tilde{e}_l^i(\mathbf{k})) \frac{\sqrt{\omega} d^3k}{i\sqrt{2}} \quad (18)$$

Here e_l are the polarization vectors: $e_l^{1,2} = \tilde{e}_l^{1,2}$, perpendicular to $\mathbf{k}$, $e_l^3 = \frac{k_l \omega_1}{|\mathbf{k}| m_1}$, $\tilde{e}_l^3 = \frac{k_l m_1}{|\mathbf{k}| \omega_1}$, $\omega_1 = \sqrt{\mathbf{k}^2 + m_1^2}$. For $\sigma(x)$ and $P_\sigma(x)$ one can write the standard decomposition.

$$\sigma(x) = (2\pi)^{-3/2} \int (\exp\{i\mathbf{kx}\} a_\sigma + \exp\{-i\mathbf{kx}\} a_\sigma^*) \frac{d^3k}{\sqrt{2\omega_2}}$$

$$P_\sigma(x) = (2\pi)^{-3/2} \int (\exp\{i\mathbf{kx}\} a_\sigma - \exp\{-i\mathbf{kx}\} a_\sigma^*) \frac{\sqrt{\omega_2}}{2} d^3k$$

$$\omega_2 = \sqrt{\mathbf{k}^2 + m_2^2} \tag{19}$$

Substitution of these representations to the eq.(17) gives the following expression for the free Hamiltonian

$$\int d^3x H_0(x) = \int d^3k [\Sigma_{i=1}^{3} (a_i^b)^*(\mathbf{k}) a_i^b(\mathbf{k}) \omega_1 + a_\sigma^*(\mathbf{k}) a_\sigma(\mathbf{k}) \omega_2] \qquad (20)$$

It follows from the eq.(20) that the spectrum of the free Hamiltonian is the sum of harmonic oscillator Hamiltonians and it describes three polarisations of the vector field with the mass m_1 and one scalar meson with the mass m_2. The scattering matrix may be written in the form

$$S((a_i^b)^*, a_i^b, a_\sigma^*, a_\sigma) = \int \exp\{i \int \tilde{L}(x)\} \prod_x dA_i(x) d\sigma(x) \quad (21)$$

where $\tilde{L}$ is obtained by substituting the expression for A_0 which follows from the constraint to the Lagrangian (14).

The boundary conditions are the following: for $t \to +\infty, A_l^a \to (A_l^a)_{out}$ and $\sigma \to \sigma_{out}$, for $t \to -\infty, A_l^a \to (A_l^a)_{in}$ and $\sigma \to \sigma_{in}$. In the initial state the negative frequencies of A and σ are fixed; in the final state the positive frequencies of A and σ are fixed.

The problem of quantization is solved, however the expression for the scattering matrix is not convenient. First of all manifest Lorentz invariance is lost. The effective Lagrangian $\tilde{L}$ is known only as a perturbation series.

To avoid these problems let us reintroduce the integration over A_0^a. Then the expression for the scattering matrix will look as follows:

$$S = \int \exp\{i \int L(x)dx\} \prod_x (m_1 + \frac{g\sigma}{2})^3 dA_\mu^a d\sigma \qquad (22)$$

Indeed, the integral over A_0^a is equal to

$$\int \exp\{i \int dx 1/2(m_1 + \frac{g\sigma}{2})^2 (A_0^a A_0^a) - A_0^a (D_k F_{0k})^a\} dA_0^a \quad (23)$$

This integral is Gaussian and may be calculated explicitly

$$\exp\{-i \int dx (D_k F_0 k)^a (m_1 + \frac{g\sigma}{2})^{-2} (D_k F_0 k)^a\} \prod_x (m_1 + \frac{g\sigma}{2})^{-3} \quad (24)$$

One sees that the integration over A_0^a leads to replacement of A_0^a in the equation which determines L by the solution of the constraint equation, that is $L \to \tilde{L}$, and the factor $\Pi_x(m_1 + \frac{g\sigma}{2})^3$ is compensated by the determinant.

The free Green function of the vector field is determined by the path integral

$$Z(J_\mu) = \int \exp\{i \int dx[-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{m_1^2}{2}A^a_\mu A^a_\mu + J^a_\mu A^a_\mu]\} \prod_x dA^a_{\mu(x)} d\sigma(x) \quad (25)$$

This function looks as follows

$$D^{ab}_{\mu\nu}(x-y) = \frac{1}{(2\pi)^4}\int \exp\{ik(x-y)\}(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_1^2})\frac{1}{k^2 - m_1^2 + i\varepsilon}dk \quad (26)$$

Its Fourie transform does not decrease, when $k \to \infty$. This leads to the nonrenormalizable divergencies of separate terms of perturbation theory. On the other hand in this gauge there are no unphisical excitations. All the particles present in the Lagrangian have positive energy and may be observed. For that reason this gauge is called a unitary gauge.

From the point of view of real calculations the renormalizabi-

lity is the very important property of a theory. Using the gauge invariance of the shifted Lagrangian (12)we may pass from the unitary, but nonrenormalizable gauge to some manifestly renormalizable gauge, for example to the Lorentz gauge $\partial_\mu A_\mu = 0$. Of course in this gauge some unphysical excitations are present, but as the renormalizable theory was obtained by the identical transformation of the unitary scattering matrix, these excitations decouple.

Let us note that in the exponent for the scattering matrix (22)is present the Lagrangian, which is obtained from the gauge invariant Lagrangian (12)by imposing the unitary gauge condition. It is convenient to introduce the variables B^i, σ

$$\varphi_1 = \frac{iB_1 + B_2}{\sqrt{2}}; \quad \varphi_2 = \mu + \frac{\sigma - iB_3}{\sqrt{2}} \tag{27}$$

In terms of the fields B_i, σ the gauge transformations have a form

$$\delta\sigma = -\frac{g}{2}(B^a\xi^a); \quad \delta B^a = -m_1\xi^a - \frac{g}{2}\varepsilon^{abc}B^b\xi^c - \frac{g}{2}\sigma\xi^a \tag{28}$$

The scattering matrix may be written as follows

$$S = \int \exp\{i\int L(A_\mu^a, B_a, \sigma)dx\}\prod_x \delta(B^a)(m_1 + \frac{g}{2}\sigma)^3 dA_\mu^a dB^a d\sigma \tag{29}$$

where L is a gauge invariant Lagrangian.

Note that the factor $(m_1 + \frac{g\sigma(x)}{2})^{-3} = \Delta^{-1}(B)_{B=0}$ may be written as follows

$$\Delta^{-1}(B)_{B=0} = \int \delta(B^{\Omega}) d\Omega_{B=0} \tag{30}$$

where integration goes over invariant measure on the group. Indeed, at least in the perturbation theory

$$\Delta^{-1}(B)_{B=0} = \int \delta(m_1 \xi^a + \frac{g\sigma(x)\xi^a(x)}{2}) \prod_x d\xi^a(x) \tag{31}$$

The factor $\Delta(B)$ is obviously gauge invariant.

Let us multiply the scattering matrix (29) by "1"

$$1 = \Delta_L(A) \int \delta(\partial_\mu A_\mu^\Omega) d\Omega \tag{32}$$

and change the variables $A_\mu^\Omega = A'_\mu, B^\Omega = B', \sigma^\Omega = \sigma'$. Taking into account the equation (31), and the gauge invariance of the factor $\Delta(B)$ we get

$$\begin{aligned} S = \int \exp\{i \int L(x) dx\} \prod_x \Delta(B) \delta(B'^\Omega) \delta(\partial_\mu A'_\mu) \Delta_L(A'_\mu) dA' dB' d\sigma' d\Omega \\ = \int \exp\{i \int L(A_\mu, B, \sigma) dx\} \prod_x \delta(\partial_\mu A_\mu) \Delta_L(A) dA dB d\sigma \end{aligned} \tag{33}$$

Now the free Green function of the field A_μ^a is given by the integral

$$Z(J_\mu) = \int \exp\{i \int [-\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) + \frac{m_1^2}{2} A_\mu^a A_\mu^a + J_\mu^a A_\mu^a] dx\} \prod_x \delta(\partial_\mu A_\mu^a) dA_\mu^a \quad (34)$$

It looks as follows

$$D_{\mu\nu}^{ab}(x) = \frac{\delta^{ab}}{(2\pi)^4} \int \exp\{ikx\}(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \frac{1}{k^2 - m_1^2 + i\varepsilon} dk \quad (35)$$

This function decreases at $k \to \infty$

The propagators of the fields B^a and σ have the standard form

$$D_B^{ab} = \frac{\delta^{ab}}{k^2 + i\varepsilon}; \quad D_\sigma = \frac{1}{k^2 - m_2^2 + i\varepsilon} \tag{36}$$

Finally we should determine the factor $\Delta_L(A)$. At the surface $\partial_\mu A_\mu = 0$ it is the usual Faddeev-Popov determinant

$$\Delta_L^{-1}(A) = \int \delta(\partial^2 \xi^a + \varepsilon^{acb} A_\mu^c \partial_\mu \xi^b) \prod_x d\xi^a(x) =$$
$$\int \exp\{i \int [\partial_\mu \bar{c}^a(x) D_\mu c^a(x)] dx\} \prod_x d\bar{c}(x) dc(x) \tag{37}$$

The theory is obviously renormalizable.

In this gauge many unphysical excitations are present. They are: zero components of the field A^a_μ, Goldstone bosons B^a, Faddeev-Popov ghosts $\bar{c}, c$. However as our transformation from unitary gauge to the Lorentz gauge was identical, the scattering matrix is unitary in the subspace which includes only physical excitations. It can be demonstrated explicitly in the Lorentz gauge, by using the BRST quantization technique.

One can also pass to some other renormalizable gauge,for example to so called α gauge, when to the gauge invariant Lagrangian the term $\frac{1}{2\alpha}(\partial_\mu A_\mu)^2$ and the Faddeev-Popov ghosts are added. These gauges however are not convenient because they lead to the mixture of the fields A_μ^a and B^a. In these gauges the quadratic terms $\partial_\mu B^a A_\mu^a$ are present. More convenient are the gauges proposed by G.t'Hooft.

Let us consider the t'Hooft's proposal in more detailes.

We introduce the functional Δ_α by the following equation

$$\Delta_\alpha \int \delta(\partial_\mu A_\mu^\Omega + \alpha m_1 B^\Omega - f(x)) d\Omega = 1 \qquad (38)$$

In this equation $f(x)$ is arbitrary function and the integration measure is invariant with respect to the group Ω.That means that the functional $\Delta_\alpha(A_\mu^a, B^a, \sigma)$ is gauge invariant: $\Delta_\alpha(A_\mu^\Omega, B^\Omega, \sigma^\Omega) = \Delta_\alpha(A_\mu, B, \sigma)$. Let us multiply the functional describing the scattering matrix in the unitary gauge by"1".

Then we get

$$S = \int \exp\{i \int L(x)dx\}\delta(B)\Delta(B)\Delta_\alpha(A_\mu, B, \sigma)\delta(\partial_\mu A_\mu^\Omega + \alpha m_1 B^\Omega - f(x))d\Omega dA_\mu dB d\sigma. \quad (39)$$

In this equation we change the variables with the help of gauge transformation: $A_\mu^\Omega, B^\Omega, \sigma^\Omega \to A_\mu, B, \sigma$, and integrate over Ω.

By the same reasonings as before we get for the scattering matrix the following representation

$$S = \int \text{exp}\{i \int L(x)dx\} \Delta_\alpha(A_\mu, B, \sigma)\delta(\partial_\mu A_\mu^a + \alpha m_1 B^a - f^a(x)) \prod_x dA_\mu(x)dB(x)d\sigma(x) \quad (40)$$

By construction the scattering matrix does not depend on the arbitrary function $f^a(x)$. Therefore we can multiply it by the factor $\int \exp\{\int dx \frac{(f^a(x))^2}{2\alpha}\} \prod_x df^a(x)$. Integrating the resulting expression over $f^a(x)$ we get the final expression for the scattering matrix in the t'Hooft gauge

$$S = \int \exp\{i \int [L(x) + \frac{1}{2\alpha}(\partial_\mu A^a_\mu + \alpha m_1 B^a)^2] dx\} \Delta_\alpha(A_\mu, B, \sigma) \prod_x dA_\mu(x) dB(x) d\sigma(x) \quad (41)$$

The gauge fixing term also produces mixing of the fields A_μ^a and B^a. The gauge is chosen in such a way that these terms exactly compensate the mixing terms present in the Lagrangian.

It remains only to determine the explicit form of the factor $\Delta_\alpha(A_\mu, B, \sigma)$. This factor was defined by the eq.(38)

$$\Delta_\alpha \int \delta(\partial_\mu A_\mu^\Omega + \alpha m_1 B^\Omega - f(x)) d\Omega = 1 \tag{42}$$

To determine the factor $\Delta_\alpha(A_\mu, B, \sigma)$ we should find the root of the argument of δ-function. Obviously $\Omega = 1, (\xi = 0)$ nullifies the argument of δ-function. It is easy to see that in the framework of perturbation theory it is the only root.

Hence we may perform the integration with the result

$$\Delta_\alpha = \det(\delta^{ac}(\partial^2 - \alpha m_1^2) - g\partial_\mu \varepsilon^{abc} A_\mu^b - \frac{\alpha g m_1}{2}\varepsilon^{abc} B^b - \delta^{ac}\frac{\alpha m_1}{2}\sigma) = \int \exp\{i \int \overline{c}^a(x) F^{ac}(A_\mu, B, \sigma) c^c(x) dx\} \prod_x d\overline{c}(x) dc(x) \quad (43)$$

In the previous discussion we completely ignored the necessity of existence of the ultraviolet regularization preserving the symmetry of the theory. We were dealing with the divergent integrals as if they were convergent. It may lead to some problems. Usually people working with the divergent integrals assume the existence of the dimensional regularization preserving the symmetry of the theory. Sometimes however the dimensional regularization does not work. Example is given by the supersymmetric theories to which the dimensional regularization is not applicable.

Another important example which are closer to our present topic is given by the theories, including the matrix γ_5. In the framework of dimensional regularization a consistent definition of γ_5 matrix is impossible. It is exactly the case for the Weinberg-Salam model of electroweak interactions. The simplest way out in this situation is to use the regularization with higher derivatives (A.A.S). For the Yang-Mills theory one introduces the following regularized Lagrangian

$$L_{YM} \to L^{\Lambda}_{YM} = -\frac{1}{4}[F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{\Lambda^4}D^2F^a_{\mu\nu}D^2F^a_{\mu\nu}] \qquad (44)$$

The new terms, appearing in the Lagrangian (44), force the free propagator to decrease faster than original one

$$D^{ab}_{\mu\nu} = \delta^{ab}(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\frac{1}{k^2 + \Lambda^{-4}k^6} - \frac{\alpha k_\mu k_\nu}{k^4 f^2(-k^2)}) \quad (45)$$

In the Lorentz gauge the propagator for large k decreases as k^{-6}. Otherwise we choose the function f accordingly. However to preserve the symmetry we are forced to introduce not the ordinary derivatives, but the covariant ones.

Calculating the divergency index of the arbitrary diagram we have

$$\omega \leq 6 - 2\Pi - n_3 - 2n_4 - 3n_5 - 4n_6 - 5n_7 - 6n_8 \qquad (46)$$

one sees that the divergent may be only the one loop diagrams. As our regularization preserved the gauge invariance it follows that the anomalies may arise only in one loop diagrams.

However a general receipt for the one loop diagrams is absent, and in some cases (including Weinberg-Salam model for leptons) one can prove that it is impossible to regularize in the invariant way the one loop diagrams. The well known example is given by the triangle one loop diagram in the $U(1)$ chiral theory,described by the Lagrangian

$$L = -\frac{1}{4}(\partial_\nu A_\mu - \partial_\mu A_\nu)^2 + i\bar{\psi}\gamma_\mu(\partial_\mu - igA_\mu\gamma_5)\psi \qquad (47)$$

This Lagrangian is gauge invariant and the divergency of the one loop diagram with three external vector line must be equal to zero. However the explicit calculations give

$$i(p+q)_\alpha \Gamma_{\mu\nu\alpha}(p,q) = -\frac{g^3}{6\pi^2}\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta \tag{48}$$

The most general expression for renormalized three leg vertex function is

$$\tilde{\Gamma}_{\mu\nu\alpha} = \Gamma_{\mu\nu\alpha}(p,q) + c_1 \varepsilon_{\mu\nu\alpha\beta} p^\beta + c_2 \varepsilon_{\mu\nu\alpha\beta} q^\beta \qquad (49)$$

where $\Gamma_{\mu\nu\alpha}(p,q)$ is the symmetric vertex function, which satisfies the equation (48). $\tilde{\Gamma}_{\mu\nu\alpha}$ also should be the symmetric function of the arguments $(\mu, p), (\nu, q), (\alpha, -(p+q))$. Hence we conclude that $c_1 = c_2 = 0$. Anomaly cannot be eliminated by the renormalization freedom.

However, as one can see from the expression (48) for the anomalous divergency of the vertex function,it is the function of g^3. Therefore if we had the similar interaction of the particles with the opposite charge, the anomalies would compensate each other. It is the case for the Weinberg-Salam model. The anomaly which are due to electron and electron neutrino are compensated by the corresponding anomaly, produced by u and d quarks.

But we have also the anomaly produced by the muon and muonic neutrino. At that time only three quarks were introduced. For the anomaly compensation the total charge of leptons (-2) should be equal to the total charge of quarks and should have the opposite charge $(2/3 \times 3 = 2)$. In this way the charmed quark was predicted. This prediction was confirmed experimentally. At the present time we know two more leptons and two more quarks. They also lead to the compensation of anomalies and therefore to renormalizability of Weinberg-Salam model.

All these considerations with nonessential changes are applicable also to the electro-weak models in which the neutrino have nonzero mass. Therefore spontaneous breaking of symmetry allows to describe the electro-weak interactions of leptons and hadrons in the framework of consistent renormalizable model. For QCD section of the Standard Model the situation is much worse. Although we have no experimental facts contradicting the Standard Model, consistent analytic explanation of color confinement is still absent.