



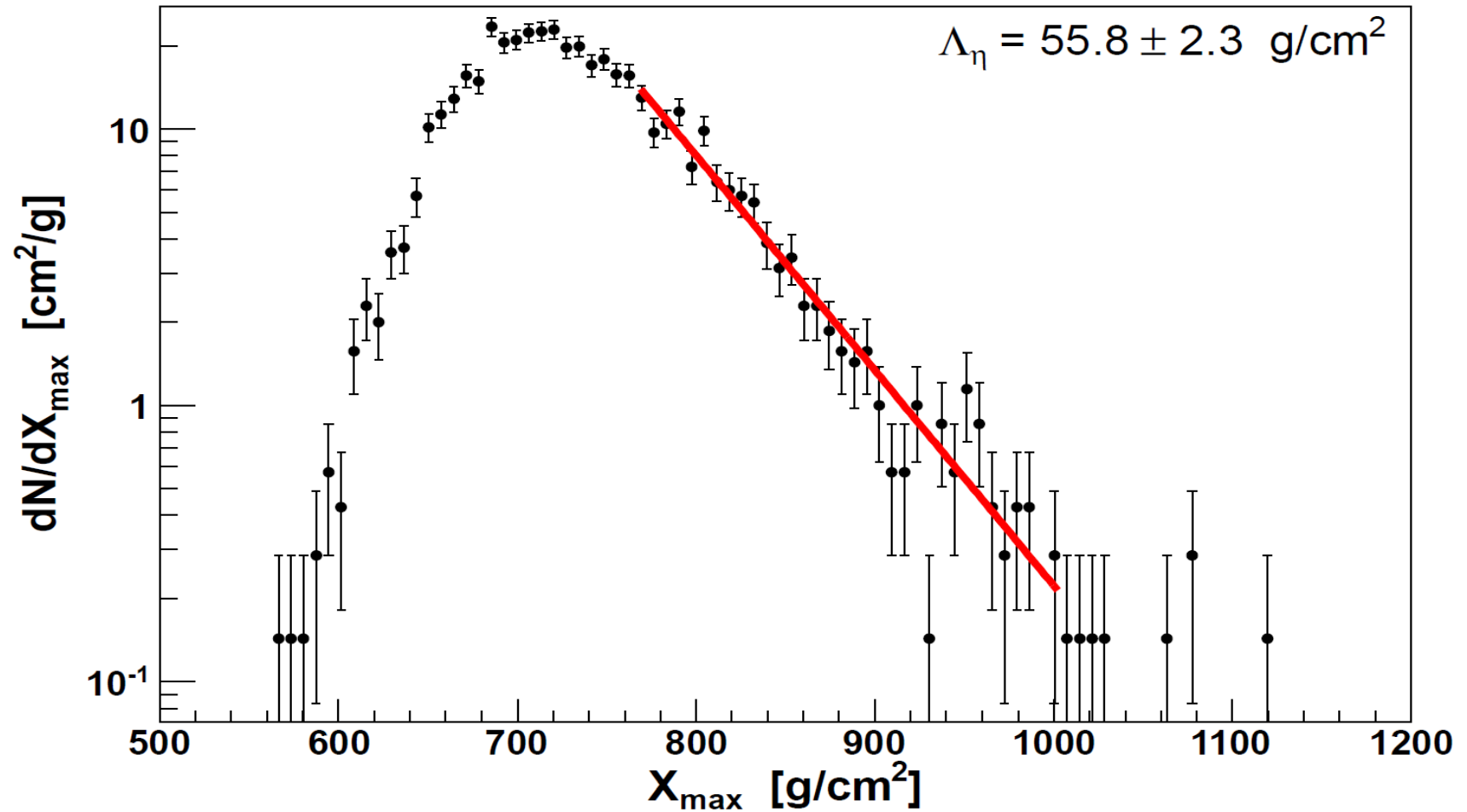
Outline

- Overview of AUGER's measurement of σ_{prod}^{p-air} at 57 TeV
- pp & pA scattering single channel model by Durand & Pi
- A minijet model with soft gluon resummation for σ_{tot}^{pp}
- Predictions for the inelastic cross section
- Extensions for p-air - first application of this model

Measurement of $\sigma_{p\text{-air}}^{\text{prod}}$ with the Pierre Auger Observatory

P. Abreu et al, *Phys.Rev.Lett.* 109 (2012) 062002

X_{max} -distribution $dN/dX_{\text{max}} \propto \exp(-X_{\text{max}}/\Lambda_{\eta})$



$$\Lambda_m = k\lambda_{p\text{-air}} = k \frac{14.4m}{\sigma_{p\text{-air}}^{\text{prod}}} \longrightarrow \sigma_{p\text{-air}}^{\text{prod}} \text{ - for } \underline{\text{particle emission}} \text{ in the primary interaction}$$

Measurement of $\sigma_{p\text{-air}}^{\text{prod}}$ with the Pierre Auger Observatory

P. Abreu et al, *Phys.Rev.Lett.* 109 (2012) 062002

Model dependence: extrapolation of MC interaction codes to cosmic ray energy region

$$f(E, f_{19}) = 1 + (f_{19} - 1) \frac{\lg(E/10^{15} \text{ eV})}{\lg(10^{19} \text{ eV}/10^{15} \text{ eV})}$$

Predictions for p-air cross section

at 57 TeV:

QSJet01 – 523.7 mb

QSJetII – 502.9 mb

SIBYLL – 496.7 mb

EPOS – 497.7 mb

Average – 505 mb

Sta. Unc. - 22 mb

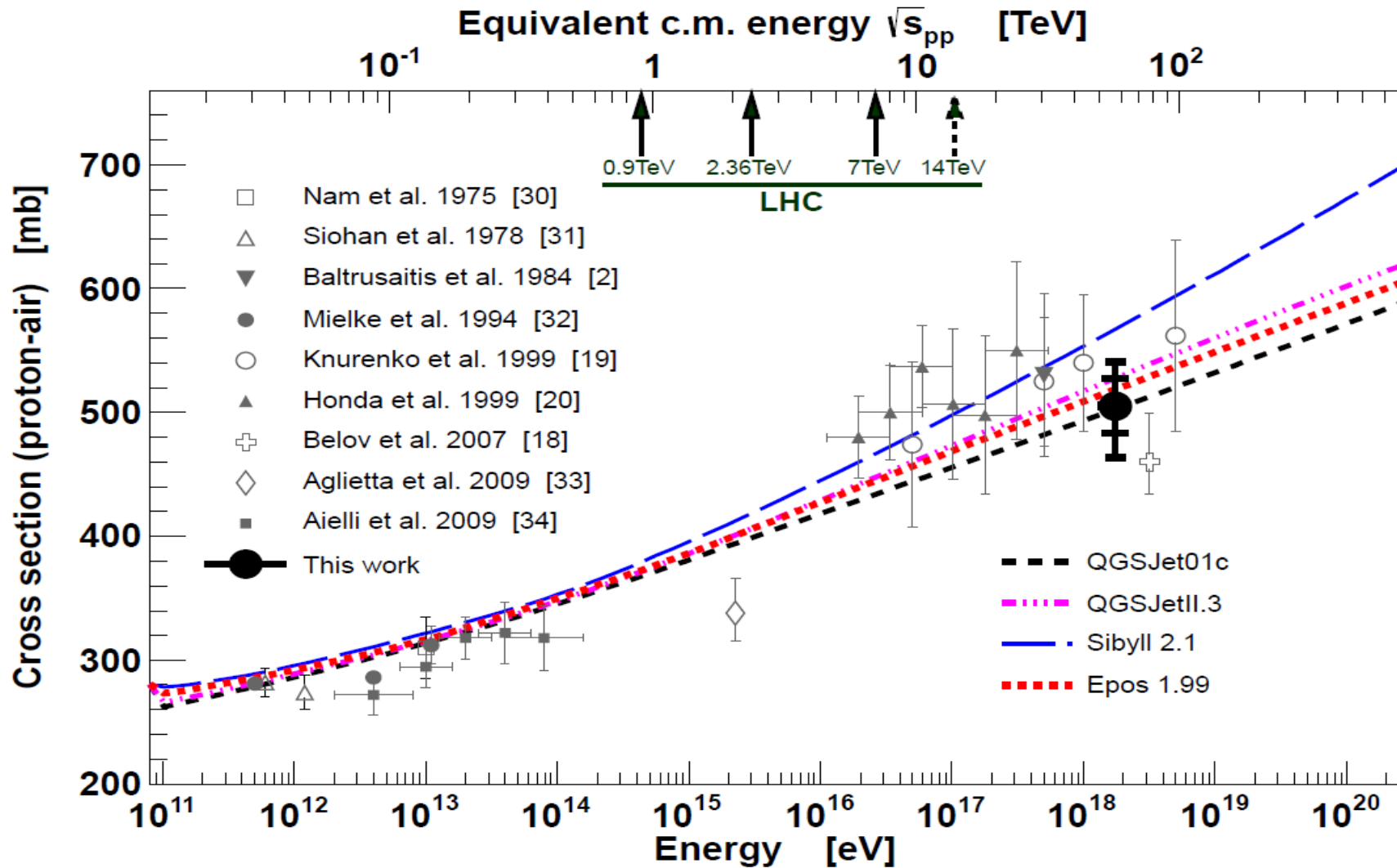
Summary of the systematic uncertainties.

Description	Impact on $\sigma_{p\text{-air}}^{\text{prod}}$
Λ_η systematics	± 15 mb
Hadronic interaction models	$^{+19}_{-8}$ mb
Energy scale	± 7 mb
Conversion of Λ_η to $\sigma_{p\text{-air}}^{\text{prod}}$	± 7 mb
Photons, <0.5 %	< +10 mb
Helium, 10 %	-12 mb
Helium, 25 %	-30 mb
Helium, 50 %	-80 mb
Total (25 % helium)	-36 mb, +28 mb

uncertainty in the mass composition

Measurement of $\sigma_{p\text{-air}}^{\text{prod}}$ with the Pierre Auger Observatory

[P. Abreu et al, *Phys.Rev.Lett.* 109 \(2012\) 062002](#)



$$\sigma_{p\text{-air}}^{\text{prod}} = [505 \pm 22(\text{stat}) \pm_{-36}^{+28}(\text{sys})] \text{ mb}$$

The Durand & Pi model for pp scattering

L. Durand and H. Pi, Phys.Rev. D38 (1988) 78-84

A single-channel eikonal model for pp scattering at high energies:

$$F(s, t) = i \int_0^\infty b db J_0(b\sqrt{-t}) \Gamma(s, b)$$

$$\sigma_{\text{tot}} = 4\pi \int db b (1 - e^{-\chi(b, s)})$$

$$\chi_{pp}(b, s) = \chi_{pp}^{\text{soft}}(b, s) + \chi_{pp}^{\text{QCD}}(b, s)$$

$$\sigma_{\text{el}} = 2\pi \int db b (1 - e^{-\chi(b, s)})^2$$

$$\chi_{pp}^{\text{soft}} = C_0 A(b) = \frac{1}{2} \sigma_0 A(b)$$

$$\sigma_{\text{inel}} = 2\pi \int db b (1 - e^{-2\chi(b, s)})$$

$$\chi_{pp}^{\text{QCD}}(b, s) = \frac{1}{2} n_{pp}(b, s) = \frac{1}{2} A(b) \sigma_{\text{QCD}}(s)$$

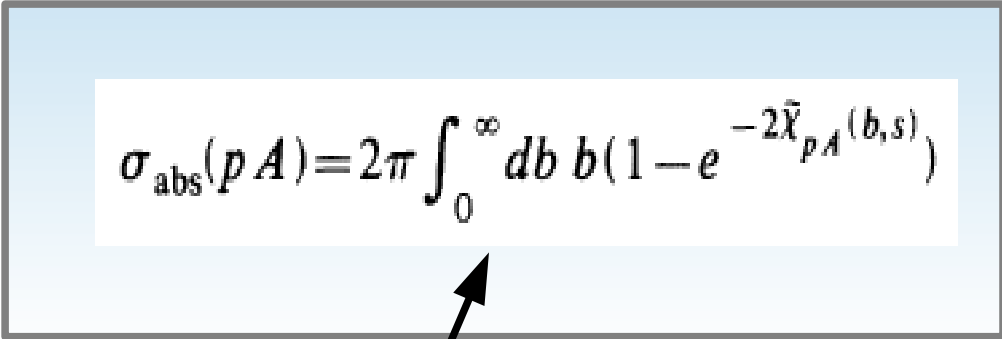
Average number of collisions *semi-hard* parton-parton collisions modelled after pQCD

$$\sigma_{\text{QCD}}(s) = 2 \int_{2v/\sqrt{s}} dx_1 \int_{x_1}^1 dx_2 \int_{Q_{\text{min}}^2}^{x_1 x_2 s/2} d|\hat{t}| \frac{9\pi\alpha_s^2(\hat{t})}{2\hat{t}^2} F(x_1, \hat{t}) F(x_2, \hat{t})$$

The Durand & Pi model for pA scattering

[L. Durand and H. Pi, Phys.Rev. D38 \(1988\) 78-84](#)

A single-channel eikonal model for pA scattering at high energies:


$$\sigma_{\text{abs}}(pA) = 2\pi \int_0^\infty db b (1 - e^{-2\tilde{\chi}_{pA}(b,s)})$$

$$\chi_{pA}^{\text{QCD}}(b,s) = \frac{1}{2} \tilde{A}(b) \sigma_{\text{QCD}}(s)$$

$$\tilde{A}(b) = \int d^2r_\perp dz \rho_A(\mathbf{r}_\perp, z) A(|\mathbf{b} - \mathbf{r}_\perp|)$$

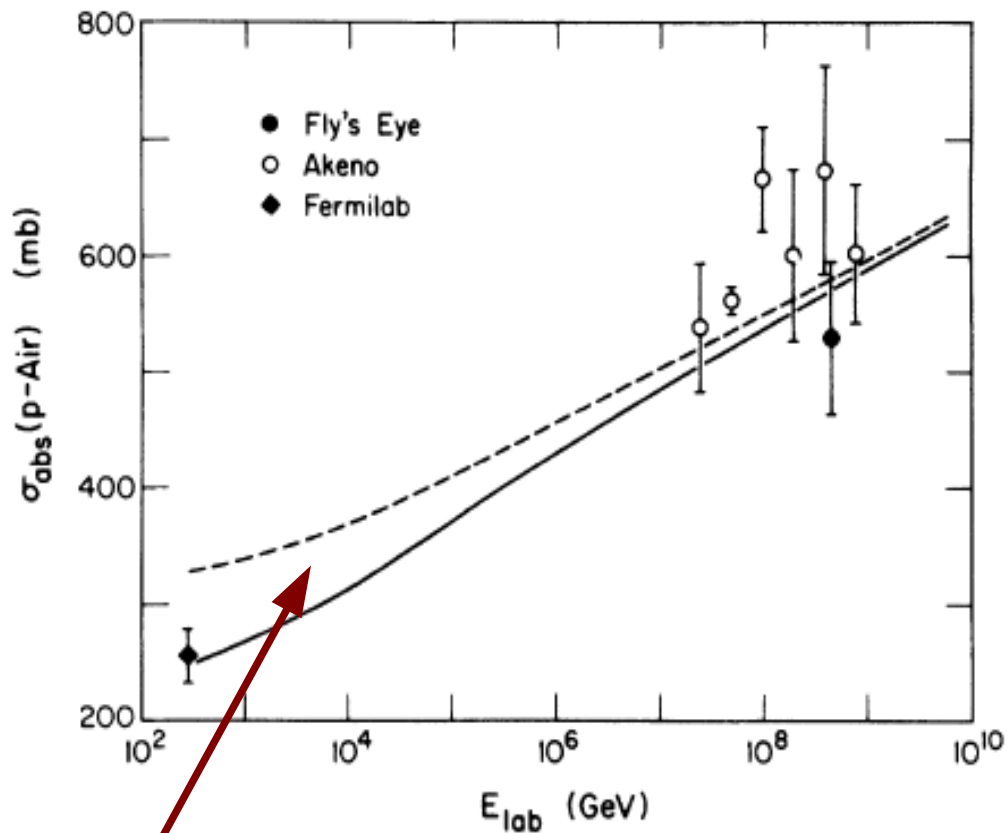
$$\tilde{\chi}_{pA}(b,s) = \frac{1}{2} (\sigma_0 + \sigma_{\text{QCD}}) \tilde{A}(b)$$

The often called “absorption” cross section gives the pA production cross section.

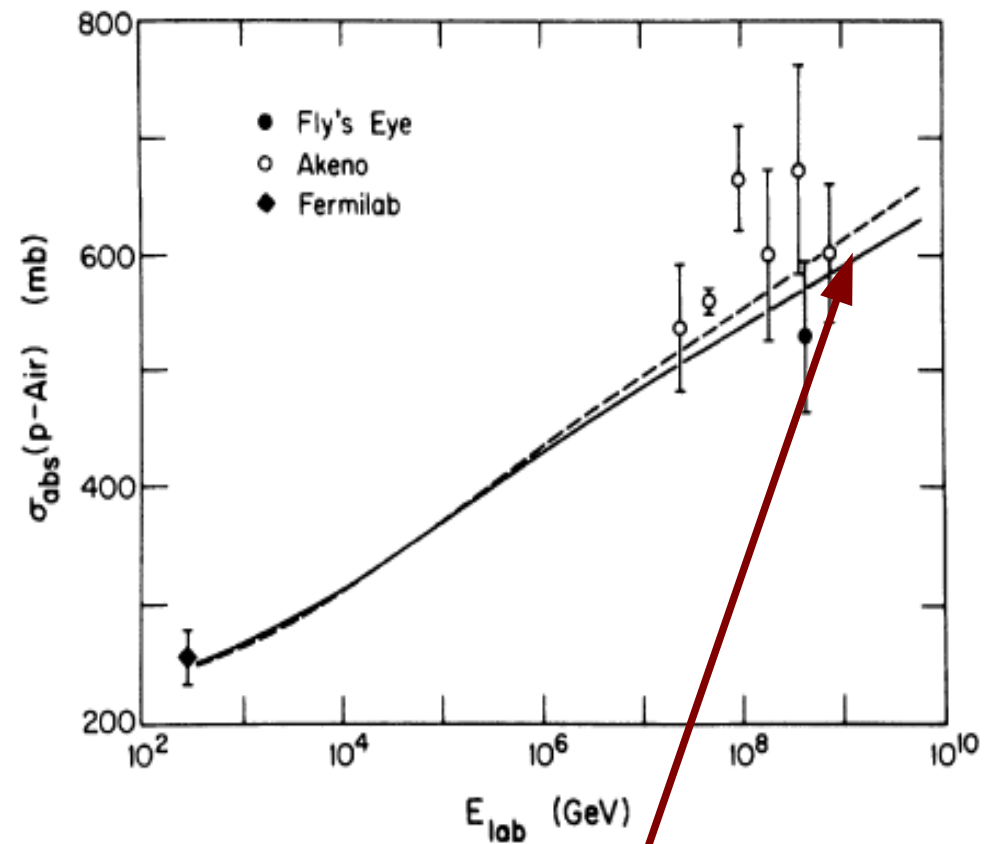
Instead, the **inelastic pA cross section** includes the **quasi-elastic scattering** of the incident particle from a target nucleon which breaks up the nucleus, without particle production.

QCD-inspired models – the Durand & Pi model

[L. Durand and H. Pi, Phys.Rev. D38 \(1988\) 78-84](#)



σ_0 effect at lower energies



Nuclear density effect - $\tilde{A}(b)$

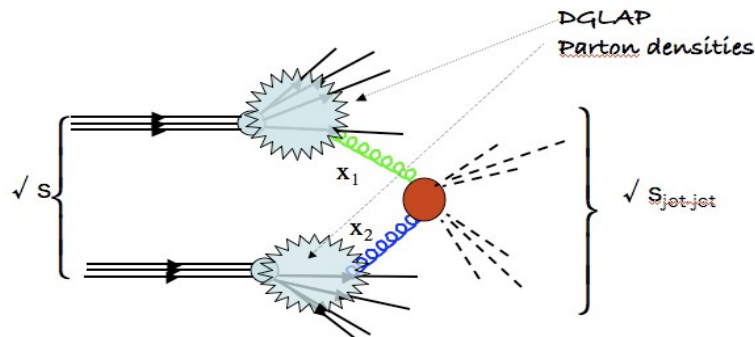
Bloch-Nordsieck model for the pp total cross section

[A. Corsetti et al., Phys.Lett. B382 \(1996\) 282;](#)

[R. Godbole et al., Phys.Rev. D72 \(2005\) 076001](#)

Key ingredients:

1) minijet cross section with PDFs evolved by DGLAP

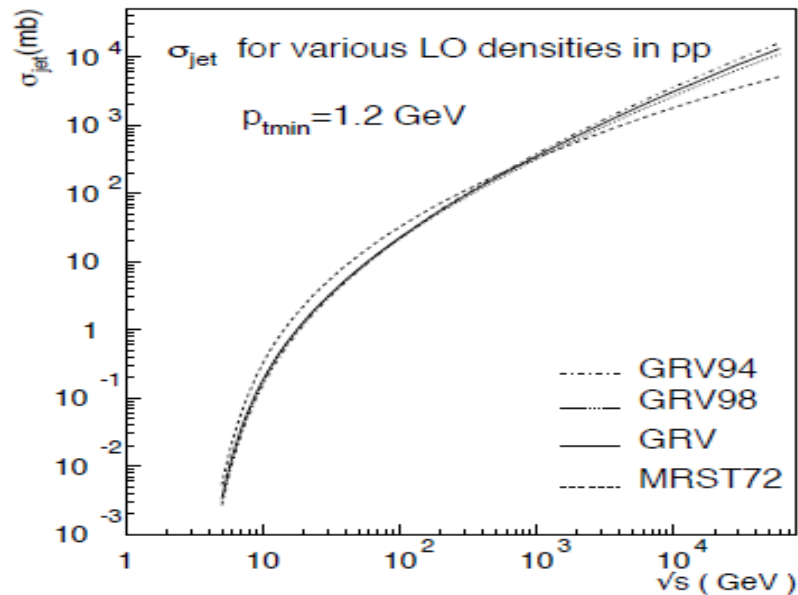


$$\sigma_{jet} \sim s^{\epsilon_{PDF}}, \quad \epsilon_{PDF} \sim 0.3 - 0.4$$

Multiple semi-hard parton-parton interactions:

$$\sigma_{total} = 2 \int d^2\vec{b} [1 - e^{-\chi_I(b,s)}]$$

$$\sigma_{inel} = \int d^2\mathbf{b} [1 - e^{-2\chi_I(b,s)}]$$



$$2\chi_I(b,s) = n_{soft}^{pp}(b,s) + n_{jet}^{pp}(b,s) = A_{FF}\sigma_{soft}^{pp}(s) + A_{BN}^{pp}(p, PDF; b,s)\sigma_{jet}(PDF, p_{tmin}; s)$$

Bloch-Nordsieck model for the pp total cross section

[A.Corsetti et al., Phys.Lett. B382 \(1996\) 282;](#)

[R.Godbole et al., Phys.Rev. D72 \(2005\) 076001](#)

2) soft kt - resummation, which tames the rise by introducing a cutoff in b -space

$$A_{\text{BN}} = \frac{e^{-h(b,s)}}{\int d^2\vec{b} e^{-h(b,s)}}$$

Fixed by single gluon emission
kinematics – energy dependent
scale

$$h(b, s) = \frac{8}{3\pi} \int_0^{q_{\text{max}}} \frac{dk}{k} \alpha_s(k^2) \ln\left(\frac{q_{\text{max}} + \sqrt{q_{\text{max}}^2 - k^2}}{q_{\text{max}} - \sqrt{q_{\text{max}}^2 - k^2}}\right) [1 - J_0(kb)]$$

Resummation function – emission
from valence quarks

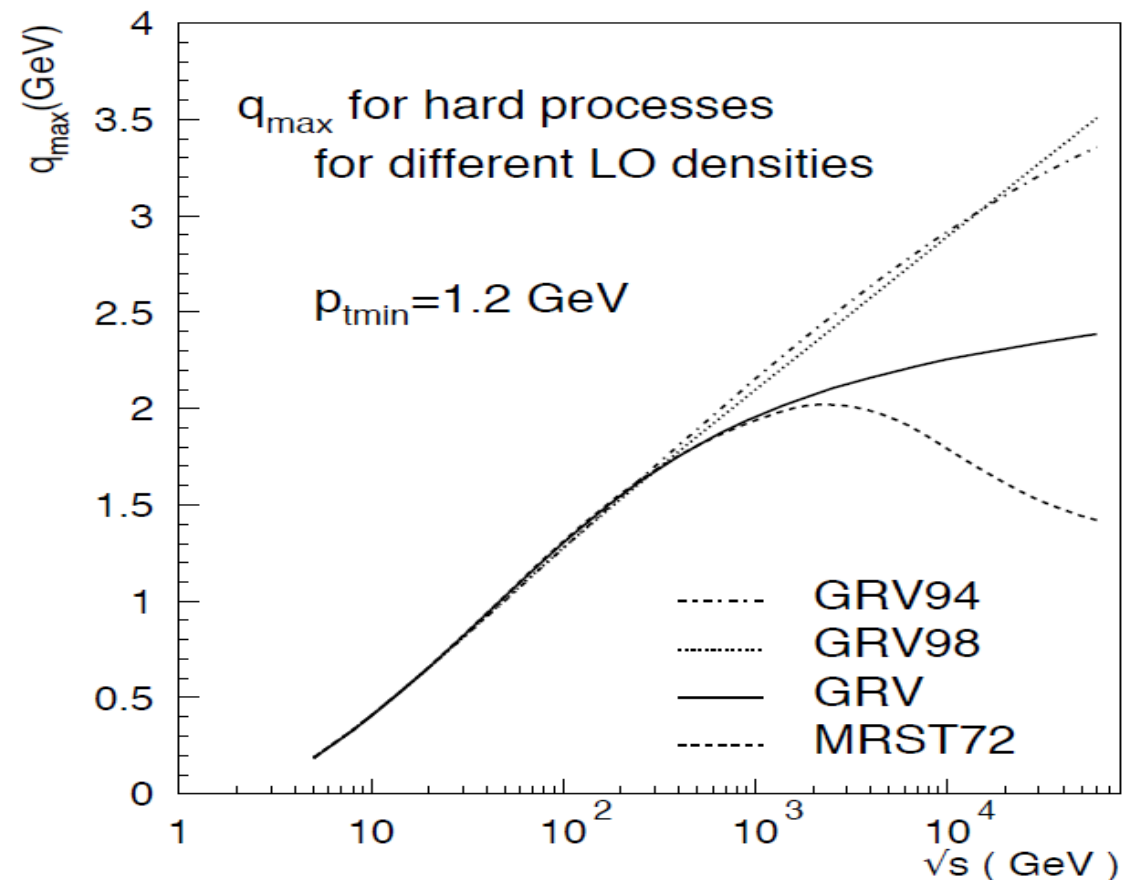
Bloch-Nordsieck model for the pp total cross section

R.Godbole et al., Phys.Rev. D72 (2005) 076001

$$q_{\max}(s; p_{t\min}) = \sqrt{\frac{s}{2}} \frac{\sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{z_{\min}}^1 dz f_i(x_1) f_j(x_2) \sqrt{x_1 x_2} (1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{z_{\min}}^1 dz f_i(x_1) f_j(x_2)}$$

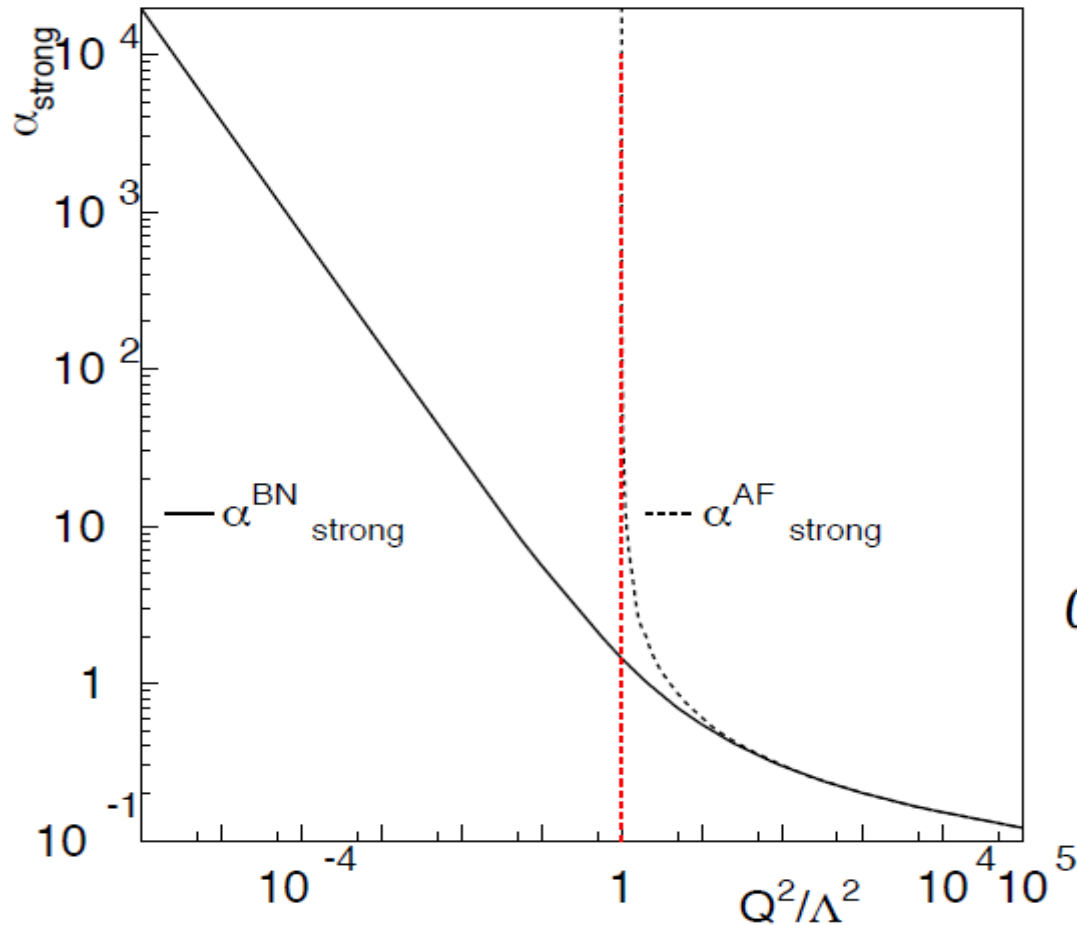
$$z_{\min} = 4p_{t\min}^2 / (sx_1 x_2)$$

- 1) Maximum transverse momentum allowed to single gluon emission
- 2) Emissions in the initial state
- 2) Average over PDFs
- 3) s and $p_{t\min}$ dependent



Bloch-Nordsieck model for the pp total cross section

[R.Godbole et al., Phys.Rev. D72 \(2005\) 076001;](#)



$$\alpha_s(k_t^2) = \frac{p}{b_0 \ln[1 + p(\frac{k_t^2}{\Lambda^2})^p]}$$

$$\alpha_s(k_t^2) \rightarrow \frac{1}{b_0} \left(\frac{k_t}{\Lambda}\right)^{-2p} \quad k_t^2 \ll \Lambda^2$$

$$\alpha_s(k_t^2) \rightarrow \alpha_s^{AF}(k_t^2) = \frac{1}{b_0 \ln[\frac{k_t^2}{\Lambda^2}]} \quad k_t^2 \gg \Lambda^2$$

Regularizes the integrated soft gluon spectrum as long as $p < 1$.

Bloch-Nordsieck model for the pp total cross section

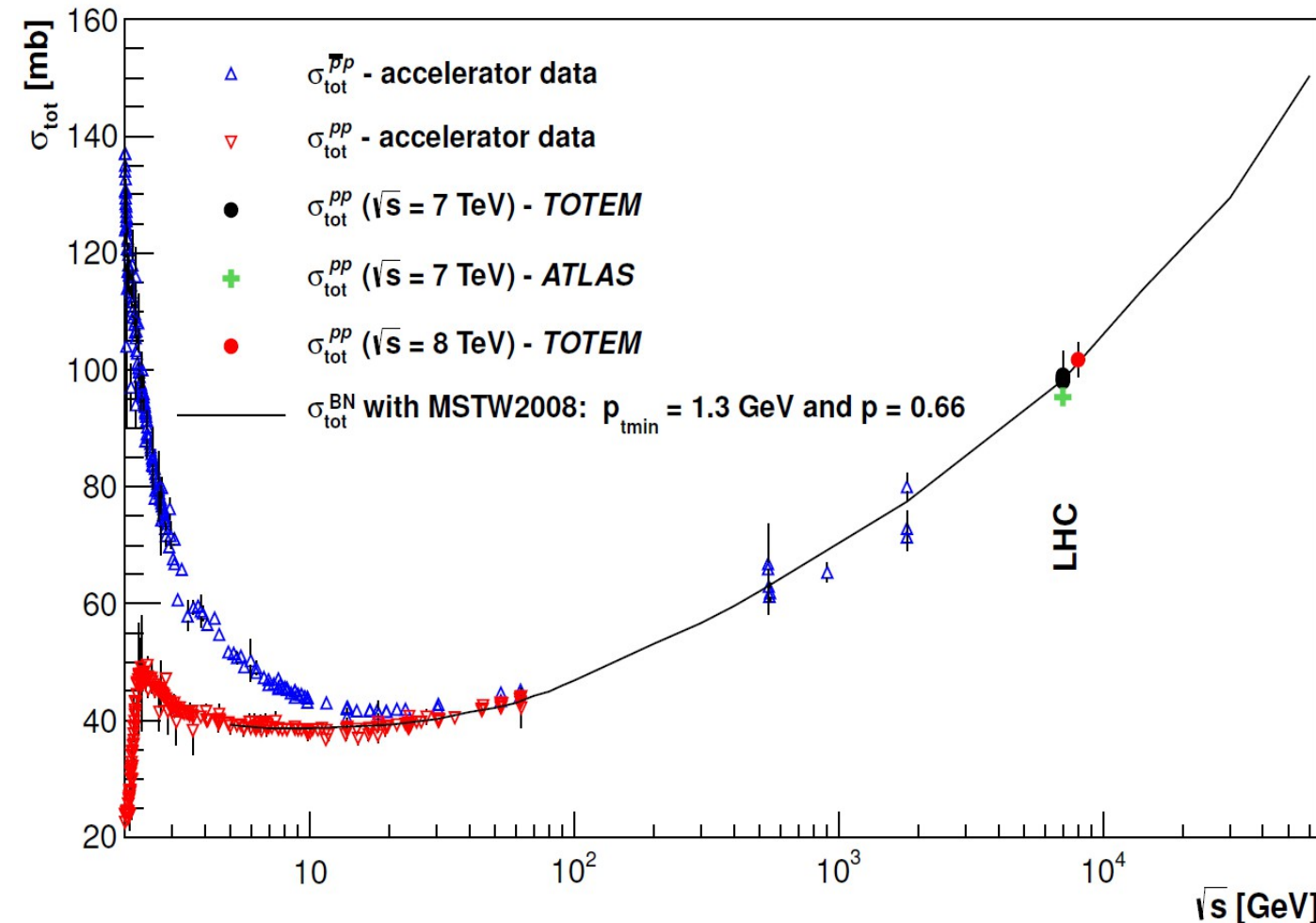
A. Achilli, Phys.Rev. D84 (2011) 094009;

D.A. Fagundes et al., arXiv:1408.2921;

For applications to πp , $\pi\pi$, γp , $\gamma\gamma$:

Phys.Lett. B693 (2010) 456-461;

Phys.Lett. B435 (1998) 441-448;



Asymptotic behaviour:

$$\sigma_{\text{tot}}^{pp} \sim \frac{2\pi}{(\bar{\Lambda})^2} [\epsilon \log s]^{1/p}$$

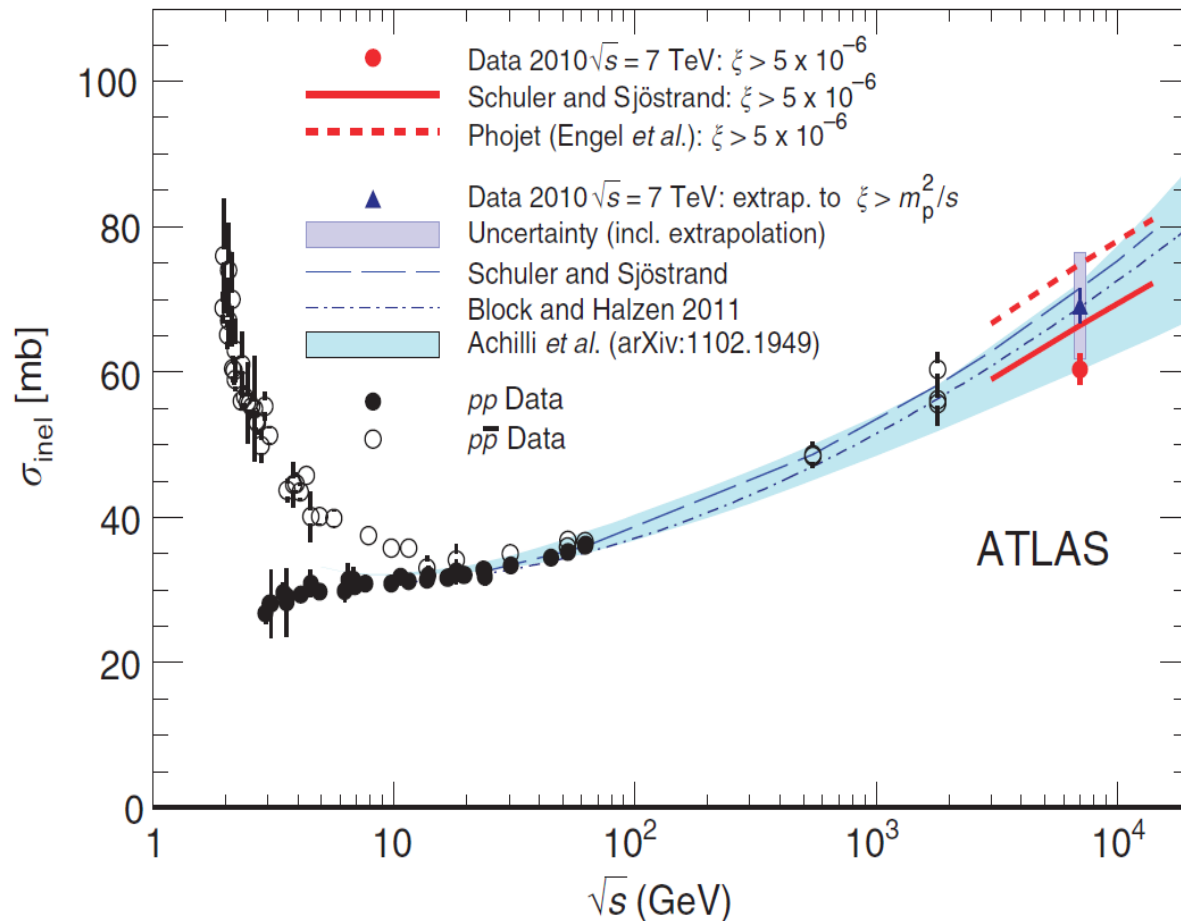
$$1/2 < p < 1$$

Parameters tuned to describe LHC7 and LHC8 data – only accelerator data is used

Predictions for the inelastic cross section - pre-LHC

G. Aad (ATLAS Collaboration), *Nature Commun.* 2 (2011) 463;

A. Achilli, *Phys.Rev. D84* (2011) 094009



$$p = 0.66 \rightarrow \sigma_{inel} = 60mb$$

$$p = 0.5 \rightarrow \sigma_{inel} = 75mb$$

Table 2 | Comparisons of the inelastic cross-section with predictions.

$\sigma(\xi > 5 \times 10^{-6})$ (mb)	
ATLAS Data 2010	60.33 ± 2.10 (exp.)
Schuler and Sjöstrand	66.4
Phojet	74.2
Ryskin <i>et al.</i>	51.8-56.2
$\sigma(\xi > m_p^2/s)$ (mb)	
ATLAS Data 2010	69.1 ± 2.4 (exp.) ± 6.9 (extr.)
Schuler and Sjöstrand	71.5
Phojet	77.3
Block and Halzen	69.0 ± 1.3
Ryskin <i>et al.</i>	65.2-67.1
Gotsman <i>et al.</i>	68
Achilli <i>et al.</i>	60-75

Measurement and theoretical predictions of the inelastic cross-section for the restricted kinematic range, $\xi > 5 \times 10^{-6}$, and for the full kinematic range, $\xi > m_p^2/s$. The experimental uncertainty (exp.) includes the statistical, systematic and luminosity uncertainties. The extrapolation uncertainty (extr.) only applies to the full kinematic range and is listed separately.

Bloch-Nordsiek model for the pp total cross section

[A.Corsetti et al., Phys.Lett. B382 \(1996\) 282;](#)

[R.Godbole et al., Phys.Rev. D72 \(2005\) 076001](#)

Yes, we can get the inelastic as well as the total. On the other hand, let us recall that in the approximation we're using only non-correlated part of the inelastic is accounted

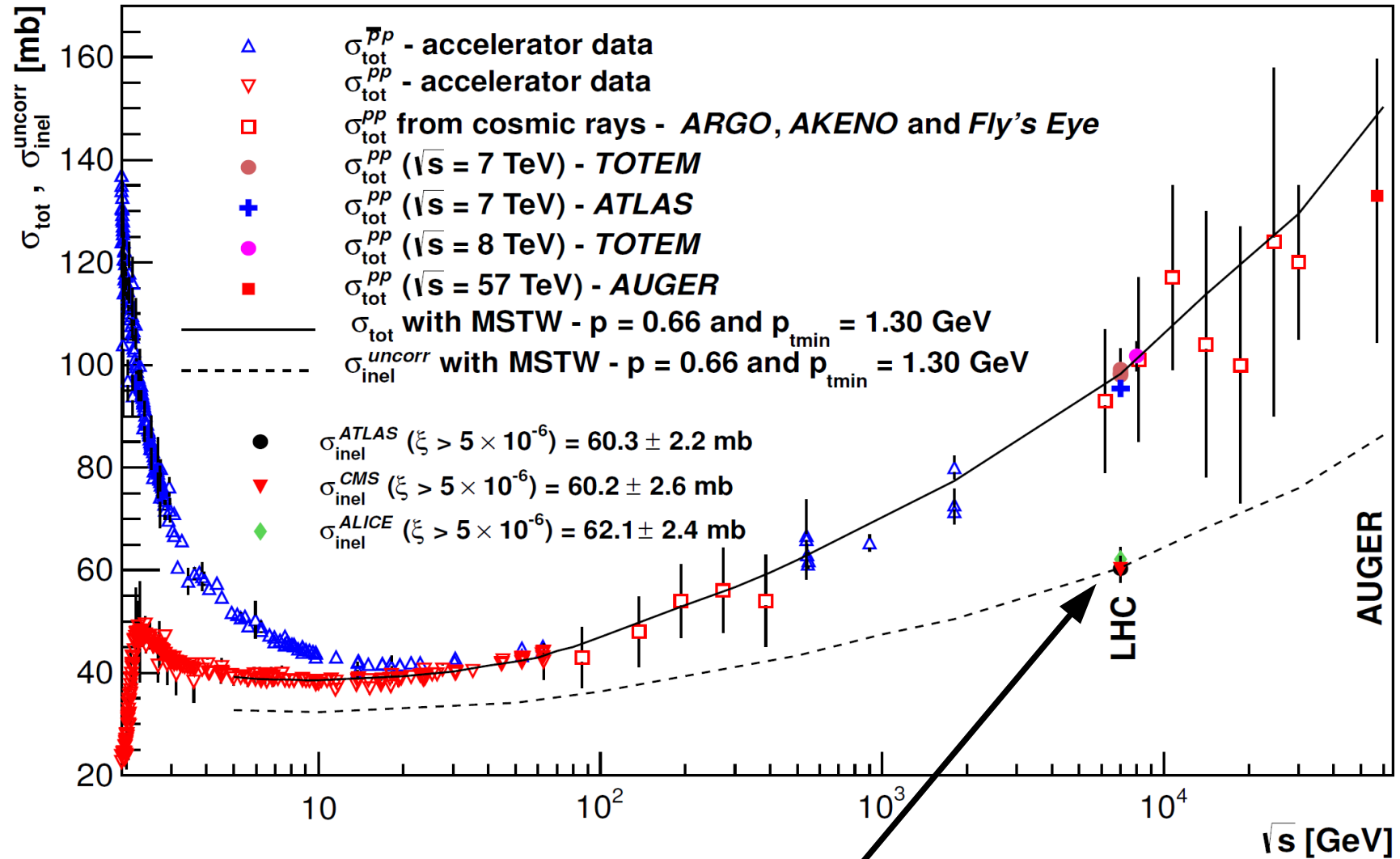
$$P_{inel}(s, b) = \sum_1^{\infty} \frac{(\bar{n})^n e^{-\bar{n}(b,s)}}{n!} = 1 - e^{-\bar{n}(b,s)}$$

Thus, the formalism is tied to give only the non-diffractive part of the inelastic

$$\sigma_{inel}^{uncorr} = \int d^2b [1 - e^{-\bar{n}(s,b)}]$$

Bloch-Nordsieck model for the pp total cross section

D.A. Fagundes et al., arXiv:1408.2921



With the same p that describes the total, the nondiffractive inelastic cross section is obtained

Glauber modelling of the p -air production cross section

N.N. Nikolaev, Phys.Rev. D48 (1993) 1904-1906;

B.Z. Kopeliovich, Phys.Rev. C68 (2003) 044906;

E. Gotsman, E. Levin, U. Maor, Phys.Rev. D88 (2013) 114027.

In the simplest Glauber formalism the production cross section can be calculated as

$$\begin{aligned}\sigma_{\text{abs}}(p\text{-air}) &= \sigma_{\text{tot}}(p\text{-air}) - \sigma_{\text{el}}(p\text{-air}) - \sigma_{\text{Qel}}(p\text{-air}) = \int d^2\mathbf{b} \left\{ 1 - \left[1 - \frac{1}{A} \sigma_{\text{in}}(p\text{-}p) T(b) \right]^{2A} \right\} \\ &\approx \int d^2\mathbf{b} \{ 1 - \exp[-\sigma_{\text{in}}(p\text{-}p) T(b)] \} .\end{aligned}$$

Screened cross section

$$\sigma_{\text{in}} = \sigma_{\text{tot}} - \sigma_{\text{el}} - \sigma_{\text{dif}} \rightarrow \sigma_{\text{inel}}^{\text{nondiff.}}$$

Nuclear profile function

C.W. De Jagier, H. De Vries and C. De Vries, *At. Data Nucl. Data Tables* 14, 479 (1974)

Gaussian:

$$T_A(b) = \frac{A}{\pi R_N^2} e^{-b^2 / R_N^2}$$

$$A = 14.5, \quad R_N = (1.1 \text{ fermi}) A^{1/3}$$

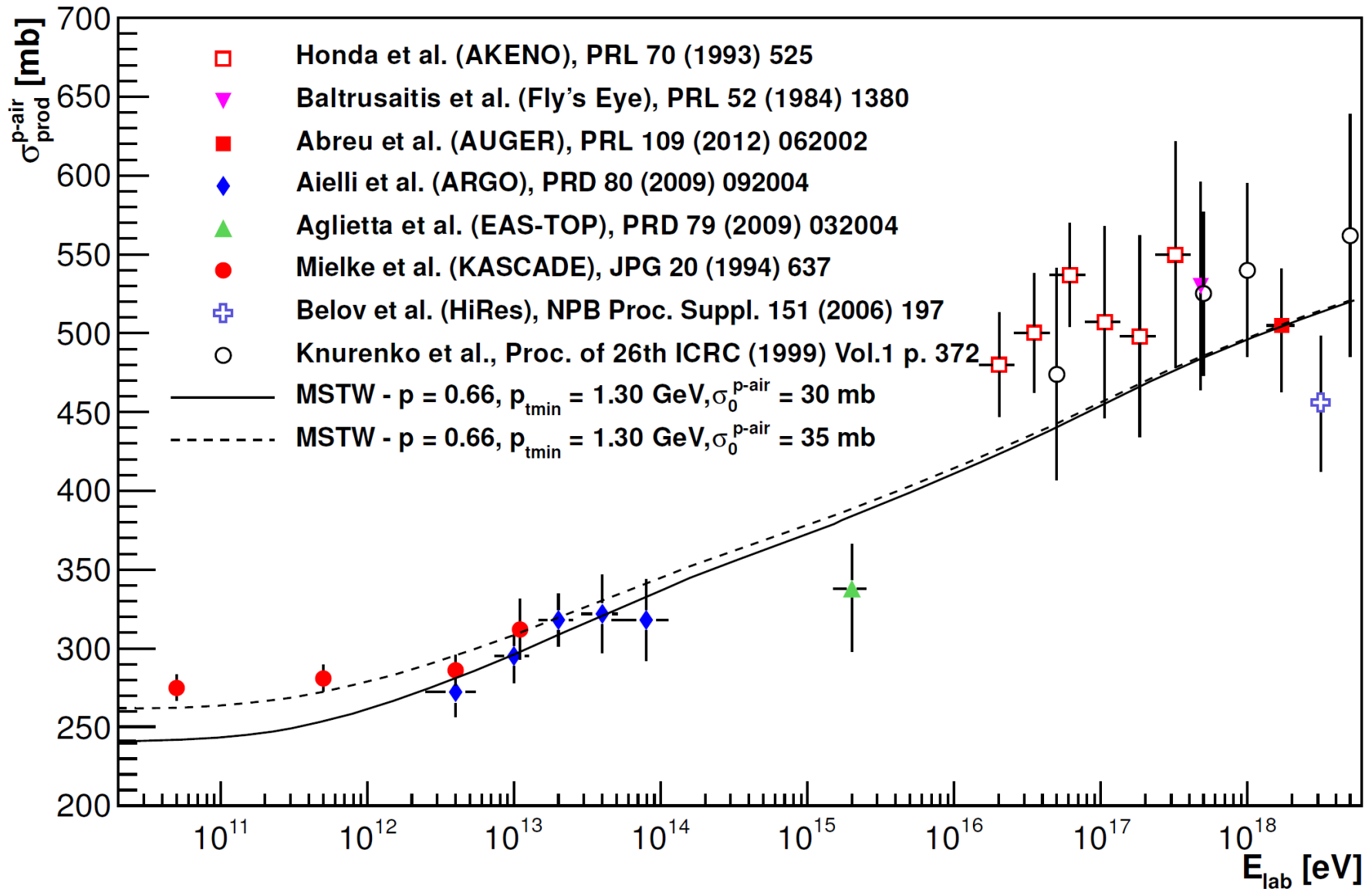
Woods-Saxon:

$$T_A(b) = \int_{-\infty}^{\infty} dz \frac{\rho_0}{1 + \exp\left(\frac{\sqrt{z^2 + b^2} - R_A}{h}\right)}$$

Normalization

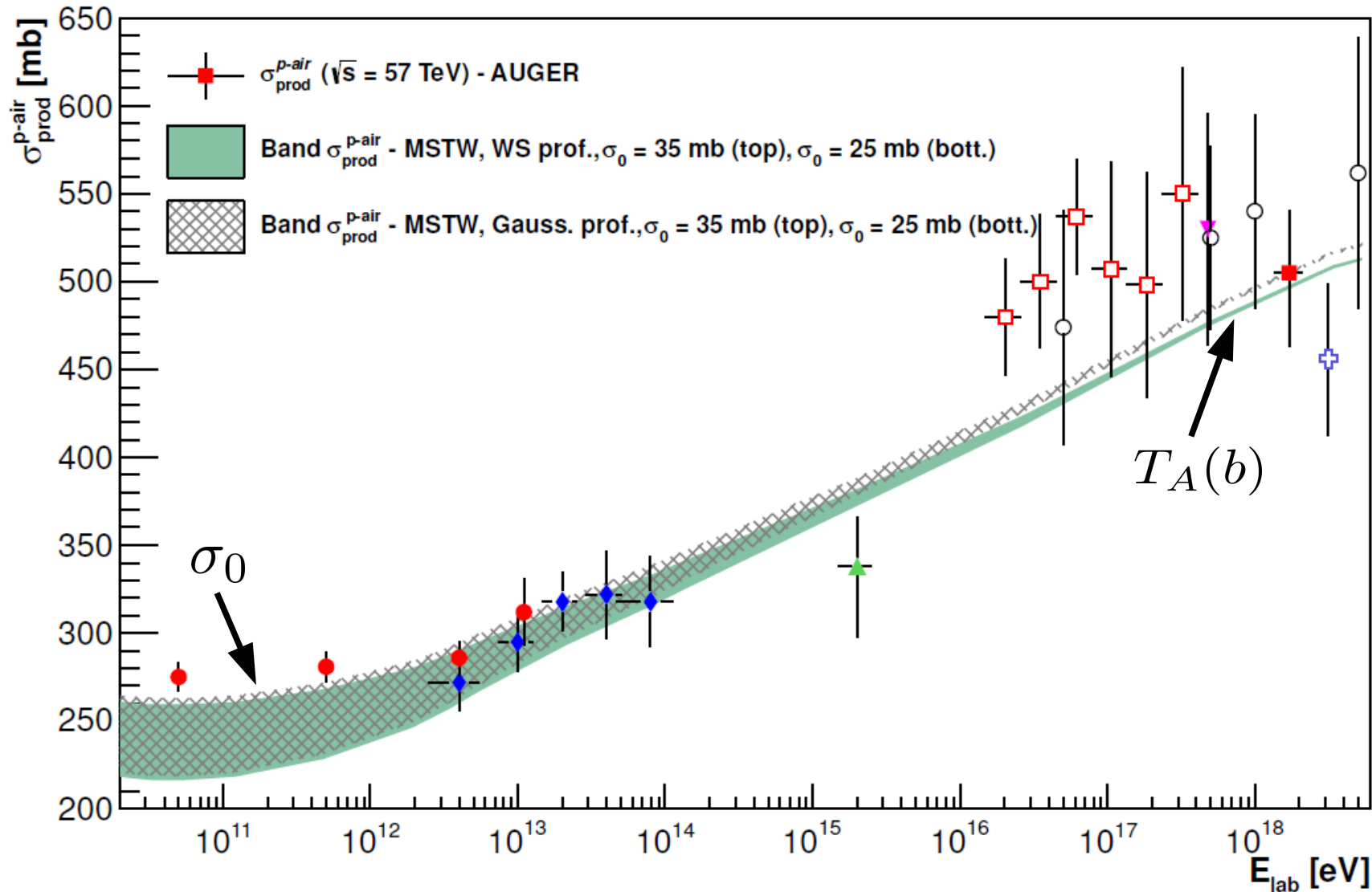
$$\int d^2 \mathbf{b} T_N(b) = A$$

Glauber modelling of the p -air production cross section



Good description of cosmic ray data, specially Auger's

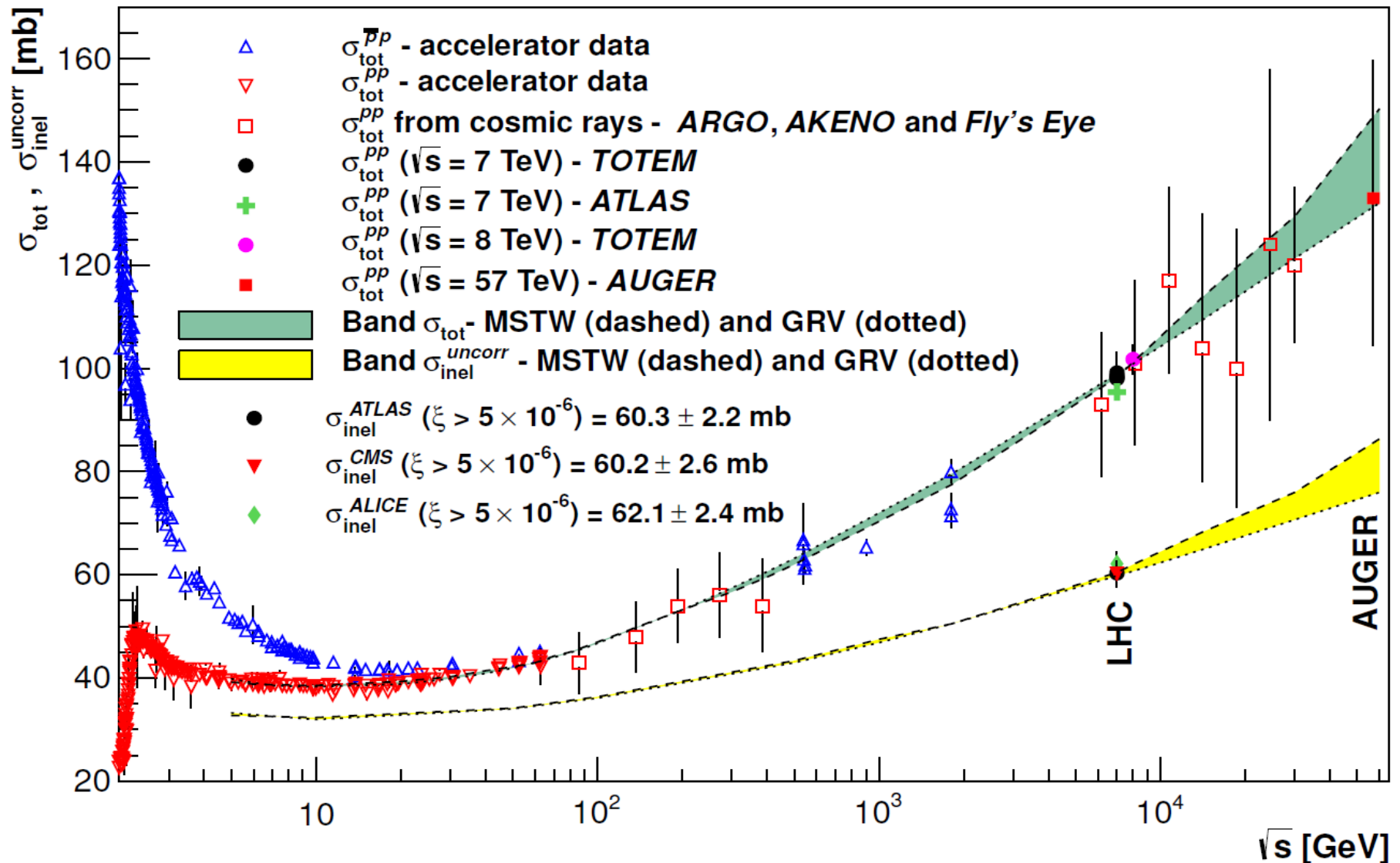
Glauber modelling of the p -air production cross section



At very high energy not very sensitive to the nuclear profile input

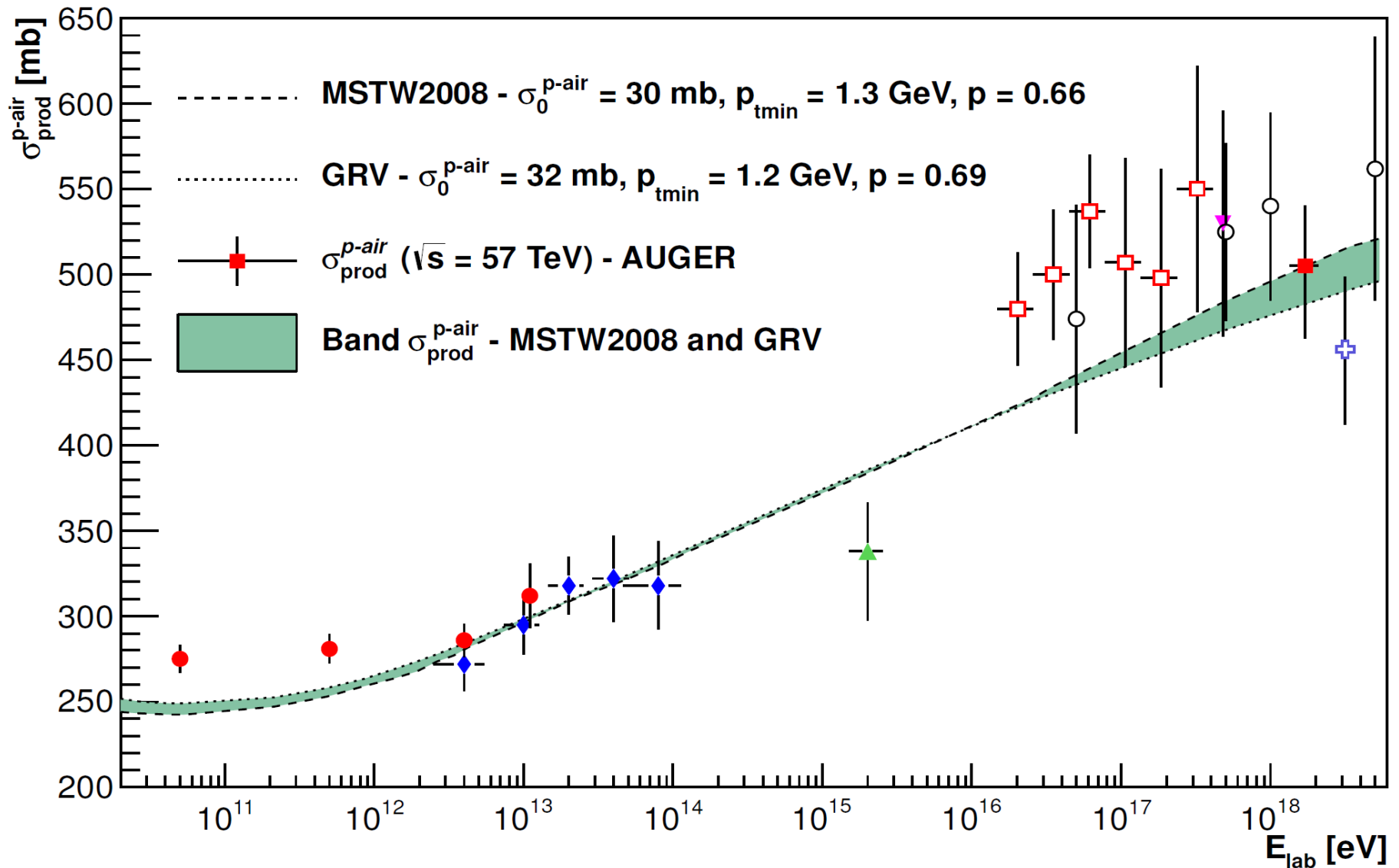
Low-x behaviour of (LO) PDFs – MSTW2008 vs GRV

D.A. Fagundes et al., arXiv:1408.2921;



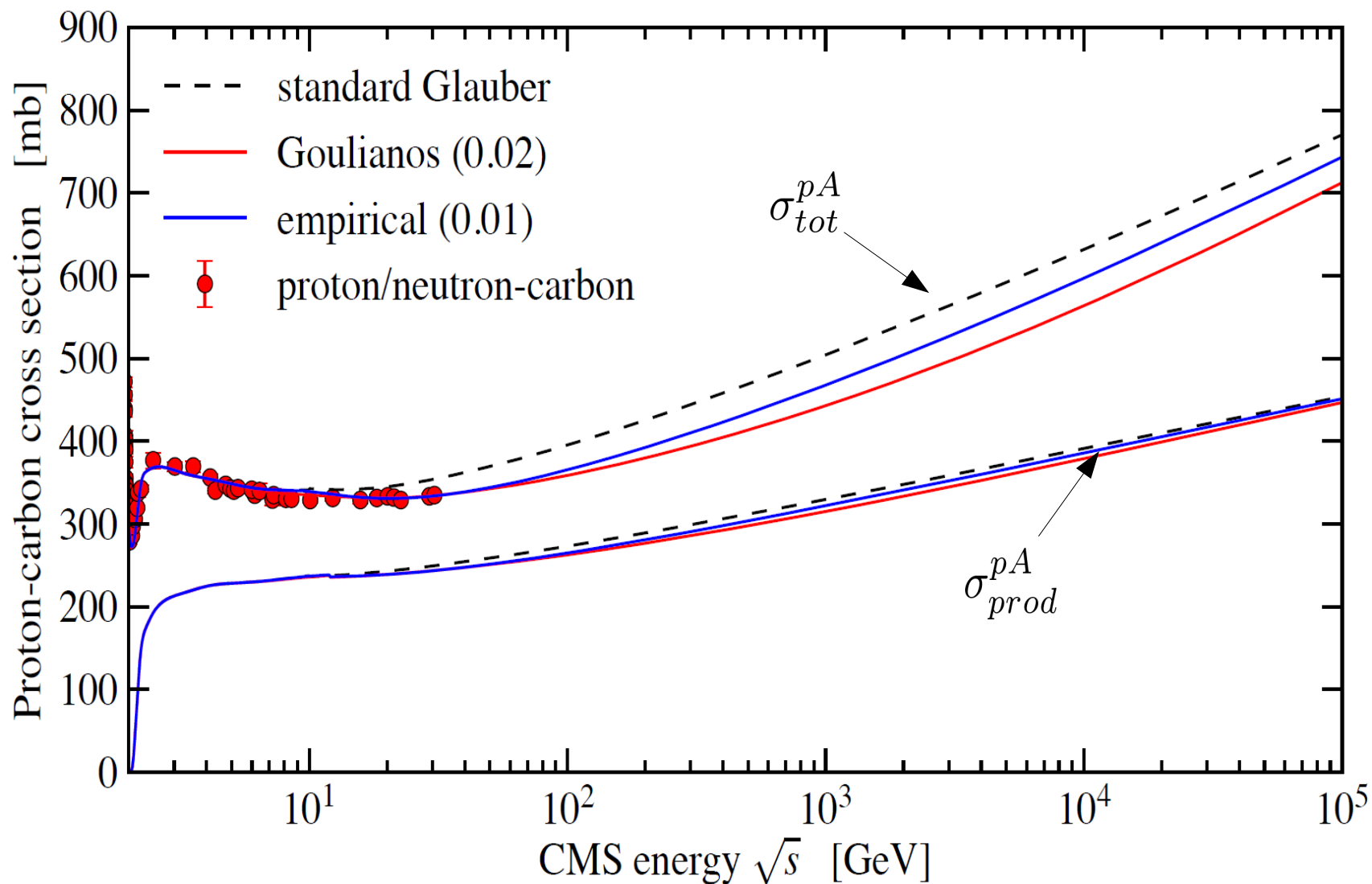
Bands at very high energy dictated by the low-x behaviour of PDFs

Uncertainties in $\sigma_{p\text{-air}}^{\text{prod}}$ at very high energy



Inelastic screening - total vs production cross sections

R.Engel and R.Ulrich, Pierre Auger Internal Note (unpublished), GAP-2012

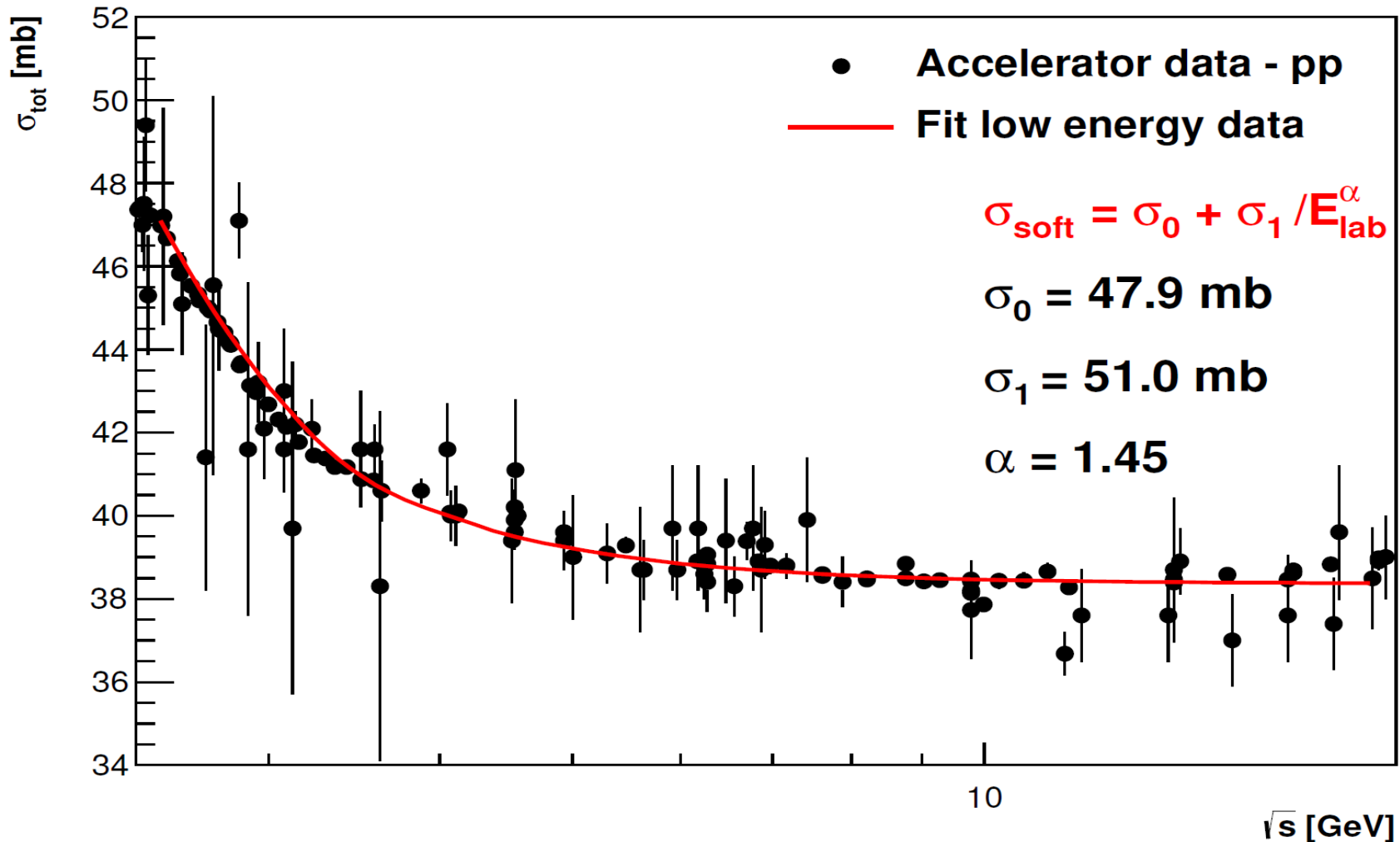


Very important for the total, less so for the production one.

Summary and Outlook

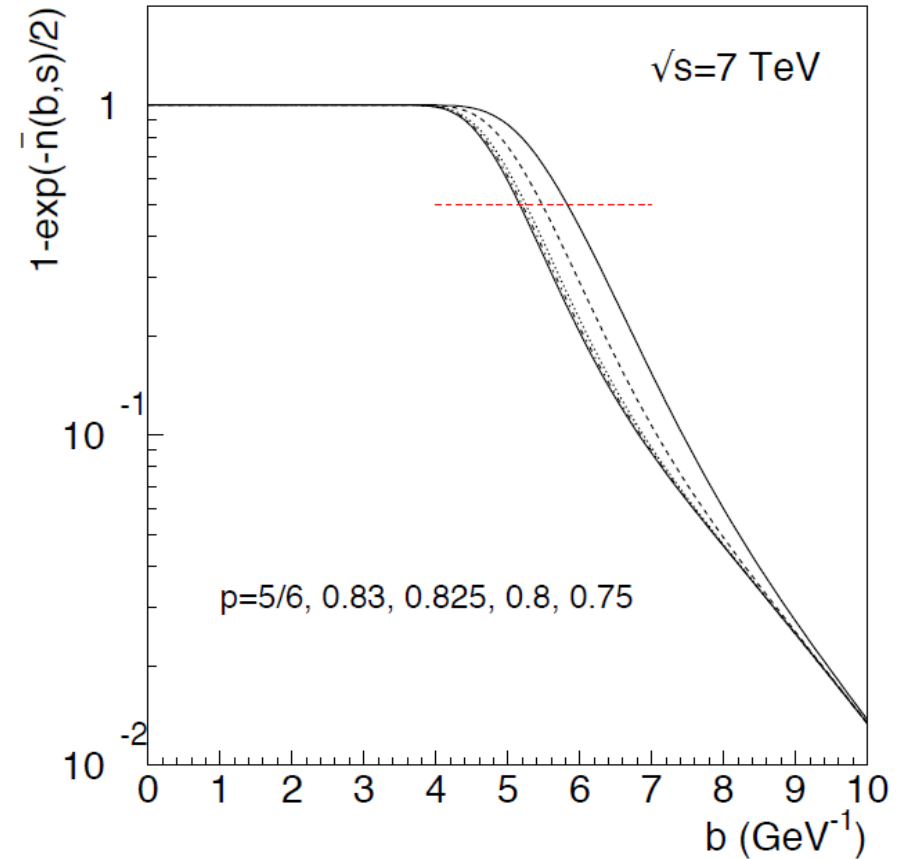
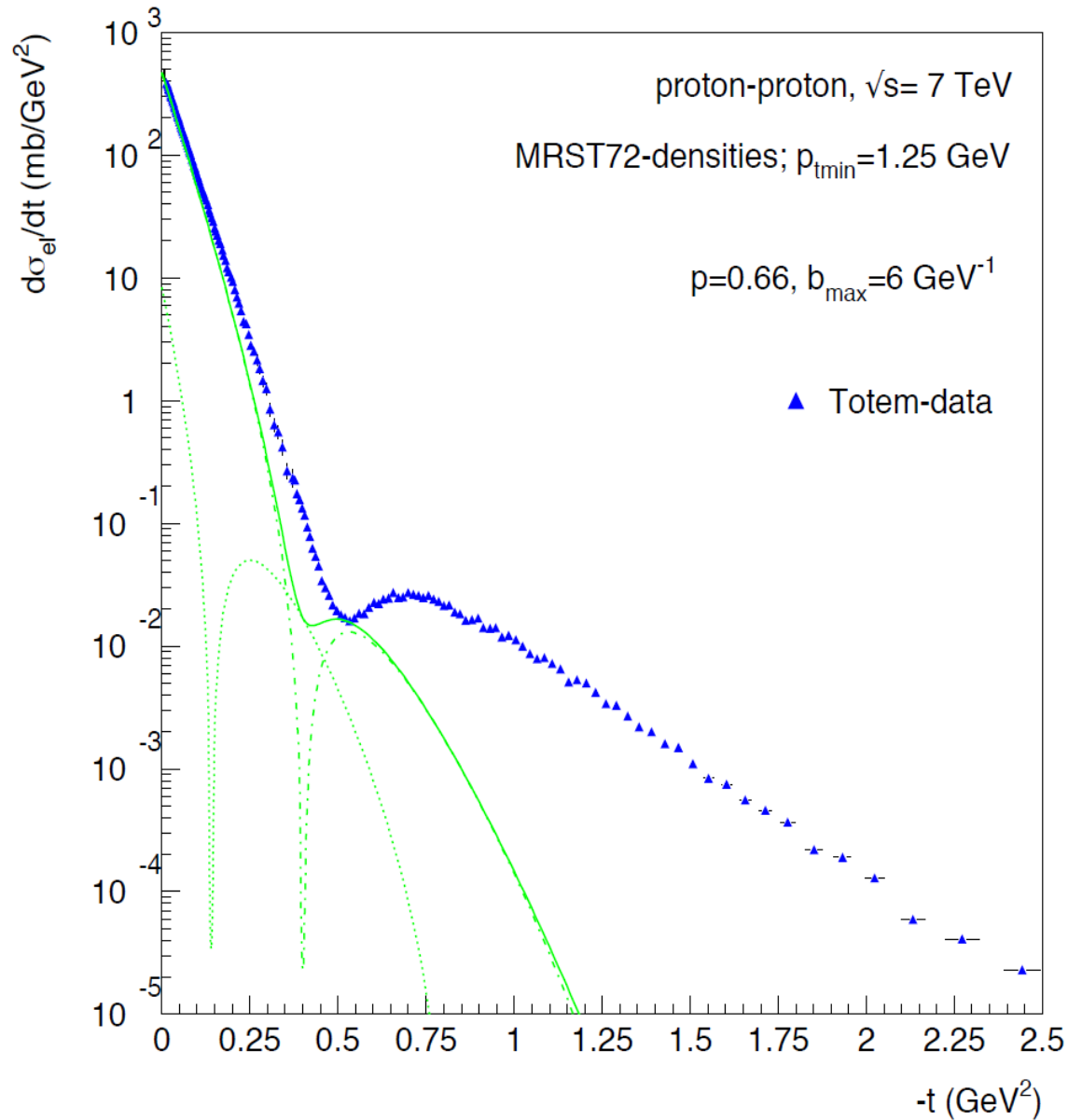
- Despite the limitations of the one-channel approach there are some interesting features in this model, namely:
 - (1) describes the rise of all total cross section: $pp, \pi^\pm p, \pi^\pm \pi^\pm, \gamma p, \gamma\gamma$
 - (2) it gives the nondiffractive (uncorrelated) inelastic cross section;
 - (3) in simplest Glauber picture, the p-air production cross section is obtained as long as the problem of subtracting the diffractive cross section is circumvented;
- However, several aspects deserve further investigation:
 - (1) how to get the elastic differential?;
 - (2) real part of the amplitude?;
 - (3) GW decomposition to include low mass diffraction;
 - (4) higher order correction from soft gluon emission?;
- While the model needs improvements, the fundamental problem of having a comprehensive and widely accepted formalism to treat soft diffractive interactions in terms of QCD remains open

The low energy contribution



$$A_{FF}(b) = \frac{1}{(2\pi)^2} \int d^2\vec{b} e^{iq\cdot b} \left[\frac{\nu^2}{q^2 + \nu^2} \right]^4 = \frac{\mu^2}{96\pi} (\mu b)^3 K_3(\mu b)$$

Differential elastic cross section



$$A_{el}(s, b) = 1/2 \rightarrow b_{max}$$

$$b_{max} \sim 6 \text{ GeV}^{-1}$$