Spin-1 Particles within Light-Front Approach

New Trend in High-Energy Physics and QCD

IIP - UFRN - NATAL - 2014

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November 6, 2014

Light-Front Motivations

- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- **Constituent Picture and Unanbiguous Partons Content of the Hadronic System**
- **Light-Front Wavefunctions: Representation of Composite Systems in QFT**
- **Invariant Under Boosts**
- **Light-Front Vacuum is Trivial**
- **After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)**
- **LF Lorentz Invariant Hamiltonian:** $P^2 = P^+P^- - P_\perp^2$

Light-Front Coordinates

Four-Vector $\implies$ $x^\mu = (x^0, x^1, x^2, x^3) = (x^+, x^-, x_\perp)$

$x^+ = t + z$ $\quad$ $x^+ = x^0 + x^3$ $\implies$ **Time**

$x^- = t - z$ $\quad$ $x^- = x^0 - x^3$ $\implies$ **Position**

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$p^+ = p^0 + p^3$, $p^- = p^0 - p^3$, $p^\perp = (p^1, p^2)$

Dirac Matrix and Electromagnetic Current

$\gamma^+ = \gamma^0 + \gamma^3 \implies$ **Electr. Current** $J^+ = J^0 + J^3$
$\gamma^- = \gamma^0 - \gamma^3 \implies$ **Electr. Current** $J^- = J^0 - J^3$
$\gamma^\perp = (\gamma^1, \gamma^2) \implies$ **Electr. Current** $J^\perp = (J^1, J^2)$

$$p^\mu x_\mu = \frac{p^+ x^- + p^- x^+}{2} - \vec{p}_\perp \vec{x}_\perp$$

$$x^+, x^-, \vec{x_\perp} \implies p^+, p^-, \vec{p}_\perp$$

$p^- \implies$ **Light-Front Energy**

$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \implies \quad p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$

On-shell

Bosons $\implies \quad S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

Fermions $\implies \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

Review Papers:

- **Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky**
- **A. Harindranath, Pramana, Journal of Indian Academy of Sciences Physics Vol.55, Nos 1 & 2, (2000) 241.**
- **An Introduction to Light-Front Dynamics for Pedestrians**

Avaroth Harindranath

Light-Front book organizers: James Vary and Frank Wolz,(1997)

Pole Dislocation Method

$p^+ \Longrightarrow p'^+ = p^+ + \delta$

Boson Eletromagnetic Current

Breit Frame $\Longrightarrow$ $q^- = 0$, $q^+ \Longrightarrow 0_+$, $\vec{q}_\perp \neq 0$

$J^+ = J^- +$ **restoration covariance term**

$J_\perp \propto q^+ \Rightarrow 0$

J. de Melo, Sales and T.Frederico Nucl. Phys. B631, (1998) 574.

Ward-Takahashi Identity $\Longrightarrow$ **Pair Contribuition**

Naus, de Melo and Frederico

Few-Body Syst. 24, 1998, 99-107

•Chang e Yan, Phys. Rev. D7 (73) 1147, Phys. Rev. D7 (73) 1780.

• Sawicki, Phys. Rev. D44 (91) 433, Phys. Rev. D46 (92) 474.

General Electromagnetic Current: Spin-1

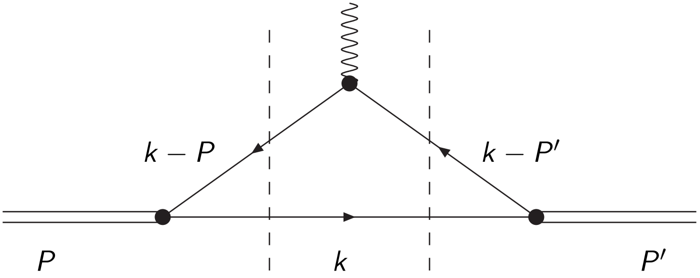
$$J^{\mu}_{\alpha\beta} = [F_1(q^2) g_{\alpha\beta} - F_2(q^2) \frac{q_\alpha q_\beta}{2m_\rho^2}] p^\mu - F_3(q^2)(q_\alpha g^\mu_\beta - q_\beta g^\mu_\alpha) \ ,$$

- Polarization Vectors

$\epsilon^{\mu}_x = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0)$, $\epsilon^{\mu}_y = (0, 0, 1, 0)$, $\epsilon^{\mu}_z = (0, 0, 0, 1)$,
$\epsilon^{'\mu}_x = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0)$, $\epsilon^{'\mu}_y = \epsilon_y$, $\epsilon^{'\mu}_z = \epsilon_z$,
where $\eta = q^2/4m_\rho^2$

- Breit Frame:

$p^{\mu}_i = (p^0, -q_x/2, 0, 0)$ (Initial) where $p^0 = m_\rho\sqrt{1+\eta}$.
$p^{\mu}_f = (p^0, q_x/2, 0, 0)$ (Final)



$$J_{ji}^{+} = \imath \int \frac{d^4k}{(2\pi)^4} \frac{Tr[\epsilon_j^{'\beta}\Gamma_\beta(k,k-p_f)(\not{k}-\not{p}_f+m)}{((k-p_i)^2-m^2+\imath\epsilon)(k^2-m^2+\imath\epsilon)}$$
$$\times \frac{\gamma^+(\not{k}-\not{p}_i+m)\epsilon_i^\alpha\Gamma_\alpha(k,k-p_i)(\not{k}+m)]\Lambda(k,p_f)\Lambda(k,p_i)}{((k-p_f)^2-m^2+\imath\epsilon}$$

$$J_{ji}^{+} = \imath \int \frac{d^4k}{(2\pi)^4} \frac{Tr[\epsilon_j^{'\beta}\Gamma_\beta(k,k-p_f)(\not{k}-\not{p}_f+m)}{((k-p_i)^2-m^2+\imath\epsilon)(k^2-m^2+\imath\epsilon)} \times \frac{\gamma^+(\not{k}-\not{p}_i+m)\epsilon_i^\alpha\Gamma_\alpha(k,k-p_i)(\not{k}+m)]\Lambda(k,p_f)\Lambda(k,p_i)}{((k-p_f)^2-m^2+\imath\epsilon}$$

- Regulator Function

$$\Lambda(k,p_{i(f)}) = N/((p-k)^2-m_R+\imath\epsilon)^2$$

- ρ-Meson Vertex

$$\Gamma^\mu(k,p) = \gamma^\mu - \frac{m_\rho}{2}\frac{2k^\mu - p^\mu}{p.k + m_\rho m - \imath\epsilon}$$

- Mass Squared $\left(x = \frac{k^+}{P^+} \implies 0 < x < 1\right)$

$$M^2(m_a, m_b) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_b^2}{1 - x} - p_\perp^2$$

- Free Mass $M_0^2(m, m)$ and Function $M_R^2(m, m_R)$

The function M_R^2 is given by

$$M_R^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_R^2}{1 - x} - p_\perp^2$$

$$M_0^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m^2}{1 - x} - p_\perp^2$$

- Wave Function

$$\Phi_i(x, \vec{k}_\perp) = \frac{N^2}{(1-x)^2(m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2} \vec{\epsilon}_i . [\vec{\gamma} - \frac{\vec{k}}{\frac{M_0}{2} + m}]$$

Refs.

- Phy.Rev. **C55** (1997) 2043 J.P.B. C. de Melo and T. Frederico
- Phy.Lett. **B708** (2012) 87 J.P.B. C. de Melo and T. Frederico
- Few.Body.Syst. **52**(2012) 403 J.P.B. C. de Melo and T. Frederico

- Instant-Form Spin Base

$$J^+_{ji} = \frac{1}{2}\begin{pmatrix} J^+_{xx} + J^+_{yy} & \sqrt{2}J^+_{zx} & J^+_{yy} - J^+_{xx} \\ -\sqrt{2}J^+_{zx} & 2J^+_{zz} & \sqrt{2}J^+_{zx} \\ J^+_{yy} - J^+_{xx} & -\sqrt{2}J^+_{zx} & J^+_{xx} + J^+_{yy} \end{pmatrix}$$

- Light-Front

$$I^+_{m'm} = \begin{pmatrix} I^+_{11} & I^+_{10} & I^+_{1-1} \\ -I^+_{10} & I^+_{00} & I^+_{10} \\ I^+_{1-1} & -I^+_{10} & I^+_{11} \end{pmatrix}$$

$\Longrightarrow \quad R^\dagger_M I^+ R^\dagger_M = J^+ \quad \Longleftrightarrow$ **Melosh**

- **Matrix Elements**

$$
\begin{aligned}
I^{+}_{11} &= \frac{J^{+}_{xx} + (1+\eta)J^{+}_{yy} - \eta J^{+}_{zz} - 2\sqrt{\eta}J^{+}_{zx}}{2(1+\eta)} \\
I^{+}_{10} &= \frac{\sqrt{2\eta}J^{+}_{xx} + \sqrt{2\eta}J^{+}_{zz} - \sqrt{2}(\eta-1)J^{+}_{zx}}{2(1+\eta)} \\
I^{+}_{1-1} &= \frac{-J^{+}_{xx} + (1+\eta)J^{+}_{yy} + \eta J^{+}_{zz} + 2\sqrt{\eta}J^{+}_{zx}}{2(1+\eta)} \\
I^{+}_{00} &= \frac{-\eta J^{+}_{xx} + J^{+}_{zz} - 2\sqrt{\eta}J^{+}_{zx}}{(1+\eta)}
\end{aligned}
$$

$$
\begin{aligned}
J_{xx}^{+} &= \frac{1}{1+\eta}[I_{11}^{+} + 2\sqrt{2\eta}I_{10}^{+} - \eta I_{00}^{+} - I_{1-1}^{+}] \\
J_{zx}^{+} &= \frac{\sqrt{2}}{1+\eta}[\frac{\sqrt{2\eta}}{2}I_{11}^{+} + (\eta-1)I_{10}^{+} + \sqrt{\frac{\eta}{2}}I_{00}^{+} \\
&\quad -\frac{\sqrt{2\eta}}{2}I_{1-1}^{+}] \\
J_{yy}^{+} &= I_{11}^{+} + I_{1-1}^{+} \\
J_{zz}^{+} &= \frac{1}{1+\eta}[-\eta I_{11}^{+} + 2\sqrt{2\eta}I_{10}^{+} + I_{00}^{+} + \eta I_{1-1}^{+}]
\end{aligned}
$$

$$\Delta(q^2) = (1+2\eta)I^+_{11} + I^+_{1-1} - \sqrt{8\eta}I^+_{10} - I^+_{00} = (1+\eta)(J^+_{yy} - J^+_{zz}) = 0$$

- **Angular Condition: Violation !!**

$$q_x \Longrightarrow J^+_{yy} = J^+_{zz} \begin{cases} \textit{Parity} \\ + \\ \textit{Rotations} \end{cases}$$

$$\Delta(q^2) \neq 0$$

- **Ref:**
- **Sov. J. Nucl. Phys. 39 (1984) 198**

I.Grach and L.A. Kondratyku

- **Phy. Rev. Lett. 62 (1989) 387**

L.L. Frankfurt, I.Grach, L.A. Kondratyku and M. Strikman

Prescriptions

$$\begin{cases} FFS\ (Frederico, Frankfurt, Strikman) \\ GK\ (Grach, Kondratyku) \\ CCKP\ (Coester, Chung, Keister, Polyzou) \\ BH\ (Brodsky, Hiller) \end{cases}$$ **vs** **COVARIANT**

- **Breit Frame** $\Longrightarrow P^{+} = P'^{+}$, $P^{-} = P'^{-}$, $\vec{P'}_{\perp} = -\vec{P}_{\perp} = \vec{q}/2$
- **B.F:** $q^{+} = q^{0} + q^{3} = 0$
- J^{+}_{ρ} $\begin{cases} 4\ \textit{Current Elements} \\ 3\ \textit{Form Factors}\quad G_0\ ,\ G_1\ and\ G_2 \end{cases}$

Inna Grach Prescription: I_{00}^{+}

$$
\begin{aligned}
G_0^{GK} &= \frac{1}{3}[(3-2\eta)I_{11}^{+} + 2\sqrt{2\eta}I_{10}^{+} + I_{1-1}^{+}] = \\
&\quad \frac{1}{3}[J_{xx}^{+} + \eta J_{zz}^{+}(2-\eta)J_{yy}^{+}] \\
G_1^{GK} &= 2[I_{11}^{+} - \frac{1}{\sqrt{2\eta}}I_{10}^{+}] = J_{yy}^{+} - J_{zz}^{+} - \frac{J_{zx}^{+}}{\sqrt{\eta}} \\
G_2^{GK} &= \frac{2\sqrt{2}}{3}[-\eta I_{11}^{+} + \sqrt{2\eta}I_{10}^{+} - I_{1-1}^{+}] = \\
&\quad \frac{\sqrt{2}}{3}[J_{xx}^{+} + J_{yy}^{+}(-1-\eta) + \eta J_{zz}^{+}] \ .
\end{aligned}
$$

CCKP

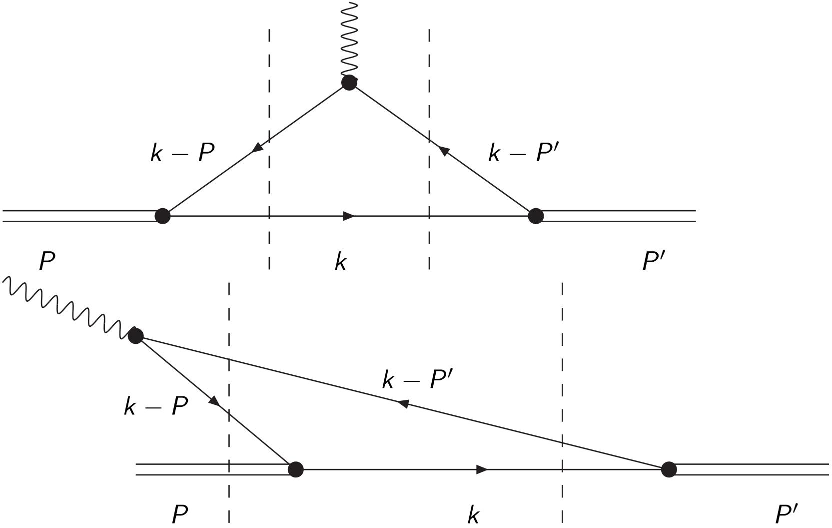
$$
\begin{aligned}
G_0^{CCKP} &= \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I_{11}^{+}+I_{00}^{+})+5\sqrt{2\eta}I_{10}^{+}+(2\eta-\frac{1}{2})I_{1-1}^{+}] \\
&= \frac{1}{6}[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}] \\
G_1^{CCKP} &= \frac{1}{(1+\eta)}[I_{11}^{+}+I_{00}^{+}-I_{1-1}^{+}-\frac{2(1-\eta)}{\sqrt{2\eta}}I_{10}^{+}] = -\frac{J_{zx}^{+}}{\sqrt{\eta}} \\
G_2^{CCKP} &= \frac{\sqrt{2}}{3(1+\eta)}[-\eta I_{11}^{+}-\eta I_{00}^{+}+2\sqrt{2\eta}I_{10}^{+}-(\eta+2)I_{1-1}^{+}] = \\
& \frac{\sqrt{2}}{3}[J_{xx}^{+}-J_{yy}^{+}]
\end{aligned}
$$

Brodsky-Hiller - (BH) - I_{11}^{+}

$$G_0^{BH} = \frac{1}{3(1+2\eta)}[(3-2\eta)I_{00}^{+} + 8\sqrt{2\eta}I_{10}^{+} + 2(2\eta-1)I_{1-1}^{+}]$$
$$= \frac{1}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) + J_{yy}^{+}(2\eta-1) + J_{zz}^{+}(3+2\eta)]$$
$$G_1^{BH} = \frac{2}{(1+2\eta)}[I_{00}^{+} - I_{1-1}^{+} + \frac{(2\eta-1)}{\sqrt{2\eta}}I_{10}^{+}]$$
$$= \frac{1}{(1+2\eta)}[\frac{J_{zx}^{+}}{\sqrt{\eta}}(1+2\eta) - J_{yy}^{+} + J_{zz}^{+}]$$
$$G_2^{BH} = \frac{2\sqrt{2}}{3(1+2\eta)}[\sqrt{2\eta}I_{10}^{+} - \eta I_{00}^{+} - (\eta+1)I_{1-1}^{+}]$$
$$= \frac{\sqrt{2}}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) - J_{yy}^{+}(1+\eta) - \eta J_{zz}^{+}]$$

FFS

$$
\begin{aligned}
G_0^{FFS} &= \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I_{11}^{+}+I_{00}^{+})+5\sqrt{2\eta}I_{10}^{+}+(2\eta-\frac{1}{2})I_{1-1}^{+}] \\
&= \frac{1}{6}[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}] \\
G_1^{FFS} &= G_1^{CCKP} \\
G_2^{FFS} &= G_2^{CCKP}
\end{aligned}
$$



Light-front time-ordered diagrams for the current: Triangle Diagram and Pair Terms

- **Vertex** $\Gamma(\gamma^{\mu}, \gamma^{\nu})$

$$Tr[gg]_{ji} = Tr[\not\epsilon_f^{\alpha}(\not k - \not p' + m)\gamma^{+}(\not k - \not p + m)\not\epsilon_i^{\alpha}(\not k + m)]$$

- **+Z (Pair Terms)** $Tr[gg]_{ji}^{+Z} = \frac{k^{-}}{2} R_{gg}$

$$R_{gg} = Tr[\not\epsilon_f^{\alpha}(\not k - \not p' + m)\gamma^{+}(\not k - \not p + m)\not\epsilon_i^{\alpha}\gamma^{+}]$$

- **Fact:** $\rightarrow k^{-(m+1)} \, (p^{+} - k^{+})^{n}$

  No Pair Terms Contribution if $m < n$

Ref. Nucl. Phys. A660 (1999) 219, De Melo, Frederico, Naus and Sauer

- **Simplification**: $[\gamma^\mu, \gamma^\nu]$ **Dirac Trace**:

$$
\begin{aligned}
Tr[gg]^{+Z}_{xx} &= -\eta \; Tr[gg]^{+Z}_{zz} \\
Tr[gg]^{+Z}_{zx} &= -\sqrt{\eta} \; Tr[gg]^{+Z}_{zz} \\
Tr[gg]^{+Z}_{zz} &= R_{gg}
\end{aligned}
$$

- **Also:**

$$Tr[gg]^{+Z}_{yy} = 4k^-(p^+ - k^+)^2$$

- **Pair Terms**

$$J^{+Z}_{xx} = \lim_{\delta^+\to 0}\int d^3K \frac{Tr[J^{+Z}_{xx}]}{[1][2][4][5][6][7]} \neq 0$$

$$J^{+Z}_{zx} = \lim_{\delta^+\to 0}\int d^3K \frac{Tr[J^{+Z}_{zx}]}{[1][2][4][5][6][7]} \neq 0$$

$$J^{+Z}_{zz} = \lim_{\delta^+\to 0}\int d^3K \frac{Tr[J^{+Z}_{zz}]}{[1][2][4][5][6][7]} \neq 0$$

$$J^{+Z}_{yy} = \lim_{\delta^+\to 0}\int d^3K \frac{Tr[J^{+Z}_{yy}]}{[1][2][4][5][6][7]} = 0$$

- **Basis $I^{+}_{m'm}$:**

$$I^{+Z}_{11} = 0,\ I^{+Z}_{10} = 0$$
$$I^{+Z}_{1-1} = 0,\ I^{+Z}_{00} = (1+\eta)J^{+}_{zz} \neq 0$$

- **Pair Term Contribution:** <u>**only: I^{+Z}_{00} !!**</u>
- **Inna Grach:** **Elimination** I^{+}_{00}

$$
\begin{aligned}
G_0^{GK} &= \frac{1}{3}[J^+_{xx} + 2J^+_{yy} - \eta J^+_{yy} + \eta J^+_{zz}] \\
G_1^{GK} &= J^+_{yy} - J^+_{zz} - \frac{J^+_{zx}}{\sqrt{\eta}} \\
G_2^{GK} &= \frac{\sqrt{2}}{3}[J^+_{xx} + J^+_{yy}(-1-\eta) + \eta J^+_{zz}]
\end{aligned}
$$

$$G_0^{GK\ (+Z)} = \frac{1}{3}\left(J_{xx}^{(+Z)}[gg] + \eta J_{zz}^{+Z}[gg]\right) = \frac{1}{3}\left(-\eta J_{zz}^{+Z}[gg] + \eta J_{zz}^{+Z}[gg]\right) = 0$$

$$G_1^{GK\ (+Z)} = \left(-J_{zz}^{+Z}[gg] - \frac{J_{zx}^{+Z}[gg]}{\sqrt{\eta}}\right) = -J_{zz}^{+Z}[gg] + \sqrt{\eta}\frac{J_{zz}^{+Z}[gg]}{\sqrt{\eta}} = 0$$

$$G_2^{GK\ (+Z)} = \frac{\sqrt{2}}{3}\left(J_{xx}^{+Z}[gg] + \eta J_{zz}^{+Z}[gg]\right) = \frac{\sqrt{2}}{3}\left(-\eta J_{zz}^{+} + \eta J_{zz}^{+Z}\right) = 0$$

Cross term with γ^μ and derivatives

$$Tr[dg]_{ji} = \epsilon'_j \cdot (2k - p')\ Tr[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\not\epsilon_i(\not k + m)]$$

- Terms with $m \geq n$:

$$Tr[dg]^Z_{ji} = \epsilon'^+_j \epsilon^+_i R_{dg} - 4m\ k^- k^+ \epsilon'^+_j\ \vec{\epsilon}_{i\perp} \cdot \vec{q}_\perp$$

•$R_{dg} = 4m\ k^- \left(k^-(k^+ - p^+) + (\vec{k}_\perp - \vec{p}'_\perp) \cdot (\vec{k}_\perp - \vec{p}_\perp) + q_\perp \cdot k_\perp + m^2\right)$

•The **Z-modes to yy is zero** $\rightarrow$ $\epsilon^+_y = 0$ **and** $\epsilon'^+_y = 0$

- **Term with k^- and $k^{-2}(k^+ - p^+)$**
- **Interval $0 < k^+ - p^+ < \delta^+$ with γ^μ and derivative couplings:**

$$J_{ji}^{+Z}[dg] = \lim_{\delta^+ \to 0_+} \int [d^4k]^Z \frac{Tr[dg]_{ji}^Z}{\{1\}\{2\}\{3\}} \frac{1}{\{4\}^2} \frac{1}{\{5\}^2} \frac{m_v}{2(p' \cdot k + m\, m_v - i\epsilon)}$$

- **Zero of $\{3\}$ is dislocated by using $p'^+ = p^+ + \delta^+$**
- **Cauchy integration in k^- with k^+ in the interval $0 < k^+ - p^+ < \delta^+$**
- **Two poles: one from the dislocated denominator $\{3\} = 0$, and**

$$k^- = \frac{1}{p^+}\left(2\vec{p}_\perp^{\,\prime} \cdot \vec{k}_\perp - k^+ p^- - 2\, m\, m_v + \imath\epsilon\right) .$$

- **The residue from the zero of $\{3\}$ is $\mathcal{O}[(\delta^+)^2]$**
- **The residue from the pole gives a contribution $\mathcal{O}[(\delta^+)^0]$.**

After integration in k^-, one has that:

$$J_{ji}^{+Z}[dg] \sim \mathcal{O}[\delta^+] .$$

Direct term with derivative couplings

$$Tr[dd]_{ji} = \left[A_{dd}\ \frac{k^-}{2} + B_{dd}\right] \epsilon'_j \cdot (2k - p')\ \epsilon_i \cdot (2k - p)\ .$$

$$\begin{aligned} A_{dd} &= Tr[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\gamma^+] = 8(p^+ - k^+)^2 \\ B_{dd} &= Tr\left[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\left(\frac{\gamma^-}{2}k^+ - \vec{\gamma}_\perp . \vec{k}_\perp + m\right)\right]\ . \end{aligned}$$

$$\begin{aligned} J^{+Z}_{ji}[dd] &= \lim_{\delta^+ \to 0_+} \int [d^4k]^Z \frac{Tr[dd]_{ji}}{\{1\}\{2\}\{3\}\{5\}^2\{6\}^2} \\ &\times \frac{m_v^2}{4(p \cdot k + m\, m_v - i\epsilon)(p' \cdot k + m\, m_v - i\epsilon)} \end{aligned}$$

$\Longrightarrow\ J^{+Z}_{ij}[dd] \sim\ \mathcal{O}[\delta^+]$

$I^{+Z}_{11} = 0\ ,\ I^{+Z}_{10} = 0\ ,\ I^{+Z}_{1-1} = 0$ **and** $I^{+Z}_{00} = (1+\eta)J^{+Z}_{zz}$ **with** $\lim_{\delta^+ \to 0_+} J^{+Z}_{zz} \neq 0$

Final Result:

No Zero Modes or Pair Terms Contribution with Inna Grach prescp.!!

REf.:

- **J.P.B.C. de Melo and T. Frederico,Phys. Lett. B708, (2012) 87**
- **J.P.B.C. de Melo and T. Frederico,Few Body Syst. 52 (2012) 403**
- **Similar Results are found by Ji, Bakker and Choi:**
- **Phy.Rev.D65 (2002) 116001**
- **Phy.Rev.D70 (2004) 053015**

Elimination of Zero Modes

- **VIP:**

$$
\begin{aligned}
J^{+z}_{xx} &= -\eta J^{+z}_{zz} \\
J^{+z}_{zx} &= -\sqrt{\eta} J^{+z}_{zz} \\
J^{+z}_{yy} &= 0 \ .
\end{aligned}
$$

- **Also:**

$$
J^{+Z}_{zz} = J^{+V}_{yy} - J^{+V}_{zz}
$$

- **J.P.B.C. de Melo and T. Frederico, Phys. Lett. B708, (2012) 87**

Electromagetic Form Factors Free of Zero Modes

$$G_0^{CCKP} = \frac{1}{6}[2J^+_{xx} + J^+_{yy} + 3J^+_{zz}] = \frac{1}{3}[J^{+V}_{xx} + (2-\eta)J^{+V}_{yy} + \eta J^{+V}_{zz}] = G_0^{GK}$$

$$G_1^{CCKP} = -\frac{J^+_{zx}}{\sqrt{\eta}} = [J^{+V}_{yy} - \frac{J^{+V}_{zx}}{\sqrt{\eta}} - J^{+V}_{zz}] = G_1^{GK}$$

$$G_2^{CCKP} = \frac{\sqrt{2}}{3}[J^+_{xx} - J^+_{yy}] = \frac{\sqrt{2}}{3}[J^{+V}_{xx} - (1+\eta)J^{+V}_{yy} + \eta J^{+V}_{zz}] = G_2^{GK}$$

$$G_0^{FFS} = \frac{1}{3}[J^+_{xx} + 2J^+_{yy}] = \frac{1}{3}[J^{+V}_{xx} - (-2-\eta)J^{+V}_{yy} + \eta J^{+V}_{zz}] = G_0^{GK}$$

$$G_1^{FFS} = G_1^{CCKP} = G_1^{GK}$$

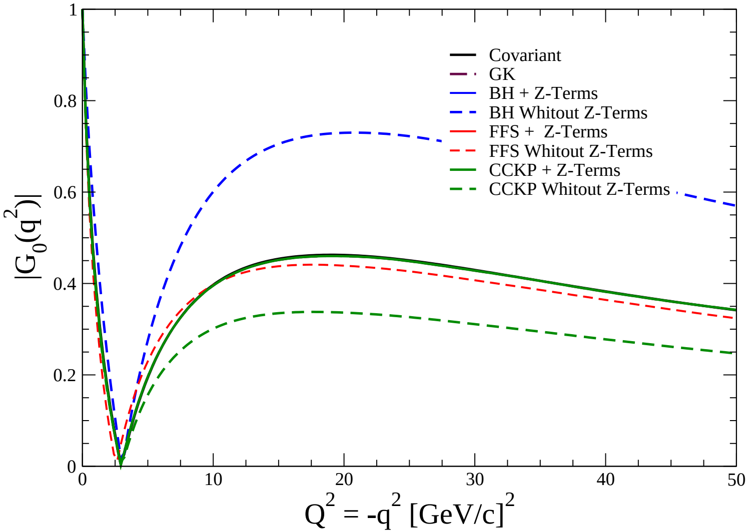
$$G_2^{FFS} = G_2^{CCKP} = G_2^{GK}$$

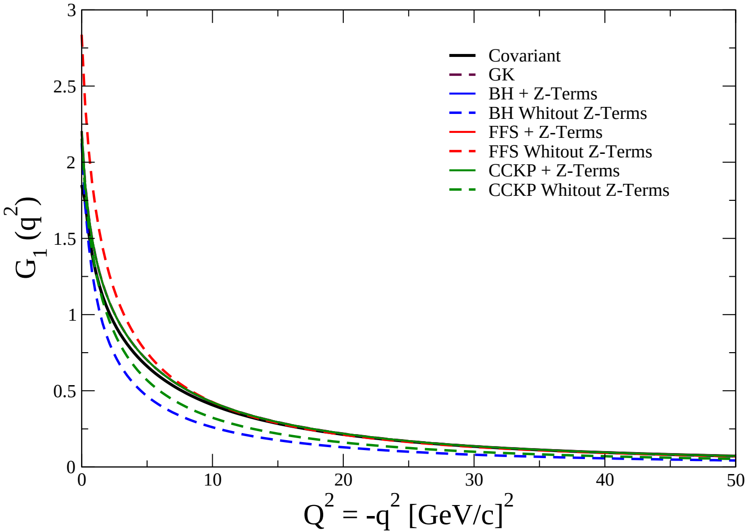
$$
\begin{aligned}
G_0^{BH} &= \frac{1}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) + J_{yy}^{+}(2\eta-1) + J_{zz}^{+}(3+2\eta)] \\
&= \frac{1}{3}[J_{xx}^{+V} + (2-\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}] = G_0^{GK} \\
G_1^{BH} &= \frac{1}{(1+2\eta)}[\frac{J_{zx}^{+}}{\sqrt{\eta}}(1+2\eta) - J_{yy}^{+} + J_{zz}^{+}] \\
&= [J_{yy}^{+V} - \frac{J_{zx}^{+V}}{\sqrt{\eta}} - J_{zz}^{+V}] \; = \; G_1^{GK} \\
G_2^{BH} &= \frac{\sqrt{2}}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) - J_{yy}^{+}(1+\eta) - \eta J_{zz}^{+}] \\
&= \frac{\sqrt{2}}{3}[J_{xx}^{+V} - (1+\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}] \; = \; G_2^{GK}
\end{aligned}
$$

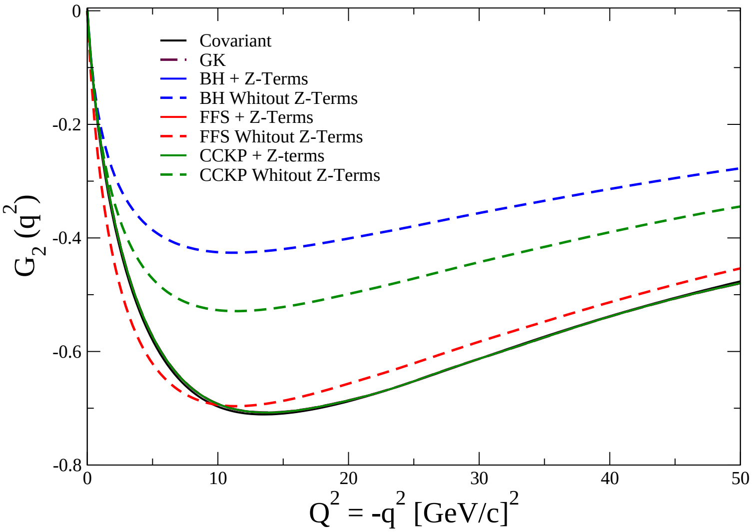
Important: **All Prescriptions Given the Same Electr. Form Factors**

Angular Condition: Free of Zero Modes

$$\begin{aligned}\Delta(q^2) &= (1+2\eta)I^+_{11} + I^+_{1-1} - \sqrt{8\eta}I^+_{10} - I^+_{00} = 0 \\ &= (1+\eta)(J^{+V}_{yy} - J^+_{zz}) = 0\end{aligned}$$

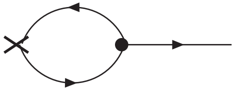

Covariant
GK
BH + Z-Terms
BH Whitout Z-Terms
FFS + Z-Terms
FFS Whitout Z-Terms
CCKP + Z-Terms
CCKP Whitout Z-Terms
$|G_0(q^2)|$
$Q^2 = -q^2$ [GeV/c]^2
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50


Covariant
GK
BH + Z-Terms
BH Whitout Z-Terms
FFS + Z-Terms
FFS Whitout Z-Terms
CCKP + Z-Terms
CCKP Whitout Z-Terms
$G_1(q^2)$
$Q^2 = -q^2$ [GeV/c]^2
0
0.5
1
1.5
2
2.5
3
0
10
20
30
40
50



Covariant
GK
BH + Z-Terms
BH Whitout Z-Terms
FFS + Z-Terms
FFS Whitout Z-Terms
CCKP + Z-terms
CCKP Whitout Z-Terms
0
-0.2
-0.4
-0.6
-0.8
0
10
20
30
40
50
$G_2(q^2)$
$Q^2 = -q^2$ [GeV/c]^2

Decay Constant



$$< 0|J^\mu(0)|p,\lambda >= \imath\sqrt{2} f_V M \epsilon_\lambda^\mu$$

- $\epsilon_\lambda^\mu \implies$ **Polarization** $\rightarrow \quad \epsilon_z^+ \; = \; 1$

Dirac Trace:

$$Tr[O^+] \;=\; [-4k^{+2} + 4k_\perp^2 + 4k^+p^+ + 4m^2] - \frac{m_V}{2}\frac{[4m(2k^+ - p^+)(k^- \; - \; k^+)]}{\frac{p^+k^- + p^-k^+}{2} + m_v m - \imath\epsilon}$$

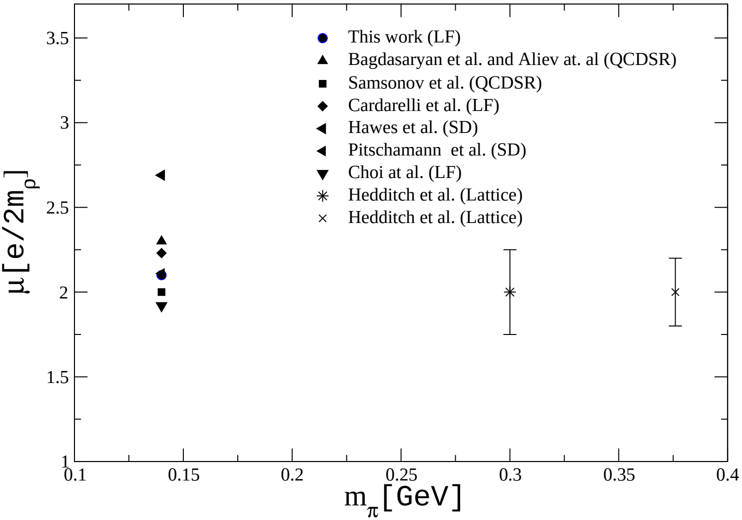
- **Because the denominator:** <u>**the zero mode is cancel out!!**</u>

Observables

m_q / m_R [GeV]	f_v [GeV]	$< r^2 >$ [fm^2]	μ	Q_d [e/m^2]
0.430 / 3.0	0.154	0.267	2.10	-0.898
[1]	0.134 / 0.151	0.296	2.10	-0.910
[2]	0.130	0.312	2.11	-0.850
[3]	0.207	0.540	2.01	-0.410
PDG	0.153 ± 0.008			

[1] **Phy.Rev.D65 (2002) 116001, B. Bakker, H. M. Choi and C. R. Ji ; / Phys.Rev. D89 (2014) 033011**
[2] Phy.Rev.C83 (2011) 065206, H. L. Roberts, A. Bashir, L.X.G. Guerrero, C. Roberts,
[3] Phy.Rev.C77 (2008) 025203, M. S. Bhagwat and P. Maris

See the poster by Clayton Santos Mello $\Longrightarrow$ Dependence with the parameters $\Longleftrightarrow$ ($< r^2 >$, μ and Q_2)


This work (LF)
Bagdasaryan et al. and Aliev at. al (QCDSR)
Samsonov et al. (QCDSR)
Cardarelli et al. (LF)
Hawes et al. (SD)
Pitschamann et al. (SD)
Choi at al. (LF)
Hedditch et al. (Lattice)
Hedditch et al. (Lattice)
3.5
3
2.5
2
1.5
1
0.1
0.15
0.2
0.25
0.3
0.35
0.4
$\mu[e/2m_\rho]$
m_π[GeV]

Rho Meson Form Factors						
	$Q^2 = 1\ GeV^2$		//	$Q^2 = 2\ GeV^2$		
	G_c	G_M	G_Q	G_c	G_M	G_Q
This Work	0.408	1.438	-0.233	0.156	1.152	-0.375
Ref.1	0.220	0.570	-0.110	0.080	0.27	-0.050
Ref.2	0.170	0.850	-0.511	-0.040	0.45	-0.320
Ref.3	0.380	0.930	-0.230	0.180	0.59	-0.150
Ref.4	0.250	0.580	-0.490	0.130	0.280	-0.240

Ref.

[1] M.S. Bhagwat and P. Maris, Phy. Rev. C77 (2008) 025203 .
[2] F. T. Hawes and M. A. Pichowsky, Phys. Rev. C59 (1999) 1743.
[3] Choi and Ji, Phy. Rev. D70 (2004) 053015
[4] T.M. Aliev and M. Savci, Phy. Rev. D70 (2004) 094007.

- Light-Front $\Longrightarrow \begin{cases} Bound\ \ States \\ Covariance \end{cases}$
- Rotational Invariance Broken $\Longrightarrow k^-$ Problematic
- Terms $\begin{cases} -\ Good\ \ /\ \ Covariant \\ -\ Z\ \ \ Terms\ \ /\ \ Non-Valence\ \ Components \end{cases}$
- Electromagnetic Current:
- $\begin{cases} -\ \mathbf{Present\ \ Work}:\ \ \mathbf{J}^+\ \ \mathbf{Component} \\ -\ \mathbf{Future\ \ Works}:\ \ \mathbf{J}^-\ \ \mathbf{and}\ \ \mathbf{J}_\perp \end{cases}$
- Take New Informations about Bound States
- $\begin{cases} -\ \mathbf{Correlations}\ \ |\mathbf{q\bar{q}}> \\ -\ \mathbf{Rho\ Meson\ Decay} \\ -\ \mathbf{Deuteron} \end{cases}$

Support by LFTC and Brazilian Agencies

- FAPESP , CNPq and CAPES

Thanks (Obrigado)!!