Impact Factors for Reggeon-Gluon Transitions

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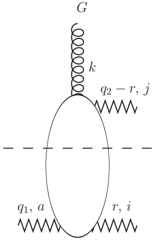
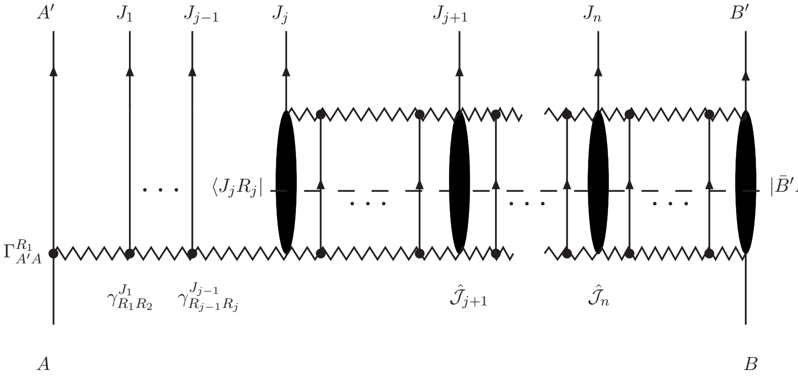
Reminder

The impact factors for Reggeon-gluon transitions describe transitions of Reggeons(Reggeized gluons) into ordinary gluons due to interaction with Reggeized gluons.



Reminder

These impact factors enter as an integral part in the s_i discontinuities of multi-Regge amplitudes



I discussed three different applications of these discontinuites.

Reminder

1. For the proof of the Reggeized form of multi-particle amplitudes.

There is an infinite set of the bootstrap relations which follow from the requirement of compatibility of the s_i-channel unitarity with the Reggeized form of multi-particle amplitudes.

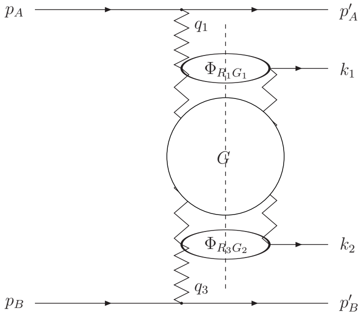
Fulfillment of these relations guarantees the multi–Regge form of scattering amplitudes.

All bootstrap relations are fulfilled if several bootstrap conditions imposed on the Reggeon vertices and the trajectory are satisfied.

All these conditions are proved to be satisfied in the NLO.

2. For a simple demonstration of violation of the BDS ansatz M^{BDS} for MHV amplitudes in N=4 SYM in the planar limit.

Energy behaviour of the discontinuity of $A_{2\to4}$ in the s_2- channel (s_2 is the invariant mass square of produced gluons)

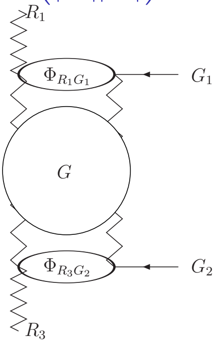


is not given by the Regge factor $\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)^{\omega(t_2)}$, as in the BDS ansatz.

Reminder

Instead, it is determined by the matrix element

$$\langle G_1 R_1 | e^{\hat{K} \ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3 \rangle = \left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)^{\omega(t_2)} \langle G_1 R_1 | e^{\hat{K}_m \ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3$$



where $\hat{K}_m = \hat{K} - \omega(t)$ – modified BFKL kernel.

Reminder

3. For the check of validity of hypotheses about the reminder function R to the BDS ansatz:

Factorization hypothesis, which states the correct amplitude can be presented as the product M^{BDS} times the remainder function R, where M^{BDS} contains all infrared divergencies and R depends only on the anharmonic ratios of kinematic invariants. This property is called dual conformal (Möbius) invariance.

This hypothesis is not proved.

the hypothesis that the remainder functions are given by expectation values of Wilson loops in N=4 SYM.

This hypothesis is also not proved.

To check these hypotheses we performed the calculation of the matrix element $\langle G_1 R_1 | e^{\hat{K}_m \ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3 \rangle$.

Reminder

The infrared safety of the modified kernel $\hat{K}_m$ in the NLO was proved.

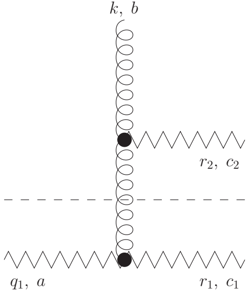
The eigenvaues of $\hat{K}_m$ were found, assuming existence of conformal invariant representation $\hat{K}_c$ of the modified kernel.

Existence of the conformal invariant representation $\hat{K}_c$ of the modified kernel was proved.

The simularity transformation between the standard form $\hat{K}_m$ of the modified kernel and the the conformal invariant representation $\hat{K}_c$ was found in an explicit form.

Impact factor in the bootstrap scheme

In the Born approximation only gluon can be in the intermediate state



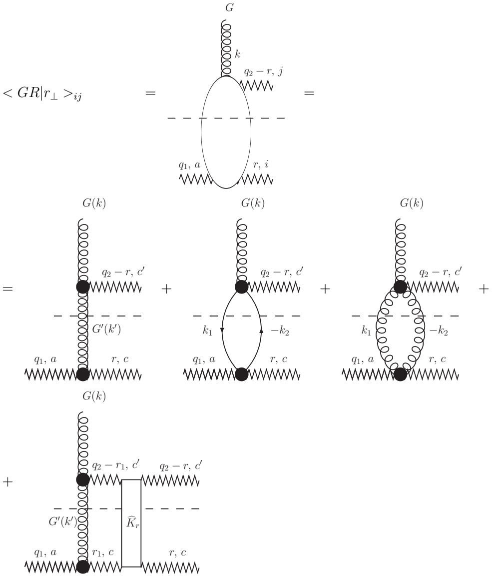
$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle^{(B)} = 2g^2\delta(\vec{q}_1 - \vec{k} - \vec{r}_1 - \vec{r}_2)\left(T^aT^b\right)_{c_1c_2}\ \vec{e}^{\,*}\vec{C}_1,$$

$\vec{e}^{\,*}$ is the conjugated transverse part of the polarization vector $e(k)$ in the gauge $e(k)p_2 = 0$ with the lightcone vector p_2 close to the vector p_B,

$$\vec{C}_1 \ = \ \vec{q}_1 - (\vec{q}_1 - \vec{r}_1)\frac{\vec{q}_1^{\,2}}{(\vec{q}_1 - \vec{r}_1)^2}.$$

Impact factor in the bootstrap scheme

In the NLO



Impact factor in the bootstrap scheme

In N=4 SYM the NLO impact factor contains gluon, fermion and scalar contributions. These contributions were found
M.G. Kozlov, A.V. Reznichenko, V.S. F. 2012,
M.G. Kozlov, A.V. Reznichenko, V.S. F. 2013,
for Yang-Mills theories with any number of fermions and scalars in arbitrary representations of the gauge group.

In general, the impact factors contain two parts with different colour structure. In the planar limit only parts with Born colour structure remain, so that the impact factor can be written as

$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle = g^2\delta(\vec{q}_1-\vec{k}-\vec{r}_1-\vec{r}_2)\left(T^aT^b\right)_{c_1c_2}\ \vec{e}^{\,*}\left[2\vec{C}_1+\bar{g}^2\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1}\right].$$

The results of
M.G. Kozlov, A.V. Reznichenko, V.S. F. 2012,
M.G. Kozlov, A.V. Reznichenko, V.S. F. 2013,
are in dimensional regularization, which differs from the dimensional reduction used in supersymmetric theories.

Impact factor in the bootstrap scheme

To take into account this difference one has to take the number of the scalar fields n_S equal to $6-2\epsilon$ ($\epsilon=(D-4)/2$, D is the space-time dimension).

With account of this, in the planar N=4 SYM

$$\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1*} = \vec{C}_1\left(\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{k}^{\,2}}\right)\ln\left(\frac{\vec{r}_2^{\,2}}{\vec{k}^{\,2}}\right)+\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2\vec{q}_1^{\,2}}{\vec{k}^{\,4}}\right)\ln\left(\frac{\vec{r}_1^{\,2}}{\vec{q}_1^{\,2}}\right)\right.$$

$$\left.-4\frac{(\vec{k}^{\,2})^{\epsilon}}{\epsilon^2}+6\zeta(2)\right)+\vec{C}_2\left(\ln\left(\frac{\vec{k}^{\,2}}{\vec{r}_2^{\,2}}\right)\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{r}_2^{\,2}}\right)\right.$$

$$\left.+\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\ln\left(\frac{\vec{k}^{\,2}}{\vec{q}_2^{\,2}}\right)\right)-2\left[\vec{C}_1\times\left(\left[\vec{k}\times\vec{r}_2\right]I_{\vec{k},\vec{r}_2}-\left[\vec{q}_1\times\vec{r}_1\right]I_{\vec{q}_1,-\vec{r}_1}\right)\right]$$

$$\left.+2\left[\vec{C}_2\times\left(\left[\vec{k}\times\vec{r}_2\right]I_{\vec{k},\vec{r}_2}+\left[q_1\times\vec{k}\right]I_{\vec{q}_1,-\vec{k}}\right)\right]\right\}.$$

Here $\bar{g}^2 = g^2\Gamma(1-\epsilon)/(4\pi)^{2+\epsilon}$,

$$\vec{C}_2 \;=\; \vec{q}_1 - \vec{k}\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}},$$

$\Gamma(x)$ is the Euler gamma-function, $\zeta(n)$ is the Riemann zeta-function ($\zeta(2)=\pi^2/6$), $\left[\vec{a}\times c\left[\vec{b}\times\vec{c}\right]\right]$ is a double vector product, and

$$I_{\vec{p},\vec{q}} = \int_0^1 \frac{dx}{(\vec{p}+x\vec{q})^2}\ln\left(\frac{\vec{p}^{\,2}}{x^2\vec{q}^{\,2}}\right)\;,\quad I_{\vec{p},\vec{q}} = I_{-\vec{p},-\vec{q}} = I_{\vec{q},\vec{p}} = I_{\vec{p},-\vec{p}-\vec{q}}\;.$$

The NLO correction $\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1*}$ is obtained after huge cancellations between gluon, fermion and scalar contributions. In particular, solely due to these cancellations only two vector structures ($\vec{C}_1$ and $\vec{C}_2$) remain; each of the contributions separately contains three independent vector structures.

Impact factor in the bootstrap scheme

As it is known, NLO corrections are scheme dependent. The scheme used in the derivation of $\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1*}$ was adjusted simplifying the verification of the bootstrap conditions (we call it bootstrap scheme). It is different from the standard scheme defined in
V.S. F., R. Fiore, M.G. Kozlov, A.V. Reznichenko, 2006.
The impact factors in these schemes are connected by the transformation

$$\langle GR_1|_* = \langle GR_1|_s - \langle GR_1|^{(B)}\widehat{\mathcal{U}}_k,$$

where subscript s means the standard scheme and the the operator $\widehat{\mathcal{U}}_k$ is defined by the matrix elements

$$\langle \mathcal{G}_1'\mathcal{G}_2'|\widehat{\mathcal{U}}_k|\mathcal{G}_1\mathcal{G}_2\rangle = \frac{1}{2}\ln\left(\frac{\vec{k}^{\,2}}{(\vec{r}_1-\vec{r}_1{}')^2}\right)\langle \mathcal{G}_1'\mathcal{G}_2'|\widehat{\mathcal{K}}_r^B|\mathcal{G}_1\mathcal{G}_2\rangle.$$

Here $\widehat{\mathcal{K}}_r^B$ is the part of the LO BFKL kernel related with the real gluon production:

Transformation to the standard scheme

$$\langle \mathcal{G}_1' \mathcal{G}_2' | \widehat{\mathcal{K}}_r^B | \mathcal{G}_1 \mathcal{G}_2 \rangle = \delta(\vec{r}_1{}' + \vec{r}_2{}' - \vec{r}_1 - \vec{r}_2) \frac{g^2}{(2\pi)^{D-1}} T^i_{c_1 c_1'} T^i_{c_2' c_2}$$

$$\times \left(\frac{\vec{r}_1{}^2 \vec{r}_2{}'^{\,2} + \vec{r}_2{}^2 \vec{r}_1{}'^{\,2}}{\vec{l}^{\,2}} - \vec{q}_2{}^2 \right),$$

where $\vec{l} = \vec{r}_1 - \vec{r}_1{}' = \vec{r}_2{}' - \vec{r}_2, \quad \vec{q}_2 = \vec{r}_1 + \vec{r}_2 = \vec{r}_1{}' + \vec{r}_2{}'$.
At large N_c we can write

$$\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1 s} = \vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1 *} + \vec{\mathcal{I}}_1,$$

$$\vec{\mathcal{I}}_1 = \int \frac{d\vec{l}}{\Gamma(1-\epsilon)\pi^{1+\epsilon}} \vec{C}_1{}' \frac{1}{\vec{r}_1{}'^{\,2} \vec{r}_2{}'^{\,2}} \left(\frac{\vec{r}_1{}^2 \vec{r}_2{}'^{\,2} + \vec{r}_2{}^2 \vec{r}_1{}'^{\,2}}{\vec{l}^{\,2}} - \vec{q}_2^{\,2} \right) \ln\left(\frac{\vec{k}^{\,2}}{\vec{l}^{\,2}} \right),$$

where

$$\vec{C}_1{}' = \vec{q}_1 - \vec{q}_1{}^2 \frac{(\vec{q}_1 - \vec{r}_1{}')}{(\vec{q}_1 - \vec{r}_1{}')^2}.$$

Transformation to the standard scheme

The integration gives

$$\vec{\mathcal{I}}_1 = \frac{1}{2}\vec{C}_1\Bigg[\ln\left(\frac{\vec{r}_2^{\,2}}{\vec{k}^{\,2}}\right)\ln\left(\frac{\vec{k}^{\,4}}{(\vec{q}_1-\vec{r}_1)^2\vec{r}_2^{\,2}}\right)+\ln\left(\frac{\vec{r}_1^{\,2}}{\vec{k}^{\,2}}\right)\ln\left(\frac{\vec{k}^{\,2}\vec{q}_1^{\,2}}{(\vec{q}_1-\vec{r}_1)^2\vec{r}_1^{\,2}}\right)$$

$$-\ln\left(\frac{\vec{k}^{\,2}}{(\vec{q}_1-\vec{r}_1)^2}\right)\ln\left(\frac{\vec{k}^{\,2}\vec{q}_1^{\,2}}{(\vec{q}_1-\vec{r}_1)^4}\right)+4\frac{(\vec{k}^{\,2})^{\epsilon}}{\epsilon^2}-4\zeta(2)\Bigg]$$

$$-\frac{1}{2}\vec{C}_2\Bigg[\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{r}_2^{\,2}}\right)\ln\left(\frac{\vec{k}^{\,4}}{\vec{r}_1^{\,2}\vec{r}_2^{\,2}}\right)+\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{q}_1^{\,2}}\right)\ln\left(\frac{\vec{k}^{\,4}}{\vec{r}_1^{\,2}(\vec{q}_1-\vec{r}_1)^2}\right)\Bigg]$$

$$+\left[\vec{C}_1\times\left(\left[\vec{k}\times\vec{r}_2\right]I_{\vec{k},\vec{r}_2}-\left[\vec{q}_1\times\vec{r}_1\right]I_{\vec{q}_1,-\vec{r}_1}\right)\right]$$

$$+\left[\vec{C}_2\times\left(\left[\vec{r}_1\times\vec{r}_2\right]I_{\vec{r}_1,\vec{r}_2}+\left[\vec{q}_1\times\vec{r}_1\right]I_{\vec{q}_1,-\vec{r}_1}\right)\right],$$

Transformation to the standard scheme

and the one-loop correction to the impact factor in the standard scheme

$$\vec{\Phi}^{\mathcal{G}_1\mathcal{G}_2}_{GR_1(s)} = \frac{1}{2}\vec{C}_1\left[\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{r}_1^{\,2}}\right)\ln\left(\frac{\vec{k}^{\,2}\vec{r}_1^{\,2}}{\vec{q}_1^{\,4}}\right)+\ln\left(\frac{\vec{r}_2^{\,2}}{\vec{k}^{\,2}}\right)\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{r}_2^{\,2}}\right)\right.$$

$$\left.+\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^2}{\vec{k}^{\,2}}\right)\ln\left(\frac{\vec{k}^{\,4}\vec{r}_1^{\,2}}{(\vec{q}_1-\vec{r}_1)^4\vec{q}_1^{\,2}}\right)-4\frac{(\vec{k}^{\,2})^{\epsilon}}{\epsilon^2}+8\zeta(2)\right]$$

$$+\left[\vec{C}_1\times\left(\left[\vec{q}_1\times\vec{r}_1\right]I_{\vec{q}_1,-\vec{r}_1}-\left[\vec{k}\times r_2\right]I_{\vec{k},\vec{r}_2}\right)\right]$$

$$+\frac{1}{2}\vec{C}_2\left[\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\ln\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,2}}{\vec{q}_2^{\,4}}\right)+\ln\left(\frac{\vec{r}_2^{\,2}}{(\vec{q}_1-\vec{r}_1)^2}\right)\ln\left(\frac{\vec{r}_2^{\,2}\vec{q}_1^{\,2}}{\vec{r}_1^{\,2}(\vec{q}_1-\vec{r}_1)^2}\right)\right]$$

$$+\left[\vec{C}_2\times\left(\left[\vec{r}_1\times\vec{r}_2\right]I_{\vec{r}_1,\vec{r}_2}+\left[\vec{q}_1,\vec{r}_1\right]I_{\vec{q}_1,-\vec{r}_1}+2\left[\vec{k},\vec{r}_2\right]I_{\vec{k},\vec{r}_2}+2\left[\vec{q}_1,\vec{k}\right]I_{\vec{q}_1,-\vec{k}}\right)\right].$$

The impact factor in the Möbius scheme

This correction corresponds to the standard kernel and the energy scale $|\vec{k}_1||\vec{k}_2|$, where $\vec{k}_{1,2}$ are the transverse momenta of produced gluons in the two impact factors connected by the Green function of the two interacting Reggeons.

$$\langle G_1 R_1 | e^{\hat{K}_m \ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3 \rangle$$

Our goal is the impact factor in the Möbius scheme, that means the impact factor for Reggeon-gluon transition which can be used for the calculation of the remainder function with conformal invariant kernel and energy evolution parameter. To obtain the impact factor in the Möbius scheme we have to perform two transformations.

The impact factor in the Möbius scheme

As it was shown, the conformal invariant $\hat{\mathcal{K}}_c$ and standard $\hat{\mathcal{K}}_m$ forms of the modified kernel are connected by the similarity transformation

$$\hat{\mathcal{K}}_c = \hat{\mathcal{K}}_m + \frac{1}{4}\left[\hat{\mathcal{K}}^B, \left[\ln\left(\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}\right), \hat{\mathcal{K}}^B\right]\right] ,$$

where $\hat{\mathcal{K}}^B$ is the usual LO kernel and $\hat{\vec{q}}_{1,2}$ are the operators of Reggeon momenta. Note that in the commutator there is no difference between usual and modifided kernels, so that $\hat{\mathcal{K}}^B$ is taken instead of $\hat{\mathcal{K}}^B_m$. The corresponding transformation for the impact factor is

$$\langle GR_1|_t = \langle GR_1|_s - \frac{1}{4}\langle GR_1|^{(B)}\left[\ln\left(\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}\right), \hat{\mathcal{K}}^{(B)}\right] ,$$

where the subscript t means transformed to fit the conformal kernel.

The impact factor in the Möbius scheme

For the NLO correction we obtain

$$\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1 t} = \vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1 s} + \vec{\mathcal{I}}_2,$$

$$\vec{\mathcal{I}}_2 = \int \frac{d\vec{l}}{\Gamma(1-\epsilon)\pi^{1+\epsilon}} \vec{C}_1' \frac{1}{\vec{r}_1^{\,\prime\,2}\vec{r}_2^{\,\prime\,2}} \left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,\prime\,2} + \vec{r}_2^{\,2}\vec{r}_1^{\,\prime\,2}}{\vec{l}^{\,2}} - \vec{q}_2^{\,2} \right) \ln\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,2}}{\vec{r}_1^{\,\prime\,2}\vec{r}_2^{\,\prime\,2}} \right).$$

This integral is infrared finite and can be calculated in two-dimensional space, with the help of the decomposition used before, the decomposition

$$\frac{1}{\vec{r}_1^{\,\prime\,2}\vec{r}_2^{\,\prime\,2}} \left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,\prime\,2} + \vec{r}_2^{\,2}\vec{r}_1^{\,\prime\,2}}{\vec{l}^{\,2}} - \vec{q}_2^{\,2} \right) = -\left(\frac{1}{(r_1 - l)^+} + \frac{1}{l^+} \right)$$

$$\times \left(\frac{1}{(r_2 + l)^-} - \frac{1}{l^-} \right) - \left(\frac{1}{(r_1 - l)^-} + \frac{1}{l^-} \right) \left(\frac{1}{(r_2 + l)^+} - \frac{1}{l^+} \right).$$

The transformed impact factor takes the form:

$$\vec{\Phi}^{\mathcal{G}_1\mathcal{G}_2}_{GR_1t} = \vec{C}_1\left[\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\ln\left(\frac{\vec{q}_2^{\,4}(\vec{q}_1-\vec{r}_1)^4}{\vec{q}_1^{\,2}\vec{r}_2^{\,2}\vec{k}^{\,2}\vec{r}_1^{\,2}}\right) - \ln\left(\frac{\vec{q}_2^{\,2}(\vec{q}_1-\vec{r}_1)^2}{\vec{q}_1^{\,2}\vec{r}_2^{\,2}}\right)\right.$$

$$\left.\times\ln\left(\frac{\vec{q}_2^{\,2}(\vec{q}_1-\vec{r}_1)^2}{\vec{k}^{\,2}\vec{r}_1^{\,2}}\right)\frac{3}{4}\ln^2\left(\frac{\vec{k}^{\,2}\vec{r}_1^{\,2}}{\vec{q}_1^{\,2}\vec{r}_2^{\,2}}\right)-\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-2\frac{(\vec{k}^{\,2})^{\epsilon}}{\epsilon^2}+4\zeta(2)\right]$$

$$+\frac{1}{4}\vec{C}_2\ln\left(\frac{\vec{q}_2^{\,2}(\vec{q}_1-\vec{r}_1)^2}{\vec{q}_1^{\,2}\vec{r}_2^{\,2}}\right)\ln\left(\frac{(\vec{q}_1-\vec{r}_1)^4\vec{q}_1^{\,2}\vec{k}^{\,2}\vec{r}_1^{\,2}}{\vec{r}_2^{\,6}\vec{q}_2^{\,4}}\right)$$

$$+\frac{3}{2}\left[\vec{C}_2\times\left(\left[\vec{r}_1\times\vec{r}_2\right]I_{\vec{r}_1,\vec{r}_2}+\left[\vec{q}_1\times\vec{r}_1\right]I_{\vec{q}_1,-\vec{r}_1}+\left[\vec{k}\times\vec{r}_2\right]I_{\vec{k},\vec{r}_2}+\left[\vec{q}_1\times\vec{k}\right]I_{\vec{q}_1,-\vec{k}}\right)\right].$$

The impact factor in the Möbius scheme

Second, we have to use the energy scale s_0 chosen in such a way that the ratio (energy evolution parameter) $s/s_0 = s\vec{q}_2^{\,2}/\sqrt{\vec{q}_1^{\,2}\vec{q}_3^{\,2}\vec{k}_1^{\,2}\vec{k}_2^{\,2}}$ is Möbius invariant. This energy scale differs from the energy scale used in the standard definition of the impact factor which is equal $|\vec{k}_1||\vec{k}_2|$. To adjust the impact factor to the energy scale $\sqrt{\vec{q}_1^{\,2}\vec{q}_3^{\,2}\vec{k}_1^{\,2}\vec{k}_2^{\,2}}/\vec{q}_2^{\,2}$, we need to perform an additional transformation:

$$\langle GR_1|_t \to \langle GR_1|_c = \langle GR_1|_t - \frac{1}{2}\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\langle GR_1|^{(B)}\hat{\mathcal{K}}_m^{(B)}|\mathcal{G}_1\mathcal{G}_2\rangle\,,$$

The impact factor in the Möbius scheme

where the subscript c means transformed to fit the conformal energy scale and $\hat{\mathcal{K}}_m^{(B)}$ is the modified LO kernel. Let

$$\vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1 c} \;=\; \vec{\Phi}^{\mathcal{G}\mathcal{G}_2}_{GR_1 t} + \vec{\mathcal{I}}_3,$$

then the integral for $\vec{\mathcal{I}}_3$ can be written as

$$\vec{\mathcal{I}}_3 = -\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\int\frac{d\vec{l}}{\pi}\left(\vec{C}_1' - \vec{C}_1\right)\frac{1}{\vec{r}_1^{\,\prime\,2}\vec{r}_2^{\,\prime\,2}}\left(\frac{\vec{r}_1^{\,2}\vec{r}_2^{\,\prime\,2}+\vec{r}_2^{\,2}\vec{r}_1^{\,\prime\,2}}{\vec{l}^{\,2}} - \vec{q}_2^{\,2}\right),$$

where instead of $\vec{C}_1'$ the difference $\left(\vec{C}_1' - \vec{C}_1\right)$ is taken and instead of the full modified kernel only its part related to real gluon production is kept. Moreover, the integral is written in two-dimensional transverse space.

The impact factor in the Möbius scheme

This result gives

$$\vec{\Phi}^{\mathcal{G}_1\mathcal{G}_2}_{GR_1c} = \vec{C}_1\left[-\ln\left(\frac{\vec{q}_2^{\,2}(\vec{q}_1-\vec{r}_1)^2}{\vec{r}_1^{\,2}\vec{k}^{\,2}}\right)\ln\left(\frac{\vec{q}_2^{\,2}(\vec{q}_1-\vec{r}_1)^2}{\vec{q}_1^{\,2}\vec{r}_2^{\,2}}\right)-\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right.$$

$$\left.-\frac{3}{4}\ln^2\left(\frac{\vec{k}^{\,2}\vec{r}_1^{\,2}}{\vec{q}_1^{\,2}\vec{r}_2^{\,2}}\right)-2\frac{(\vec{k}^{\,2})^\epsilon}{\epsilon^2}+4\zeta(2)\right]$$

$$+\frac{1}{4}\vec{C}_2\ln\left(\frac{\vec{q}_2^{\,2}(\vec{q}_1-\vec{r}_1)^2}{\vec{q}_1^{\,2}\vec{r}_2^{\,2}}\right)\ln\left(\frac{\vec{q}_2^{\,4}(\vec{q}_1-\vec{r}_1)^4\vec{r}_1^{\,2}\vec{k}^{\,2}}{\vec{r}_2^{\,6}\vec{q}_1^{\,6}}\right)$$

$$+\frac{3}{2}\left[\vec{C}_2\times\left(\left[\vec{r}_1\times\vec{r}_2\right]I_{\vec{r}_1,\vec{r}_2}+\left[\vec{q}_1\times\vec{r}_1\right]I_{\vec{q}_1,-\vec{r}_1}+\left[\vec{k}\times\vec{r}_2\right]I_{\vec{k},\vec{r}_2}+\left[\vec{q}_1\times\vec{k}\right]I_{\vec{q}_1,-\vec{k}}\right)\right].$$

This expression gives us the NLO correction to the impact factor for Reggeon-gluon transition in the scheme with conformal kernel and energy evolution parameter.

The impact factor in the Möbius scheme

However, it is the impact factor for the full amplitude, not for the remainder function. To obtain the impact factor for the remainder function we have to take the impact factor with the correction $\Phi^{\mathcal{G}_1\mathcal{G}_2}_{GR_1c}$ and with the polarisation vector $\vec{e}^{\,*}$ of definite helicity, and to extract from it the piece included in the BDS ansatz.

Let us consider, for definiteness, the production of a gluon with positive helicity, $\vec{e}^{\,*} = (\vec{e}_x - i\vec{e}_y)/\sqrt{2}$. Then,

$$\vec{e}^{\,*}\vec{C}_1 = -\frac{q_1^- r_1^+}{\sqrt{2}(q_1 - r_1)^+}, \quad \vec{e}^{\,*}\vec{C}_2 = -\frac{q_1^- q_2^+}{\sqrt{2}k^+}, \quad \frac{\vec{e}^{\,*}\vec{C}_2}{\vec{e}^{\,*}\vec{C}_1} = 1 - z,$$

where the ratio $z = -q_1^+ r_2^+/(k^+ r_1^+)$ is conformal invariant, i.e. invariant with respect to Möbius transformations of complex variables p_i such that
$r_1^+ = p_1 - p_2, r_2^+ = p_2 - p_3, -q_1^+ = p_3 - p_4, k_1^+ = p_4 - p_1)$.

The impact factor in the Möbius scheme

As the result, after some algebra we obtain

$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle = \langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle^{(B)}\left\{1+\frac{\bar{g}^2}{8}\left[(1-z)\right.\right.$$

$$\times\left(\ln\left(\frac{|1-z|^2}{|z|^2}\right)\ln\left(\frac{|1-z|^4}{|z|^6}\right)-3Li_2(z)+3Li_2(z^*)-\frac{3}{2}\ln|z|^2\ln\frac{1-z}{1-z^*}\right)$$

$$-4\ln|1-z|^2\ln\frac{|1-z|^2}{|z|^2}-3\ln^2|z|^2$$

$$\left.\left.-4\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-8\frac{(\vec{k}^{\,2})^\epsilon}{\epsilon^2}+16\zeta(2)\right]\right\}.$$

The NLO discontinuity of the $A_{2\to 4}$ amplitude

Therefore, with the NLO accuracy we have

$$\langle G_1 R_1 | \mathbf{G}_1 \mathbf{G}_2 \rangle = -\sqrt{2} g^2 \delta(\vec{q}_1 - \vec{k}_1 - \vec{r}_1 - \vec{r}_2) \frac{q_1^- r_1^+}{(q_1 - r_1)^+} [1 + \bar{g}^2 I(z_1)]$$

$$\times \left[1 + \bar{g}^2 \left(-\frac{1}{2} \ln^2 \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) - \frac{(\vec{k}^{\,2})^{\epsilon}}{\epsilon^2} + 2\zeta(2) \right) \right],$$

$$\langle \mathbf{G}'_1 \mathbf{G}'_2 | G_2 R_3 \rangle = \sqrt{2} g^2 \delta(\vec{r}_1^{\,\prime} + \vec{r}_2^{\,\prime} - \vec{q}_3 - \vec{k}_2) \frac{q_3^+ r_1^{\prime -}}{(r'_1 - q_3)^-} [1 + \bar{g}^2 I^*(z_2)]$$

$$\times \left[1 + \bar{g}^2 \left(-\frac{1}{2} \ln^2 \left(\frac{\vec{q}_3^{\,2}}{\vec{q}_2^{\,2}} \right) - \frac{(\vec{k}^{\,2})_2^{\epsilon}}{\epsilon^2} + 2\zeta(2) \right) \right],$$

where $z_1 = -q_1^+ r_2^+ / (k_1^+ r_1^+)$, $z_2 = q_3^+ r_2^{\prime +} / (k_2^+ r_1^{\prime +})$

$$I(z) = (1-z)\left(\ln\left(\frac{|1-z|^2}{|z|^2}\right)\ln\left(\frac{|1-z|^4}{|z|^6}\right) - 6Li_2(z) + 6Li_2(z^*)\right.$$

$$\left. -3\ln|z|^2\ln\frac{1-z}{1-z^*}\right) - 4\ln|1-z|^2\ln\frac{|1-z|^2}{|z|^2} - 3\ln^2|z|^2 \,.$$

In terms of eigenstates $|\nu, n\rangle$ and eigenvalues $\omega(\nu, n)$ of $\hat{K}_c$

$$\langle G_1 R_1 | e^{\hat{K}_m \ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3\rangle =$$

$$\times \sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\nu\, \langle G_1 R_1 | \nu, n\rangle e^{\omega(\nu,n)\ln\left(\frac{s_2 \vec{q}_2^{\,2}}{|\vec{q}_1||\vec{q}_3||\vec{k}_1||\vec{k}_2|}\right)} \langle \nu, n | G_2 R_3\rangle \,.$$

The eigenfunctions are well known. The eigenvalues $\omega(\nu, n)$ with the NLO accuracy were found.

In an explicit form

$$\langle G_1 R_1 | e^{\hat{K}_m \ln\left(\frac{s_2}{|\vec{k}_1||\vec{k}_2|}\right)} | G_2 R_3 \rangle = \delta(\vec{q}_1 - \vec{k}_1 - \vec{k}_2 - \vec{q}_3) g^2 \gamma_{R_1 R_2}^{G_1(B)} \gamma_{R_2 R_3}^{G_2(B)}$$

$$\times \left[1 + \bar{g}^2 \left(-\frac{1}{2} \ln^2 \left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) - \frac{(\vec{k}_1^{\,2})^\epsilon}{\epsilon^2} - \frac{1}{2} \ln^2 \left(\frac{\vec{q}_3^{\,2}}{\vec{q}_2^{\,2}} \right) - \frac{(\vec{k}_2^{\,2})^\epsilon}{\epsilon^2} + 4\zeta(2) \right) \right]$$

$$\times \frac{1}{2} \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\nu \, e^{\omega(\nu,n) \ln\left(\frac{s_2 \vec{q}_2^{\,2}}{|\vec{q}_1||\vec{q}_3||\vec{k}_1||\vec{k}_2|}\right)} w^{\frac{n}{2}+i\nu} (w^*)^{-\frac{n}{2}+i\nu}$$

$$\int \frac{dz_1}{\pi |z_1|^2} \frac{1}{1 - z_1} \left(1 + \bar{g}^2 I(z_1) \right) z_1^{\frac{n}{2}+i\nu} (z_1^*)^{-\frac{n}{2}+i\nu}$$

$$\int \frac{dz_2}{\pi |z_2|^2} \frac{1}{1 - z_2^*} \left(1 + \bar{g}^2 I^*(z_2) \right) (z_2^*)^{\frac{n}{2}-i\nu} z_2^{-\frac{n}{2}-i\nu}$$

where $w = k_2^+ q_1^+ / (k_1^+ q_3^+)$.

Summary

- The impact factors for Reggeon-gluon transition are an integral part of the discontinuities of multi-Regge amplitudes.
- Formal expressions for s_i-channel discontinuities of MRK amplitudes in the NLA are known since 2006.
- Fulfilment of all bootstrap conditions is proved.
- The LLA discontinuities are in an evident contradiction with the BDS anzatz for $2 \to 2 + n$ amplitudes at $n \geq 2$ even in the LLA.
- The NLO impact factors are known now in Yang-Mills theories with any number of fermions and scalars in arbitrary representations of the gauge group.
- The discontinuity in invariant mass of two produced gluons is calculated in the NLA in planar $N = 4$ SYM.
- It's compatibility with the BDS ansatz corrected by the remainder factor is under consideration.