From Colour Glass Condensate to Quark–Gluon Plasma

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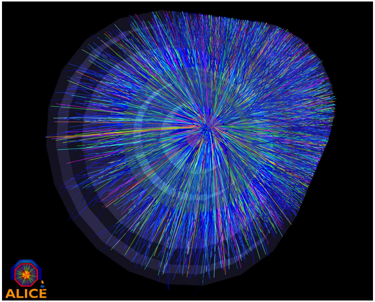
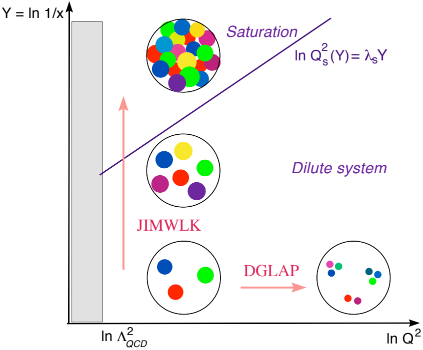
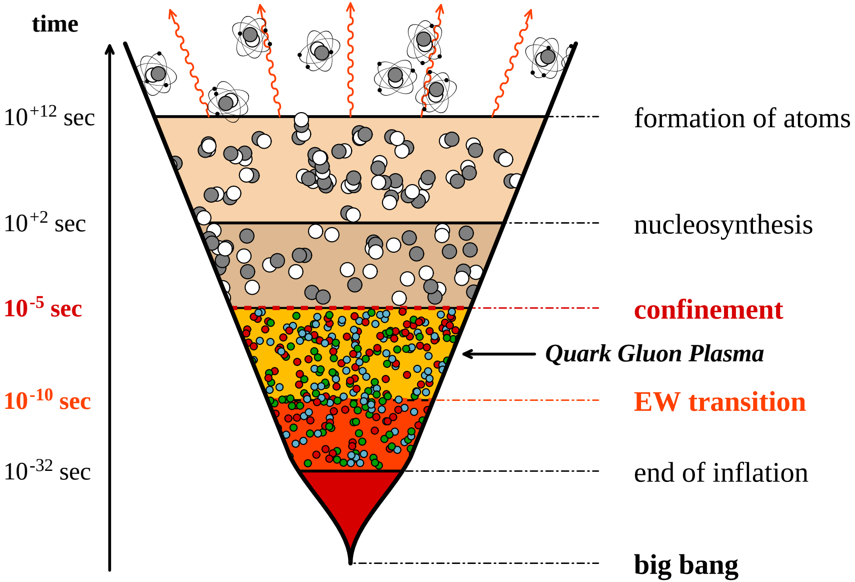




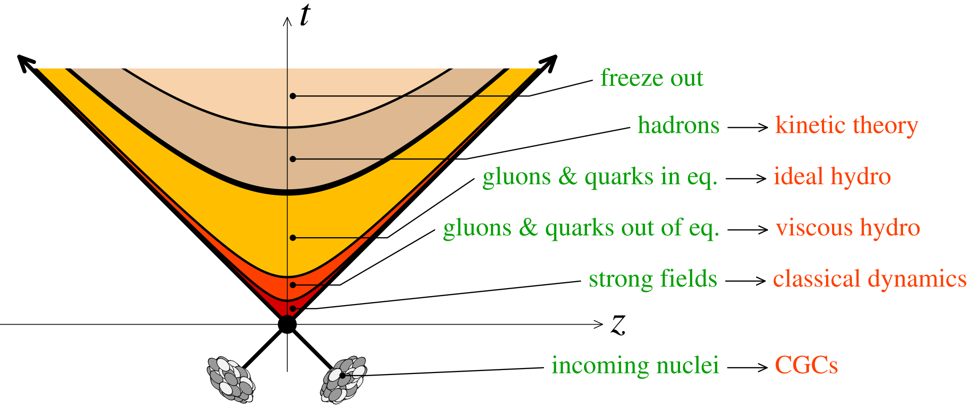
The Big Bang



time
10^{+12} sec
10^{+2} sec
10^{-5} sec
10^{-10} sec
10^{-32} sec
formation of atoms
nucleosynthesis
confinement
Quark Gluon Plasma
EW transition
end of inflation
big bang

The Little Bang

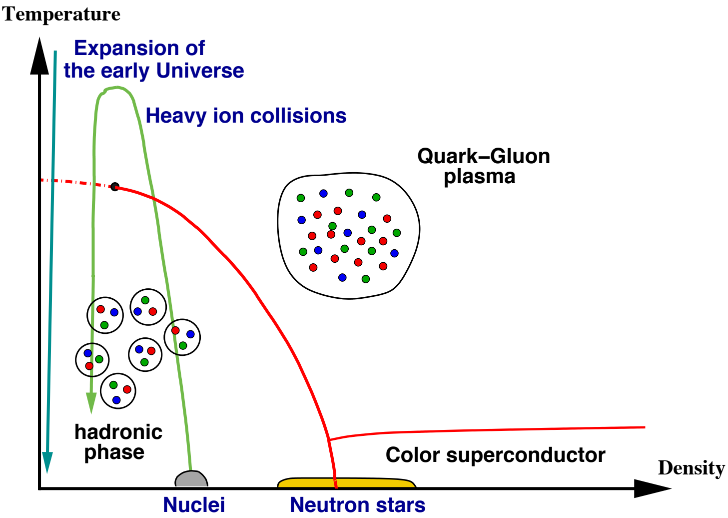
- A space–time picture of a heavy ion collision (HIC)



- 'Initial singularity' : the collision between the two incoming nuclei
- The QGP is re-created in the intermediate stages

Phase–diagram for QCD

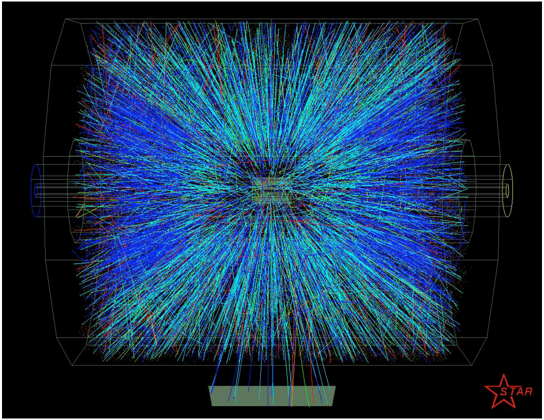
- ... as explored by the expansion of the Early Universe ...



- ... and in the ultrarelativistic heavy ion collisions.

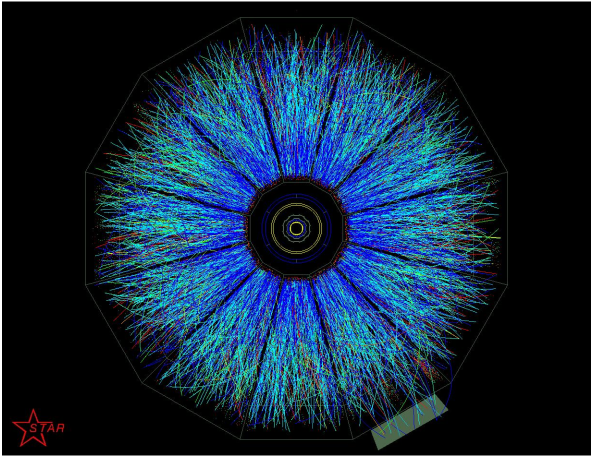
Heavy Ion Collisions @ RHIC & the LHC

Au+Au collisions at RHIC



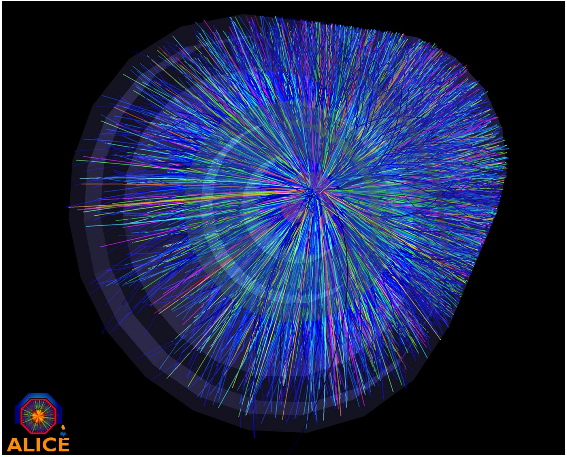
- Au+Au collision at STAR: longitudinal projection
- ~ 7000 produced particles streaming into the detector
- Collision energy (COM frame) : $\sqrt{s} = 200$ GeV/nucleon

Au+Au collisions at RHIC

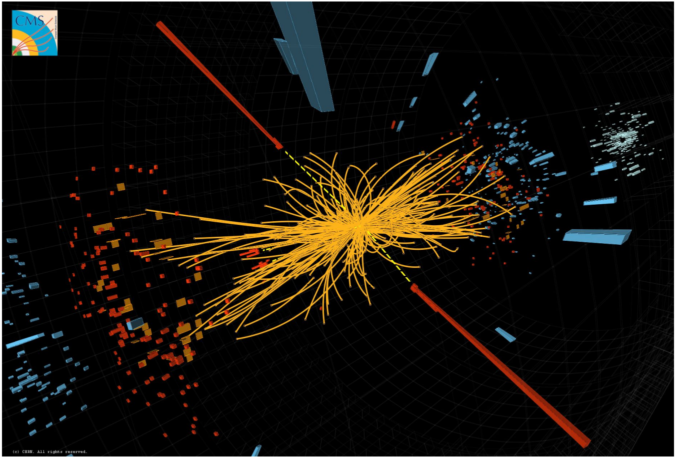


- Au+Au collision at STAR: transverse projection

Pb+Pb collisions at the LHC: ALICE

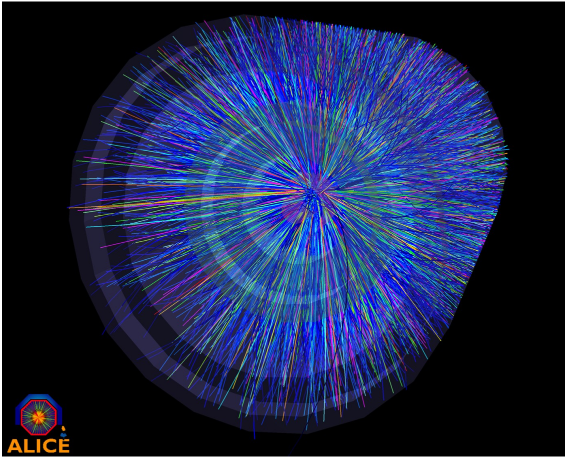


- Pb+Pb collision at ALICE: $\sqrt{s} = 2760$ GeV/nucleon
- $\gtrsim$ 20,000 hadrons in the detectors
- Is that much ?



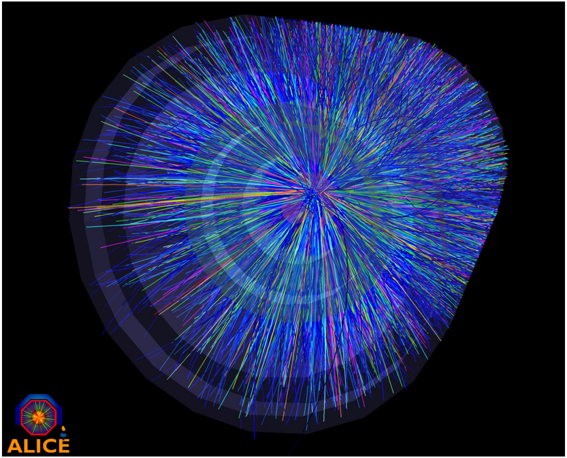
- p+p collision at 7 TeV: candidate event for $H \to \gamma\gamma$
- Less than 50 tracks/hadrons in the final state

Pb+Pb collisions at the LHC: ALICE



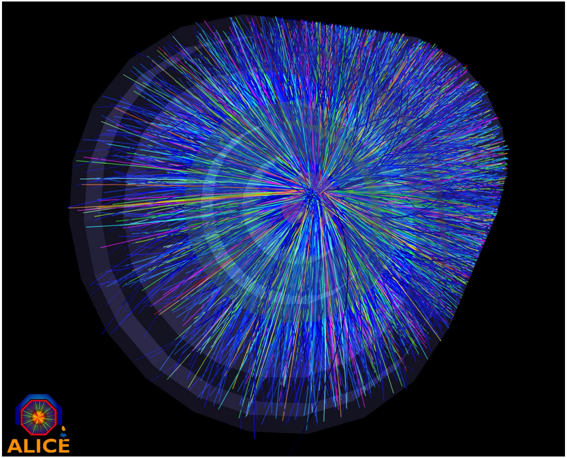
- Where are all these (> 10000) hadrons coming from ?
- How to trace back their history ?
- How to understand that from first principles (QCD) ?

Pb+Pb collisions at the LHC: ALICE



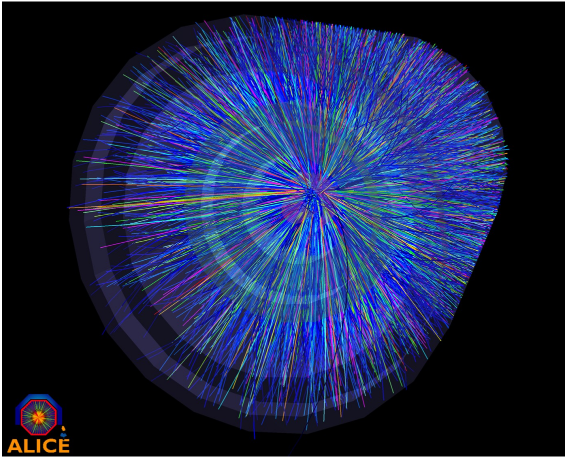
- Partons which have been liberated by the collision.
- How to trace back their history ?
- How to understand that from first principles (QCD) ?

Pb+Pb collisions at the LHC: ALICE



- Partons which have been liberated by the collision.
- They leave imprints on the hadron distribution in the final state.
- How to understand that from first principles (QCD) ?

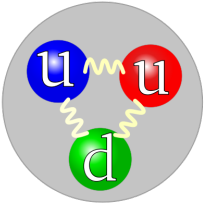
Pb+Pb collisions at the LHC: ALICE

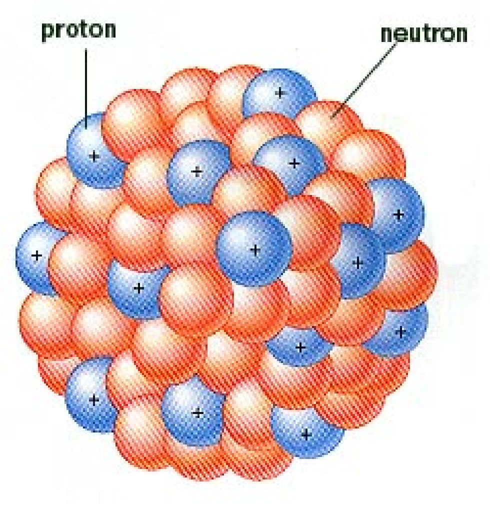


- Partons which have been liberated by the collision.
- They leave imprints on the hadron distribution in the final state.
- Build effective theories for the relevant degrees of freedom.

QCD matter: from hadrons ...

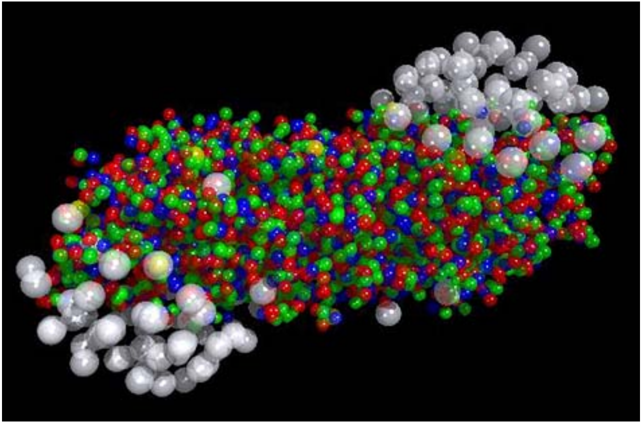
Quark composition of a pion





- At low energies, QCD matter exists only in the form of hadrons (mesons, baryons, nuclei) ... as a consequence of confinement

QCD matter: ... to partons

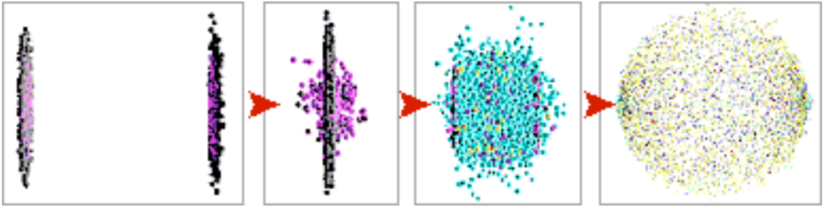


- At high energies, the relevant d.o.f. are partonic (quarks & gluons)

  ▷ interactions occur over distances much shorter than the confinement scale

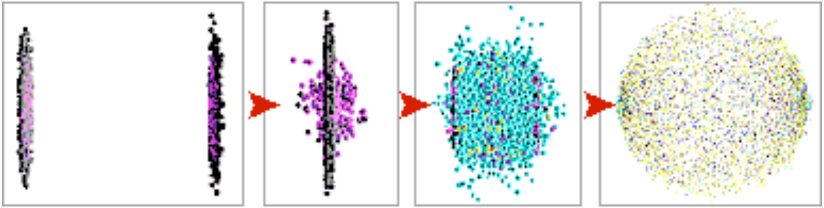
- The HIC's give us access to dense forms of partonic matter

New forms of QCD matter produced in HIC



- Prior to the collision: 2 Lorentz–contracted nuclei ('pancakes')
  - 'Color Glass Condensate' : highly coherent form of gluonic matter
- Right after the collision: non–equilibrium partonic matter
  - 'Glasma' : color fields break into partons
- At later stages ($\Delta t \gtrsim 1$ fm/c) : local thermal equilibrium
  - 'Quark–Gluon Plasma' (QGP)
- Final stage ($\Delta t \gtrsim 10$ fm/c) : hadrons
  - 'final event', or 'particle production'

New forms of QCD matter produced in HIC



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  - 'Quark–Gluon Plasma' (QGP)
- My focus here: the partonic phases at early and intermediate stages

Outline

- The wavefunctions of the incoming hadrons:

  Color Glass Condensate

- Particle production at early stages :

  proton-proton (pp), proton-nucleus (pA), nucleus-nucleus (AA)

- AA collisions : Glasma & thermalization

- Flow and hydrodynamics

- Thermodynamics of the Quark Gluon Plasma

- Hard probes of the QGP: jet quenching

▷ Main emphasis: What do heavy ion collisions teach us about QCD

Instead of references ...

- For more references, formulae, cartoons and hand-waving arguments, have a look at this review paper

arXiv.org > hep-ph > arXiv:1205.0579

Search or Artic

High Energy Physics – Phenomenology

QCD in heavy ion collisions

Edmond Iancu

(Submitted on 2 May 2012)

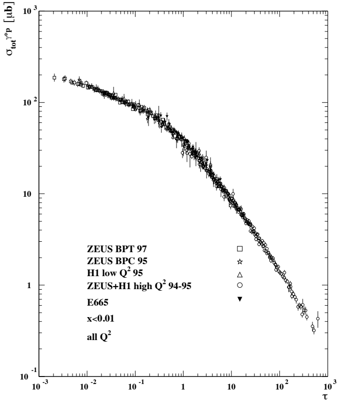
These lectures provide a modern introduction to selected topics in the physics of ultrarelativistic heavy ion collisions which shed light on the fundamental theory of strong interactions, the Quantum Chromodynamics. The emphasis is on the partonic forms of QCD matter which exist in the early and intermediate stages of a collision -- the colour glass condensate, the glasma, and the quark-gluon plasma -- and on the effective theories that are used for their description. These theories provide qualitative and even quantitative insight into a wealth of remarkable phenomena observed in nucleus-nucleus or deuteron-nucleus collisions at RHIC and/or the LHC, like the suppression of particle production and of azimuthal correlations at forward rapidities, the energy and centrality dependence of the multiplicities, the ridge effect, the limiting fragmentation, the jet quenching, or the dijet asymmetry.

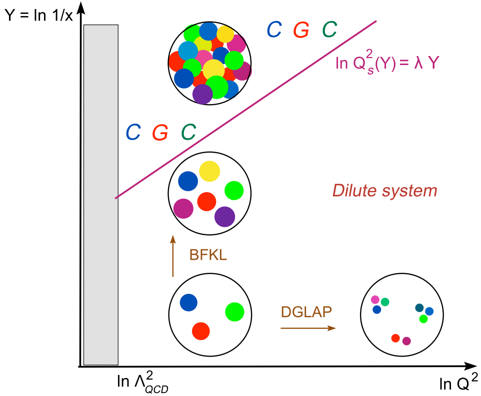
Comments: Based on lectures presented at the 2011 European School of High-Energy Physics, 7-20 September 2011, Cheile Gradistei, Romania. 73 pages, many figures

Subjects: **High Energy Physics – Phenomenology (hep-ph)**; High Energy Physics – Experiment (hep-ex); Nuclear Theory (nucl-th)

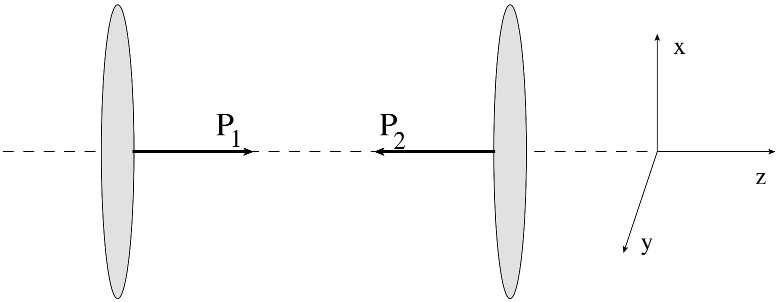
Cite as: **arXiv:1205.0579 [hep-ph]**
(or **arXiv:1205.0579v1 [hep-ph]** for this version)

Color Glass Condensate



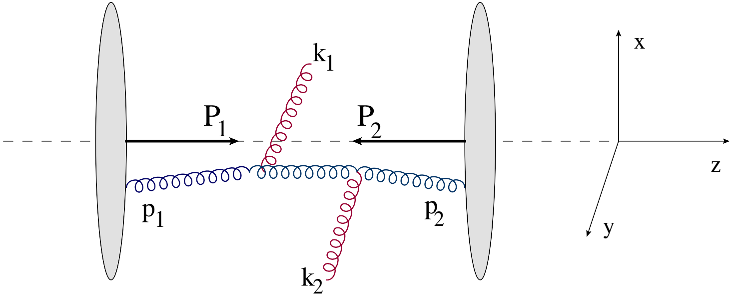


A hadron–hadron collision



- pp or nucleon–nucleon (NN) pair from a pA or AA collision
- z : longitudinal (or 'beam') axis; $\boldsymbol{x}_\perp = (x, y)$: transverse plane
- Center-of-mass frame : $P_1^\mu = (E, 0, 0, E)$, $P_2^\mu = (E, 0, 0, -E)$
- Center-of-mass energy squared : $s = (P_1 + P_2)^2 = 2P_1 \cdot P_2 = 4E^2$

The partonic subcollision (1)

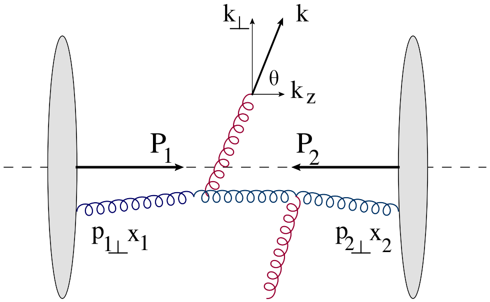


- 2 → 2 subcollision : $g(p_1) + g(p_2) \;\rightarrow\; g(k_1) + g(k_2)$
- Initial partons : nearly collinear with the incoming hadrons

$$p_1^\mu = (x_1 E,\, \boldsymbol{p}_{1\perp},\, x_1 E)\,, \qquad p_2^\mu = (x_2 E,\, \boldsymbol{p}_{2\perp},\, -x_2 E)$$

  - longitudinal momentum fraction $x = |p_z|/E$
  - transverse momentum $\boldsymbol{p}_\perp = (p_x,\, p_y)$
  - $p_\perp \ll p_z \Longrightarrow$ nearly on–shell gluons: $p^2 = -p_\perp^2 \approx 0$

The partonic subcollision (2)



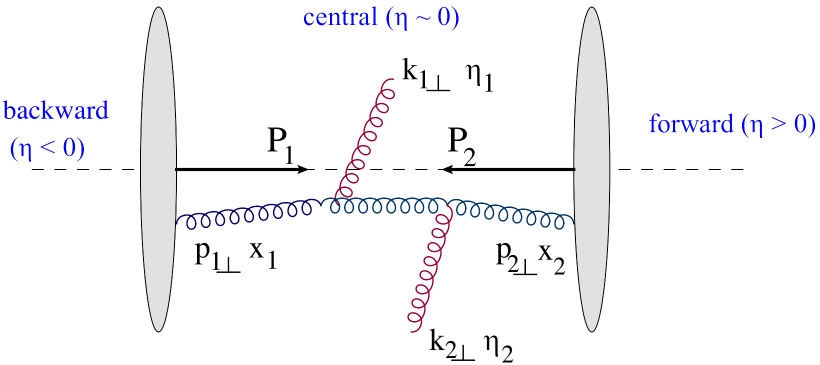
$$\eta = \frac{1}{2} \ln \frac{k + k_z}{k - k_z}$$

$$\eta = - \ln \tan \frac{\theta}{2}$$

$$\cos\theta = \frac{k_z}{k}$$

- Final (or 'produced') partons are on–shell : $k_i^2 = 0$
  - transverse momentum $\boldsymbol{k}_\perp$
  - polar angle θ or (pseudo) rapidity $\eta \equiv - \ln \tan(\theta/2)$
- No a priori hierarchy between k_z and $k_\perp$
  - "central rapidity" : $\eta \approx 0 \;\Leftrightarrow\; \theta \approx \pi/2 \;\Leftrightarrow\; |k_z| \ll k_\perp \approx k$
  - "forward/backward rapidities" : $\theta \approx 0$ or $\pi \iff |k_z| \approx k \gg k_\perp$

The partonic subcollision (3)

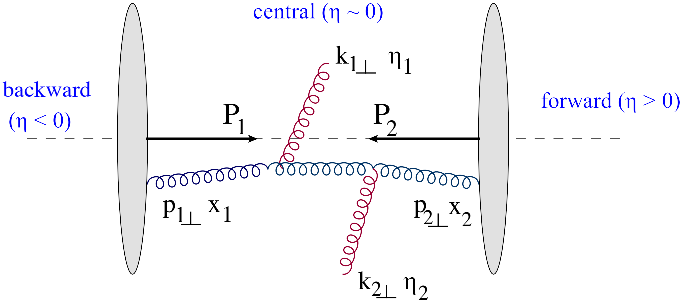


- Energy–momentum conservation ⇒ $\boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp} = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$

$$x_1 = \frac{k_{1\perp}}{\sqrt{s}}\,\mathrm{e}^{\eta_1} + \frac{k_{2\perp}}{\sqrt{s}}\,\mathrm{e}^{\eta_2}\,, \qquad x_2 = \frac{k_{1\perp}}{\sqrt{s}}\,\mathrm{e}^{-\eta_1} + \frac{k_{2\perp}}{\sqrt{s}}\,\mathrm{e}^{-\eta_2}$$

- Exercice ! Hint: use $E = \sqrt{s}/2$, $k = k_\perp \cosh\eta$, $k_z = k_\perp \sinh\eta$

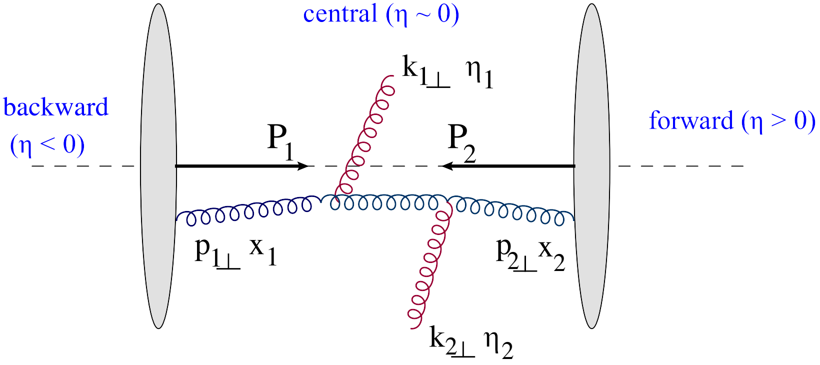
Particle production



$$x_1 = \frac{k_{1\perp}}{\sqrt{s}}\,\mathrm{e}^{\eta_1} + \frac{k_{2\perp}}{\sqrt{s}}\,\mathrm{e}^{\eta_2}\,, \qquad x_2 = \frac{k_{1\perp}}{\sqrt{s}}\,\mathrm{e}^{-\eta_1} + \frac{k_{2\perp}}{\sqrt{s}}\,\mathrm{e}^{-\eta_2}$$

- Particle production $\mathrm{d}N/\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}_\perp$ probes the wave functions of the incoming hadrons (their parton distributions in x)
- High–energy regime: $k_\perp/\sqrt{s} \ll 1 \longleftrightarrow$ small–x partons: $x \ll 1$
- Forward production : η_1, η_2 positive and large $\Longrightarrow x_2 \ll x_1$

Collinear factorization

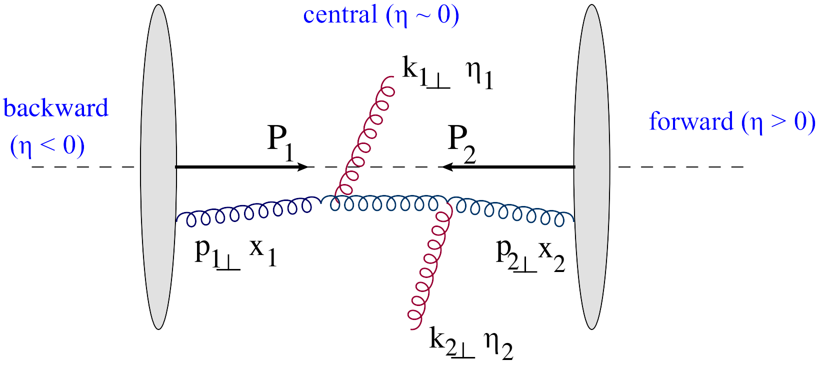


- $p_{1\perp}, p_{2\perp} \approx 0$ (truly of order $\Lambda_{\rm QCD}$) $\Longrightarrow \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp} = 0$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2k_{1\perp}\mathrm{d}^2k_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} = \sum_{ij} x_1 f_i(x_1,\mu^2)\, x_2 f_j(x_2,\mu^2)\, \delta^{(2)}(\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp})\frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}k_\perp^2}$$

- μ^2 : factorization scale (of order $k_\perp^2 \gg \Lambda_{\rm QCD}^2$)

Collinear factorization

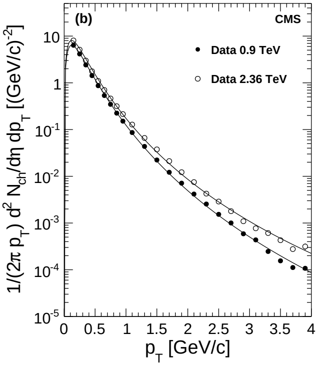


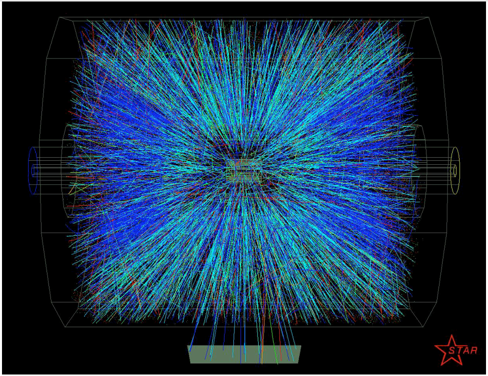
- $p_{1\perp},\, p_{2\perp} \;\approx\; 0$ (truly of order $\Lambda_{\rm QCD}$) $\Longrightarrow$ $\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp} = 0$

$$\frac{d\sigma}{d^2k_{1\perp}d^2k_{2\perp}d\eta_1 d\eta_2} = \sum_{ij} x_1 f_i(x_1,\mu^2)\, x_2 f_j(x_2,\mu^2)\, \delta^{(2)}(\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}) \frac{d\hat{\sigma}_{ij}}{dk_\perp^2}$$

- This eventually fails when $x \ll 1$ and $k_\perp^2$ is not extremely large

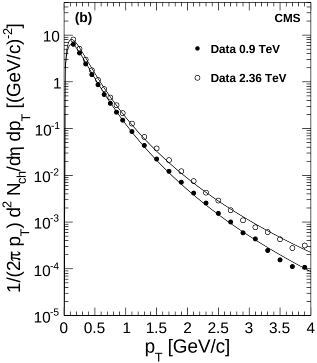
Multiplicity in pp, pA, AA : $\mathrm{d}N/\mathrm{d}\eta$

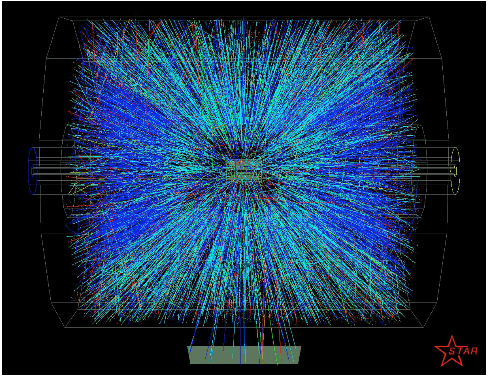




- 99% of the total multiplicity lies below $p_\perp = 2$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV & $\eta = 0$)
- $x \sim 4 \times 10^{-4}$ at the LHC ($\sqrt{s} = 5$ TeV & $\eta = 0$)
- $x_2 \sim 10^{-5}$ at the LHC & forward rapidity ($\sqrt{s} = 5$ TeV & $\eta = 4$)

Multiplicity in pp, pA, AA : $\mathrm{d}N/\mathrm{d}\eta$

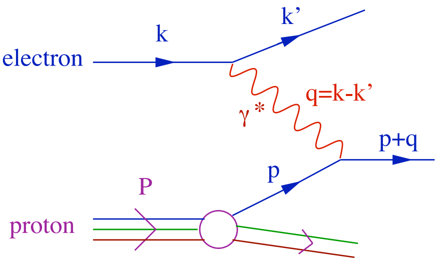




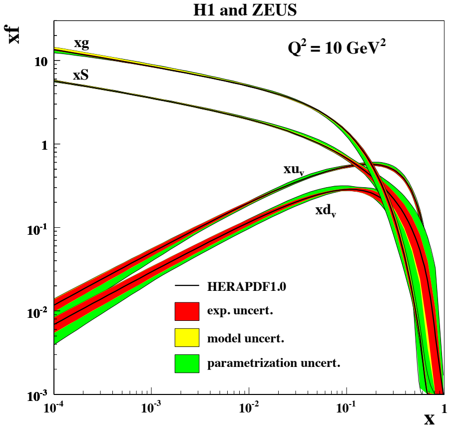
- The bulk of particle production is controlled by partons at small $x \ll 1$
- Where do all these partons come ?!

  ▷ 'a nucleon is built with 3 valence quarks, each one carrying $x \sim 1/3$'
- Need to better understand the parton structure of a hadron

Deep inelastic scattering at HERA

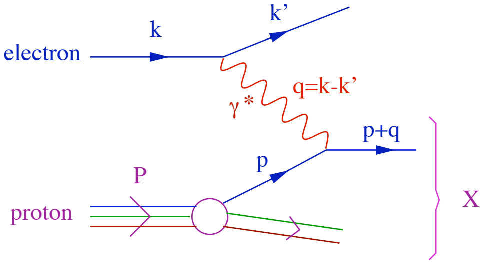


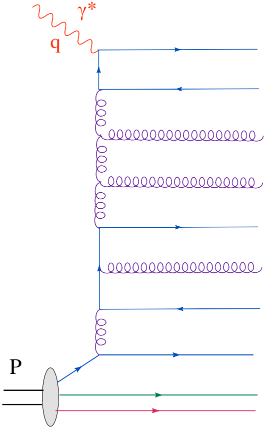
$$Q^2 = -q^\mu q_\mu > 0\,, \quad x = \frac{Q^2}{s}$$



- Parton distribution functions: $xq(x,Q^2)$, $xG(x,Q^2)$

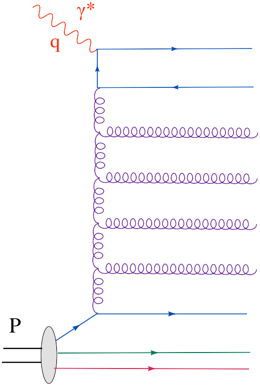
  ▷ number of partons (quark, gluons) with transverse size $\Delta x_\perp \sim 1/Q$ and longitudinal momentum fraction $x \sim Q^2/s$

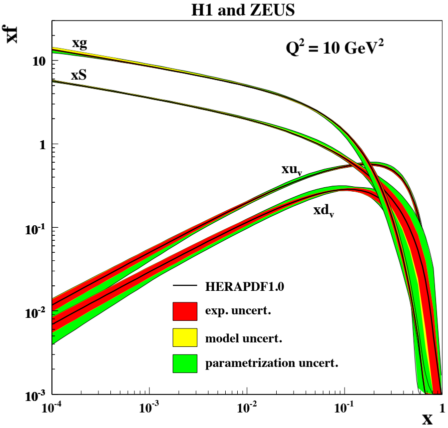




- The virtual photon γ^* couples to the (anti)quarks inside the proton
- Gluons are measured indirectly, via their effect on quark distribution
- Quantum evolution : change in the partonic content when changing the resolution scales x and Q^2, due to additional radiation

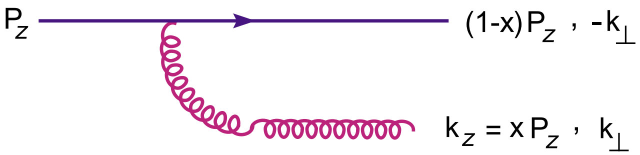
The small–x partons are mostly gluons





- For $x \leq 0.01$ the hadron wavefunction contains mostly gluons !
- The gluon distribution is rapidly amplified by the quantum evolution with decreasing x (or increasing energy s)

Bremsstrahlung



$$\mathrm{d}\mathcal{P}_{\rm Brem} \equiv \sum_{a,\lambda} \left|\mathcal{M}^a_\lambda(k_z, \boldsymbol{k}_\perp)\right|^2 \simeq \frac{\alpha_s(k_\perp^2)\, C_R}{\pi^2} \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \frac{\mathrm{d}x}{x}$$

- Phase–space enhancement for the emission of
  - collinear $(k_\perp \to 0)$
  - and/or soft (low–energy) $(x \to 0)$ gluons
- The parent parton can be either a quark or a gluon

$$C_F = t^a t^a = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}\,, \quad C_A = T^a T^a = N_c = 3$$

The gluon distribution of a single quark

- To leading order in α_s : single gluon emission by the quark $\Longrightarrow$

$$\frac{\mathrm{d}N_{\rm gluon}}{\mathrm{d}x\mathrm{d}^2k_\perp} = \frac{\mathrm{d}\mathcal{P}_{\rm Brem}}{\mathrm{d}x\mathrm{d}^2k_\perp}$$

▷ "unintegrated gluon distribution"

- The gluon distribution $xG(x,Q^2)$: # of gluons with a given energy fraction x and any transverse momentum $k_\perp \lesssim Q$

$$xG(x,Q^2) = \int^Q \mathrm{d}^2\boldsymbol{k}\, x\frac{\mathrm{d}N_{\rm gluon}}{\mathrm{d}x\mathrm{d}^2k_\perp} = \frac{\alpha_s C_F}{\pi}\int_{\Lambda^2}^{Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2} = \frac{\alpha_s C_F}{\pi}\ln\frac{Q^2}{\Lambda^2}$$

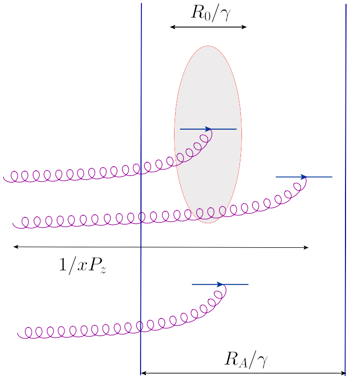
▷ logarithmic sensitivity to the confinement scale Λ

▷ the first 'transverse' logarithm of the DGLAP resummation

▷ no dependence upon energy (x) since gluon spin $j=1$: s^{j-1}

The gluon distribution of a large nucleus

- 'Large nucleus' : incoherent superposition of A nucleons, each one made with N_c valence quarks *(McLerran–Venugopalan model, 1994)*



$$xG_A(x,Q^2) \;=\; AN_c\, xG_q(x,Q^2)$$

$$xG_q(x,Q^2) \;=\; \frac{\alpha_s C_F}{\pi} \ln \frac{Q^2}{\Lambda^2}$$

$$R_A \;=\; R_0\, A^{1/3} \quad \text{(nuclear radius)}$$

$$\gamma = 100\,(RHIC) \div 1000\,(LHC)$$

- The small–x gluons are delocalized over a large longitudinal distance:

$$\Delta z \sim \frac{1}{xP_z} \gg \frac{R_A}{\gamma}$$

Gluon saturation in a large nucleus

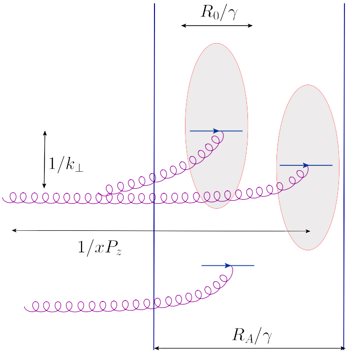
- $AN_c \sim 600$ for Au or Pb : can we simply superpose the different emissions as if they were independent from each other ?

  ▷ can one ignore gluon recombination ?

- In order to interact, gluons must overlap with each other

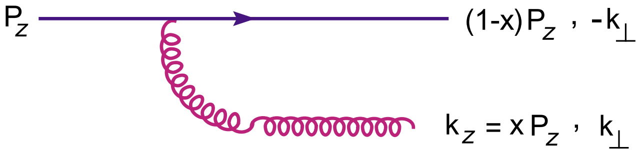
  ▷ they naturally overlap in longitudinal direction ...

  ▷ but what about their overlap in the transverse plane ?



- numerous enough : large density per unit $\perp$ area $\propto A^{1/3} \simeq 6$
- large enough : relatively small $k_\perp$
- large occupation numbers $\sim 1/\alpha_s$

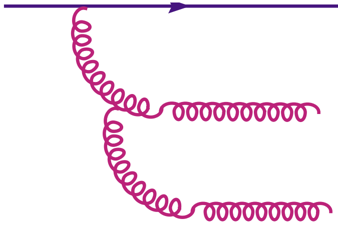
Bremsstrahlung strikes back



$$d\mathcal{P}_{\rm Brem} \simeq \frac{\alpha_s C_R}{\pi^2} \frac{d^2 k_\perp}{k_\perp^2} \frac{dx}{x} \propto \alpha_s \frac{dx}{x} = \alpha_s \, dY$$

- $Y \equiv \ln(1/x) = \eta_{\rm quark} - \eta_{\rm gluon}$:
  rapidity difference between the parent quark and the emitted gluon
- A probability of $\mathcal{O}(\alpha_s)$ to emit one gluon per unit rapidity
- If $\alpha_s Y \sim 1$, the emitted gluon can in turn emit an even softer one
- The origin of the 'BFKL cascades' (high energy evolution in QCD)

Two gluons



- The 'cost' of the additional gluon :

$$\alpha_s \int_x^1 \frac{\mathrm{d}x_1}{x_1} = \alpha_s \ln\frac{1}{x} = \alpha_s Y$$

- Formally, a process of higher order in α_s, but which is enhanced by the large available rapidity interval
- When $\alpha_s Y \gtrsim 1 \Longrightarrow$ need for resummation !

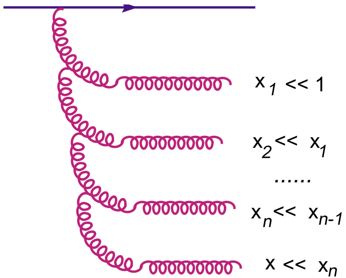
Gluon cascades

- n gluons strictly ordered in x

  $x \ll x_n \ll x_{n-1} \cdots \ll x_1 \ll 1$

- The n–gluon cascade contributes

$$\frac{1}{n!}\,(\alpha_s Y)^n$$



- Gluons are strongly ordered also in their lifetimes :

$$\Delta t \;\sim\; \frac{\hbar}{\Delta E} \;=\; \frac{2k_z}{k_\perp^2} \;=\; \frac{2xP}{k_\perp^2}$$

  ▷ the smaller x, the shorter the lifetime ! (Lorentz time dilation)

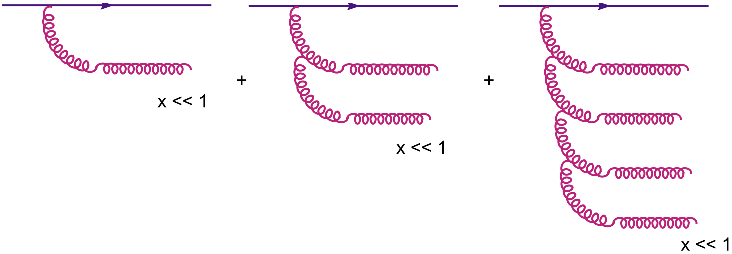
- During its short lifetime, the gluon at x overlaps with all its parent gluons at $x' \gg x$, which appear to it as frozen in some random configuration

BFKL evolution *(Balitsky, Fadin, Kuraev, Lipatov, 75–78)*

- The sum of all the cascades exponentiates :

$$\sum_n \frac{1}{n!}\,(\alpha_s Y)^n \;\propto\; \mathrm{e}^{\omega\alpha_s Y} \;\sim\; \frac{1}{x^{\omega\alpha_s}}$$

- BFKL really applies to the unintegrated gluon distribution



$$\frac{\mathrm{d}N_{\rm gluon}}{\mathrm{d}Y\,\mathrm{d}k_\perp^2} \;\simeq\; \frac{\alpha_s C_F}{\pi}\,\frac{1}{k_\perp^2}\,\mathrm{e}^{\omega\alpha_s Y}$$

BFKL evolution *(Balitsky, Fadin, Kuraev, Lipatov, 75–78)*

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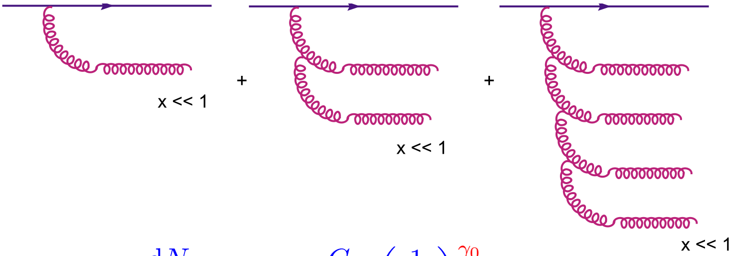
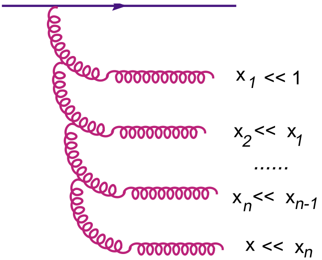
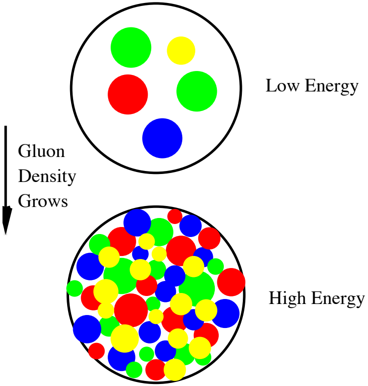


$$\frac{dN_{\text{gluon}}}{dY\, dk_\perp^2} \simeq \frac{\alpha_s C_F}{\pi}\left(\frac{1}{k_\perp^2}\right)^{\gamma_0} e^{\omega_0 \alpha_s Y}$$

- $\gamma_0 = 1/2$: 'BFKL anomalous dimension'
- $\omega_0 = 4 \ln 2 (N_c/\pi)$: 'BFKL Pomeron intercept'

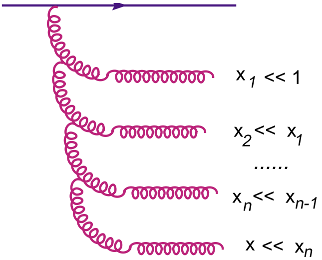
Gluon evolution at small x

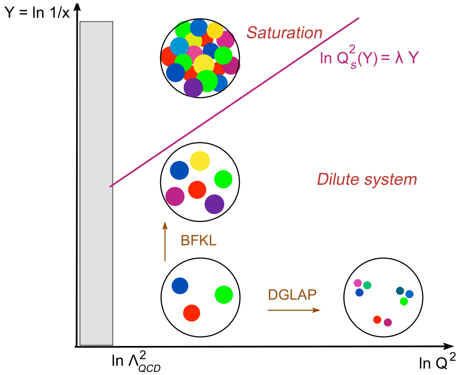




- BFKL: an evolution towards increasing density

Gluon evolution at small x





- BFKL: an evolution towards increasing density
- Non–trivial: not true for the DGLAP evolution !
  - the BFKL gluons have similar transverse momenta, hence similar transverse areas $\Longrightarrow$ they can overlap with each other
- The relevant quantity: not the gluon number, but ...

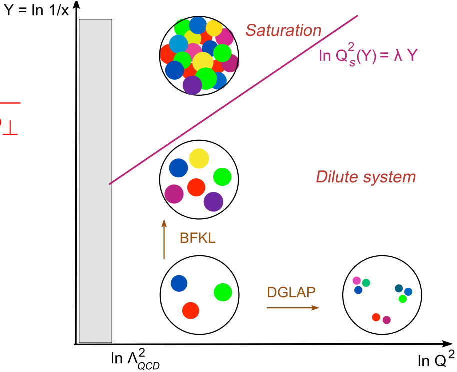
Color Glass Condensate

- The gluon occupation number (or ‘packing factor’)

$$n(x, \boldsymbol{k}_\perp) \equiv \frac{(2\pi)^3}{2(N_c^2-1)} \frac{\mathrm{d}N_{\rm gluon}}{\mathrm{d}Y\, \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}^2\boldsymbol{b}_\perp}$$

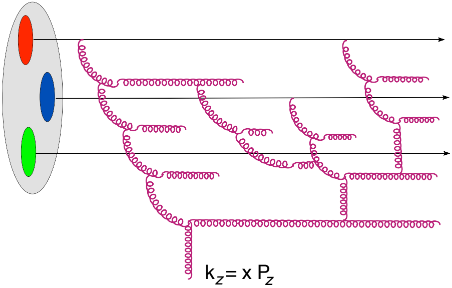
▷ $\boldsymbol{b}_\perp$: impact parameter in $\perp$ plane

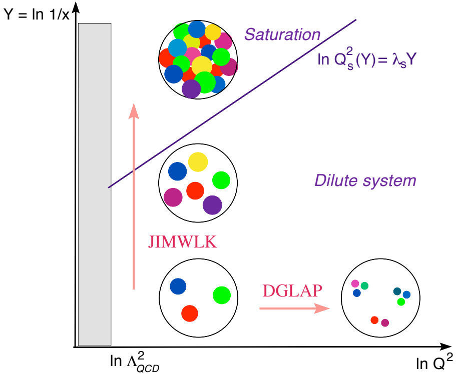
$$n(x, Q^2) \simeq \frac{\pi}{Q^2} \times \frac{xG(x,Q^2)}{\pi R^2}$$



- When $n \gtrsim 1$: gluons overlap, so they are coherent with each other
  - semi–classical description as a strong color field A_a^i : ‘condensate’
  - during the scattering, they are frozen by Lorentz dilation, but randomly distributed due to quantum fluctuations: ‘glass’

Gluon saturation





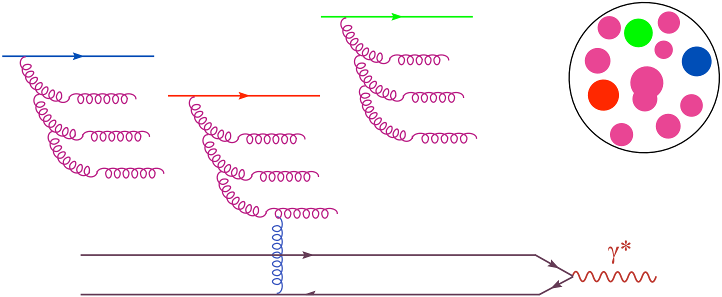
- $\alpha_s n \sim 1$: strong overlapping which compensates small coupling
- The evolution becomes non–linear :

  ▷ emissions + recombination $\Longrightarrow$ gluon saturation
- BFKL gets replaced by the non–linear Balitsky–JIMWLK equations

  Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (97–00)

A cartoon of the evolution equations : BFKL

- $n(Y, Q^2)$: gluon occupation number
- Rapidity increment $Y \to Y + \mathrm{d}Y$: a probability $\alpha_s \mathrm{d}Y$ to emit an additional gluon out of any of the preexisting ones



$$\frac{\partial n}{\partial Y} \simeq \alpha_s n \qquad \Longrightarrow \qquad n(Y) \propto \mathrm{e}^{\omega \alpha_s Y}$$

- Valid so long as $n(Y, Q^2) \ll 1/\alpha_s$ (dilute system)

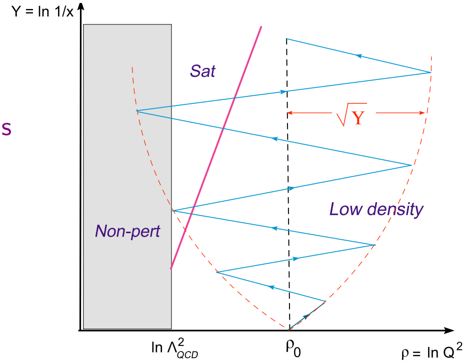
Conceptual difficulties

- Unitarity violation: $T \sim \alpha_s n$ cannot exceed 1
- Infrared diffusion : excursion through soft ($\sim \Lambda_{\rm QCD}$) momenta

  - The gluon emission vertex is non–local in $k_\perp$:

$$\partial_Y n = \alpha_s n + \alpha_s \partial_\rho^2 n$$

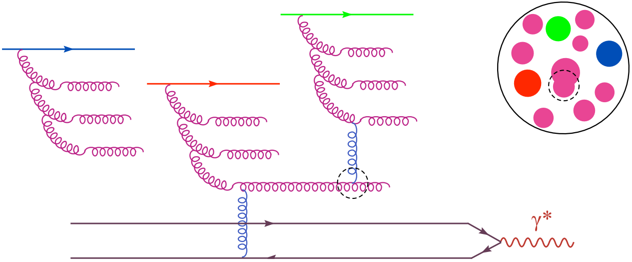
$\Longrightarrow$ diffusion in $\rho \equiv \ln k_\perp^2$



- Both problems are solved by gluon saturation

BK/JIMWLK

- High gluon density: recombination processes leading to saturation



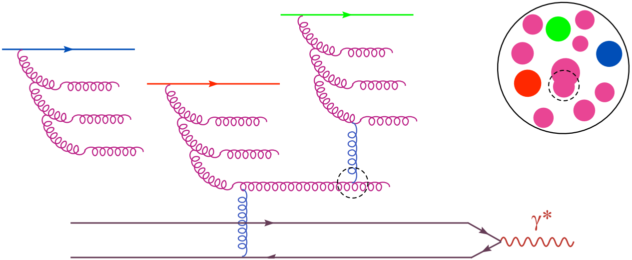
$$\frac{\partial n}{\partial Y} \simeq \alpha_s n - \alpha_s^2 n^2 = 0 \quad \text{when} \quad n \sim \frac{1}{\alpha_s} \gg 1$$

- Fixed point : the evolution stops when $\alpha_s n(Y, Q^2) \sim 1$
- The saturation condition involves Y and Q^2

  $\Longrightarrow$ saturation momentum $Q_s(Y)$

A classical stochastic process

$$\partial_Y n(\rho, Y) = \alpha_s \partial_\rho^2 n(\rho, Y) + \alpha_s n(\rho, Y) - \alpha_s^2 n^2(\rho, Y)$$



- Cartoon version of the Balitsky–Kovchegov equation
- FKPP equation for the 'reaction–diffusion' process ($A \rightleftharpoons 2A$)
  (Munier, Peschanski, 03; Iancu, Mueller, Munier 04; Pomeron loops ...)
- Mean field approximation (large–N_c) to the B–JIMWLK equations
- Known to next-to-leading-log accuracy (consistent with NLO BFKL)

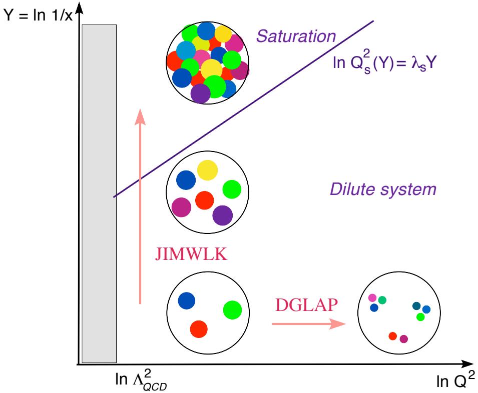
The saturation momentum

- The transverse momentum where saturation starts to be important

$$n(x,Q^2) = \frac{\pi}{Q^2} \times \frac{xG(x,Q^2)}{\pi R^2}$$

$$n(x, Q_s^2(x)) \sim \frac{1}{\alpha_s}$$

$$Q_s^2(x) \simeq \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2} \sim \frac{1}{x^{\lambda_s}}$$



- Q_s is rapidly rising with $1/x$, i.e. with the center-of-mass energy :

$\lambda_s \simeq 0.2 \div 0.3$ at NLO accuracy *(Triantafyllopoulos, 2003)*

▷ the actual 'Pomeron intercept' in the presence of saturation

The saturation momentum

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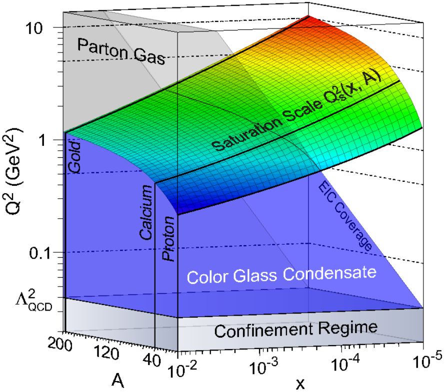
$$n\big(x, Q_s^2(x)\big) \sim \frac{1}{\alpha_s}$$

$$Q_s^2(x) \simeq \alpha_s \frac{xG_A(x,Q_s^2)}{\pi R_A^2} \sim \frac{A^{1/3}}{x^{\lambda_s}}$$



- ... and also with the atomic number A for a large nucleus $(A \gg 1)$

▷ $A^{1/3} \simeq 6$ for pA and AA collisions at RHIC and the LHC

The saturation momentum

- The transverse momentum where saturation starts to be important

$$n(x,Q^2) \;=\; \frac{\pi}{Q^2} \times \frac{xG(x,Q^2)}{\pi R^2}$$

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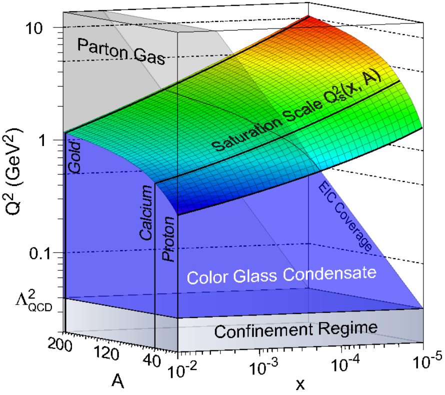

- $x \sim 10^{-5}$: $Q_s \sim 1$ GeV for proton and ~ 3 GeV for Pb or Au

  ▷ a semi–hard scale, at which perturbation theory is marginally valid

Gluon distribution & geometric scaling

- $Q_s^2(x) \propto$ the gluon density per unit transverse area
- $Q_s(x)$: the typical transverse momentum of the gluons with a given x

$$xG(x,Q^2) = \int \mathrm{d}^2 b_\perp \int^Q \mathrm{d}k_\perp \, k_\perp \, n(x, b_\perp, k_\perp)$$

$$n(Y, k_\perp) = \begin{cases} \dfrac{1}{\alpha_s} \ln \dfrac{Q_s^2(Y)}{k_\perp^2} & \text{for } k_\perp < Q_s(Y) \\[2ex] \dfrac{1}{\alpha_s} \left(\dfrac{Q_s^2(Y)}{k_\perp^2} \right)^{\gamma_s} & \text{for } k_\perp > Q_s(Y) \end{cases}$$

- $\gamma_s \simeq 0.63$: anomalous dimension at saturation
- Geometric scaling : $n(Y, k_\perp) = F\big(k_\perp / Q_s(Y)\big)$

(Iancu, Itakura, McLerran; Mueller, Triantafyllopoulos; Munier, Peschanski, 02-03)

Gluon distribution & geometric scaling

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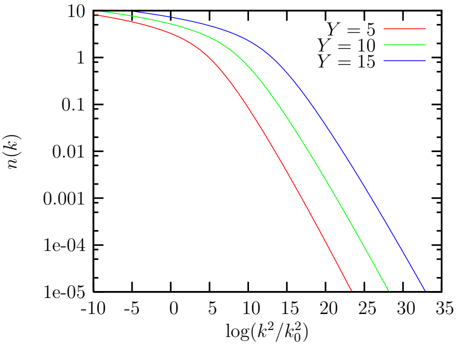


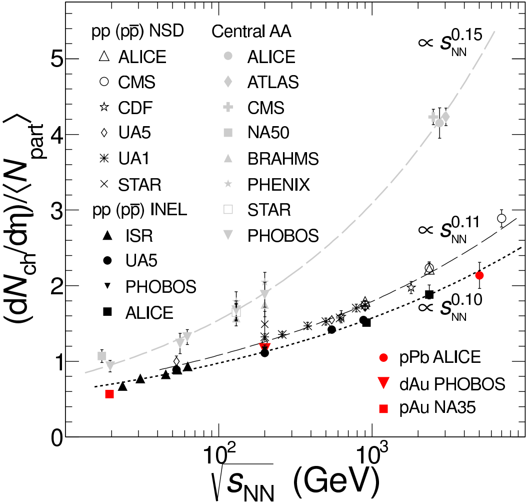
Multiplicity : energy dependence

- pp, pA, AA : the saturated gluons are released in the final state
- Particle multiplicity $\mathrm{d}N/\mathrm{d}\eta \propto xG(x, Q_s^2) \propto Q_s^2(x) \sim s^{\lambda_s/2}$

$$x \simeq \frac{k_\perp}{\sqrt{s}}$$

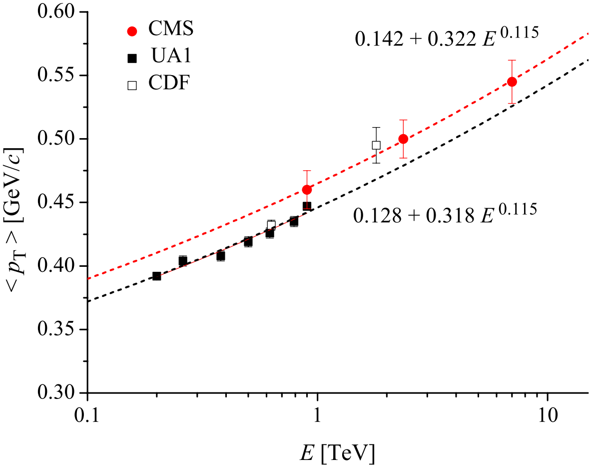
$$k_\perp \sim Q_s$$

$$\lambda_s \simeq 0.2 \div 0.3$$



Average transverse momentum in p+p

- Typical transverse momentum $\langle p_T \rangle \propto Q_s(x) \sim E^{\lambda_s/2}$ $(E \equiv \sqrt{s})$

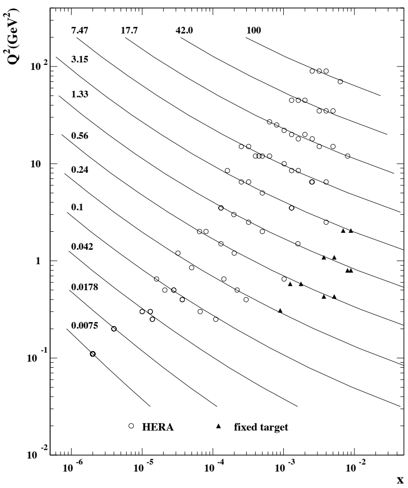


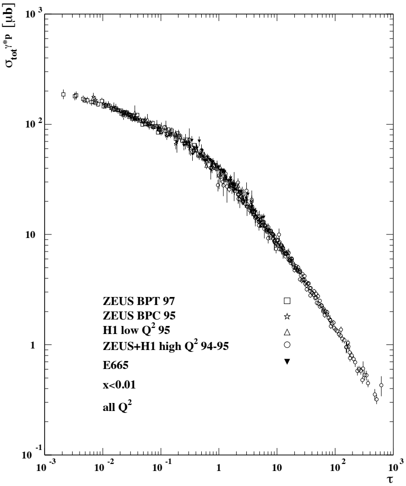
(McLerran and Praszalowicz, 2010)

Geometric scaling at HERA: F_2

- DIS cross–section at HERA *(Staśto, Golec-Biernat, Kwieciński, 2000)*

$$\sigma(x, Q^2) \text{ vs. } \tau \equiv Q^2/Q_s^2(x) \propto Q^2/x^{0.3} : x \le 0.01, \quad Q^2 \le 450 \text{ GeV}^2$$

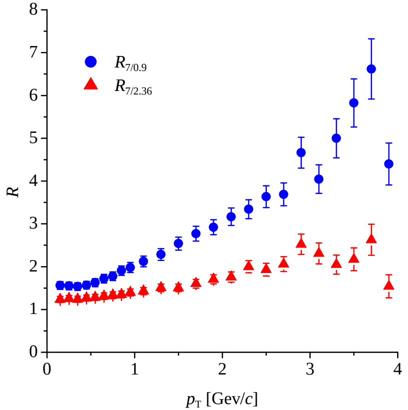


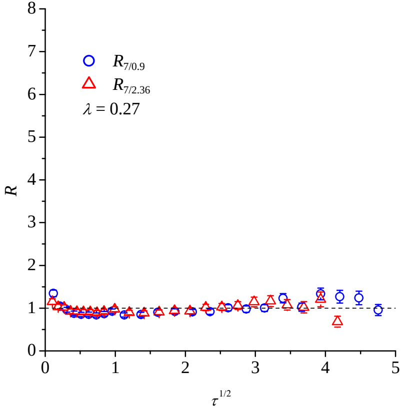


Geometric scaling in p+p at the LHC

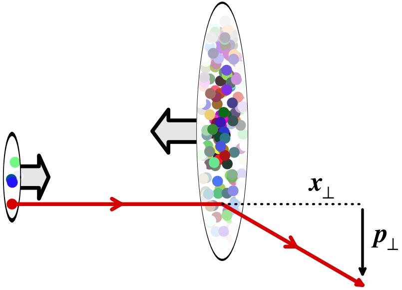
- Ratio between particle production at 2 different energies, s_1 and s_2

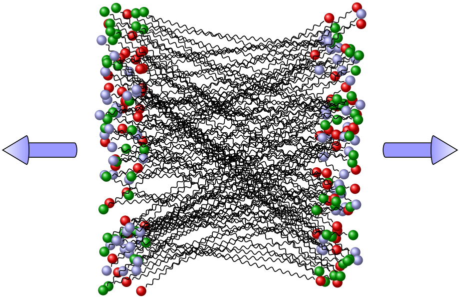
$$R_{s_1/s_2} = \frac{(\mathrm{d}N/\mathrm{d}\eta\,\mathrm{d}^2p_\perp)\big|_{s_1}}{(\mathrm{d}N/\mathrm{d}\eta\,\mathrm{d}^2p_\perp)\big|_{s_2}} \to 1 \text{ as a function of } \tau \equiv \frac{p_\perp^2}{Q_s^2(p_\perp/\sqrt{s})}$$



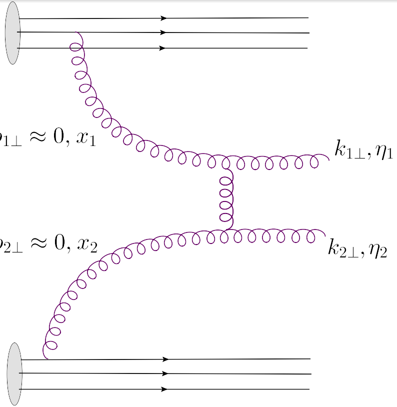


Particle production





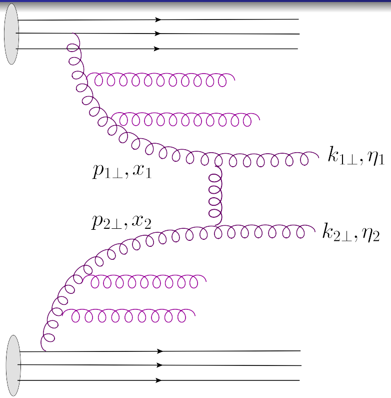
From collinear factorization ...



$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2k_{1\perp}\mathrm{d}^2k_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} = x_1G(x_1,Q^2)\, x_2G(x_2,Q^2)\, \delta^{(2)}(\boldsymbol{k}_{1\perp}+\boldsymbol{k}_{2\perp})\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}k_\perp^2}$$

$$xG(x,Q^2) = \int^Q \mathrm{d}^2\boldsymbol{p}\ \Phi(x,\boldsymbol{p}_\perp)\,, \quad \Phi(x,\boldsymbol{p}_\perp) \equiv x\,\frac{\mathrm{d}N_{\rm gluon}}{\mathrm{d}x\mathrm{d}^2\boldsymbol{p}_\perp}$$

- Assumes $p_{1\perp},\, p_{2\perp} \approx \Lambda_{\rm QCD} \ll k_\perp$

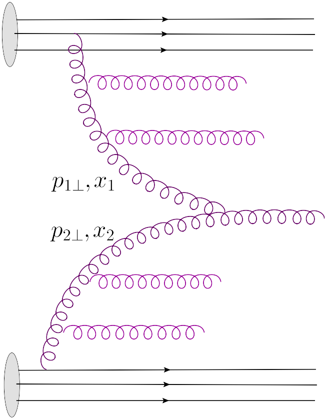


- In reality $p_{1\perp},\, p_{2\perp} \sim Q_s$ can be comparable with $k_{i\perp}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2k_{1\perp}\mathrm{d}^2k_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} = \int \mathrm{d}^2\boldsymbol{p}_{1\perp} \int \mathrm{d}^2\boldsymbol{p}_{2\perp}\, \delta^{(2)}(\boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp} - \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp}) \times \Phi(x_1, \boldsymbol{p}_{1\perp})\, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}k_{1\perp}^2} \Phi(x_2, \boldsymbol{p}_{2\perp})$$

- Consistent with the BFKL evolution *(Catani and Hautmann, 1994)*

… and "mono–jets"



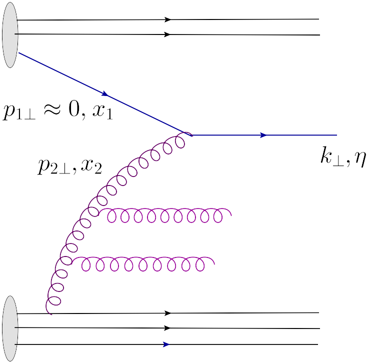
$$x_1 \sim \frac{k_\perp}{\sqrt{s}}\, e^{\eta}$$

$$x_2 \sim \frac{k_\perp}{\sqrt{s}}\, e^{-\eta}$$

- A parton with $k_\perp \lesssim Q_s$ can also be produced via the fusion of 2 initial partons : $gg \to g,\ qg \to q$

$$\frac{d\sigma}{d^2\boldsymbol{k}_\perp d\eta} \simeq \frac{\alpha_s}{k_\perp^2} \int d^2\boldsymbol{p}_\perp \Phi(x_1, \boldsymbol{p}_\perp)\, \Phi(x_2, \boldsymbol{k}_\perp - \boldsymbol{p}_\perp)$$

Forward quark production



$$x_{1,2} \sim \frac{k_\perp}{\sqrt{s}} \, e^{\pm\eta}$$

$$\eta \sim 3 \div 4$$

$$\frac{x_1}{x_2} = e^{2\eta} \sim 400 \div 3000$$

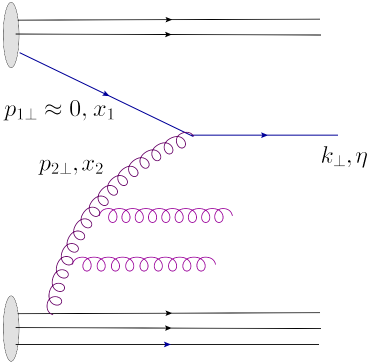
$$\text{e.g. } x_1 = 0.2 \ \& \ x_2 = 10^{-4}$$

- $p_{1\perp} \sim \Lambda_{\rm QCD} \ll k_\perp \sim Q_s(x_2) \Longrightarrow$ hybrid factorization

$$\frac{d\sigma}{d^2\boldsymbol{k}_\perp d\eta} \simeq \frac{\alpha_s}{k_\perp^2} \, x_1 q(x_1, k_\perp^2) \Phi(x_2, \boldsymbol{k}_\perp)$$

- Without saturation: $\Phi(x_2, \boldsymbol{k}_\perp) \propto 1/k_\perp^2 \Longrightarrow dN/d\eta$ is divergent !

Forward quark production



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$$\frac{x_1}{x_2} = \mathrm{e}^{2\eta} \sim 400 \div 3000$$

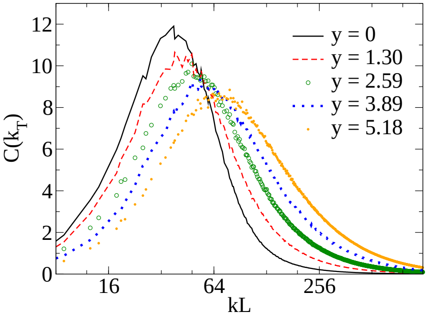
$$\text{e.g. } x_1 = 0.2 \ \& \ x_2 = 10^{-4}$$

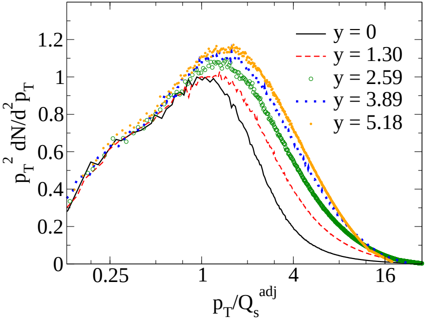
- $p_{1\perp} \sim \Lambda_{\text{QCD}} \ll k_\perp \sim Q_s(x_2) \Longrightarrow$ hybrid factorization

$$\frac{\mathrm{d}N}{\mathrm{d}\eta} = \int \mathrm{d}^2\boldsymbol{k}_\perp \frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta} \propto \int \frac{\mathrm{d}k_\perp^2}{k_\perp^4}$$

- Without saturation: $\Phi(x_2, \boldsymbol{k}_\perp) \propto 1/k_\perp^2 \Longrightarrow \mathrm{d}N/\mathrm{d}\eta$ is divergent !

The importance of saturation





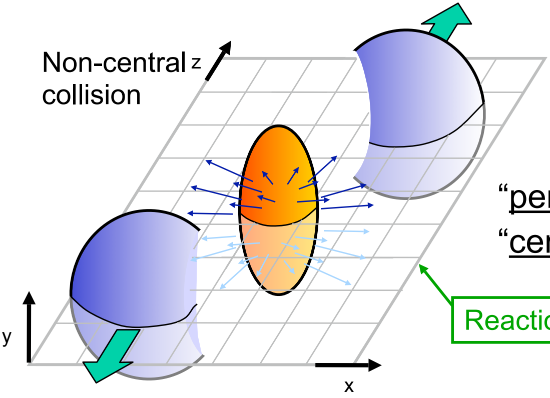
▷ *Numerical solutions to JIMWLK eq. by T. Lappi, arXiv:1105.5511 [hep-ph]*

▷ *Left: unintegrated gluon distribution for different values of* $Y = \ln(1/x)$

▷ *Right: spectrum of gluons produced in* AA *for different energies (*$y \propto \ln E$*)*

- Saturation introduces a dynamical 'infrared cutoff' which increases with energy, $Q_s^2(Y) = Q_0^2 \mathrm{e}^{\lambda_s Y}$, and is 'semi–hard' at RHIC and LHC
- Both gluon distribution and particle production can be computed from first principles (at sufficiently high energies)

The geometry of a HIC



"<u>peripheral</u>" collision ($b \sim b_{max}$)

"<u>central</u>" collision ($b \sim 0$)

Number of participants (N_{part}): number of incoming nucleons (participants) in the overlap region