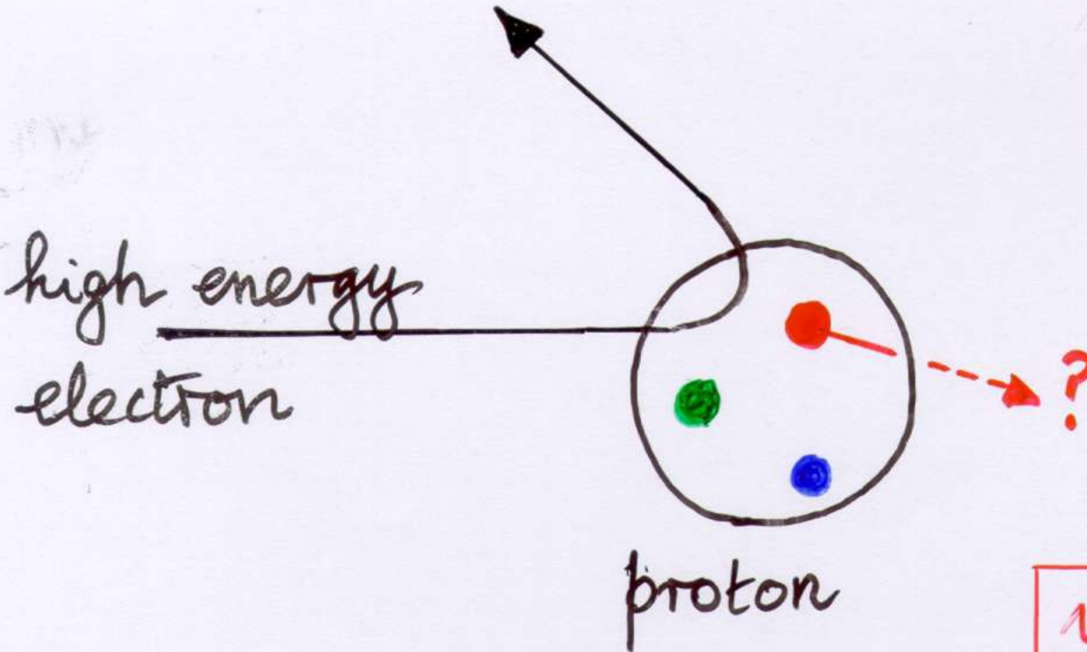
~ BIG PUZZLE ~



Quark acts as if free

yet not seen, (confined in colourless hadron)

Puzzle “explained” by QCD
non-abelian gauge theory

$\psi \to \psi\, e^{i\theta}$ GLOBAL

Probability $|\psi|^2$ unchanged

physics "

Symmetry under ~~phase~~ gauge transformation

$\psi \to \psi\, e^{i\theta(x)}$ LOCAL

γ

need photon to reconcile phase differences

→ QED

massless

Empirical Laws → Maxwell's equations → QED → local gauge symmetry

Coulomb's law
Biot-Savart law
Faraday's law
Ampere's law

for $\vec{E}, \vec{B}$

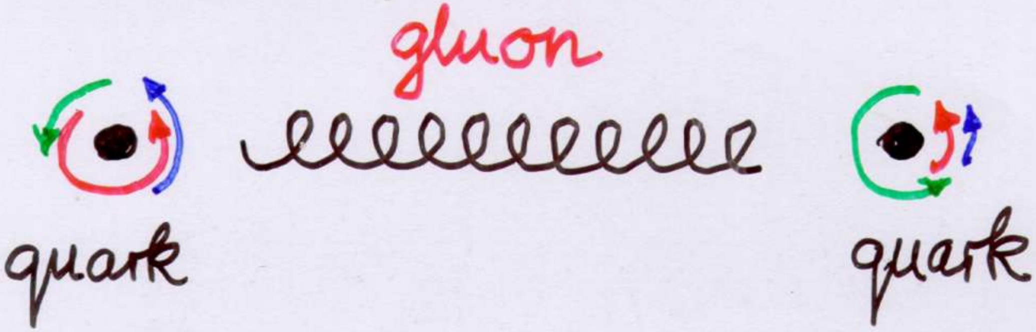
photon:
massless
spin 1

U(1)

← ← ← Impose gauge symmetry on $\psi_{electron}$

ELECTROMAGNETIC FORCE	⟷	U(1) GAUGE SYMMETRY

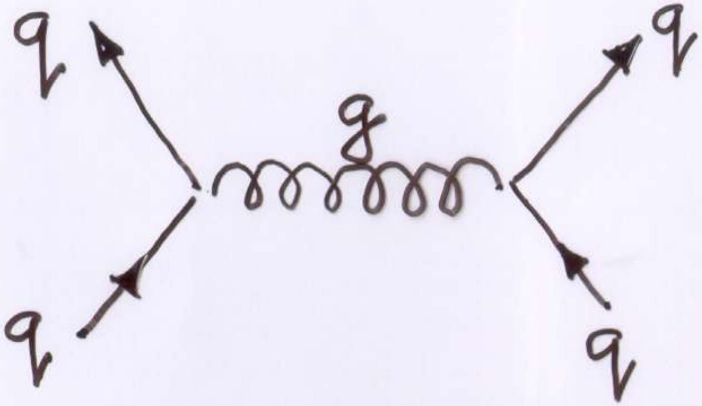
Quarks have 3 colour charges

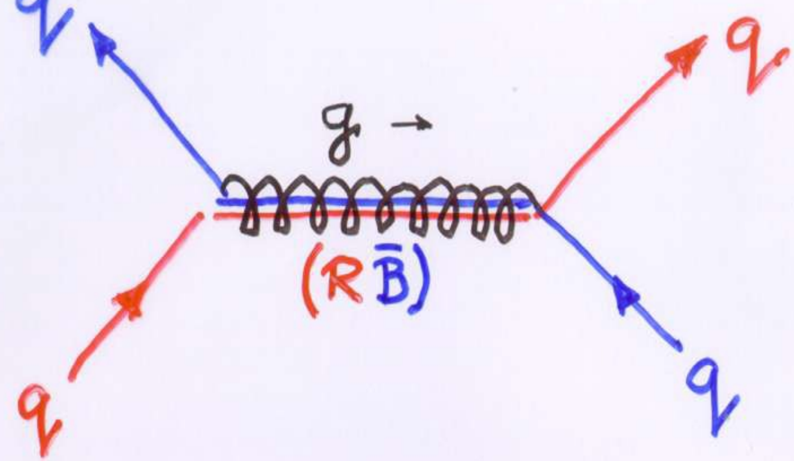


To impose gauge symmetry we need 8 coloured gluons → QCD

massless

$SU(3)_{colour}$





8 possibilities: 8 coloured gluons $R\bar{B}$, $R\bar{G}$, ...

9^{th} is colourless (colour singlet): $\frac{1}{\sqrt{3}}\left(R\bar{R}+B\bar{B}+G\bar{G}\right)$

Coupling constant renormalisation

no scale in $\mathcal{L}_{QCD}$

Consider a dimensionless QCD observable R

Naive prediction for energies $Q \gg m_i$: $R \to$ const. indep of Q

Not true in renormalizable field theory like QCD (or QED)

Scale enters when calculate $R = \sum_n c_n \alpha_s^n$ since encounter (loop) Feynman diagrams which diverge logarithmically.

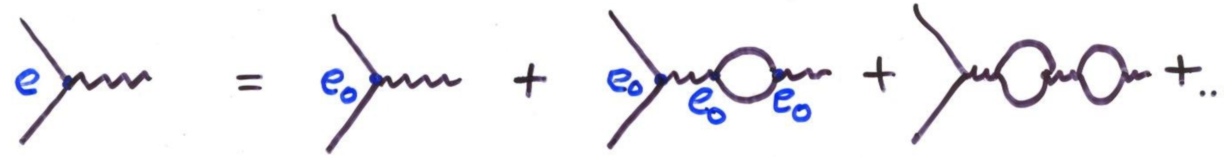
Need to renormalize (reparametrize) the theory

$\to$ introduces a renormalization scale μ

$\to$ $R\left(\log \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$

Running coupling

QED



physical or effective charge — bare charge — bare charge screened

At large Q^2

$$\alpha(Q^2) = \alpha_0 \left\{ 1 + \frac{\alpha_0}{3\pi} \log \frac{Q^2}{M^2} + \left(\frac{\alpha_0}{3\pi} \log \frac{Q^2}{M^2} \right)^2 + \ldots \right\}$$

$$= \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \log \frac{Q^2}{M^2}}$$

Loops each give infinite contribution

M = cut-off on loop momentum

To eliminate dep. on M, introduce renormalisation scale μ

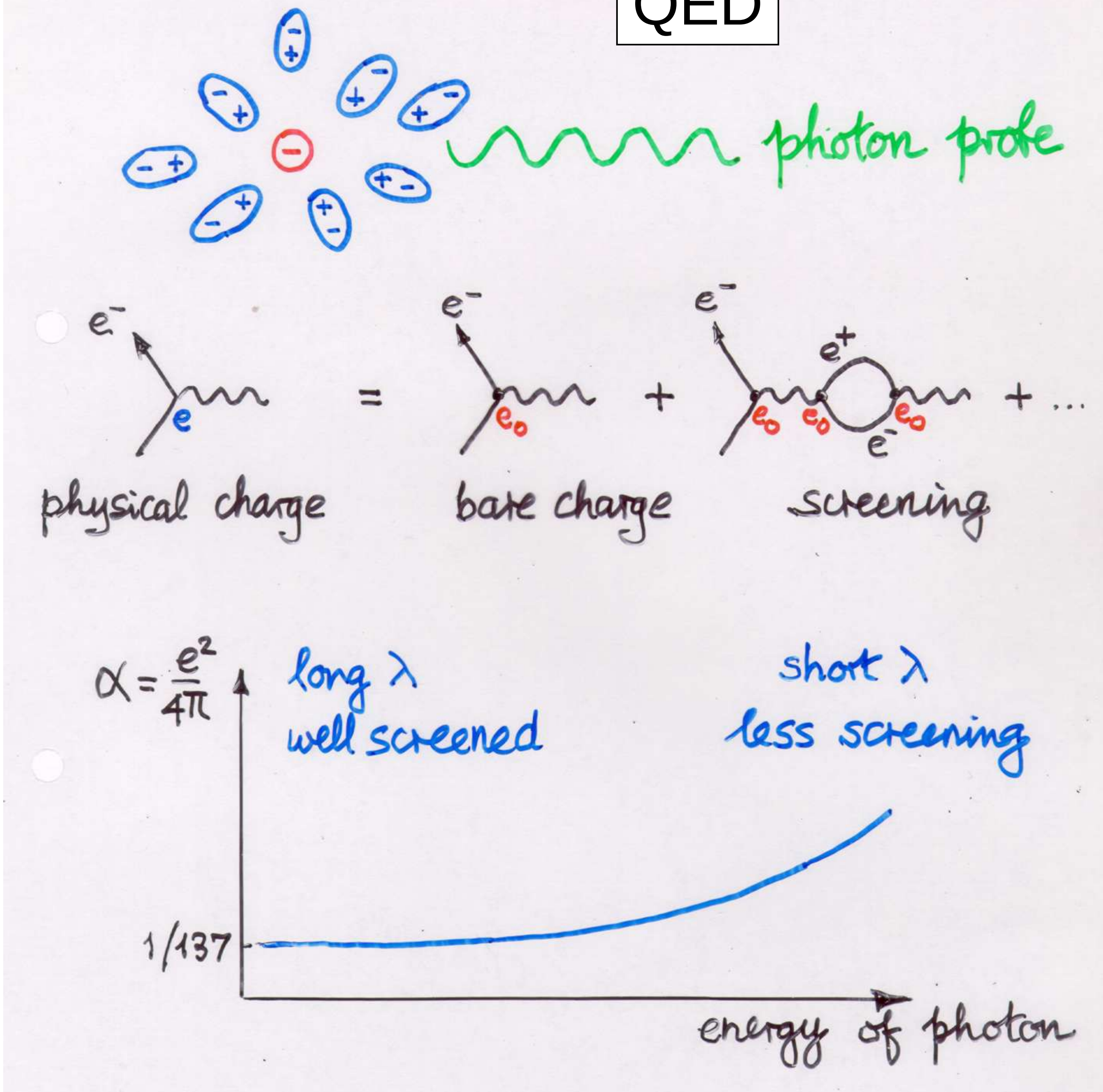
$$\left.\begin{aligned} \frac{1}{\alpha(Q^2)} &= \frac{1}{\alpha_0} - \frac{1}{3\pi} \log \frac{Q^2}{M^2} \\ \frac{1}{\alpha(\mu^2)} &= \frac{1}{\alpha_0} - \frac{1}{3\pi} \log \frac{\mu^2}{M^2} \end{aligned}\right\} \text{subtract}$$

$$\boxed{\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log \frac{Q^2}{\mu^2}}}$$

infinities removed at price of ren. scale μ

QED predicts running, expt. the absolute value.

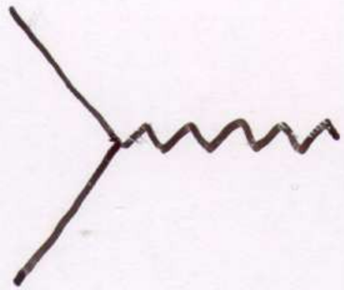
QED



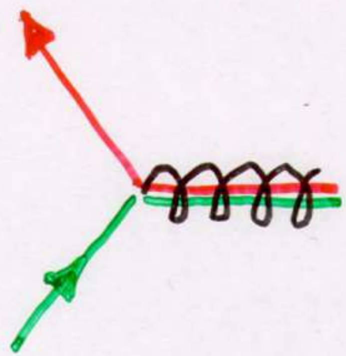
Vacuum is polarised

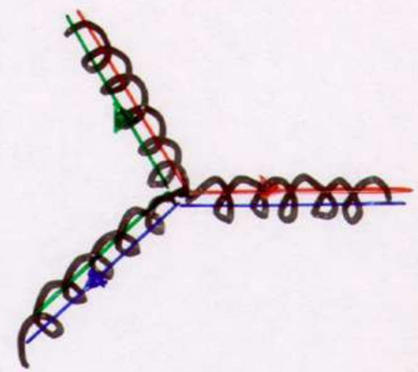
QED coupling runs

QED



QCD





VERY
INTERESTING

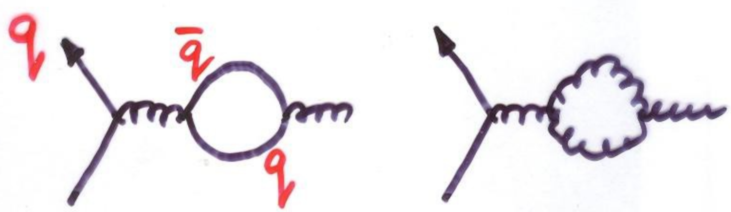
<u>Note</u>: UV divergences of 

cancel via Ward identities (basic prop of gauge FT's)

Just as well – so renormalized charge for $e, \mu, \ldots$ remains equal

QCD Running of α_s

like $-\frac{1}{3\pi}$ of QED (New)



$$b_0 = -\frac{n_f}{6\pi} + \frac{33}{12\pi}$$

screening antiscreening

$$\boxed{\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0\, \alpha_s(\mu^2) \ln \frac{Q^2}{\mu^2}}}$$

Running of QCD coupling α_s

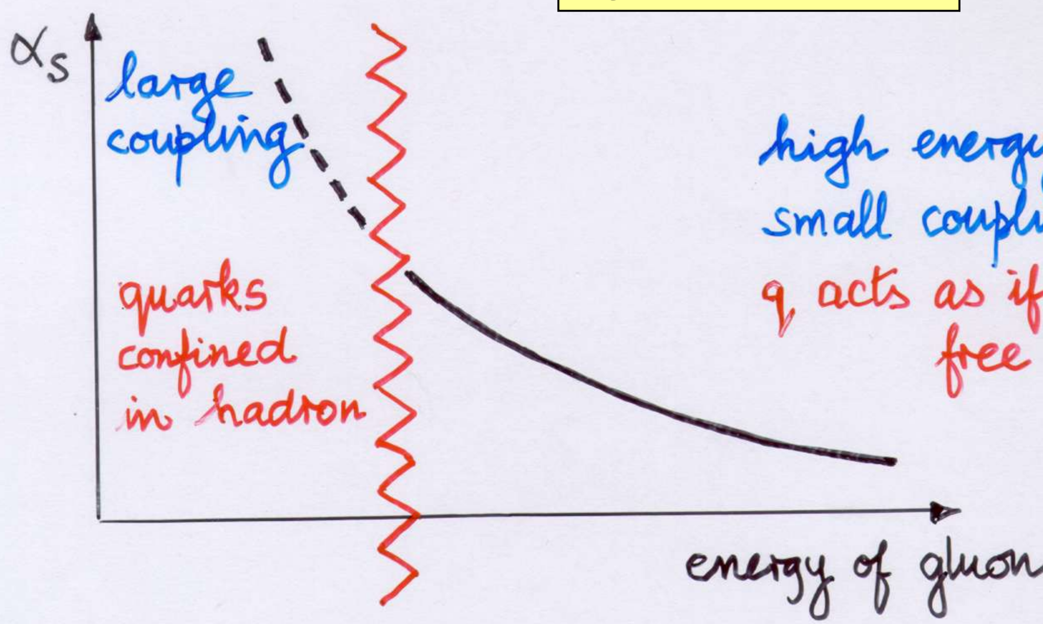
~QED

NEW

colour screening

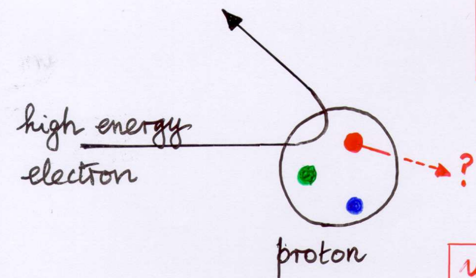
colour antiscreening

QCD -- 1973



A 'dream' theory!

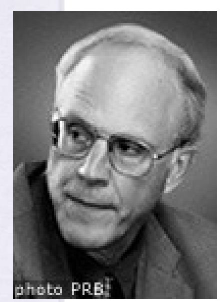
~ BIG PUZZLE ~

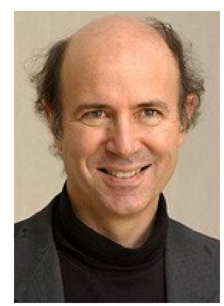


Quark acts as if free

yet not seen, (confined in colourless hadron)

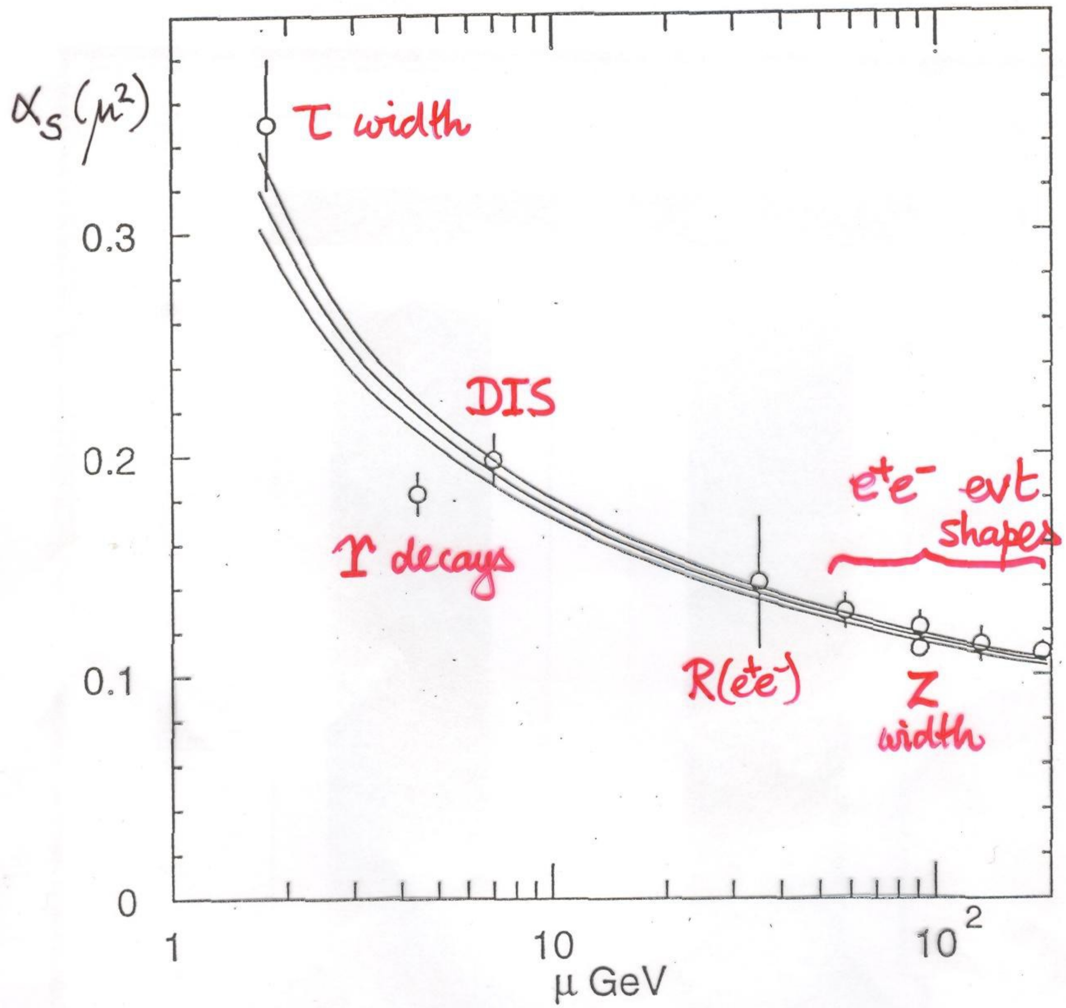
Gross Wilczek Politzer



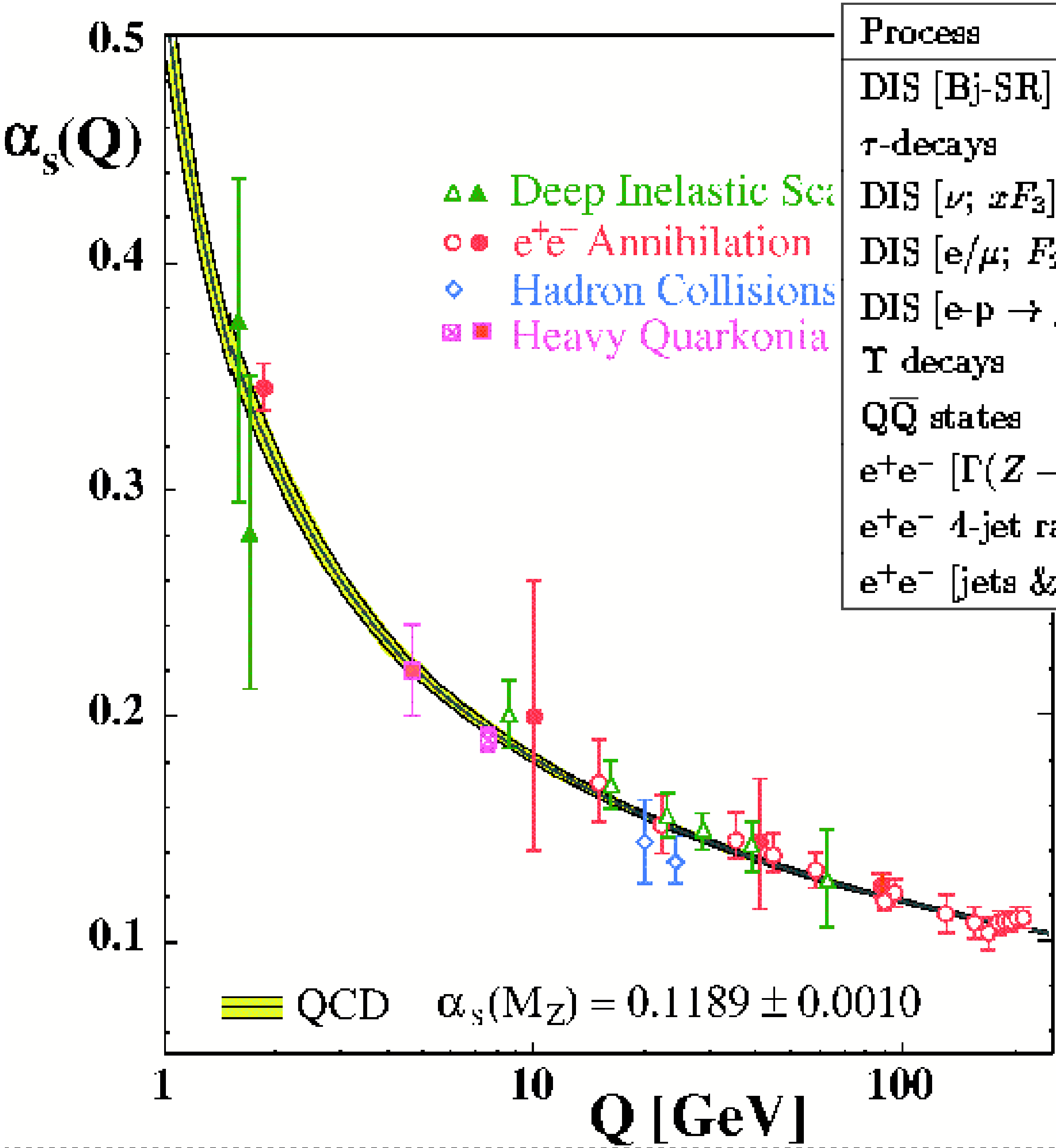


Nobel prize -- 2004

$$\frac{Z,\tau \to \text{hadrons}}{Z,\tau \to \text{leptons}} = R_{EW}\left(1 + \delta_{QCD} + \delta_{NP}\right)$$



curves correspond to $\alpha_s(M_Z^2) = 0.117_2 \pm 0.002$



Process	Q [GeV]	$\alpha_s(M_{Z^0})$
DIS [Bj-SR]	1.58	$0.121\ ^{+0.005}_{-0.009}$
τ-decays	1.78	0.1215 ± 0.0012
DIS [ν; xF_3]	2.8 - 11	$0.119\ ^{+0.007}_{-0.006}$
DIS [e/μ; F_2]	2 - 15	0.1166 ± 0.0022
DIS [e-p $\rightarrow$ jets]	6 - 100	0.1186 ± 0.0051
Υ decays	4.75	0.118 ± 0.006
$Q\overline{Q}$ states	7.5	0.1170 ± 0.0012
e^+e^- [$\Gamma(Z \rightarrow had)$]	91.2	$0.1226^{+0.0058}_{-0.0038}$
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022
e^+e^- [jets & shps]	189	0.121 ± 0.005

<u>Note</u>: UV divergences of

cancel via Ward identities (basic prop' of gauge FT's)

Just as well – so renormalized charge for $e, \mu, \ldots$ remains equal

---

Ward identities of QED → Slavnov-Taylor identities of QCD

$$\alpha_s(q\bar{q}g)_{\substack{\text{with}\\\text{loops}}} = \alpha_s(ggg)_{\substack{\text{with}\\\text{loops}}}$$

(gauge theory)

equality preserved by renormalisation

dimensionless $R \xrightarrow{\text{renormalisation}} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$

but R cannot depend on renorm scale μ: RGE $\frac{\partial R}{\partial \log \mu^2} = 0$

can show this gives $R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = R\left(1, \alpha_s(Q^2)\right)$

$\underbrace{\text{running of } \alpha_s}$ determines Q dep of R

$$\frac{\partial \alpha_s}{\partial \log \mu^2} \equiv \beta(\alpha_s)$$

$\boxed{\text{1 loop } \beta \text{ function}}$ so far only summed LL (i.e. $\left(\alpha_s \log \frac{Q^2}{\mu^2}\right)^n$)

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0\, \alpha_s(\mu^2) \log \frac{Q^2}{\mu^2}}$$

$$b_0 = -\frac{n_f}{6\pi} + \frac{33}{12\pi}$$

$$\frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s(\mu^2)} + b_0 \log \frac{Q^2}{\mu^2}$$

$$0 = -\frac{1}{\alpha_s^2(\mu^2)} \frac{d\alpha_s}{d \log \mu^2} - b_0 \quad \Rightarrow \quad \boxed{\beta(\alpha_s) = -b_0 \alpha_s^2}$$

$$\left[\frac{dg}{d \log \mu^2} \sim g^3 \quad \therefore \frac{d\alpha_s}{d \log \mu^2} \sim g^4 \sim \alpha_s^2 \right]$$

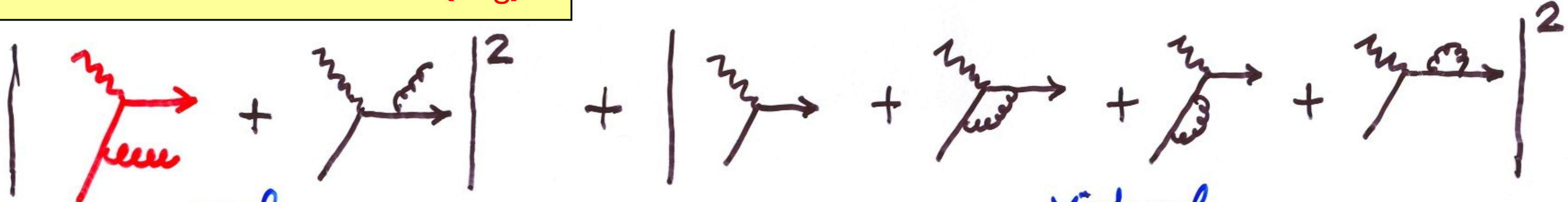
Return to DIS

we left QPM at

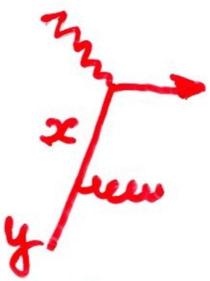
Not^{n}: $\xi \to y$
(not $y = Q^2/xs$)

$$\frac{F_2(x)}{x} = \sum_q \int_0^1 dy\, f_q(y)\, e_q^2\, \delta(y-x) = \sum_q \int_0^1 \frac{dy}{y} f_q(y)\, e_q^2\, \delta\left(1-\frac{x}{y}\right)$$

QCD corrections: O(α_s)



Calcul^{n} gives

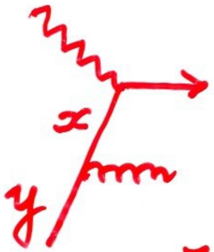


$(x < y)$

sing. $\mu \to 0$

$$\frac{F_2(x,Q^2)}{x} = \sum_q \int_x^1 \frac{dy}{y} f_q(y)\, e_q^2 \left[\delta\left(1-\frac{x}{y}\right) + \frac{\alpha_s}{2\pi}\left\{P\left(\frac{x}{y}\right) \log\frac{Q^2}{\mu^2} + C\left(\frac{x}{y}\right)\right\}\right]$$

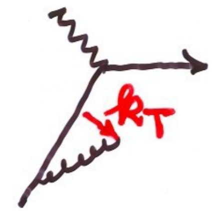
Origin of collinear singularity:



$z \equiv \frac{x}{y}$

small k_T^2 limit of Feyn. diagram (HM 10.30)

$$\frac{1}{\hat{\sigma}_0}\frac{d\hat{\sigma}}{dk_T^2} \simeq e_q^2 \, \frac{1}{k_T^2} \, \frac{\alpha_s}{2\pi} P(z)$$



$\frac{1}{\epsilon}$ pole

$P = \frac{4}{3} \, \frac{1+z^2}{1-z}$

$$\frac{\hat{\sigma}(\gamma^* q \to qg)}{\hat{\sigma}_0} \simeq e_q^2 \, P(z) \int_{\mu^2}^{Q^2} \frac{dk_T^2}{k_T^2} \, \frac{\alpha_s}{2\pi} \;=\; e_q^2 \, \frac{\alpha_s}{2\pi} P(z) \log\frac{Q^2}{\mu^2}$$

μ is artifical regulator

PDF $f_q(y)$

$$\frac{F_2(x,Q^2)}{x} = \sum_q \int_x^1 \frac{dy}{y} q_0(y)\, e_q^2 \left[\delta\left(1-\frac{x}{y}\right) + \frac{\alpha_s}{2\pi}\left\{P\left(\frac{x}{y}\right) \log\frac{Q^2}{\mu^2} + C\left(\frac{x}{y}\right)\right\}\right]$$

fac.^{n} scale

In analogy with α_s, absorb sing. in q_0. That is redefine $q_0(y) \to q_0(y, M^2)$

$$q(x,M^2) = q_0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} q_0(y) \left\{P\left(\frac{x}{y}\right)\log\frac{M^2}{\mu^2} + C_1\right\}$$

$C = C_1 + C_2$

fac. scheme

DIS: $C_2 = 0$

$\overline{MS}$: $C_2 = C_{\overline{MS}}$

so $$\frac{F_2(x,Q^2)}{x} = \sum_q e_q^2 \underline{\int_x^1 \frac{dy}{y} q(y,M^2)\left\{\delta\left(1-\frac{x}{y}\right) + \frac{\alpha_s}{2\pi}P\left(\frac{x}{y}\right)\log\frac{Q^2}{M^2} + C_2\right\}}$$

$q(x,M^2)$ finite, but no absolute pQCD pred.^{n} – depends on proton wave fn. (non-pert.)

However

$$\boxed{\frac{\partial q(x,M^2)}{\partial \log M^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} q(y,M^2)\, P\left(\frac{x}{y}\right)} + O(\alpha_s^2) \longleftrightarrow$$

DGLAP evolution eq.

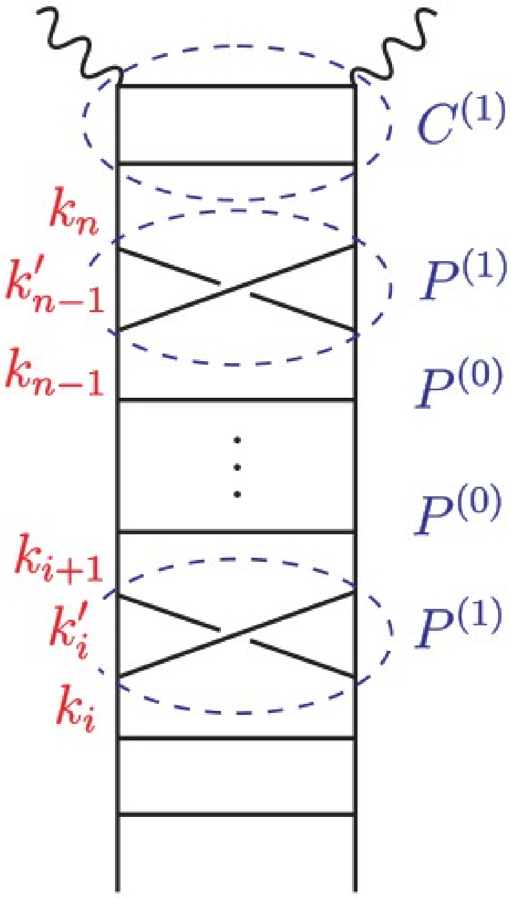
- M = factorization scale
- expt. $\to q(x,Q_0^2)$
- running of $q(x,Q^2)$ given by pQCD

<u>Analogy</u>

$$\frac{\partial \alpha_s(\mu^2)}{\partial \log\mu^2} = \beta(\alpha_s)$$

- μ = renorm. scale
- expt. $\to \alpha_s(Q_0^2)$
- running $\alpha_s(Q^2)$ given by pQCD

DGLAP evolution



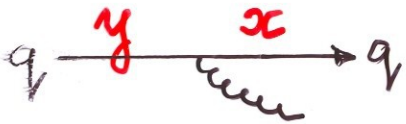
Resumms large logs

Approximation:

$$x_i > x_{i+1}$$

$$k_{i,t} \ll k_{i+1,t}$$

Further discussion of DGLAP evol^{n} and splitting functions $P(z)$



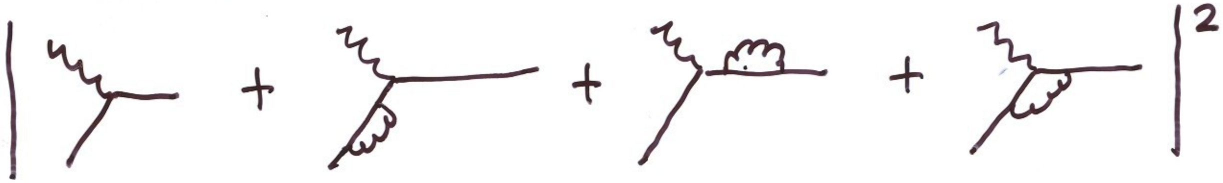
$\frac{\alpha_s}{2\pi} P_{q \leftarrow q}(z)$ is prob. of finding q with fraction x inside q with fraction y

$z \equiv x/y$

- Including virtual contrib^{n}

$$P_{qq} = \frac{4}{3} \frac{1+z^2}{1-z}$$

$z=1$ singularity (soft infrared gluon) cancelled by virtual contributions



After cancell^{n} of singularity there remains a residual $\delta(1-z)$ contrib^{n} from virtual diagrams

$$P_{qq} = \frac{4}{3} \frac{1+z^2}{(1-z)_+} + 2\,\delta(1-z)$$

easy to verify using $\int_0^1 P_{qq}(z)\, dz = 0$

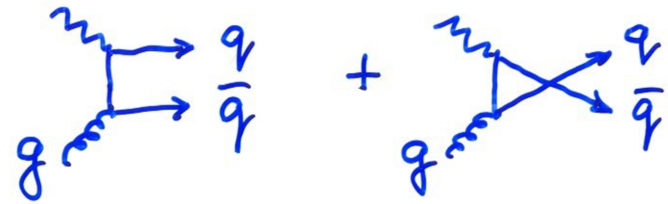
[to conserve q (i.e. baryon) number the integral of $q(x, Q^2)$ cannot dep. on Q^2]

+ prescrip^{n} ensures cancell^{n}

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} \equiv \int_0^1 dz \frac{f(z) - f(1)}{1-z}$$

- <u>Include gluon compt.</u>

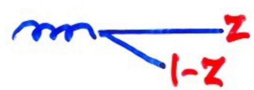
at $O(\alpha_s)$ need to include $g\gamma^* \to q\bar{q}$



$$P \otimes q \equiv \int_0^1 \frac{dy}{y}\, q(y, Q^2)\, P\left(\frac{x}{y}\right)$$

DGLAP evol^{n} eqs.

$$\frac{\partial q(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\left[P_{qq} \otimes q + P_{qg} \otimes g\right]$$



$$P_{qg} = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

similarly

$$\frac{\partial g(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\left[\sum_i P_{gq} \otimes (q_i + \bar{q}_i) + P_{gg} \otimes g\right]$$

$$P_{gq} = \frac{4}{3}\,\frac{1 + (1-z)^2}{z}$$

$$P_{gg} = 6\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \left(\frac{11}{2} - \frac{n_f}{3}\right)\delta(1-z)$$