International Centre for Theoretical Physics
South American Institute for Fundamental Research

Physics Beyond the Standard Model

Eduardo Pontón

Oct. 21 – Nov. 6, 2014

School on "New Trends on High-Energy Physics and QCD"

Three Lectures on BSM Physics

- Lecture 1: The Standard Model

  Why the SM cannot be a complete description of Nature?
  Why do we think we could find new physics at the TeV scale?

- Lecture 2: Supersymmetry as an example for new Physics at the TeV scale.

  Motivations and virtues.
  Assessment of the present status.

- Lecture 3: Elementary or composite Higgs?

  Strong dynamics as the origin of EWSB.
  The connection to extra spatial dimensions.

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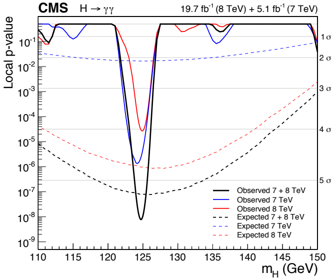
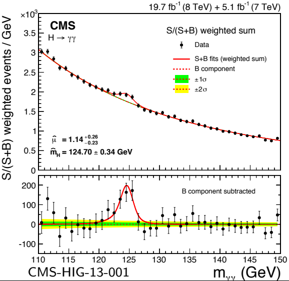
Observation in Single Channels





$$h \to \gamma\gamma$$

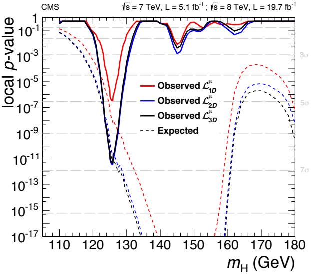
CMS: 5.7σ

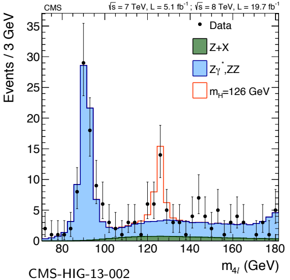
ATLAS: 7.4σ

$$h \to ZZ$$

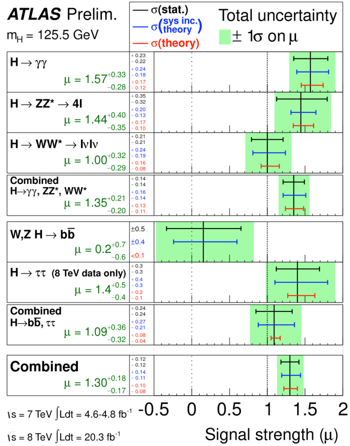
CMS: 6.8σ

ATLAS: 8.1σ

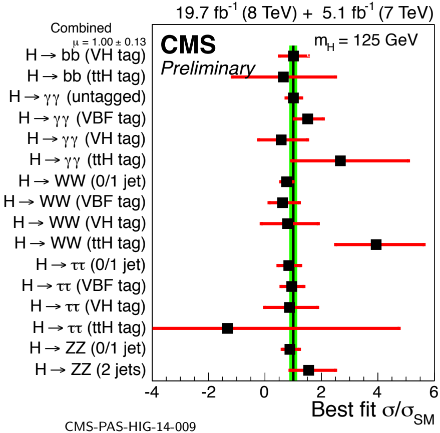




Signal Strengths



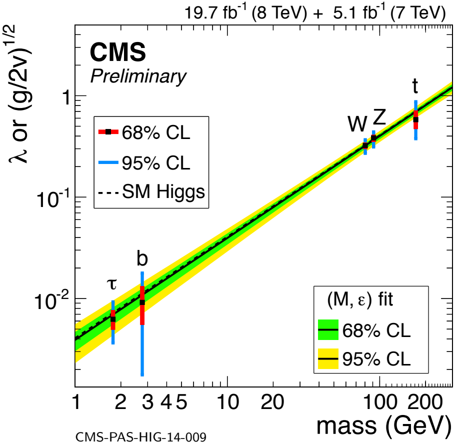
ATLAS-CONF-2014-009



CMS-PAS-HIG-14-009

CMS: $\mu_{\text{global}} = 1.00 \pm 0.09(\textbf{stat.}) \; {}^{+0.08}_{-0.07} \; (\textbf{theor.}) + 0.07 \; (\textbf{syst.})$

The Higgs Boson Couples to Mass!



Elementary or Composite?

- Within the SM, the Higgs boson is understood as an elementary d.o.f. (i.e. no substructure). The same holds true for the SM fermions and gauge bosons (and there is no evidence to the contrary).
- To the extent that the LHC measurements agree with the SM Higgs properties, one could be tempted to conclude that it is indeed elementary. Although we seriously doubt that the SM is valid down to ultrashort distances, the existence of UV completions like SUSY (where the shortcomings of the SM can be addressed in a robust framework) shows that a truly elementary scalar field could be realized in nature as such.
- Nevertheless, since the present measurements of the Higgs properties have uncertainties at roughly the 20% level (still large), it is imaginable that with more precision some Higgs substructure could be revealed.
- As it turns out, if the Higgs was a composite state this could provide an understanding of the weak scale (both of the Hierarchy Problem and the question of why the EW symmetry is broken).

Old Paradigms: Technicolor

As Emmanuel described in his lectures, historically we have progressively discovered that what seems elementary at some scale has turned out to have substructure:

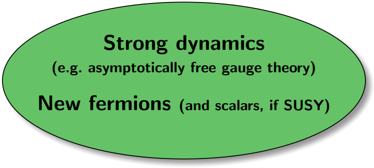
atoms $\rightarrow$ nuclei + electrons $\rightarrow$ nucleons $\rightarrow$ quarks $\rightarrow$ preons (?)

Also, we already noted in the first lecture that QCD, through the condensate $\langle \bar{q}^i_L q^j_R \rangle \sim \Lambda^3_{\rm QCD} \delta^{ij}$, would give a mass to the $W^\pm, Z$ gauge bosons, although the scale is far too small.

Could it be that our "elementary" particles have indeed substructure, that these preons are held together by a new strong interaction with a characteristic scale somewhat above the weak scale, and that those strong interactions are ultimately responsible for the breaking of the EW symmetry?

Strong Dynamics as the origin of EWSB: Old and New

BSM: Standard Model + Strongly coupled sector



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BSM: Standard Model + Strongly coupled sector

Strong dynamics
(e.g. asymptotically free gauge theory)

New fermions (and scalars, if SUSY)

Sufficiently large global symmetry G

$(SU(3)_C \times SU(2)_L \times U(1)_Y \subset G)$

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(e.g. $\langle \psi\bar{\psi} \rangle \neq 0$) ↓ **Dynamically generated scale Λ**

Set of resonances

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Set of resonances

- $SU(2)_L \times U(1)_Y \not\subset \mathcal{H}$ ⟶ **"Technicolor", no need of Higgs (disfavored)**
- $SU(2)_L \times U(1)_Y \subset \mathcal{H}$ ⟶ **Standard Model group unbroken at Λ**
  Amongst resonances: state with Higgs quantum numbers

Strong Dynamics as the origin of EWSB: New Paradigms

Elementary sector: SM gauge bosons, quarks & leptons	SM gauge interactions ⟷ Mixing	New strong sector: resonances + Higgs bound state	spin 1/2, 1, ... (in full G reps)

But why would the Higgs resonance be lighter than the rest?

Natural to interpret the composite Higgs as a (pseudo) Nambu-Goldstone boson

Higgs in $G/\mathcal{H}$

Georgi & Kaplan '84
Agashe et. al '03

Inspiration from pions in QCD (with 2 flavors): $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$

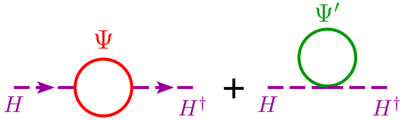
$\pi^0, \pi^\pm$ are NGB's of spontaneous breaking

Acquire masses from explicit breaking:

- $m_q \neq 0 \;\Rightarrow\; m_\pi^2 \simeq m_q B_0$
- $e \neq 0 \;\Rightarrow\; m_{\pi^\pm}^2 - m_{\pi^0}^2 \sim \frac{e^2}{16\pi^2} \Lambda^2$

Strong Dynamics as the origin of EWSB: New Paradigms

- How is this a solution to the Hierarchy Problem?
  Short answer: The SM would cease to be a full description at the scale of the strong resonances!
- We should be able to understand in detail how the pNGB Higgs is somewhat lighter than those resonances. It turns out that the cancellations occur between fields of the same spin (normal symmetries, as opposed to supersymmetry!)



Strong Dynamics as the origin of EWSB: New Paradigms

- The possibility that the weak scale is generated dynamically in analogy to QCD remains intriguing. Even if a scaled up version of QCD would not work, it may be possible that other strongly interacting theories display the desired features.
- Answering this question in detail remains extremely challenging, but if there were any hints for Higgs compositeness, this would certainly boost the effort to understand strongly interacting theories beyond QCD!
- For the time being one can try to parametrize the low-energy possibilities (à la the Chiral Lagrangian) to try to learn what could be expected at LHC energies, if the origin of the EW scale was connected to some new strong dynamics.
- Observation: strongly interacting theories may be amenable to analysis via a duality with certain higher-dimensional theories! (through the AdS/CFT correspondence and generalization of this conjecture; see also Dmitry's lectures.) **This has opened new model-building horizons.**

Extra Dimensions

- I will therefore switch gears slightly and describe a few relevant features of extra-dimensional constructions.
- The idea of new compact spatial dimensions has a long history (and we have already heard about them in the context of string theory in Gary's lectures). More recently there was renewed interest, motivated as an alternative to SUSY as a solution to the Hierarchy Problem. It was not immediately realized that many of this constructions can be interpreted as 4D strong dynamics. Nowadays we are again thinking about 4D extensions, even if we use 5D setups as a computational tool...

The Kaluza-Klein Decomposition

Quantum fields in $4+n$ dimensions: for definiteness, consider a scalar field

$$\Phi(x^\mu, y^i) \qquad (\mu = 0,1,2,3; \quad y^i \text{ parametrize the compact space})$$

We can expand any field configuration in any complete set of functions $\{f_n(y^i)\}$

$$\Phi(x^\mu, y^i) = \frac{1}{\sqrt{V}} \sum_n \phi^{(n)}(x^\mu)\, f_n(y^i)$$

$\phi^{(n)}(x^\mu)$: "n-th KK mode"

Life is easier if the basis is orthonormal:

$$\langle f_n | f_m \rangle = \delta_{nm} \longrightarrow$$

allows to think of the $\phi^{(n)}$ as independent d.o.f.

$\langle f_n | f_m \rangle$: Normally defined in terms of an integral

How do we choose a convenient basis? $\longrightarrow$ Depends on the model under consideration

The Kaluza-Klein Decomposition

To understand the logic, replace the KK expansion back in the free scalar Lagrangian:

$$-\int d^4x\, d^ny\, \frac{1}{2}\Phi\left(\partial_\mu\partial^\mu + \partial_i\partial^i + M^2\right)\Phi$$

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= \quad -\frac{1}{V}\int d^4x\, d^ny \sum_{n,n'} \frac{1}{2}\phi^{(n)} f_n \left[f_{n'}\partial_\mu\partial^\mu\phi^{(n')} + \phi^{(n')}(\partial_i\partial^i + M^2) f_{n'}\right]
\end{aligned}
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$$= -\sum_{n,n'}\left(\frac{1}{V}\int d^ny\, f_n f_{n'}\right)\int d^4x\, \frac{1}{2}\phi^{(n)}\left(\partial_\mu\partial^\mu + m^2_{n'}\right)\phi^{(n')}$$

where to reach the last line we chose the f_n's to satisfy $\left(\partial_i\partial^i + M^2\right) f_n = m_n^2 f_n$.

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- \int d^4x\, d^ny\, \frac{1}{2} \Phi \left(\partial_\mu \partial^\mu + \partial_i \partial^i + M^2 \right) \Phi \\
&= -\frac{1}{V} \int d^4x\, d^ny \sum_{n,n'} \frac{1}{2} \phi^{(n)} f_n \left[f_{n'} \partial_\mu \partial^\mu \phi^{(n')} + \phi^{(n')} (\partial_i \partial^i + M^2) f_{n'} \right] \\
&= -\sum_{n,n'} \left(\frac{1}{V} \int d^ny\, f_n f_{n'} \right) \int d^4x\, \frac{1}{2} \phi^{(n)} \left(\partial_\mu \partial^\mu + m_{n'}^2 \right) \phi^{(n')}
\end{aligned}
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Choosing appropriate boundary conditions, this KK equation of motion implies the orthogonality relations $\frac{1}{V} \int d^ny\, f_n f_{n'} = \delta_{n,n'}$.

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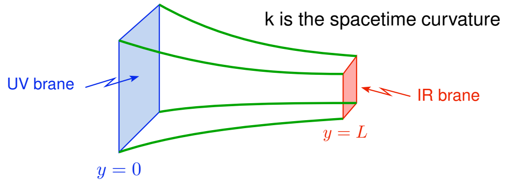
The upshot is that the theory can be rewritten as: $-\sum_n \int d^4x\, \frac{1}{2}\phi^{(n)}\left(\partial_\mu\partial^\mu + m_n^2\right)\phi^{(n)}$

Warped Extra Dimensions

Fast forward to 1999: phenomenological applications of warping in Xdim...

Inspired by string constructions, Randall and Sundrum realized that field theories in a curved 5D background would have rather interesting properties:

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2,$$



The Magic of Curvature (Warping and the RS Scenario)

To illustrate, consider the RS background $ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$, and a scalar field ϕ_{IR}, localized on the IR brane:

$$S \supset \int d^4x\, dy \sqrt{-G} \left\{ \delta(y-L) \left[\frac{1}{2} G^{\mu\nu} \partial_\mu \phi_{IR} \partial_\nu \phi_{IR} - \lambda_{IR} \left(\phi_{IR}^2 - v_{IR}^2 \right)^2 \right] \right\}$$

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&\overset{\tilde{\phi}_{IR} = e^{-kL}\phi_{IR}}{=} \int d^4x \left\{\frac{1}{2}\partial_\mu\tilde{\phi}_{IR}\partial^\mu\tilde{\phi}_{IR} - \lambda_{IR}\left(\tilde{\phi}_{IR}^2 - v_{IR}^2 e^{-2kL}\right)^2\right\}
\end{aligned}
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\overset{\tilde{\phi}_{IR} = e^{-kL}\phi_{IR}}{=} & \int d^4x \left\{ \frac{1}{2} \partial_\mu \tilde{\phi}_{IR} \partial^\mu \tilde{\phi}_{IR} - \lambda_{IR} \left(\tilde{\phi}_{IR}^2 - v_{IR}^2 e^{-2kL} \right)^2 \right\}
\end{aligned}
$$

Now repeat the exercise with a UV brane-localized field, ϕ_{UV}: all warp factors are 1!

$$
V = \lambda_{UV} \left(\tilde{\phi}_{UV}^2 - v_{UV}^2 \right)^2 + \lambda_{IR} \left(\tilde{\phi}_{IR}^2 - v_{IR}^2 e^{-2kL} \right)^2
$$

We end up with a 4D theory characterized by exponentially different vev's!

KK Decompositions in an Arbitrary 5D Background

For 5D theories preserving 4D Lorentz invariance: $ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$

For scalars:

$$\Phi(x^\mu, y) = \frac{a(y)^{-1}}{\sqrt{L}} \sum_n \phi^{(n)}(x^\mu) f_n(y)$$

KK eq. of motion:

$$f_n'' + 2\frac{a'}{a} f_n' - \left[\frac{a''}{a} + 2\frac{a'^2}{a^2} + M^2\right] f_n = -m_n^2 a^{-2} f_n$$

Solution for $a(y) = e^{-ky}$ and $M^2 = \left[c_s^2 + c_s - \frac{15}{4}\right] k^2$:

$$f_n(y) = N_n e^{ky} \left[J_{|c_s+1/2|}(m_n e^{ky}/k) + b_n\, Y_{|c_s+1/2|}(m_n e^{ky}/k)\right]$$

For fermions:

$$\Psi_{L,R}(x^\mu, y) = \frac{a(y)^{-3/2}}{\sqrt{L}} \sum_n \psi^{(n)}_{L,R}(x^\mu) f_n^{L,R}(y)$$

KK eq. of motion:

$$f_{n,L}' - (c_f - 1/2)\frac{a'}{a} f_{n,L} = m_n a^{-1} f_{n,R}$$

$$-f_{n,R}' - (c_f - 1/2)\frac{a'}{a} f_{n,R} = m_n^* a^{-1} f_{n,L}$$

Solution for $a(y) = e^{-ky}$ and $M = c_f k$:

$$f_n(y) = N_n e^{ky} \left[J_{|c_f+1/2|}(m_n e^{ky}/k) + b_n Y_{|c_f+1/2|}(m_n e^{ky}/k)\right]$$

KK Decompositions in an Arbitrary 5D Background

For gauge fields with a gauge-fixing term $\frac{1}{2\xi}\left\{\eta^{\mu\nu}\partial_\mu A_\nu - \xi\,\partial_y\left[a(y)^2 A_5\right]\right\}^2$:

$$A_\mu(x^\mu, y) = \frac{1}{\sqrt{L}}\sum_n A_\mu^{(n)}(x^\mu) f_n(y)$$

KK eq. of motion:

$$f_n''(y) + 2\frac{a'}{a}f_n'(y) = -m_n^2 a^{-2} f_n(y)$$

Solution for $a(y) = e^{-ky}$:

$$f_n(y) = N_n e^{ky}\left[J_1(m_n e^{ky}/k) + b_n\, Y_1(m_n e^{ky}/k)\right]$$

All KK wavefunctions normalized according to:

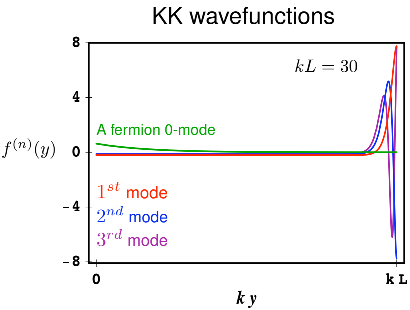
$$\frac{1}{L}\int d^n y\, f_n f_{n'} = \delta_{n,n'}$$

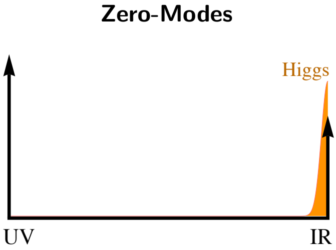
These wave functions reflect the strength of the interactions at each point y

The boundary conditions fix the constants b_n and the spectrum m_n.

Localization and Partial Compositeness

Let us get acquainted with the KK wavefunction localization properties.





"Zero-modes" play a special role. They should correspond to the observed particles!

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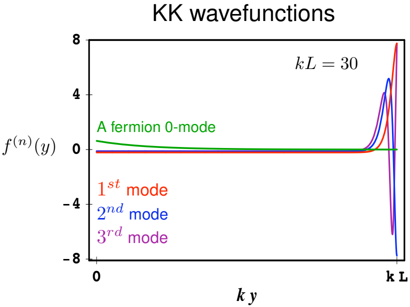

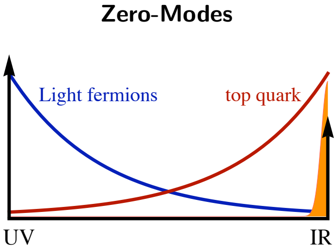

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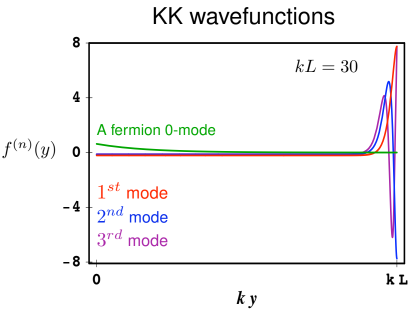

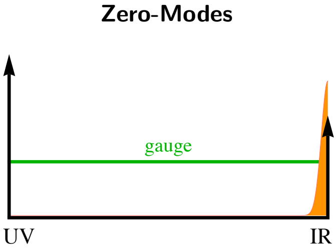

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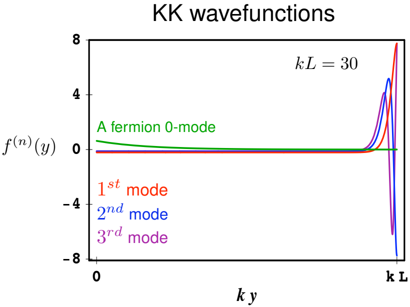

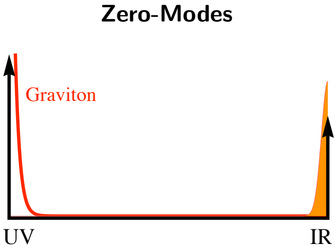

"Zero-modes" play a special role. They should correspond to the observed particles!

Connection to 4D Strong Dynamics

There is a second rewriting of the 5D theory as a 4D theory, the holographic approach:

- One treats the UV brane in a special way. By separating the bulk d.o.f. from the values of the fields on the UV brane, one can identify two sectors:
  1. The bulk d.o.f. can be identified with the states of the 4D strongly interacting sector.
  2. The (UV) brane d.o.f. can be identified as the "elementary" fields, which do not participate of the strong dynamics.
- The Kaluza-Klein modes are then understood as admixtures of the elementary and composite d.o.f. This realizes the paradigm of "partial compositeness", first proposed by D.B.Kaplan (1991).

The dictionary can be made very concrete. For further details see:

- T. Gherghetta, "TASI Lectures on a Holographic View of Beyond the Standard Model Physics," arXiv:1008.2570
- E. Pontón, "TASI 2011: Four Lectures on TeV Scale Extra Dimensions," arXiv:1207.3827

Connection to 4D Strong Dynamics

The previous comments are part of a general dictionary that allows to interpret 5D constructions in terms of 4D language:

- UV localized modes are understood to be mostly elementary.
- IR localized fields are understood to be mostly composite: hence a solution to the hierarchy problem, with the Higgs localized near the IR brane, maps into a composite Higgs scenario.
- KK modes are the physical resonances. These are almost pure resonances of the 4D strong sector, but they have a small admixture of the elementary sector.
- Bulk gauge symmetries correspond to *global* symmetries of the 4D strongly coupled sector. Hence, if you are interested in a global symmetry like $SO(5)$ you can focus on an $SO(5)$ gauge symmetry in 5D (Minimal Composite Higgs Models, MCHM$_X$).

The dictionary can be made very concrete. For further details see:

- T. Gherghetta, “TASI Lectures on a Holographic View of Beyond the Standard Model Physics,” arXiv:1008.2570
- E. Pontón, “TASI 2011: Four Lectures on TeV Scale Extra Dimensions,” arXiv:1207.3827

Outlook

What can we expect in these scenarios?

- The most striking feature would be the appearance of a tower of resonances. These can be interpreted either as a KK tower or simply as a tower of bound states ("mesons" and "baryons") of a strongly coupled theory.
- Current bounds from EW Precision Tests indicate that such resonances cannot be lighter than a few TeV. LHC searches for KK resonances ("bulk RS Model") are not yet competitive, but nevertheless provide important information.
- Due to the rather heavy states, the Higgs properties need not deviate by huge amounts from the SM. However, one expects parametrically lighter fermionic resonances that could induce observable deviations. Thus, a hint of the new physics may be revealed by future Higgs precision studies. The nature of the deviations can hold clues about the properties of the microscopic d.o.f.

End of Third Lecture

Thank you!