BFKL Phenomenology

Grigorios Chachamis, IFT (UAM-CSIC)
Madrid

New Trends in High Energy Physics and QCD, Natal, Brazil
October, 2014

What about these?

- BFKL
- Pomeron (soft/hard)
- Regge limit
- Regge trajectory
-
-
-
-
-
-
-
-

Or these?

- BFKL
- Pomeron (soft/hard)
- Regge limit
- Regge trajectory
- Intercept
- Gluon Green's function
- Impact factor
-
-
-
-
-

What about that?

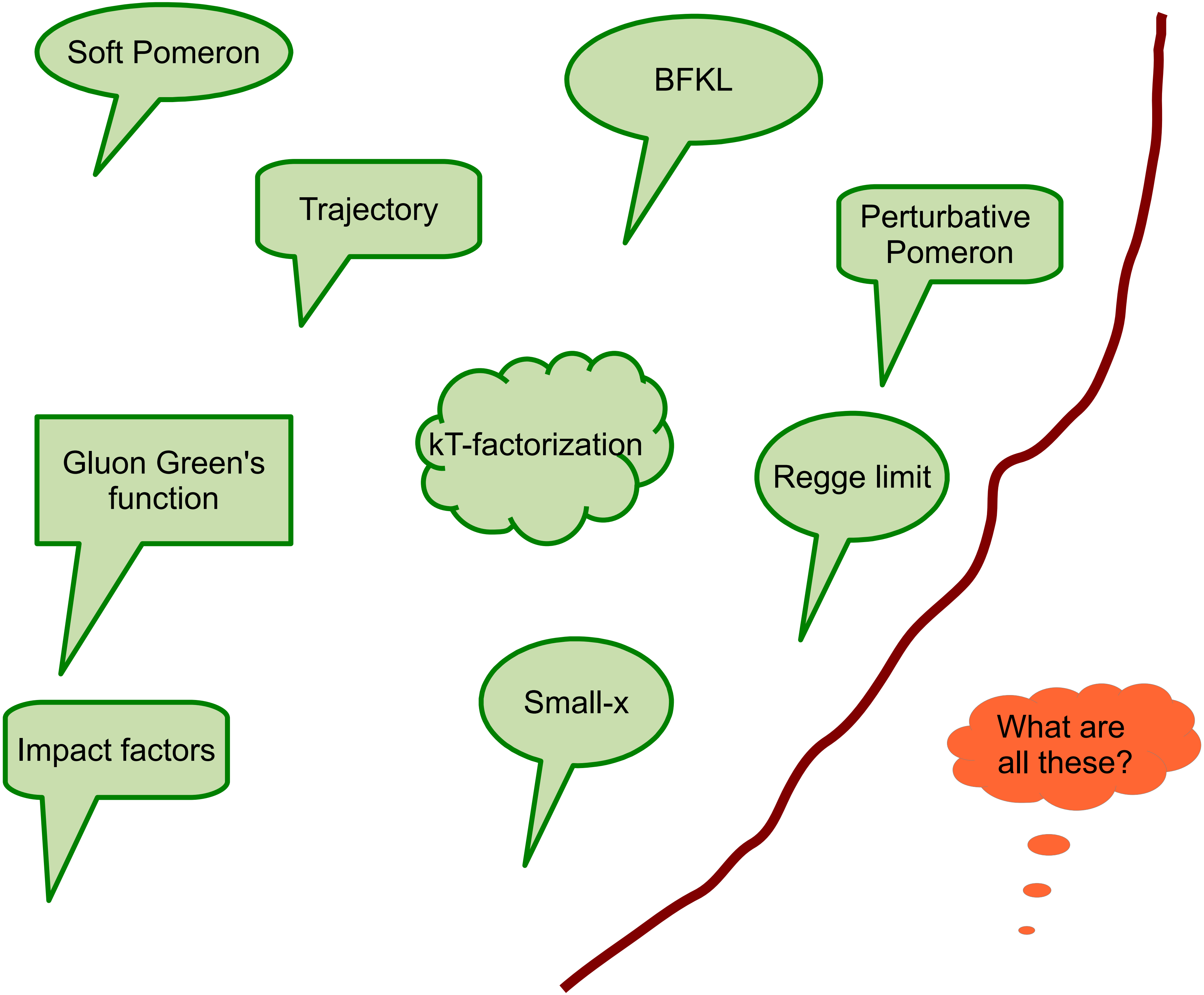
- BFKL
- Pomeron (soft/hard)
- Regge limit
- Regge trajectory
- Intercept
- Gluon Green's function
- Impact factor
- Fixed order calculations VS resummation
-
-
-
-

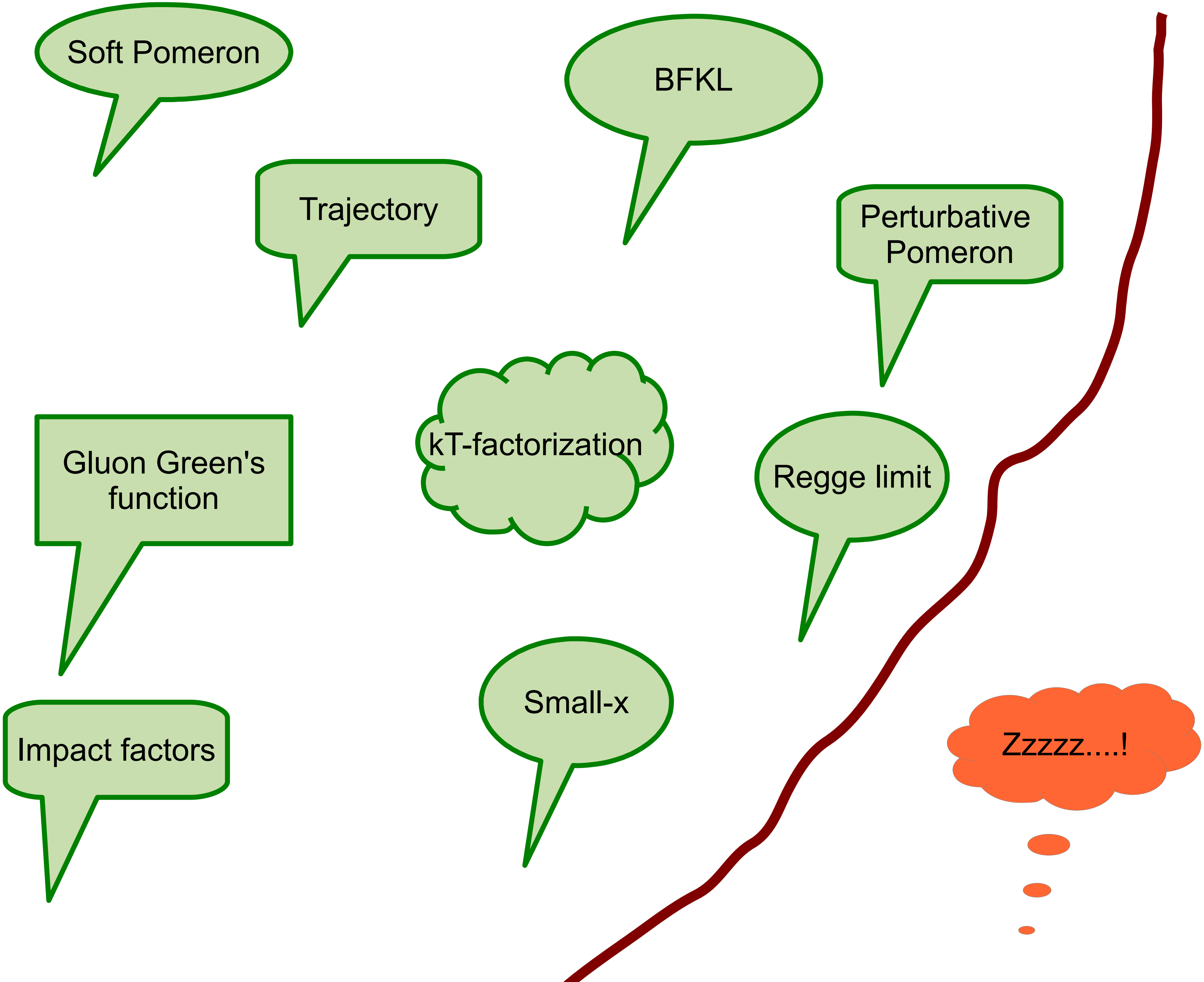
Maybe these?

- BFKL
- Pomeron (soft/hard)
- Regge limit
- Regge trajectory
- Intercept
- Gluon Green's function
- Impact factor
- Fixed order calculations VS resummation
- S-Matrix
- Sudakov variables
- Optical theorem
- Unitarity

OK

- BFKL
- Pomeron (soft/hard)
- Regge limit
- Regge trajectory
- Intercept
- Gluon Green's function
- Impact factor
- Fixed order calculations VS resummation
- S-Matrix
- Sudakov variables
- Optical theorem
- Unitarity


Soft Pomeron
BFKL
Trajectory
Perturbative Pomeron
kT-factorization
Gluon Green's function
Regge limit
Small-x
Impact factors
What are all these?



Soft Pomeron
BFKL
Trajectory
Perturbative Pomeron
kT-factorization
Gluon Green's function
Regge limit
Small-x
Impact factors
Zzzzz....!

Many people complain that it is not easy to follow a small-x physics talk. Well,...

- It is something different to the usual collinear factorization approach.
- The field has lots of internal conventions.
- Particular methods-notation-jargon.
- Different subfields in small-x: hard core BFKL, dipole picture, light cone picture, BK, Color Glass Condensate, saturation...
- Many times, speakers or authors in works on small-x, assume (the present speaker not excluded) that everybody else shares their knowledge on the field.

Why bother at all?

- Small-x physics studies only a part of the phase space, a certain limit, the limit of scattering at very high energies
- There is a plethora or things though to be learnt from studying that limit, to mention but a few:

 - Integrability

 - Gravity

 - AdS/CFT

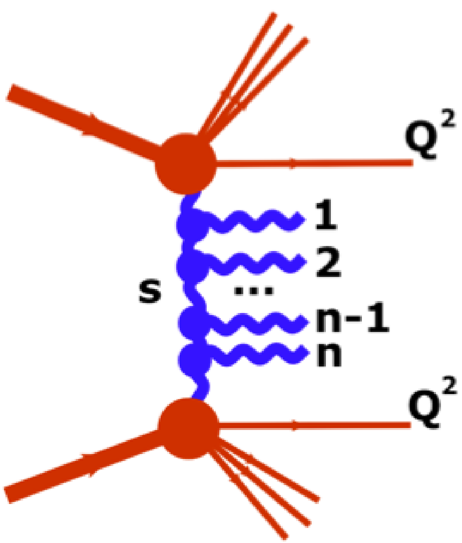
 - BDS amplitudes

- And this is only from the 'pure' theory point of view

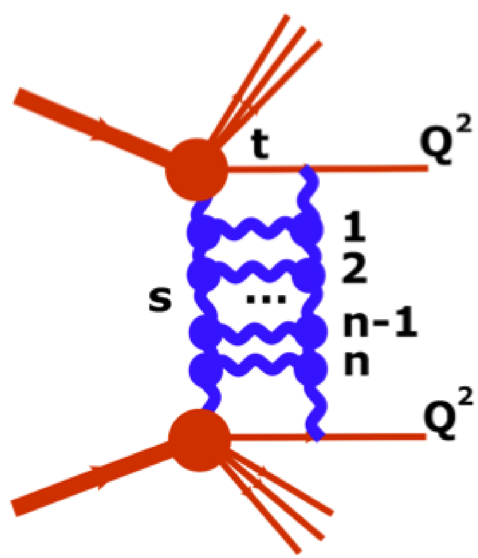
Why bother at all?

Rich Phenomenology:

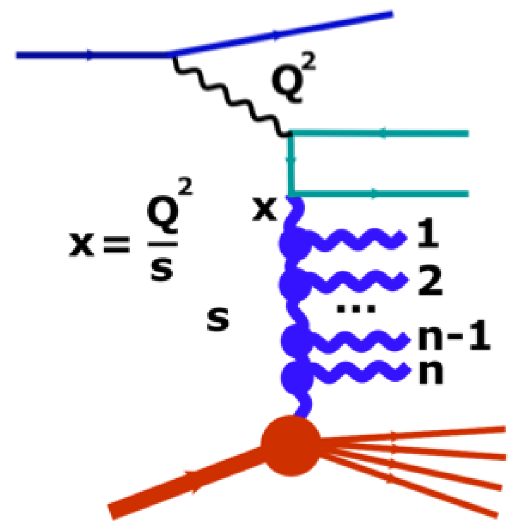
Multijets



Rapidity Gaps



Deep Inelastic Scattering



For these we obtain $\sigma \simeq s^{0.3}$

Hadron-Hadron (Soft Pomeron is Non-Perturbative): $\sigma \simeq s^{0.08}$

We have calculated a Perturbative or Hard (BFKL) Pomeron.

And now, for the time being, forget that you've heard about BFKL, Pomeron

List of prerequisites

- S-matrix approach
- Cutkosky rules / Optical theorem (used a lot in the Regge theory but also in BFKL)
- Sudakov parametrization
- Again, which is the kinematical limit we are (which part of the phase space)? Can we picture it? **Regge limit**

arXiv:1410.4746 [pdf, other]

Three-point correlators from string amplitudes: Mixing and Regge spins
Joseph A. Minahan, Raul Pereira
Comments: 24 pages, 1 figure
Subjects: **High Energy Physics - Theory (hep-th)**

arXiv:1407.1394 [pdf, ps, other]

Regge Quantum Gravity Solution to the Cosmological Constant Problem
Aleksandar Mikovic, Marko Vojinovic
Comments: arXiv admin note: substantial text overlap with arXiv:1407.1124
Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; High Energy Physics - Phenomenology (hep-ph); Hig

arXiv:1407.1124 [pdf, ps, other]

Cosmological Constant in a Regge State-sum Model of Quantum Gravity
Aleksandar Mikovic, Marko Vojinovic
Comments: 16 pages, typos corrected
Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; High Energy Physics - Theory (hep-th)

arXiv:1406.1285 [pdf, ps, other]

The Appell Function F_1 and Regge String Scattering Amplitudes
Jen-Chi Lee, Yi Yang
Comments: 12 pages,no figure
Subjects: **High Energy Physics - Theory (hep-th)**

arXiv:1405.3658 [pdf, ps, other]

Heptagon Amplitude in the Multi-Regge Regime
J. Bartels, V. Schomerus, M. Sprenger
Comments: 14 pages
Subjects: **High Energy Physics - Theory (hep-th)**

hep-th

arXiv:1405.1731 [pdf, ps, other]

Regge behavior in effective field theory
John F. Donoghue, Basem Kamal El-Menoufi, Grigory Ovanesyan
Comments: 18 pages, 6 figures
Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Theory (hep-th); Nuclear Theory

arXiv:1404.6506 [pdf, other]

Wilson loop OPE, analytic continuation and multi-Regge limit
Yasuyuki Hatsuda
Comments: 36 pages, 5 figures, v2: references added
Subjects: **High Energy Physics - Theory (hep-th)**

arXiv:1407.1655 [pdf, ps, other]

The concept of phenomenological light-front wave functions -- Regge improved diquark model predictions

Dieter Müller, Dae Sung Hwang

Comments: 104 pages, 7 figures, 2 tables

Subjects: **High Energy Physics - Phenomenology (hep-ph)**

arXiv:1407.1394 [pdf, ps, other]

Regge Quantum Gravity Solution to the Cosmological Constant Problem

Aleksandar Mikovic, Marko Vojinovic

Comments: arXiv admin note: substantial text overlap with arXiv:1407.1124

Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; High Energy Physics - Phenomenology (hep-ph); High Energy Physics - Theory (hep-th)

arXiv:1406.4370 [pdf, other]

Mass spectra and Regge trajectories of light mesons in the Bethe-Salpeter approach

Christian S. Fischer, Stanislav Kubrak, Richard Williams

Comments: 11 pages, 8 figures, 2 tables. Minor typos corrected; accepted in EPJA

Subjects: **High Energy Physics - Phenomenology (hep-ph)**; Nuclear Theory (nucl-th)

hep-ph

arXiv:1405.1731 [pdf, ps, other]

Regge behavior in effective field theory

John F. Donoghue, Basem Kamal El-Menoufi, Grigory Ovanesyan

Comments: 18 pages, 6 figures

Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Theory (hep-th); Nuclear Theory (nucl-th)

arXiv:1404.6058 [pdf, ps, other]

The non-ordinary Regge behavior of the $f_0(500)$ meson

J.R.Peláez, J.T.Londergan, J. Nebreda, A.P.Szczepaniak

Comments: 6 pages. To appear in the proceedings of the "Excited QCD 2014", Bjelasnica Mountain, Sarajevo, Bosnia-Herzegovina, 2-8 February,2014

Subjects: **High Energy Physics - Phenomenology (hep-ph)**; Nuclear Theory (nucl-th)

arXiv:1403.4454 [pdf, ps, other]

Past, Present, and Future Multi-Regge Theory

Alan R. White

Subjects: **High Energy Physics - Phenomenology (hep-ph)**

arXiv:1403.3988 [pdf, ps, other]

Non-ordinary nature of the f_0(500) resonance from its Regge trajectory

J. Nebreda, J. T. Londergan, J. R. Pelaez, A. P. Szczepaniak

Comments: 5 pages, 2 figures, to appear in the proceedings of the "XV International Conference on Hadron Spectroscopy-Hadron 2013". arXiv admin note: substa
with arXiv:1403.2790

Subjects: **High Energy Physics - Phenomenology (hep-ph)**

No worries! We will have to go through some important things of small-x physics first

- Small-x physics is not only BFKL, BFKL physics is in its essence small-x physics.
- BFKL though is essential if we want to understand things related to small-x physics (things, for example, that Edmond was discussing in his lectures).
- Moreover, in order to understand the basics of BFKL physics, one would need to go through many things that were there before (e.g. old Regge theory.
- Before we go any further in introducing the BFKL dynamics, we will take some time to review some prerequisite material and also dive back to history to make a connection to the times before QCD.

NOTE: small-x $\rightarrow$ Very high energies

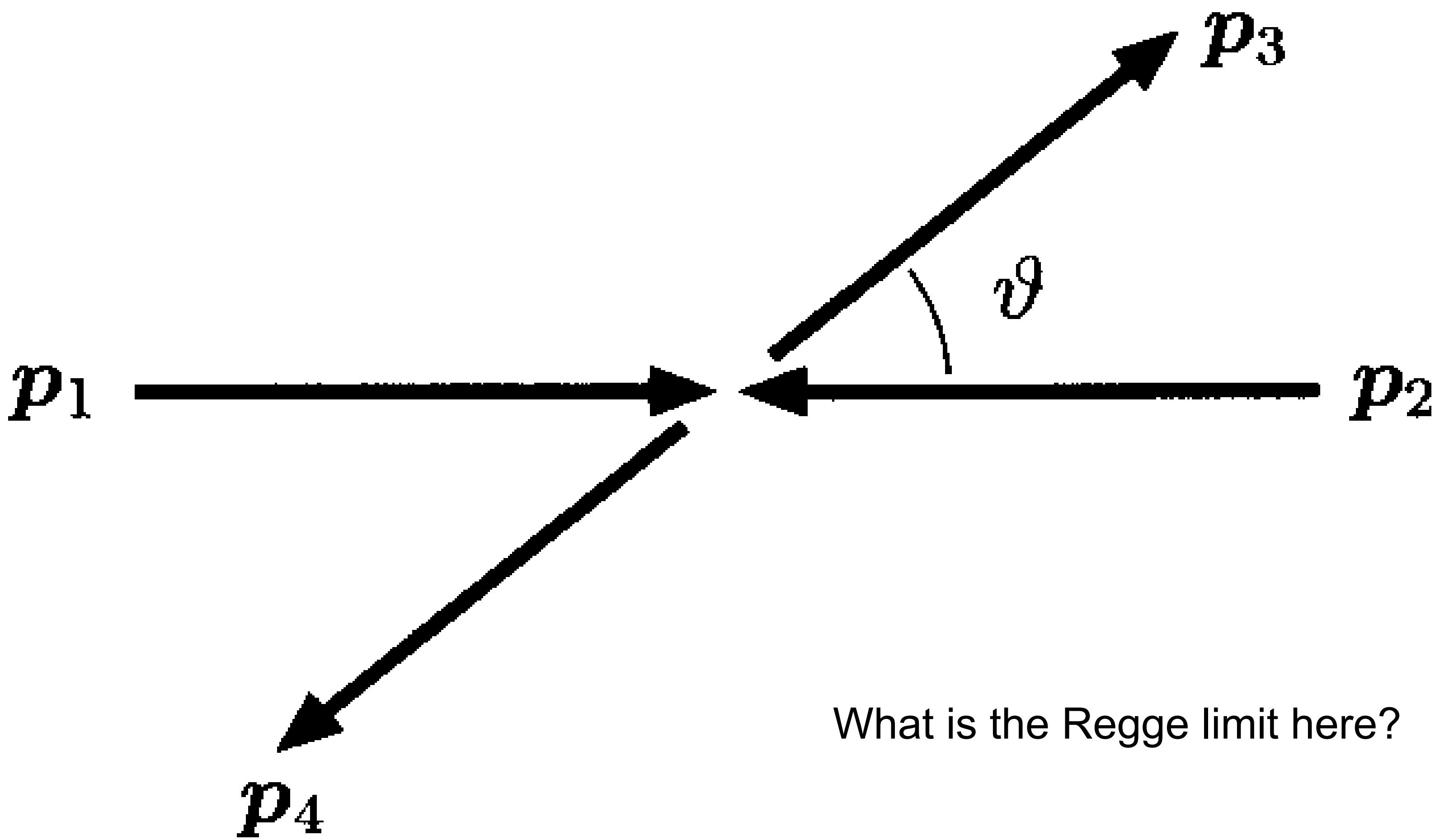
SETTING UP THE STAGE

Scattering amplitudes

The stage of the play is called scattering: take particles and smash them against each other with huge velocities

- It is the main way we have to test our theories against real life (or vice versa)
- The trend is to go to higher colliding energies to see more (probe deeper the internal structure)
- Maybe there is a better way but for the time we do not know it

A picture that should be familiar



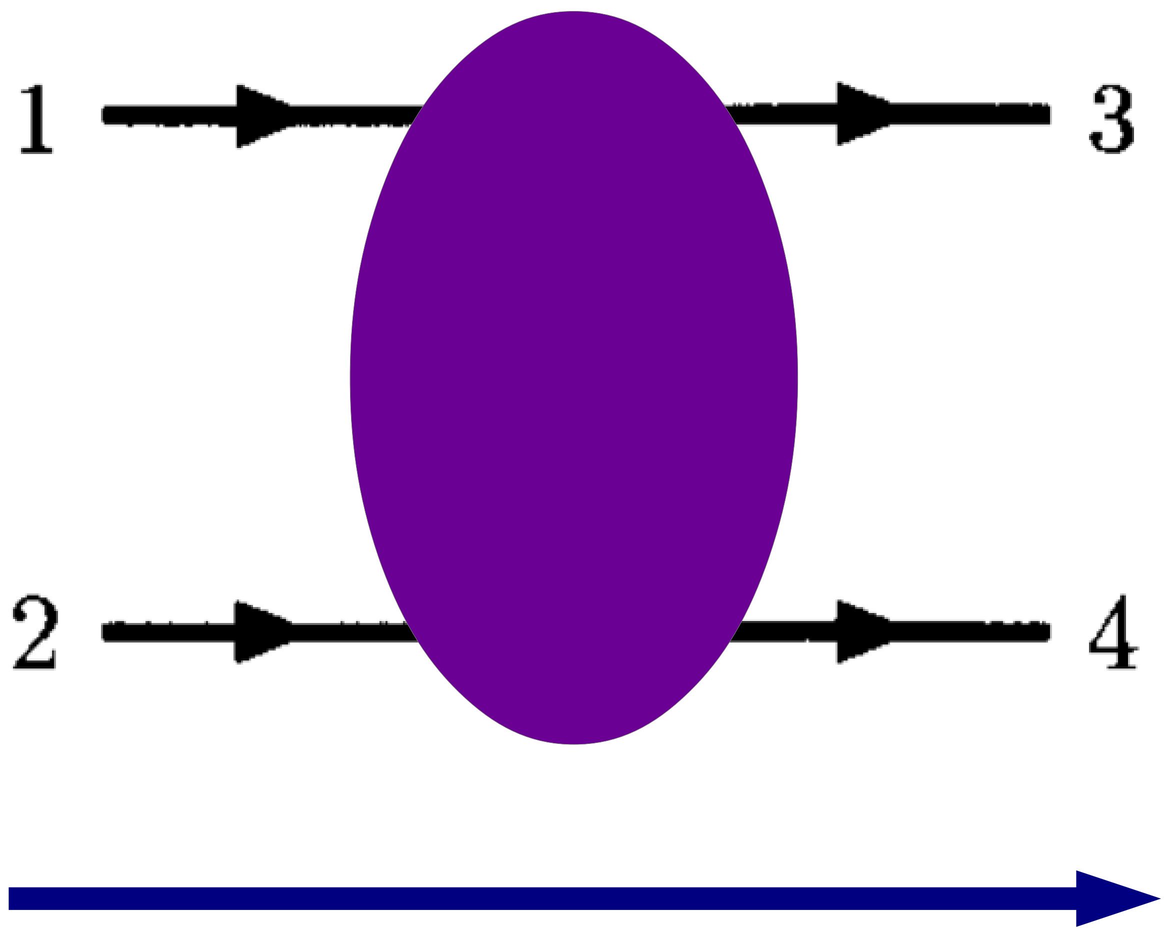
What is the Regge limit here?

Scattering in our language of Feynman Calculus?

We assume that we have a theory that tells us what is going on inside the blob.

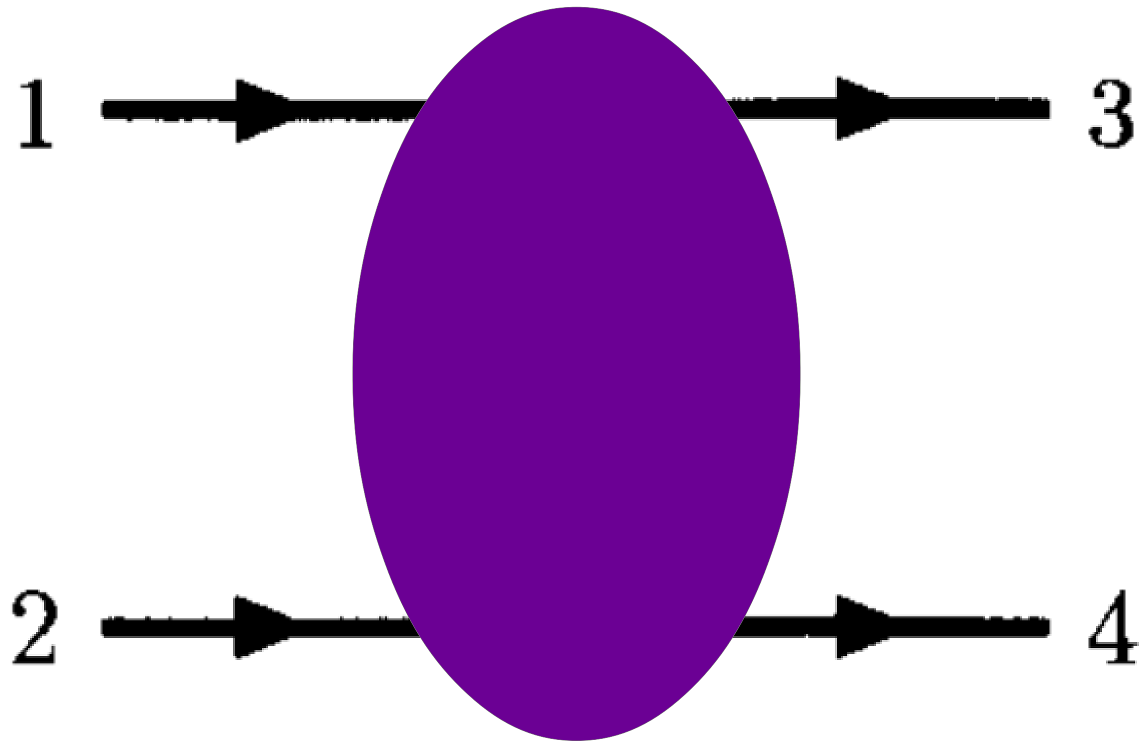
Maybe we cannot solve the theory exactly and then we should do a series expansion around a small parameter.

To compare with experimental data, we may have to consider 2, 3 or maybe more terms in our perturbative expansion



Time

s, t, u



$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \, ,$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \, ,$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \, .$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

Fixed order calculations

On the whiteboard...

PREREQUISITES

S-matrix

Before the advent of QCD, the S-matrix approach was what people used to describe hadronic interactions.

The idea was that there is a linear operator called scattering matrix or S-matrix that transforms the initial state of the particles into the final state.

$$\text{initial state } |i\rangle \xrightarrow{\quad S \quad} \text{final state } |f\rangle$$

$$S\,|i\rangle = |f\rangle$$

$$P_{i\to f} = |\langle f|S|i\rangle|^2$$

Clearly, knowing the S-matrix would allow the description of the scattering process.

S-matrix

Postulate 1: The S-matrix is Lorentz invariant

$$a + b \rightarrow c + d$$

2 → 2 scattering process

$$\begin{aligned} s &= (p_a + p_b)^2 \\ t &= (p_a - p_c)^2 \\ u &= (p_a - p_d)^2 \end{aligned}$$

Kinematical invariants

The amplitude will be a function of s and t (and of the masses of the particles)

S-matrix

Postulate 2: The S-matrix is unitary

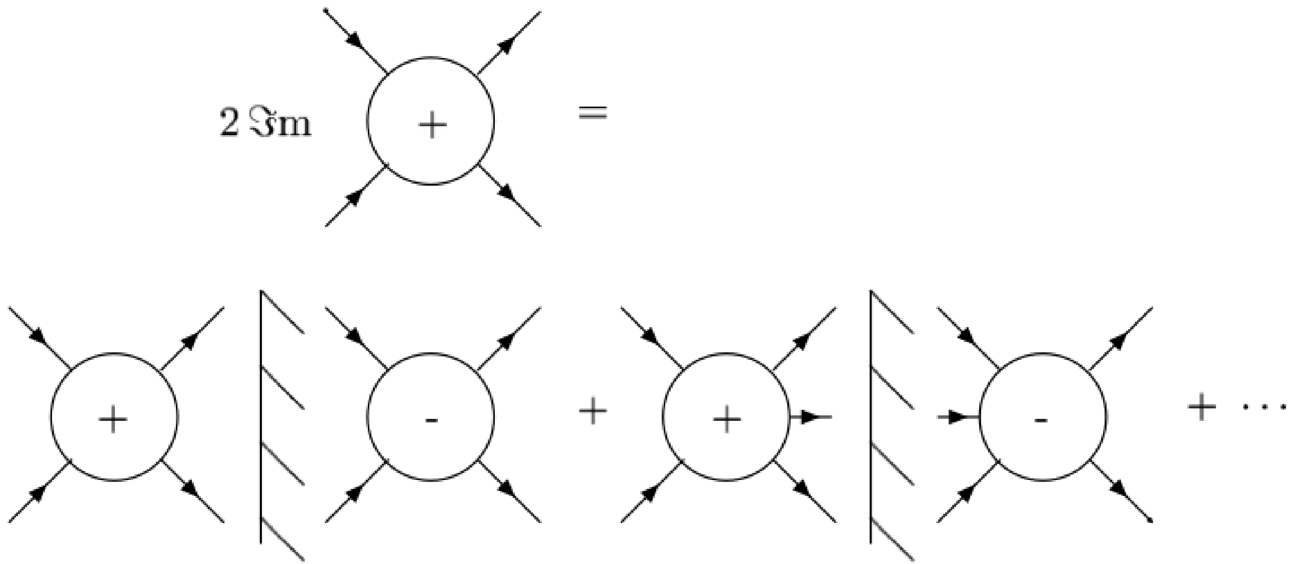
$$\sum_k P_{i\to k} = \sum_k |\langle k|S|i\rangle|^2$$
$$= \sum_k \langle i|S^\dagger|k\rangle\langle k|S|i\rangle$$
$$= \langle i|S^\dagger S|i\rangle = 1\ ,$$

$$SS^\dagger = S^\dagger S = \mathbb{1}$$

A statement of conservation of probability: The probability of an initial state to end up in a particular final state summed over all possible final states has to be unity.

A small digression: the Cutkosky rules

$$| a\rangle \rightarrow | b\rangle \qquad S_{ab} = \delta_{ab} + i(2\pi)^4\delta^4\left(\sum_a p_a - \sum_b p_b\right)\mathcal{A}_{ab}$$



$$2\Im\mathrm{m}\,\mathcal{A}_{ab} = (2\pi)^4\delta^4\left(\sum_a p_a - \sum_b p_b\right)\sum_c \mathcal{A}_{ac}\mathcal{A}^{\dagger}_{cb}$$

A small digression: … and the optical theorem

$$2\Im\mathrm{m}\,\mathcal{A}_{ab} = (2\pi)^4\delta^4\left(\sum_a p_a - \sum_b p_b\right)\sum_c \mathcal{A}_{ac}\mathcal{A}^{\dagger}_{cb}$$

Forward scattering,
in state |α> = out state |b>

$$2\Im\mathrm{m}\mathcal{A}_{aa}(s,0) = (2\pi)^4\sum_n \delta^4\left(\sum_f p_f - \sum_a p_a\right)|\mathcal{A}_{a\to n}|^2 = F\,\sigma_{\mathrm{tot}}$$

S-matrix

Postulate 3: The analyticity of S-matrix

The S-matrix is an analytic function of Lorentz invariants (these seen as complex variables) with only those singularities as required by unitarity.

This postulate is a consequence of causality.

Another property of the S-matrix, namely the crossing symmetry is a result of the analyticity.

Sudakov parametrization

$$A^\mu = (A^0, A^1, A^2, A^3) = (A^0, \boldsymbol{A}_\perp, A^3) = (A^0, \boldsymbol{A})$$

$$A^\pm = \frac{1}{\sqrt{2}} (A^0 \pm A^3)$$

$$A^\mu = (A^+, A^-, \boldsymbol{A}_\perp)$$

Light-cone components

$$p^\mu = \frac{1}{\sqrt{2}} (\Lambda, 0, 0, \Lambda),$$
$$n^\mu = \frac{1}{\sqrt{2}} (\Lambda^{-1}, 0, 0, -\Lambda^{-1})$$

$$p^2 = n^2 = 0, \quad p \cdot n = 1, \quad n^+ = p^- = 0$$

$$\begin{aligned} A^\mu &= \alpha\, p^\mu + \beta\, n^\mu + A_\perp^\mu \\ &= (A \cdot n)\, p^\mu + (A \cdot p)\, n^\mu + A_\perp^\mu \end{aligned}$$

$$A^2 = 2\,\alpha\beta - \boldsymbol{A}_\perp^2$$

BACK TO “BEFORE QCD”

Let us add a historical perspective

- The problem of scattering amplitudes
- Old Regge theory - soft Pomeron (60's, 70's)
- Advent of pQCD - mid 70's achievements – BFKL and the perturbative Pomeron
- HERA @ DESY: input from the experimental side / boom in the field of small-x physics (90's)
- Collinear / k_T-factorization scheme (90's)
- Connection of BFKL dynamics with other fields (last two decades but mainly the last few years)
- Phenomenology at the LHC (recent) and linear colliders (since much earlier)

On Phenomenology

- Philosophy

Phenomenology (from Greek: phenomenon = "that which appears" and logos = "study") is the philosophical study of the structures of subjective experience and consciousness.

- Particle Physics (our familiar QCD phenomenology, for example)

Use assumed fundamental laws to produce theoretical estimates for physical observables and then compare against experimental data to validate or falsify the assumed laws

- Science in general

Observe "that which appears", a collection of phenomena that share a unifying principle, and try to find patterns to describe it. The patterns might or might not be of fundamental nature or they might be up to a certain degree.

THE REGGE THEORY

The old Regge* theory

- Regge theory was the alternative to QFT for describing strong interactions in the 60's, 70's
- **Reggeon, Regge pole, Regge trajectory** were **really** familiar phrases

"Indeed, Regge theory has been included in good undergraduate textbooks for more than a decade"

E. Leader, Nature, 1978

"In no time at all, Regge pole (pole in the mathematical sense) had become a household expression. Indeed, there is the story of a party at which the charming wife of an American physicist, on being introduced to 'Regge', exclaimed:
<< A Mr. Pole, I'm so pleased to meet you at last >>"

E. Leader, Nature, 1978

*Tulio Regge, Italian physicist, 1931

The old Regge theory

- The main objects under study are scattering amplitudes
- One can use (familiar from Quantum Mechanics) the "partial wave expansion":

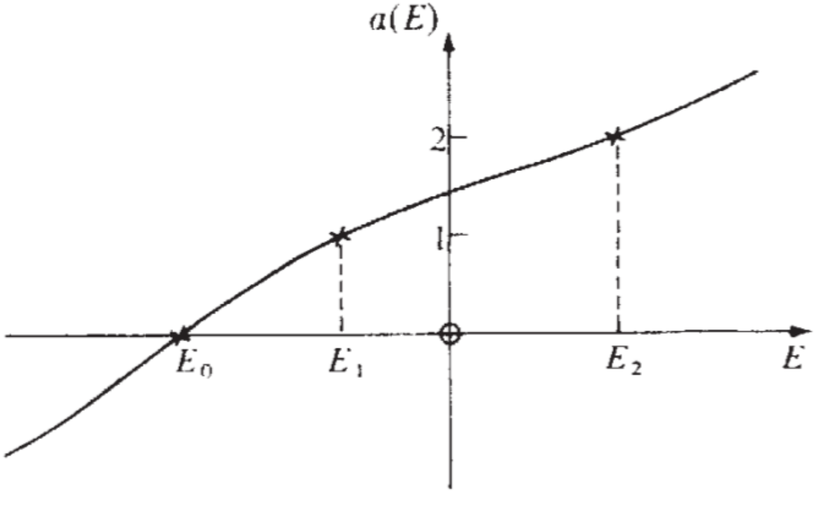
$$\mathcal{A}_{ab\to cd}(s,t) = \sum_{l=0}^{\infty}(2l+1)\, a_l(t)\, P_l\,(1+2s/t)$$

- Regge's idea: complexify energy and angular momentum and focus on the function $\alpha_{(l)}(t)$

Note that s, t and u are the Mandelstam variables and cos(θ) = 1 + 2 t/s

The old Regge theory

- The function $\alpha_{(l)}(t)$, the trajectory function, has the property that if an energy, E, exists such that $\alpha(E)$ is a positive integer L, then a bound state would exist at energy E with angular momentum, *l*=L



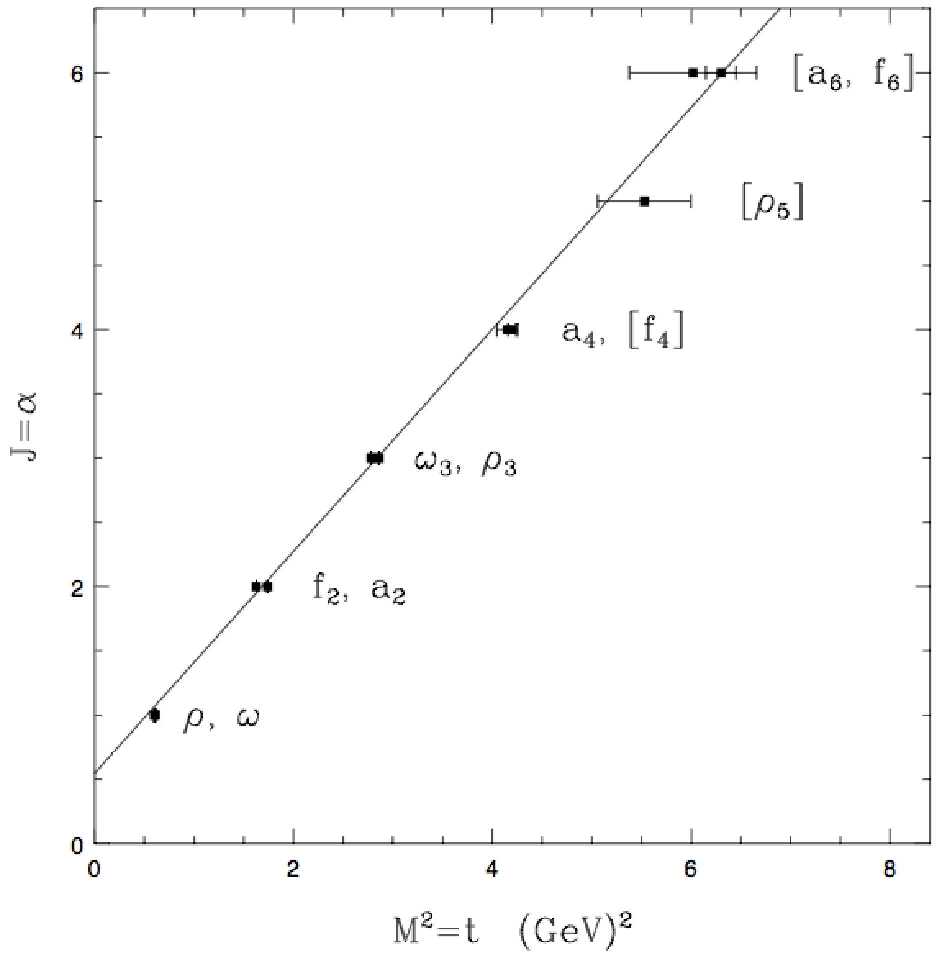
The old Regge theory

Chew and Frautschi ('61, '62) plotted the spins of low lying mesons against mass squared and noticed that they lie in a straight line.

$$\alpha(t) = \alpha(0) + \alpha' t$$

$$\begin{aligned} \alpha(0) &= 0.55 \\ \alpha' &= 0.86 \text{ GeV}^{-2} \end{aligned}$$

$$\frac{d\sigma}{dt} \propto s^{(2\alpha(0) - 2\alpha' t - 2)}$$



How to use this fact?

The old Regge theory

- First, find a hadronic process that can be "characterized" by the particles that lie in the trajectory.
- Second, extrapolate the trajectory to negative values of t by using

$$\begin{aligned} \alpha(0) &= 0.55 \\ \alpha' &= 0.86\ \mathrm{GeV}^{-2} \end{aligned}$$

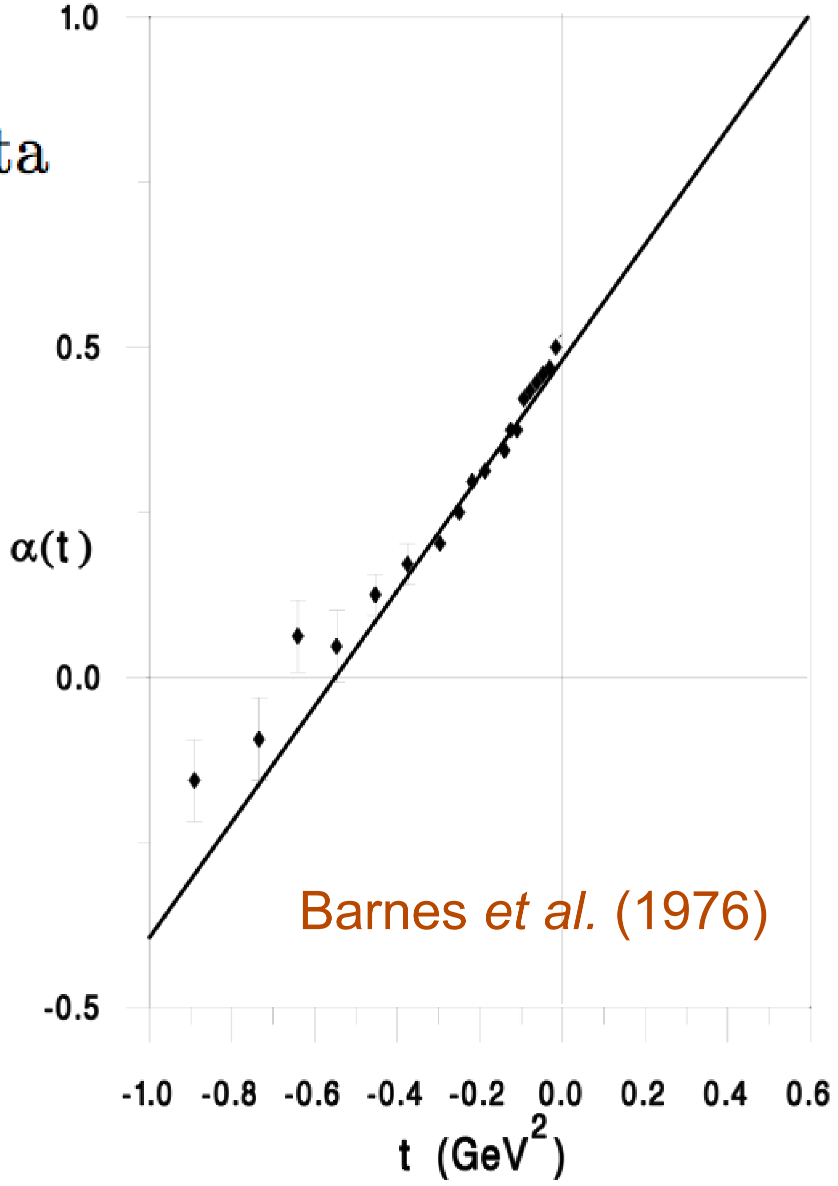
into $\frac{d\sigma}{dt} \propto s^{(2\alpha(0)-2\alpha' t-2)}$

- Plot both the extrapolated trajectory and the experimental data

The old Regge theory

$\alpha(t)$ obtained from $\pi^- p \to \pi^0 n$ data

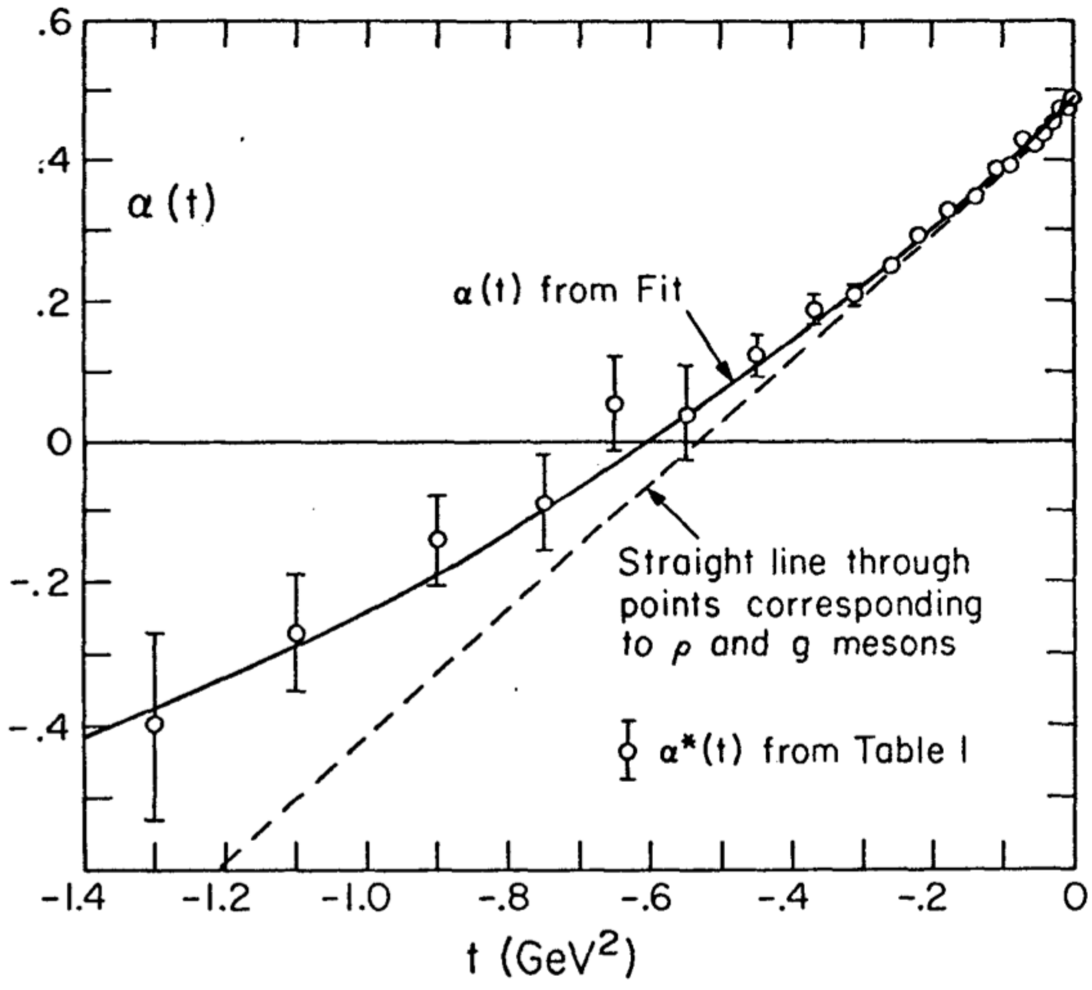
The interaction is mediated by the "exchange of a trajectory".



The old Regge theory

$\alpha(t)$ obtained from $\pi^- p \to \pi^0 n$ data

Barnes *et al.* (1976)



Actually, in the paper, they offer an effective trajectory, plotted with the continuous line whereas the dashed curve is the continuation of the ρ meson trajectory.

Toward the (soft) Pomeron

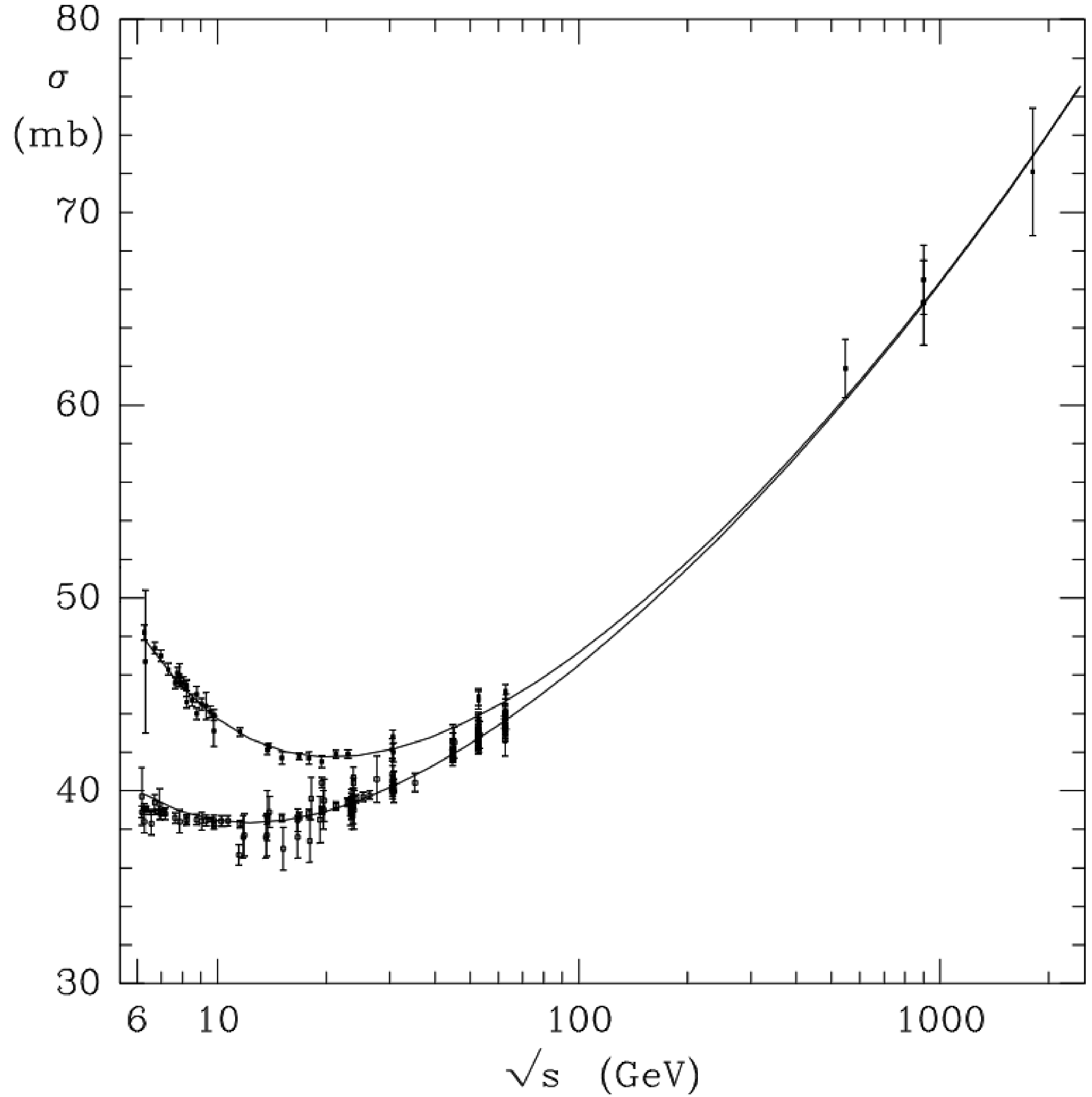
$$\sigma_{\text{tot}} \propto s^{(\alpha(0)-1)}$$

- The asymptotic behavior of the total cross section for a process can be obtained by the intercept of the Regge trajectory that dominates that process.
- Pomeranchuk and Okun &Pomeranchuk (1956) proved that if in a process there is charge exchange, then the cross section vanishes asymptotically.
- Pomeranchuk theorem: $\sigma_{\text{tot}}(ab) \underset{s\to\infty}{\simeq} \sigma_{\text{tot}}(a\bar{b})$
- Froissart-Martin bound: $\sigma_{\text{tot}} \leq C \ln^2 s \,, \quad \text{as } s \to \infty$

Toward the (soft) Pomeron

$$\sigma_{\text{tot}} \propto s^{(\alpha(0)-1)}$$

But the total cross sections do rise with energy! This is not compatible with an exponent smaller than 1.



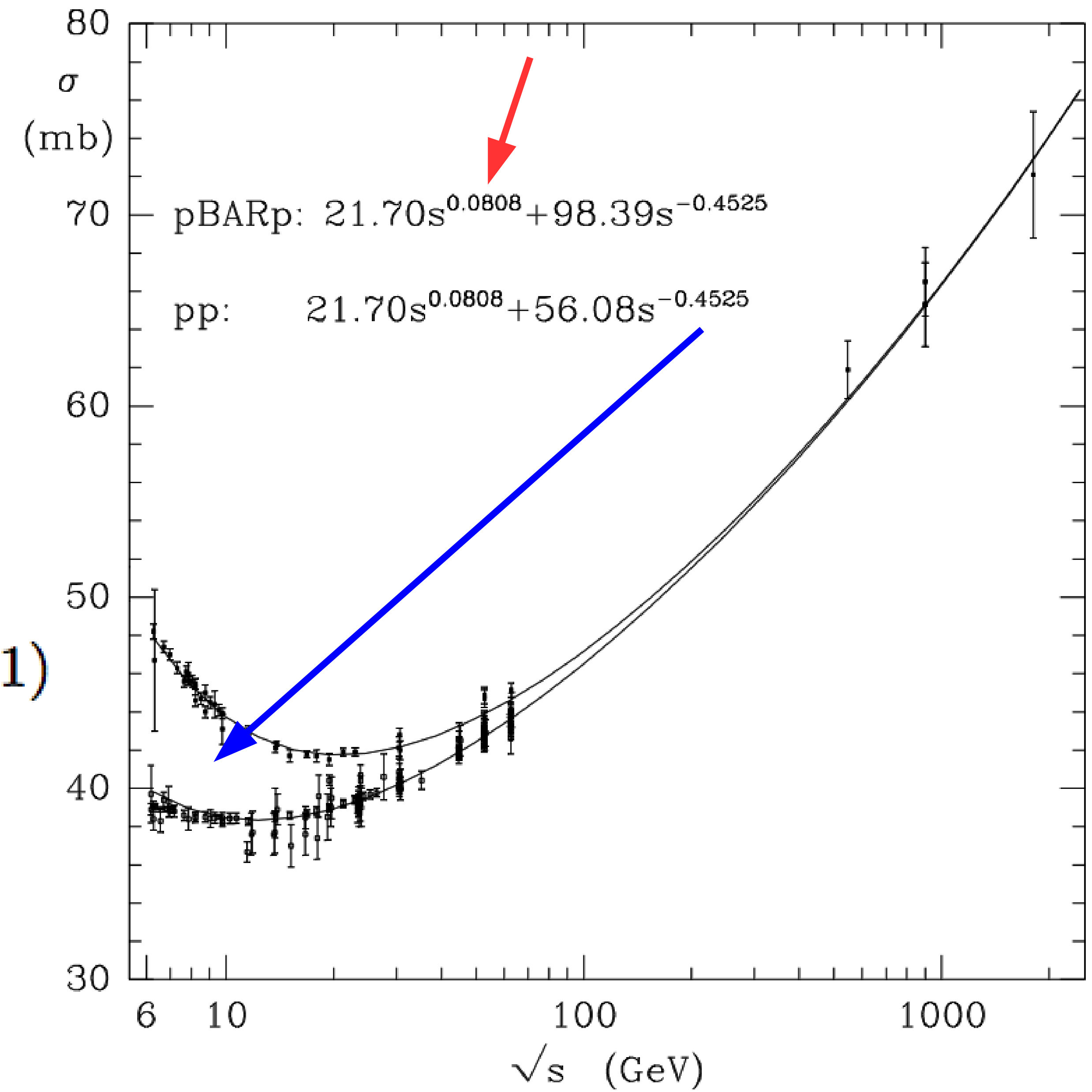
The Pomeron

- Gribov introduced (1961) a Regge trajectory with intercept 1: the Pomeron (named after Pomeranchuk)
- It does not correspond to any known particle (glueballs?)
- It carries the quantum numbers of the vacuum, C-even, P-even, Charge 0, Isospin 0.
- Intercept consistent with fits (~1.08)

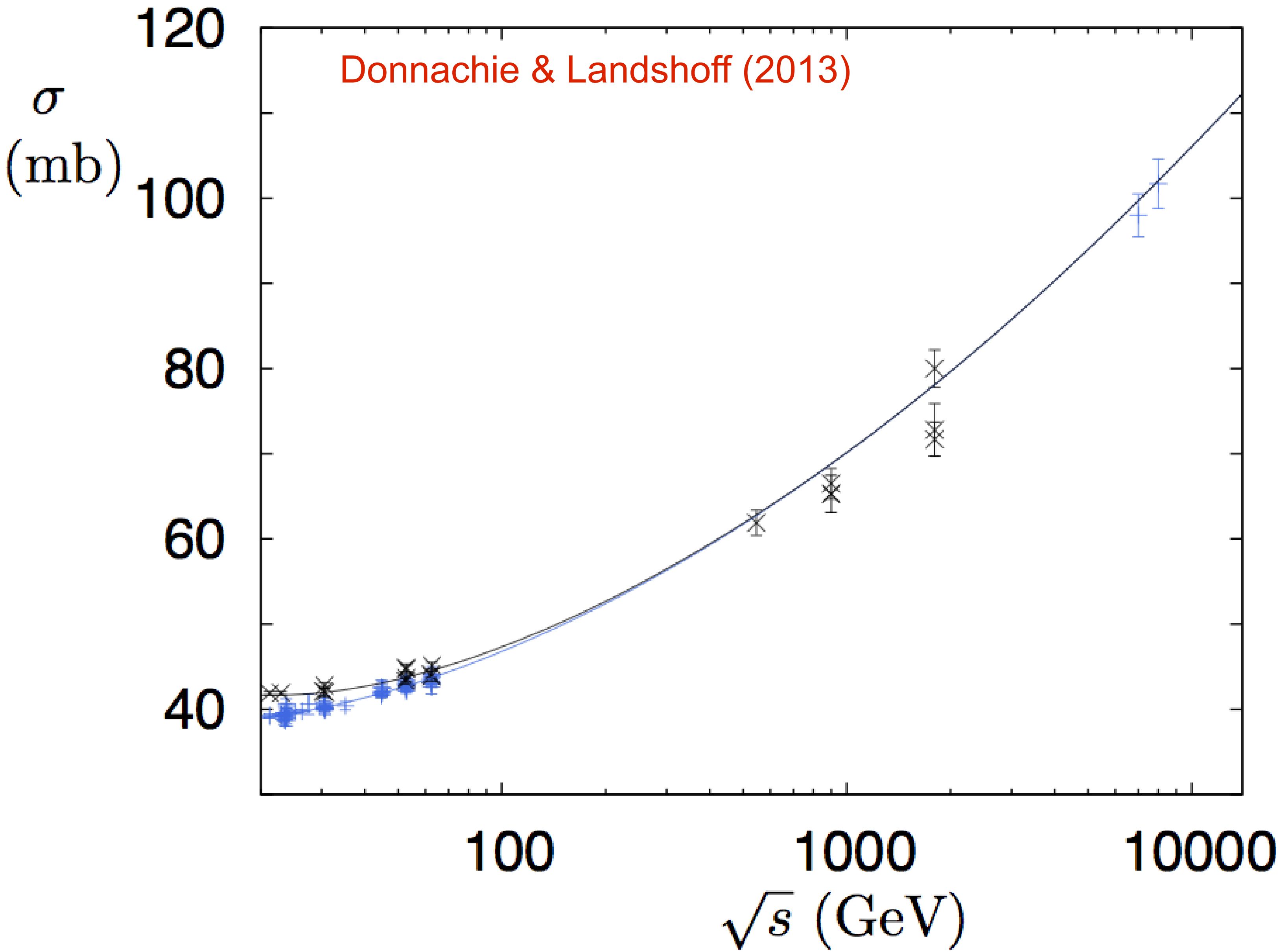
Toward the (soft) Pomeron

Fitting proton-proton & proton-antiproton scattering data.
NOTE: the parameters of the fit were determined before the measurements at Tevatron by using data below 100 c.o.m energy

$$\sigma_{\text{tot}} \propto s^{(\alpha(0)-1)}$$



Donnachie & Landshoff (1992)


Donnachie & Landshoff (2013)
σ
(mb)
120
100
80
60
40
100
1000
10000
√s (GeV)

Bibliography (not complete)

- P.D.B. Collins, *An introduction to Regge theory and high energy physics*, 1977.
- V.N. Gribov, *The Theory of Complex Angular Momenta*, 2003
- V.N. Gribov, *Strong Interactions of Hadrons at High Energies*, 2009

- M. Baker and K.A. Ter-Martirosyan, *Gribov's Reggeon Calculus: Its physical basis and implications*, Phys. Rep. C**28** (1976) 1.
- K. G. Boreskov, A. B. Kaidalov, and O. V. Kancheli, *Strong Interactions at High Energies in the Reggeon Approach*, Phys. Atomic Nuclei, 69 (2006) 1765.
- L.L. Jenkovszky, *High-energy Elastic Hadron Scattering*, Riv. Nuovo Cim. 10 (1987) 1.
- A.B.Kaidalov. *Regge poles in QCD*. arXiv:hep-ph/0103011.
- A.B.Kaidalov. *Pomeranchuk singularity and high- energy hadronic interactions*, Usp. Fiz. Nauk, 46 (2003) 1153 .
- E.M. Levin, *Everything about Reggeons*, arXiv:hep-ph/9710546.

See also a nice talk by Poghosyan: ***An introduction to Regge Field Theory***