Summer school and workshop on high energy physics at the LHC

Phenomenology and experimental aspects of SUSY and searches for extradimensions at the LHC

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Tentative outline

LECTURE 1

- Motivations and short introduction to supersymmetry (SUSY)

LECTURE 2

- minimal SUSY extensions of the standard model (SM) phenomenology
  Higgs sector, sfermions and gauginos

- direct search examples at the LHC + some indirect constraints

LECTURE 3

- dark matter

LECTURE 4

- extra dimensions searches at the LHC

Tentative outline

LECTURE 1

- Motivations and short introduction to supersymmetry (SUSY)

Motivations and short introduction to Supersymmetry

- can address the naturalness issue of the standard model (SM)

- UNIQUE extension of the Poincaré group i.e. space-time symmetry (Coleman Mandula theorem, Haag Lopuszanski Sohnius)

- bringing gravitation into the game i.e. supergravity

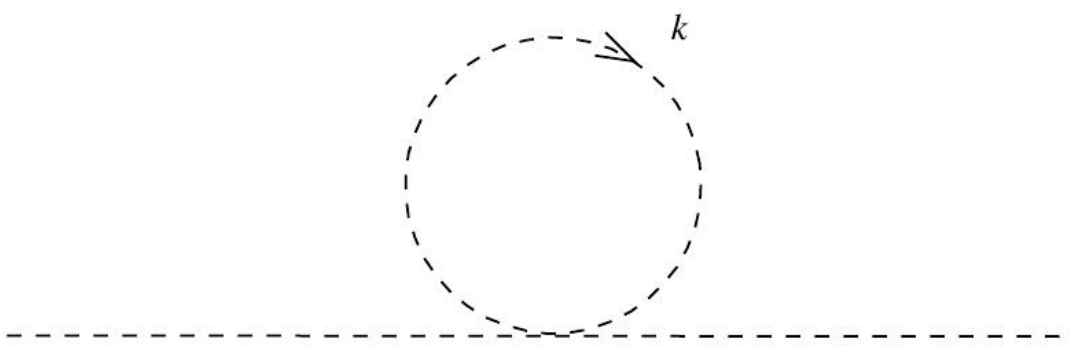
- gauge interactions unification

- dark matter candidates

- essential role in string theories

Naturalness

presence of fundamental scalar fields leads to the problem of quadratic corrections



$$\delta m^2 \propto \lambda \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2}$$

$$\lambda \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \rightarrow \text{ if new scale } \Lambda \rightarrow \lambda \int^{\Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \sim \frac{\lambda}{16\pi^2} \int^{\Lambda} dk^2$$

giving a contribution of the order to $\lambda \Lambda^2 / 16\pi^2$ to the scalar mass squared m^2

problematic if new scale if close to GUT scale or Planck Scale

Naturalness

more precisely, the SM Higgs boson receives the following 1-loop correction

$$\delta m_H^2 = \frac{3\Lambda^2}{8\pi^2 v^2}\left[\left(4\, m_t^2 - \underbrace{2\, M_W^2 - M_Z^2}_{} - m_H^2\right) + O\left(\log\frac{\Lambda}{\mu}\right)\right]$$

contribution from top quark

contribution from gauge bosons

contribution from Higgs self coupling

vanishes if the Veltman condition $4 m_t^2 = 2\, M_W^2 + M_Z^2 + m_H^2$ is fullfilled

but Veltman condition may not hold at higher orders

defining the amount of fine tuning by $\dfrac{\delta m_H^2}{m_H^2} = \dfrac{1}{f}$ then if $\delta m_H^2 = 100\ m_H^2$

one needs to fine tune the Higgs bare mass m_o^2 to the % level in order to recover

the right physical Higgs mass m_H^2

Naturalness

supersymmetry as a solution to the problem of naturalness

relate a scalar (boson) field with a fermion field with a new symmetry

supersymmetry

$\Rightarrow$ introducing one supersymmetry (SUSY) operator ***Q*** (i.e. SUSY N = 1)

$$Q \; | \, \text{boson} > \; = \; | \, \text{fermion} >$$

$$Q \; | \, \text{fermion} > \; = \; | \, \text{boson} >$$

contribution of the fermion to quadratic divergence cancels contribution of boson

Supersymmetry algebra

we have introduced the SUSY operator ***Q*** such that

$$Q\;|\,\text{boson}> \;=\; |\,\text{fermion}>$$

$$Q\;|\,\text{fermion}> \;=\; |\,\text{boson}>$$

and also

$$\bar{Q}\;|\,\text{boson}> \;=\; |\,\text{fermion}>$$

$$\bar{Q}\;|\,\text{fermion}> \;=\; |\,\text{boson}>$$

⇒ by definition the ***Q*** changes the spin of the states

spins being related to behaviour under to spatial rotations

Q is a spinor

supersymmetry is a space-time symmetry

Question : can we have such extra space-time symmetries in addition to the conventional translations and Lorentz transformations (Poincaré group) ?

Supersymmetry algebra

answer 1 : **no** ! due to the Coleman Mandula theorem

the only conserved charges which transforms as tensors under the Lorentz group are:

- P_μ the generators of translations
- $M_{\mu\nu}$ the generators of Lorentz transformations

these generators of the Poincaré group satisfy the following algebra **(i.e. with commutators)**

$$[P_\mu, P_\nu] = 0$$

$$[P_\mu, M_{\rho\sigma}] = i(\eta_{\mu\rho} P_\sigma - \eta_{\mu\sigma} P_\rho)$$

$\eta_{\mu\nu}$ is the Minkowski metric

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} - \eta_{\mu\rho} M_{\nu\sigma} + \eta_{\mu\sigma} M_{\nu\rho})$$

answer 2 : **yes** ! if one can evade the Coleman Mandula theorem

this can be done if the spinorial generator **Q** of the new symmetry obeys **anti commutation** relations i.e. $\{\boldsymbol{Q}, \bar{\boldsymbol{Q}}\} \propto P_\mu$

being a symmetric combination of two Lorentz spinors the anticommutator $\{Q, \bar{Q}\}$ is a vector (i.e. boson) under Lorentz transformations $\Rightarrow$ proportional to P_μ (due to Coleman Mandula)

schematically put : $Q\bar{Q}\,|\text{boson}> \sim P\,|\text{boson}>$ i.e. the product of 2 generators preserves the spin of the particle but the particle is translated in spacetime

Supersymmetry algebra

thus the basic supersymmetry algebra is (2-component spinor notation r,s = 1,2)

$$\left\{Q_r, \bar{Q}_s\right\} = 2\sigma^{\mu}_{rs} P_{\mu}$$

σ^{μ} are Pauli matrices

$$\left[Q_r, P^{\mu}\right] = 0$$

$$\left[Q_r, M^{\mu\nu}\right] = i\,\sigma^{\mu\nu}_{rs} Q_s$$

$\sigma^{\mu}\bar{\sigma}^{\nu} = \eta^{\mu\nu} - i\sigma^{\mu\nu}$

$\sigma^{\mu}_{rs} = \bar{\sigma}^{\mu}_{sr}$

the last relation means that Q_r transforms as a spinor in spacetime rotations

Supersymmetry algebra

- this N=1 SUSY algebra is the UNIQUE possible extension of the Poincaré group

- at this point SUSY is called GLOBAL as SUSY transformations implied by Q are independent of spacetime coordinates

- from the 1st relation one can show that $\frac{1}{4}\sum_r Q_r^2 = P_0 = H$ (i.e. Hamiltonian) i.e. in globally supersymmetric theories the energy of the states is positive

  important consequences for supersymmetry breaking

Supersymmetry algebra

- one can make the SUSY transformations dependent of spacetime coordinates

i.e. **local supersymmetry**

$\{Q_r, \bar{Q}_s\} = 2\sigma^{\mu}_{rs} P_{\mu}$ $\Rightarrow$ 2 susy transformations gives a translation

local supersymmetry $\Rightarrow$ **spacetime dependent translation**

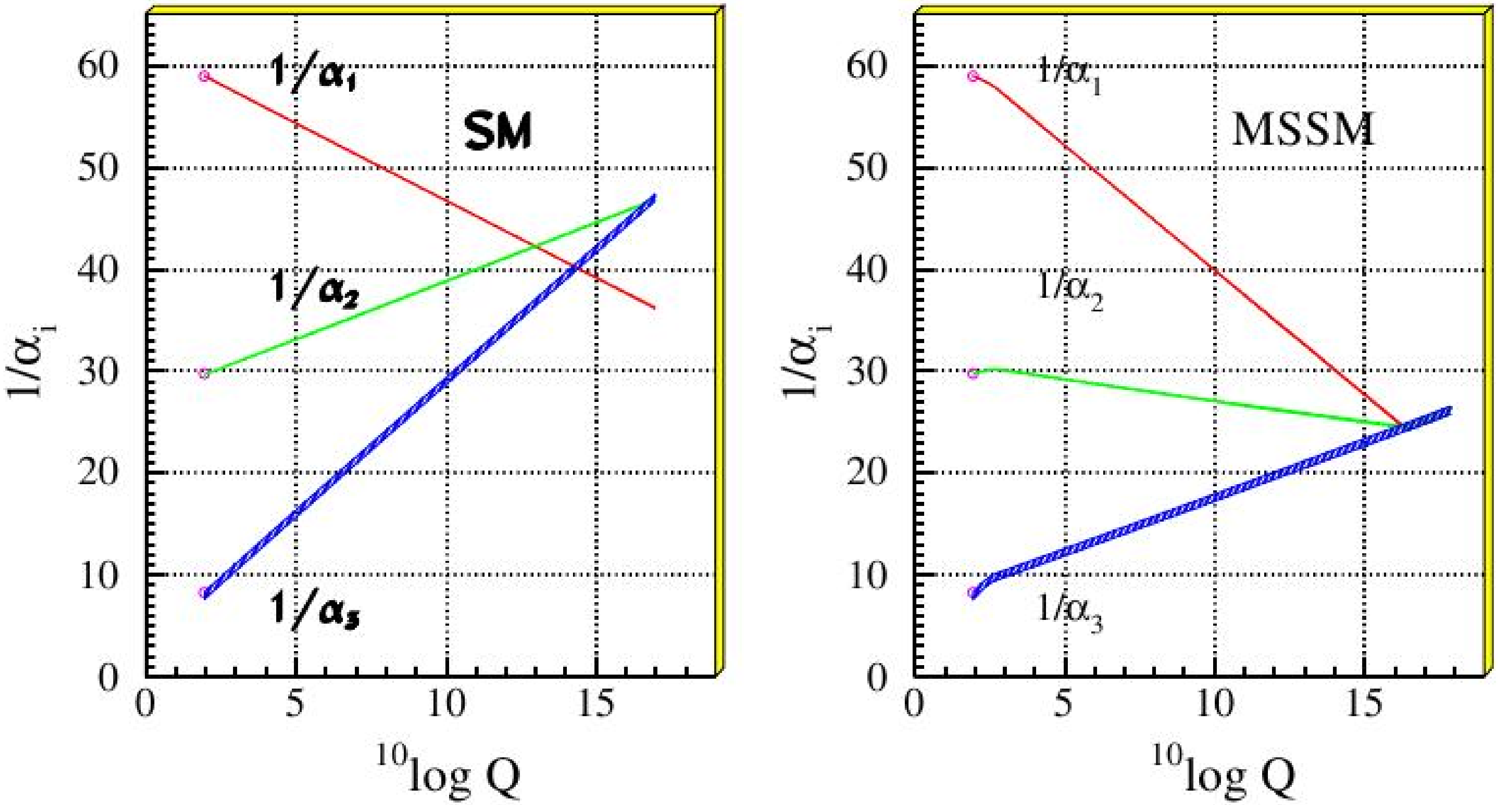
$\Rightarrow$ **general coordinate transformation**

$\Rightarrow$ expect **gravity** to appear in any locally supersymmetric theory

local supersymmetry usually called **supergravity**

Supersymmetry and Grand Unification

Unification of the Coupling Constants in the SM and the minimal MSSM



Representation of Susy algebra

one can show that :

- **the representations of the susy algebra will contain different spins**

- **in any susy multiplet the number n_B of bosons equals the number n_F of fermions**

$$n_B = n_F$$

- **the particles of a (unbroken) susy multiplet have the same mass**

Representation of Susy algebra on states

with one-particle massless state $P_\mu = (E,\ 0,\ 0,\ E)$ we have

$$\{Q_r, \bar{Q}_s\} = 2\ \sigma^\mu_{rs}\ P_\mu = 2E\ (\sigma^0 + \sigma^3)_{rs} = 4E \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{rs}$$

which implies $\{Q_2, \bar{Q}_2\} = 0$ (one can show that $Q_2 = 0$)

and $\{Q_1, \bar{Q}_1\} = 4E$ so that we can define creation and annihilation operators :

$$a = \frac{Q_1}{2\sqrt{E}}$$ (lowers helicity by $\frac{1}{2}$)

$$a^\dagger = \frac{\bar{Q}_1}{2\sqrt{E}}$$ (raises helicity by $\frac{1}{2}$)

with the anti commutation relations (sometimes called the little susy algebra) :

$$\{a, a^\dagger\} = 1 \qquad \{a, a\} = 0 \qquad \{a^\dagger, a^\dagger\} = 0$$

Representation of Susy algebra on states

since $\left[a, J^3\right] = \frac{1}{2}\left(\sigma^3\right)_{11} a = \frac{1}{2} a$ we have

$$J^3\left(a|P^\mu, \lambda>\right) = \left(aJ^3 - \left[a, J^3 \right] \right) |P^\mu, \lambda> = \left(aJ^3 - \frac{a}{2}\right) |P^\mu, \lambda> = \left(\lambda - \frac{1}{2}\right) a|P^\mu, \lambda>$$

which means that $a|P^\mu, \lambda>$ has helicity $\lambda - \frac{1}{2}$

similarly one can also show that $a^\dagger|P^\mu, \lambda>$ has helicity $\lambda + \frac{1}{2}$

Representation of Susy algebra on states

starting with a ground state $|E,\lambda>$ with minimum helicity λ with :

$$a\,|E,\lambda> = 0$$

one can build the representation by application of the operator $a^\dagger$

$$a^\dagger\,|E,\lambda> = |E,\lambda+\frac{1}{2}>$$

one also has $a\,|E,\lambda+\frac{1}{2}> = |E,\lambda>$

since $a^\dagger a^\dagger\,|E,\lambda> = 0\,|E,\lambda> = 0$ the whole multiplet consists of :

$$|E,\lambda> \quad \text{and} \quad |E,\lambda+\frac{1}{2}>$$

then one has to add the CPT conjugate to get :

$$|E,\pm\lambda> \quad \text{and} \quad |E,\pm\left(\lambda+\frac{1}{2}\right)>$$

Representation of Susy algebra on states

there are for example: chiral multiplets with $\lambda = 0, \frac{1}{2}$

vector or gauge multiplets with $\lambda = \frac{1}{2}, 1$

$\lambda = 0$ scalar	$\lambda = \frac{1}{2}$ fermion
squark	quark
slepton	lepton
Higgs	Higgsino

$\lambda = \frac{1}{2}$ fermion	$\lambda = 1$ boson
photino	photon
gluino	gluon
Wino, Zino	W, Z

as well as the gravity multiplet

$\lambda = \frac{3}{2}$ fermion	$\lambda = 2$ boson
gravitino	graviton

Representation of Susy algebra

minimal supersymmetric extension of the standard model $\Rightarrow$

doubling of the particle spectrum

SM boson and SM model fermion cannot be SUSY partners

they cannot be in the same multiplet as they belong to different representations of the SM gauge group

each SM particle has a new particle as SUSY partner

new particle as SUSY partner ↓ — ↓ in the 'fundamental' representation of the SM gauge group

chiral multiplet = (scalar $\lambda = 0$, SM fermion $\lambda = \frac{1}{2}$)

vector or gauge multiplet = (fermion $\lambda = \frac{1}{2}$, SM vector boson $\lambda = 1$)

new particle as SUSY partner ↑ — ↑ in the 'adjoint' representation of the SM gauge group

Representation of Susy algebra

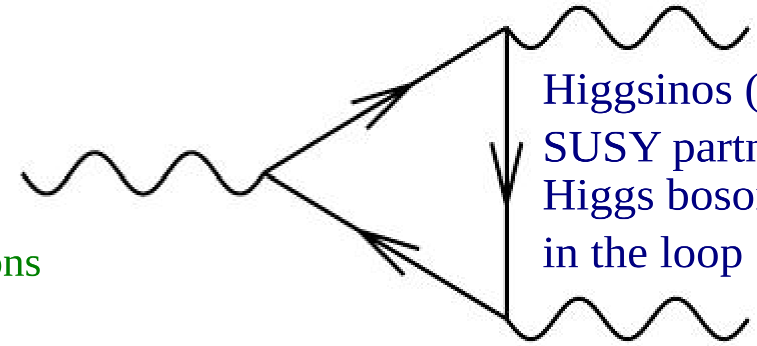
minimal supersymmetric extension of the standard model $\Rightarrow$

needs two Higgs doublets of opposite hypercharge

$$H_d = \begin{pmatrix} H_d^o \\ H_d^- \end{pmatrix} \quad \text{with } Y_{H_d} = -1 \qquad\qquad H_u = \begin{pmatrix} H_u^+ \\ H_u^o \end{pmatrix} \quad \text{with } Y_{H_u} = +1$$

- to avoid gauge anomalies

in the standard model triangle diagrams
spoil the local gauge symmetry at quantum level
but add up to 0 when one sums over all possible fermions



Higgsinos (fermions SUSY partners of Higgs bosons) in the loop

- to give mass to:

u-type quarks

d-type quarks and charged leptons

Representation of Susy algebra

we have introduced new fermions :

gauginos → supersymmetric partners of SM gauge bosons

Higgsinos → supersymmetric partners of Higgs bosons

- gauginos have vectorial coupling and do not contribute to the axial anomaly
- a unique higgsino introduces anomalies proportional to Y^3 with $Y^3 = \pm 1 \neq 0$
- introducing a second Higgs doublet of opposite hypercharge i.e. a second higgsino of opposite hypercharge, solves the problem

$$Y^3_{H_d} + Y^3_{H_u} = 0$$

Minimal SUSY extension of SM - fields content

superfields		boson fields	fermion fields	
Matter multiplets				
		sleptons	leptons	
L	leptons	$\tilde{L} = (\tilde{\nu}_L, \tilde{e}_L^-)$	(ν_L, e_L^-)	
E^C		$\tilde{E} = \tilde{e}_R^+$	e_L^c	
		squarks	quarks	
Q	quarks	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	
U^C		$\tilde{U} = \tilde{u}_R^*$	u_L^c	
D^C		$\tilde{D} = \tilde{d}_R^*$	d_L^c	
		Higgs	Higgsinos	
H_1	Higgs	(H_1^o, H_1^-)	$(\tilde{H}_1^o, \tilde{H}_1^-)_L$	
H_2		(H_2^+, H_2^o)	$(\tilde{H}_2^+, \tilde{H}_2^o)_L$	
Gauge multiplets				
G		g	$\tilde{g}$	gluino
V		W^a	$\tilde{W}^a$	wino
V'		B	$\tilde{B}$	bino

Minimal SUSY extension of SM - fields content

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E^C		$\tilde{E} = \tilde{e}_R^+$	e_L^c	
		squarks	quarks	
Q	quarks	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	
U^C		$\tilde{U} = \tilde{u}_R^*$	u_L^c	
D^C		$\tilde{D} = \tilde{d}_R^*$	d_L^c	
		Higgs	Higgsinos	
H_1	Higgs	(H_1^o, H_1^-)	$(\tilde{H}_1^o, \tilde{H}_1^-)_L$	
H_2		(H_2^+, H_2^o)	$(\tilde{H}_2^+, \tilde{H}_2^o)_L$	
Gauge multiplets				
G		g	$\tilde{g}$	gluino
V		W^a	$\tilde{W}^a$	wino
V'		B	$\tilde{B}$	bino

a=1, 2 → charginos $\tilde{X}_{i=1,2}^{\pm}$

Minimal SUSY extension of SM - fields content

superfields		boson fields	fermion fields	
Matter multiplets		sleptons	leptons	
L	leptons	$\tilde{L} = (\tilde{\nu}_L, \tilde{e}_L^-)$	(ν_L, e_L^-)	
E^C		$\tilde{E} = \tilde{e}_R^+$	e_L^c	
		squarks	quarks	
Q	quarks	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	
U^C		$\tilde{U} = \tilde{u}_R^*$	u_L^c	
D^C		$\tilde{D} = \tilde{d}_R^*$	d_L^c	
		Higgs	Higgsinos	
H_1	Higgs	(H_1^o, H_1^-)	$(\tilde{H}_1^o, \tilde{H}_1^-)_L$	
H_2		(H_2^+, H_2^o)	$(\tilde{H}_2^+, \tilde{H}_2^o)_L$	
Gauge multiplets				
G		g	$\tilde{g}$	gluino
V		W^a	$\tilde{W}^a$	wino
V'		B	$\tilde{B}$	bino

a=1, 2

neutralinos

$\tilde{X}^o_{i=1,4}$

Minimal SUSY extension of SM - fields content

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0 \ H_d^0 \ H_u^+ \ H_d^-$	$h^0 \ H^0 \ A^0 \ H^\pm$
squarks	0	−1	$\widetilde{u}_L \ \widetilde{u}_R \ \widetilde{d}_L \ \widetilde{d}_R$	(same)
			$\widetilde{s}_L \ \widetilde{s}_R \ \widetilde{c}_L \ \widetilde{c}_R$	(same)
			$\widetilde{t}_L \ \widetilde{t}_R \ \widetilde{b}_L \ \widetilde{b}_R$	$\widetilde{t}_1 \ \widetilde{t}_2 \ \widetilde{b}_1 \ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L \ \widetilde{e}_R \ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L \ \widetilde{\mu}_R \ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L \ \widetilde{\tau}_R \ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1 \ \widetilde{\tau}_2 \ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0 \ \widetilde{W}^0 \ \widetilde{H}_u^0 \ \widetilde{H}_d^0$	$\widetilde{N}_1 \ \widetilde{N}_2 \ \widetilde{N}_3 \ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm \ \widetilde{H}_u^+ \ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm \ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\widetilde{G}$	(same)

Minimal SUSY extension of SM – towards lagrangian

the quarks and leptons have the property that:

their L parts are $SU(2)_L$ doublets

their R parts are $SU(2)_L$ singlets

these weak gauge properties suggest that we should treat the L and R part separately

i.e. there will be separate susy partners for each chirality state of the massive fermion

so for example the case of the 1^{st} family there will be

4 squark flavours and "chiralities" $\tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \tilde{d}_R$

3 sleptons flavours and "chiralities" $\tilde{\nu}_{eL}, \tilde{e}_L, \tilde{e}_R$

to be repeated for the other 2 families $\longrightarrow$ in total 21 new fields

these are all complex scalar fields so the L and R labels do not mean chirality

but show which SM fermion they are partnered with

Minimal SUSY extension of SM – towards lagrangian

- gauge interaction in the MSSM can be derived from the SM ones
- SM particles and their superpartners belonging to the same supermultiplet have the same gauge properties and couple with the same strength
- usually taking

chiral (matter) supermultiplet

(Ψ, ϕ)

fermion — complex scalar

vector supermultiplet

(A, λ)

gauge vector boson — gaugino

example $Q \equiv \begin{pmatrix} U \\ D \end{pmatrix} \equiv \begin{pmatrix} u_L & \tilde{u}_L \\ d_L & \tilde{d}_L \end{pmatrix}$ $\quad (g, \tilde{g})$

- SM gauge coupling between a boson and a fermion pair is written $(A \Psi \Psi)$
  complemented in the MSSM by the couplings $(A \phi \phi)$ $(\lambda \phi \Psi)$

example : $(W^- e_L^- \nu_e)$ in the SM complemented by $(W^- \tilde{e}_L^- \tilde{\nu}_e)$ $(\tilde{W}^- e_L^- \tilde{\nu}_e)$ in the MSSM

$W^{\pm} (\tilde{W}^{\pm}) \propto g_2 \, I^3$ charged wino couples to the 3rd component of weak isospin

$\gamma (\tilde{\gamma}) \quad \propto e \, q = g_2 \, q \, \sin \theta_W$ photino couples to the electric charge

$Z^o (\tilde{Z}^o) \quad \propto g_2 (I^3 - q \, \sin^2 \theta_W)$ zino couples to same combination of isospin and charge as the Z

Minimal SUSY extension of SM – towards lagrangian

if gauge interactions in minimal SUSY extension of SM can be derived from the SM ones (as seen in previous slide) we have ~ **analog rules for Yukawa interactions**

- after electroweak symmetry breaking both Higgs fields H_u , H_d aquire vacuum expectation values (vev) : $v_u = \frac{1}{\sqrt{2}} <0|H_u|0>$, $v_d = \frac{1}{\sqrt{2}} <0|H_d|0>$

  with $v^2 = v_u^2 + v_d^2 = 4 \frac{M_W^2}{g_2^2 + g_1^2} \simeq (246 \ GeV)^2$

- masses of up-type and down-type (d-type quarks and charged leptons) fermions are obtained from Yukawa terms: $(h_u v_u u_L u_L^c)$, $(h_d v_d e_L e_L^c)$, $(h_d v_d d_L d_L^c)$

- defining the ratio of Higgs vev as $\tan\beta = \frac{v_u}{v_d}$ the Yukawa couplings are then given in terms of particle masses $h_u = \frac{m_u}{v \sin\beta}$, $h_d = \frac{m_d}{v \sin\beta}$

i.e. unlike in the SM the Yukawa coupling of minimal SUSY extension of the SM are not uniquely determined by the fermions masses but also depend on the parameter $\tan\beta$

Susy breaking

remember : particles of a ('unbroken') susy multiplet have the same mass

however SUSY particles have not been observed (yet)

this non observation requires them to be heavier than SM particles

suspersymmetry must be broken to lift mass degeneracy

between SM particles and their SUSY partners

⇒ several way to do this

Susy breaking

taking the vacuum state $|0>$ one can show that *global* supersymmetry is is *spontaneously* broken if:

$$Q\ |0> \neq 0$$

or in other word if the vacuum energy $<0|\,H\,|0>$ is strictly positive

where we remember that $H = P_0 = \frac{1}{4}\sum_r Q_r^2$ (i.e. Hamiltonian)

this can happen in two cases known as:

- type F breaking (O'Raifertaigh)

- type D breaking (Fayet Iliopoulos)

however early incarnations of these were not found to be phenomenologically viable

$\Rightarrow$ look for alternatives

Susy breaking → Susy models

alternatively one can show that *global* supersymmetry can be *explicitely* broken by adding in a supersymmetry invariant lagrangian new renormalizable, gauge invariant terms which are not supersymmetry invariant

such terms have been listed and are known as **soft SUSY breaking terms**

there are as many new parameters as there are soft susy breaking terms

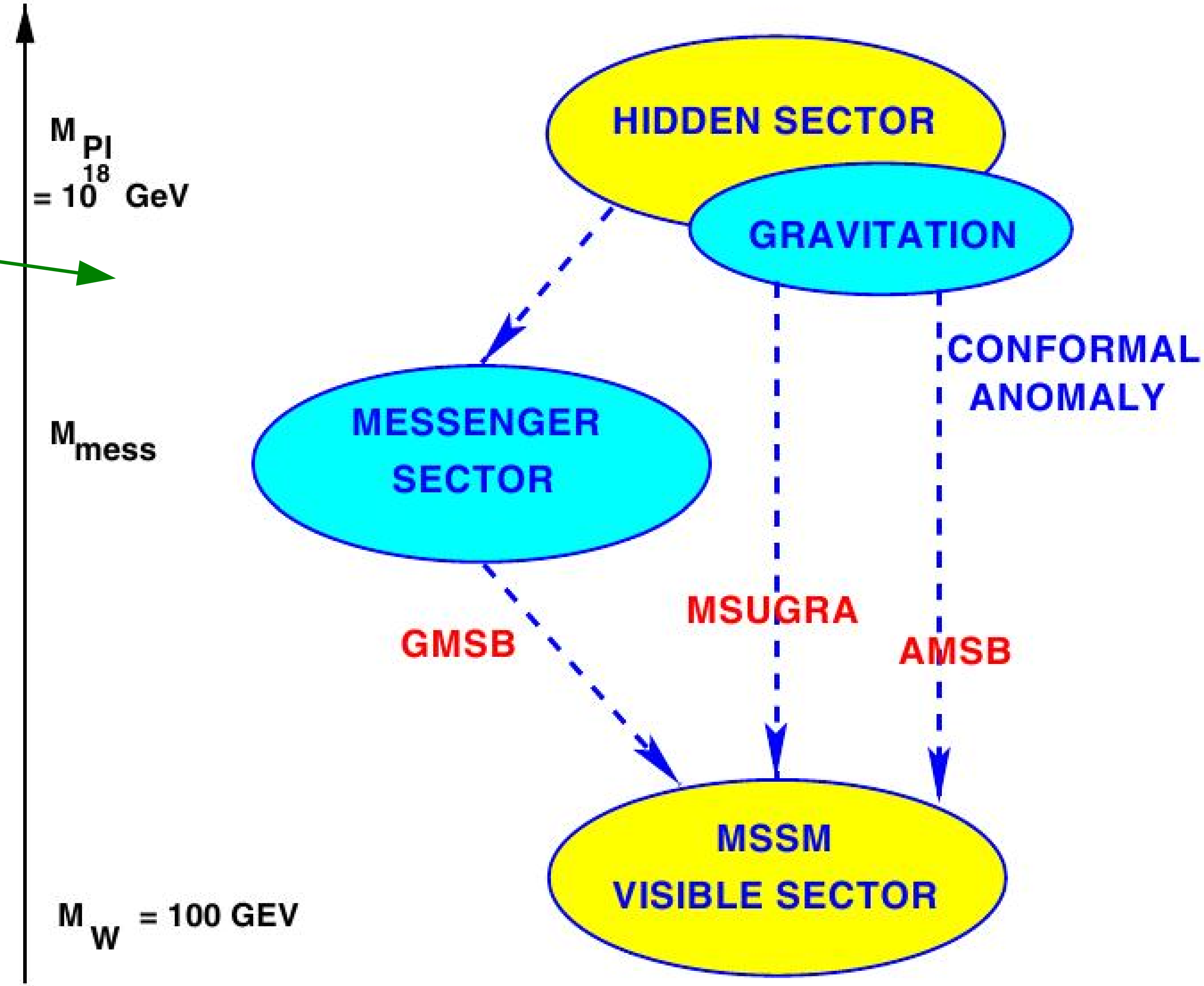
the existence of soft susy breaking terms have been justified in the context of *spontaneously* broken *local* supersymmetry i.e. spontaneouly broken supergravity

the spontaneous breaking occurs in a so called 'hidden sector' at high energy scale and is transmitted to a so called 'visible sector' at lower energy scale (SM particles and susy partners) via some interactions (messengers) :

- gravitational interactions ⇒ e.g. **MSUGRA, AMSB**
- new gauge interactions ⇒ e.g. **Gauge Mediated SUSY Breaking i.e. GMSB**

Susy breaking → Susy models

Several SUSY breaking models (non exhaustive list)



Customary to introduce a SUSY breaking scale M_{SB} **and write gravitino mass**

$$m_{3/2} = \frac{M_{SB}^2}{M_{Pl}\sqrt{3}}$$

Susy breaking → Susy models

- **underlying mechanism of SUSY breaking still to be found**

- **soft SUSY breaking terms** should be merely viewed as a **parametrization of our ignorance** and can be used as an effective lagrangian from which to derive phenomenology

- in this general form a large number of new free parameter are introduced i.e. in total 124 including the ones from the SM (which have 19)

- several parameters could induce flavour changing neutral currents (FCNC) or CP violation at an unacceptable level

- more restrictive models satisfying these requirements and relying on fewer parameters have been developed

- mass parameters introduced by the soft SUSY breaking terms lift mass degeneracy of the members of the supermultiplet

- they contribute to the radiative corrections of the Higgs mass

to avoid "return" of hierarchy problem their values should be at most a few TeV

Susy breaking → Susy models

phenomenologically more "viable" models can be defined by making some further assumptions :

- all the soft SUSY-breaking parameters are real and therefore there is no new source of CP-violation in addition to the one from the CKM matrix

- the matrices for the sfermion masses and for the trilinear couplings are all diagonal implying the absence of flavor changing neutral current (FCNC) at tree level

- the soft susy-breaking masses and trilinear couplings of the first and second sfermion generations are the same at low energy to cope with some severe constrains such as for example from $K^o - \bar{K}^o$ mixing

Susy breaking → Susy models

with these assumptions we are left with 22 parameters

$\tan\beta$	ratio of the vevs of the two Higgs doublet fields
$m^2_{H_u},\ m^2_{H_d}$	the Higgs mass parameters squared
$M_1,\ M_2,\ M_3$	the bino, wino and gluino mass parameters
$m_{\tilde{q}},\ m_{\tilde{u}_R},\ m_{\tilde{d}_R},\ m_{\tilde{l}},\ m_{\tilde{e}_R}$	the 1st / 2nd generation mass parameters
$A_u,\ A_d,\ A_e$	the 1st / 2nd generation trilinear coupling
$m_{\tilde{Q}},\ m_{\tilde{t}_R},\ m_{\tilde{b}_R},\ m_{\tilde{L}},\ m_{\tilde{\tau}_R}$	the 3rd generation sfermion mass parameters
$A_t,\ A_b,\ A_\tau$	the 3rd generation trilinear coupling

- the Higgs-higgsino supersymmetric mass parameter μ and the soft-breaking bilinear term **B** are determined through the electroweak symmetry breaking condition

- one can trade the values of $m^2_{H_u},\ m^2_{H_d}$ with the more physical pseudoscalar Higgs boson mass m_A and parameter μ

- trilinear sfermion coupling will be multiplied by fermion masses i.e. important in the case of the 3rd generation only (exceptions: electric and magnetic dipole)

Susy breaking → Susy models

- assuming these parameters to be real

→ solves potential problems with CP violation

- assuming gravity mediation underlying SUSY-breaking (supergravity i.e. SUGRA based)

→ soft breaking terms are flavor blind (like ordinary gravitational interaction)

- assuming soft breaking parameters obey a set of boundary conditions at high energy scales (GUT scale) $M_U \approx 2\cdot 10^{16}$ **GeV**

→ solves problems of the general or unconstrained MSSM

→ **universal soft breaking terms (universality)**

$$M_1(M_U) = M_2(M_U) = M_3(M_U) = \boldsymbol{m_{1/2}}$$

unification of gaugino mass

$$M_{\tilde{Q}_i}(M_U) = M_{\tilde{u}_R}(M_U) = M_{\tilde{d}_R}(M_U) = M_{\tilde{L}}(M_U) = M_{\tilde{l}_R}(M_U) = m_{H_u}(M_U) = m_{H_d}(M_U) = \boldsymbol{m_o}$$

universal scalar mass

$$A_u(M_U) = A_d(M_U) = A_l(M_U) = \boldsymbol{A_o}$$

universal trilinear coupling

Susy breaking → Susy models

- in addition to $\boldsymbol{m_{1/2}, m_o, A_o}$ the SUSY sector is described by the bilinear coupling B and Higgs mass parameter μ at high energy scales M_U

- electroweak symmetry breaking at lower energy scales results into minimization condition in the two Higgs doublets scalar potential

⟶ fix the value of μ^2 and $B\mu$

but sign of μ undetermined

- one is left with 4 free parameters and an unknown sign

$$\tan\beta\ , m_{1/2},\ m_o,\ A_o,\ sign(\mu)$$

- all soft SUSY-breaking parameters **at the weak scale** are then obtained via RGEs

- such a model is often refered to as **mSUGRA**

MSUGRA or CMSSM

at $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}\,\mathrm{GeV}$:

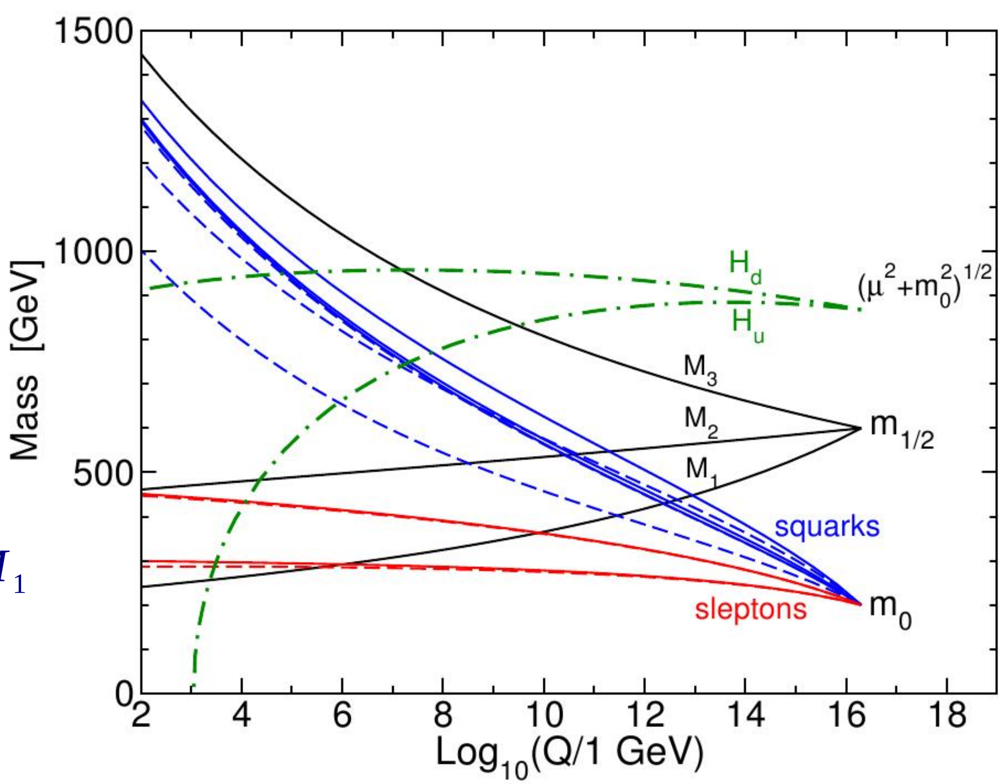
- gauginos : $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$
- scalars : $m^2_{\tilde{q}_i} = m^2_{\tilde{l}_i} = m^2_{H_b} = M^2_{H_t} = m^2_o$
- trilinear soft terms : $A_b = A_t = A_o$
- radiative EWSB : $\mu^2 = \dfrac{m^2_{H_b} - m^2_{H_t}\tan^2\beta}{\tan^2\beta - 1} - \dfrac{m^2_Z}{2}$

⇒ five independent parameters

$M_{1/2},\ m_o,\ A_o,\ \tan\beta,\ \mathrm{sgn}(\mu)$

at ~EW scale : $M_3 = \dfrac{\alpha_3}{\alpha}\sin^2\theta_W M_2 = \dfrac{3}{5}\dfrac{\alpha_3}{\alpha}\cos^2\theta_W M_1$

$M_3 : M_2 : M_1 \approx 7 : 2 : 1$



Susy breaking → Susy models

GMSB parameters :

- messenger scale **M** representing the average mass of the messengers

- scale $\Lambda = M_{SB}^2 / M$ setting the scale of the superpartner spectrum

- messenger index **N** determined by the group representation of the gauge group of the gauge mediation interactions

- trilinear couplings **A** set to zero at messenger scale and become non-zero at the electroweak scale due to RGE

- μ^2 and B are obtained by requiring correct electroweak symmetry breaking (as in the previous case)

$$\Lambda\,,\; M\,,\; N\,,\; \tan\beta\,,\; sign(\mu)$$

- soft SUSY breaking masses for the gauginos and squared masses for the sfermions arise from 1-loop and 2-loop diagrams (respectively) involving messengers fields
  suppression of FCNC and CP violation

- LSP is the gravitino which can have a mass below 1 eV

MSSM lagrangian: a reminder

additional terms allowed by gauge invariance (and renormalizability)

$$W_{R_p} = \mu_i H_u L_i + \lambda_{ijk} L_i L_j E_k^C + \lambda'_{ijk} L_i Q_j D_k^C - \lambda''_{ijk} U_i^C U_j^C D_k^C$$

violate Lepton number **L** conservation

violate Baryon number **B** conservation

since B and L
can be carried by boson fields

violations can be avoided by introducing a discrete symmetry known as R-parity

i.e. introducing a multiplicatively conserved number

$$R = (-)^{3B+L+2S}$$

$$R_{SM} = +1 \qquad R_{SUSY} = -1$$

Field	B	L	S	$3B+L+2S$
quark	$1/3$	0	$1/2$	2
squark	$1/3$	0	0	1
lepton	0	1	$1/2$	2
slepton	0	1	0	1

BACKUP

fine tuning arguments

Triviality, vacuum stability and naturalness

consider a complex scalar Φ field with lagrangian :

$$L = \partial^{\mu}\Phi^{\dagger}\partial_{\mu}\Phi - V\left(\Phi^{\dagger}\Phi\right)$$

$$V\left(\Phi^{\dagger}\Phi\right) = -m^{2}\Phi^{\dagger}\Phi + \lambda\left(\Phi^{\dagger}\Phi\right)^{2}$$

the minimization of V gives $\quad <\Phi^{\dagger}\Phi> = \dfrac{m^{2}}{2\lambda} = \dfrac{v^{2}}{2} \quad$ with $v^{2} = \dfrac{m^{2}}{\lambda}$

Φ is parameterized as $\quad \Phi = \begin{pmatrix} \phi^{+} \\ \frac{1}{\sqrt{2}}\left(v + H + i\phi^{o}\right) \end{pmatrix}$

the fields ϕ^{+} and ϕ^{o} are Goldstone bosons and the mass of the field H is

$$m_{H}^{2} = 2\mathrm{m}^{2} = 2\lambda v^{2}$$

fine tuning arguments

Triviality

in the RG context, the scalar self-coupling λ becomes a running coupling $\lambda(\mu)$ varying with the energy scale μ characteristic of the process considered

the evolution of $\lambda(\mu)$ with the scale μ at lowest order in λ can be calculated,

at one loop : $\mu \dfrac{d\lambda}{d\mu} = \dfrac{3}{2\pi^2}\lambda^2 + \ldots$ i.e. $\lambda(\mu)$ is monotically increasing

for the theory from the previous lagrangian to make sense up to scale Λ we need to impose $\lambda(\mu) < \infty$ for $\mu < \Lambda$

if Λ is known, we obtain some bound on λ at low energy, say $\lambda(v)$ in particular $\lambda(v) = 0$ for $\Lambda \to \infty$

a theory described by the previous lagrangian which would be valid at all energy scales is known as trivial i.e. it is a free field theory in the infrared low energy) regime

fine tuning arguments

Triviality

this means that at some scale Λ smaller than the scale Λ_{Landau}
where the coupling would diverge and known as the Landau pole
some new physics appears

the exact value of the laudau pole requires non perturbative calculation
but one can use the previous one-loop result to obtain a general trend

thus solving the equation one gets : $\lambda^{-1}(\mu) = \lambda^{-1} - \frac{3}{2\pi^2} \log\frac{\mu}{v}$ where $\lambda \equiv \lambda(v)$

using $\lambda^{-1}(\Lambda_{Landau}) = 0$, one obtains : $\Lambda_{Landau} \sim v\ e^{2\pi^2/(3\lambda)}$

since λ can be expressed in terms of m_H , for a given value of Λ , the Higgs mass is bounded by :

$$m_H^2 < \frac{4\pi^2 v^2}{3\log(\Lambda/v)}$$

which is a decreasing function of Λ

fine tuning arguments

Vacuum stability

when the scalar field is light (λ is small) and when it has other interactions, such as top Yukawa interactions with a (light) Higgs boson as in the Standard Model, one can obtain a decreasing $\lambda(\mu)$ as μ increases

i.e : $$\mu \frac{d\lambda}{d\mu} = -\frac{3}{8\pi^2}\lambda_t^4 + \dots ,$$ for the dominant term coming from top Yukawa interactions

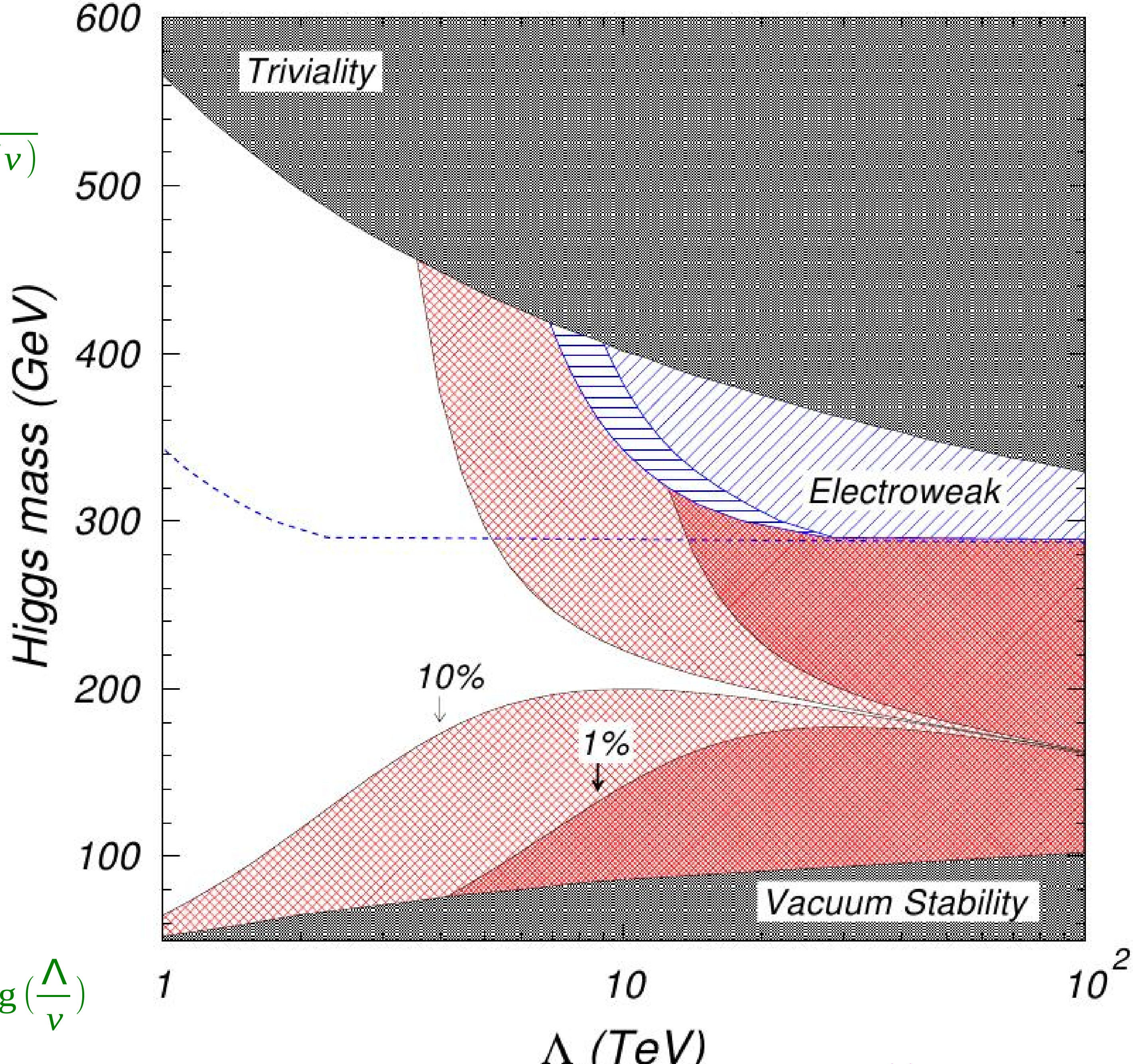
when $\lambda(\mu)$ turns negative, the potential V becomes unbounded from below

and the theory suffers from an instability (new physics should take charge)

for a given value of Λ we have $$m_H^2 > \frac{3\, m_t^4}{\pi^2 v^2} \log\left(\frac{\Lambda}{v}\right)$$

fine tuning arguments

$$m_H^2 < \frac{4\pi^2 v^2}{3\log(\Lambda/v)}$$



$$m_H^2 > \frac{3 m_t^4}{\pi^2 v^2} \log\left(\frac{\Lambda}{v}\right)$$

Kolda, Murayama, JHEP 07 (2000)35

fine tuning arguments

Unitarity

unitarity of the S-matrix from conservation of probabilities at the quantum level

imposes some constraints on the high energy behavior of scattering cross-sections

this is usually expressed in terms of partial wave expansion

if $M(s,\theta)$ is the amplitude for a $2 \to 2$ scattering process with center of mass energy $\sqrt{s}$ and diffusion angle θ, unitarity is obtained if (optical theorem) :

$$\sigma_{\text{tot}} = \frac{1}{s} \; \text{Im}\left(M(\theta=0)\right)$$

one defines the Jth partial wave as : $a_J(s) = \frac{1}{32\pi}\int d\cos\theta \; P_J(\cos\theta) \; M(s,\theta)$

where P_J is the Jth Legendre polynomial $(P_J(1)=1)$ and we have :

$$M = 16\pi \sum_{J=0}^{+\infty} (2\text{J}+1) \; P_J(\cos\theta) \; a_J$$

fine tuning arguments

Unitarity

the total cross-section for a $(2) \to (2)$ process reads :

$$\sigma_{tot} = \int \frac{|M|^2}{64\pi^2 s} d\Omega = \frac{16\pi}{s} \sum_{J=0}^{+\infty} (2J+1) |a_J|^2$$

then the unitarity condition reads $|a_J|^2 = (\mathrm{Re}\, a_J)^2 + (\mathrm{Im}\, a_J)^2 = \mathrm{Im}\, a_J$ or :

$$(\mathrm{Re}\, a_J)^2 + \left(\mathrm{Im}\, a_J - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2$$

so that one should have :

$$|\mathrm{Re}\, a_J| \leqslant \frac{1}{2}$$

in order to have unitarity

fine tuning arguments

Unitarity

applied to the physics of the standard model, in the absence of a fundamental Higgs field, the $J = 0$ tree level amplitude for $W^+_{\text{long.}} W^-_{\text{long.}} \to Z_{\text{long.}} Z_{\text{long.}}$ reads :

$$a_0(s) = \frac{G_F \sqrt{2}\, s}{16\pi}$$

the previous tree level unitarity constraint imposes that new physics apperas at a scale $\Lambda < \Lambda_U = \sqrt{8\pi/\left(G_F \sqrt{2}\right)} \sim 1.2$ TeV

the most stringent constraint comes from the mixed zero-isospin channel $2 W^+_{\text{long.}} W^-_{\text{long.}} + Z_{\text{long.}} Z_{\text{long.}}$

a constraint on the Higgs boson mass can then be derived from unitarity constraint

$$m_h < \sqrt{\frac{16\pi}{5}}\, v = 780 \text{ GeV}$$

(in terms of the EW breaking scale $v = \left(G_F \sqrt{2}\right)^{-1/2}$)

fine tuning arguments

supersymmetry as a solution to the problem of naturalness

one can check this result using an explicit example : the Wess Zumino Model

the model contains :

a complex scalar field (2 degrees of freedom) :

$$\phi = \frac{1}{\sqrt{2}}\left(A + i\,B\right)$$

a fermion field described by a Majorana spinor Ψ (2 degrees of freedom) :

$$\Psi^{c} = C\,\bar{\Psi}^{T} = \Psi$$

supersymmetry as a solution to the problem of naturalness

the WZ lagrangian decomposes into

1) a kinetic term :

$$L^{WZ}_{kin} = \partial^\mu \phi^* \partial_\mu \phi + \frac{i}{2}\bar{\Psi}\gamma^\mu \partial_\mu \Psi$$
$$= \frac{1}{2}\partial^\mu A \partial_\mu A + \frac{1}{2}\partial^\mu B \partial_\mu B + \frac{i}{2}\bar{\Psi}\gamma^\mu \partial_\mu \Psi$$

2) an interaction term expressed in terms of a single function $W(\phi)$ known a the superpotential :

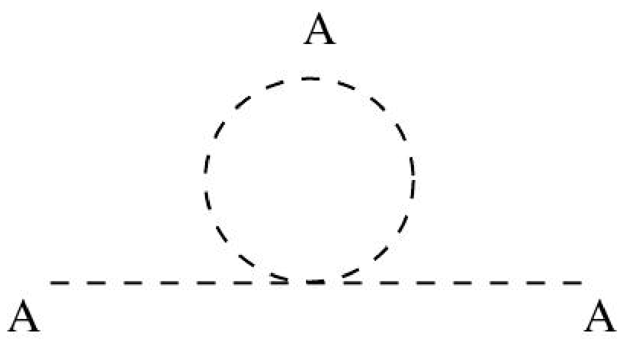
$$L^{WZ}_{\text{int}} = -\left|\frac{dW}{d\phi}\right|^2 - \frac{1}{2}\left(\frac{d^2 W}{d\phi^2}\bar{\Psi}_R \Psi_L + \frac{d^2 W^*}{d\phi^2}\bar{\Psi}_L \Psi_R\right)$$

explicitely : $W(\phi) = \frac{1}{2} m\phi^2 + \frac{1}{3}\lambda\phi^3$ and then :

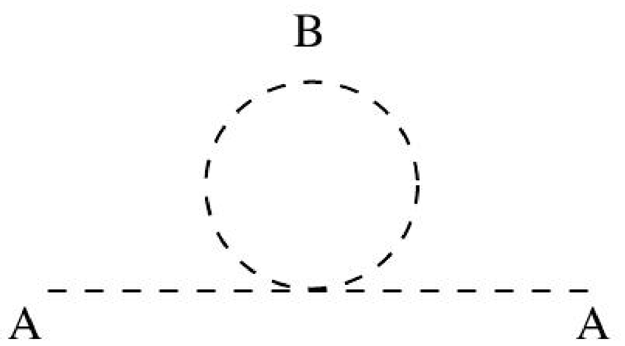
$$L^{WZ}_{\text{int}} = -\left|m\phi + \lambda\phi^2\right|^2 - \frac{1}{2}\left[m\left(\bar{\Psi}_R\Psi_L + \bar{\Psi}_L\Psi_R\right) + 2\lambda\left(\phi\bar{\Psi}_R\Psi_L + \phi^*\bar{\Psi}_L\Psi_R\right)\right]$$
$$= -\frac{1}{2}m^2\left(A^2 + B^2\right) - \frac{m\lambda}{\sqrt{2}} A\left(A^2 + B^2\right) - \frac{\lambda^2}{4}\left(A^2 + B^2\right)^2$$
$$-\frac{1}{2} m\bar{\Psi}\Psi - \frac{\lambda}{\sqrt{2}}\bar{\Psi}\left(A - iB\gamma_5\right)\Psi$$

supersymmetry as a solution to the problem of naturalness

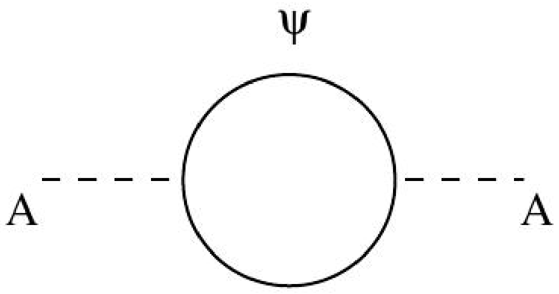
the self energy diagrams for the scalar field A contributing to the 1 loop quadratic divergence are :



$$-\frac{i\lambda^2}{4}\, 4\times 3 \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2} = 3\lambda^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m^2}$$



$$-\frac{i\lambda^2}{2}\, 2 \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2} = \lambda^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m^2}$$



$$(-)\left(-\frac{i\lambda}{\sqrt{2}}\right)^2 2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\frac{i}{\not{k}-m}\quad\frac{i}{\not{k}-\not{p}-m}\right]$$

$$= -\lambda^2 \int \frac{d^4k}{(2\pi)^4}\quad \frac{\mathrm{Tr}\,(\not{k}+m)(\not{k}-\not{p}+m)}{(k^2-m^2)\left[(k-p)^2-m^2\right]}$$

supersymmetry as a solution to the problem of naturalness

using :

$$\mathrm{Tr}\,(\not{k}+m)(\not{k}-\not{p}-m)=4\left[k.(k-p)+m^2\right]=2\left[(k^2-m^2)\left((k-p)^2-m^2\right)-p^2+4\mathrm{m}^2\right]$$

one finds a total contribution :

$$2\lambda^2\left\{\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m^2}-\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k-p)^2-m^2}+\int\frac{d^4k}{(2\pi)^4}\frac{p^2-4\mathrm{m}^2}{(k^2-m^2)\left[(p-k)^2-m^2\right]}\right\}$$

which shows that the quadratic divergences cancel

Representation of Susy algebra on states

one can define the (Casimir) operators

mass square operator $P^2 = P_\mu P^\mu$

generalized spin operator $W^2 = W_\mu W^\mu$

where W^μ is the Pauli-Lubanski vector $W^\mu = -\frac{1}{2} \epsilon^{\mu\nu\rho\sigma} P_\nu W_{\rho\sigma}$

in the rest frame of a massive state we have

$$P_\mu = (m,\ 0,\ 0,\ 0) \quad \text{and} \quad W^2 = -m^2 \boldsymbol{J}^2$$

where m^2 is the mass squared eigenvalue and $\boldsymbol{J}^2 = j(j+1)$ is the angular momentum with $J^i = \frac{1}{2} \epsilon^{ijk} M_{jk}$

one can show that $[P^2,\ Q_r] = [P^2,\ \bar{Q}_r] = 0$ but $[W^2,\ Q_r] \neq 0$

⇒ the (massive) representations of the susy algebra will contain different spins

Susy algebra

one can show that :

- the representations of the susy algebra will contain different spins

- in any supersymmetry multiplet the number n_B of bosons equals the number n_F of fermions

$$n_B = n_F$$

Representation of Susy algebra on states

for massless states with discrete helicities we have

$$W_\mu = \lambda P_\mu \qquad (\lambda \text{ half integer})$$
$$\text{and} \quad P^2 = W^2 = 0$$

a massless one-particle state can always be rotated into a standard frame where its movement is in the z-direction where $P_\mu = (E,\ 0,\ 0,\ E)$

the space-time properties of the state are determined by its energy E and helicity λ

the helicity of the one-particle state is the projection of its spin onto the direction of motion i.e. the eigenvalue of $\frac{1}{E}\, \boldsymbol{J}\cdot\boldsymbol{P}$ where $\boldsymbol{J} = (M_{23},\ M_{31},\ M_{21})$

we have $W_0 = \boldsymbol{J}\cdot\boldsymbol{P}$ and for a massless helicity eigenstate $|E,\ \lambda>$ we always have :

$$W_0 = \lambda E \quad \text{and} \quad W_\mu\, |E,\, \lambda> \;=\; \lambda\, P_\mu\, |E,\, \lambda>$$

Representation of Susy algebra on states

since $\left[a, J^3\right] = \frac{1}{2}\left(\sigma^3\right)_{11} a = \frac{1}{2} a$ we have

$$J^3\left(a|P^\mu, \lambda>\right) = \left(aJ^3 - \left[a, J^3 \right] \right) |P^\mu, \lambda> = \left(aJ^3 - \frac{a}{2} \right) |P^\mu, \lambda> = \left(\lambda - \frac{1}{2}\right) a|P^\mu, \lambda>$$

which means that $a|P^\mu, \lambda>$ has helicity $\lambda - \frac{1}{2}$

similarly one can also show that $a^\dagger|P^\mu, \lambda>$ has helicity $\lambda + \frac{1}{2}$

MSSM lagrangian: a reminder

- **the standard model is chiral**
  i.e. left-handed and right-handed fermion component transform differently under gauge interaction (correspond to separate 2-component real Weyl spinors)

- when **discussing supersymmetry** it is more convenient to **describe all fermions by spinors of the same chirality say left-handed**

- if necessary use charge conjugation i.e. trade u_R for u_L^C

- introduce for each generation of quarks and leptons

$$Q = \begin{pmatrix} U \\ D \end{pmatrix} \qquad L = \begin{pmatrix} N \\ E \end{pmatrix} \qquad U^C \quad D^C \quad E^C$$

capital letter indicates the superfield

for example U contains the left-handed quark u_L and the squark field $\tilde{u}_L$
whereas U^C contains the left-handed anti-quark u_L^c and the anti-squark $\tilde{u}_R^*$
(respectively charge conjugates of the right-handed quark u_R and of the squark $\tilde{u}_R$

one way of viewing it: $Q \equiv \begin{pmatrix} U \\ D \end{pmatrix} \equiv \begin{pmatrix} u_L & \tilde{u}_L \\ d_L & \tilde{d}_L \end{pmatrix}$

MSSM lagrangian: a reminder

Kinetic terms and gauge interactions (written here without D and F auxiliary fields)

(index **i** for fermion and sfermion flavours and chirality, index **a** for vector bosons and gauginos gauge groups

$$L_{kin} = D^{\mu}\phi^{\dagger i} D_{\mu}\phi_i + i X^{\dagger i}\bar{\sigma}^{\mu} D_{\mu} X_i - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i\lambda^{\dagger a}\bar{\sigma}^{\mu} D_{\mu}\lambda^a - \sqrt{2} g\left[(\phi^{\dagger i} T^a X_i)\lambda_a + \lambda^{\dagger}_a (X^{\dagger i} T^a \phi_i)\right]$$

- kinetic terms for the complex scalar and interactions with the gauge bosons

- kinetic terms for the 2-component fermionic matter fields and interaction with gauge fields

with covariant derivatives

$$D_{\mu}\phi_i = \partial_{\mu}\phi_i + i g A^a_{\mu}(T^a \phi)_i$$

$$D_{\mu} X_i = \partial_{\mu} X_i + i g A^a_{\mu}(T^a X)_i$$

T^a: generators of the gauge group

include trilinear coupling $(A\Psi\Psi)$, $(A\phi\phi)$ and quartic interaction $(A\ A\phi\phi)$ between scalars and gauge bosons

MSSM lagrangian: a reminder

Kinetic terms and gauge interactions **(written here without D and F auxiliary fields)**

(index **i** for fermion and sfermion flavours and chirality, index **a** for vector bosons and gauginos gauge groups

$$L_{kin} = D^{\mu} \phi^{\dagger i} D_{\mu} \phi_i + i X^{\dagger i} \bar{\sigma}^{\mu} D_{\mu} X_i - \frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu a} + i \lambda^{\dagger a} \bar{\sigma}^{\mu} D_{\mu} \lambda^{a} - \sqrt{2} g \left[(\phi^{\dagger i} T^{a} X_i) \lambda_a + \lambda^{\dagger}_{a} (X^{\dagger i} T^{a} \phi_i) \right]$$

- trilinear interactions $(\lambda \phi X)$ **between gaugino, scalar and fermion**

MSSM lagrangian: a reminder

potential of the MSSM lagrangian can be derived **from** a function called **superpotential**

form constrained by the requirement of invariance under supersymmetry

- polynomial of at most order 3 in the scalar fields with no complex conjugates fields (analytic function of the fields) with a general form (of dimension 3 in mass) :

$$W = \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k$$

determining masses and couplings of matter fields

- in MSSM :

$$W = \epsilon_{ij} \left(-L^i h_L E^C H_d^j - Q^i h_D D^C H_d^j + Q^i h_U U^C H_u^j + \mu H_u^i H_d^j \right)$$

$\epsilon_{ij} = -\epsilon_{ji} \quad (\epsilon_{12} = 1)$ i,j isospin indices

→ $h_L,\ h_D,\ h_U$ are dimensionless Yukawa coupling constants expressed as 3x3 matrices in family space

→ μ Higgs mixing parameter
the only parameter with dimension (mass) in the superpotential

MSSM lagrangian: a reminder

- contribution to the lagrangian for chiral fermions

$$L_{chir} = -\frac{1}{2}\frac{\partial^2 W}{\partial\phi_i \partial\phi_j} X_i X_j + h.c.$$

generates Yukawa interactions and fermion mass terms → for example :

$$L_{chir,e} = -\frac{h_e}{2}\left[(e_L e_L^c) H_d^o + (e_L^c \tilde{H}_d^o)\tilde{e}_L + (e_L \tilde{H}_d^o)\tilde{e}_L^c - \right]$$
$$\left[(\nu_e e_L^c) H_d^- - (e_L^c \tilde{H}_d^-)\tilde{\nu}_e - (\nu_e \tilde{H}_d^-)\tilde{e}_L^c \right] + h.c.$$

contains the trilinear Yukawa interaction between fermions, scalars and Higgs or Higgsinos

1^{st} term is the familiar SM Yukawa interaction generating mass for the fermions after electroweak symmetry breaking, other terms correspond to new interactions

- in addition : contribution from the mu-term in the superpotential

$$\mu \left(\tilde{H}_u^o \tilde{H}_d^0 - \tilde{H}_u^+ \tilde{H}_d^- \right) + c.c.$$

providing off-diagonal elements in the mass matrices for the higgsino fermions
physical states will be mixtures of the higgsino fields

MSSM lagrangian: a reminder

- scalar potential including two contributions :

$$F_i = \frac{\partial W}{\partial \phi_i}$$

chiral contribution or F-terms (from equation of motion of auxiliary fields F)

$$D^a = g\,\phi_i^{\dagger} T^a_{ij} \phi_j$$

gauge contribution or D-terms (from equation of motion of auxiliary field D)

$$V(\phi) = F^i F_i^{\dagger} + \frac{1}{2} D^a D_a$$

scalar potential (in compact form)

Full lagrangian density for an unbroken supersymmetry

$$L = L_{kin} + L_{chir} + V(\phi)$$

MSSM lagrangian: a reminder

previous superpotential is not the most general one

additional terms allowed by gauge invariance (and renormalizability)

$$W_{R_p} = \mu_i H_u L_i + \lambda_{ijk} L_i L_j E_k^C + \lambda'_{ijk} L_i Q_j D_k^C - \lambda''_{ijk} U_i^C U_j^C D_k^C$$

violate Lepton number **L** conservation

violate Baryon number **B** conservation

since B and L
can be carried by boson fields

violations can be avoided by introducing a discrete symmetry known as R-parity

i.e. introducing a multiplicatively conserved number

$$R = (-)^{3B + L + 2S}$$

$$R_{SM} = +1 \qquad R_{SUSY} = -1$$

Field	B	L	S	$3B + L + 2S$
quark	$1/3$	0	$1/2$	2
squark	$1/3$	0	0	1
lepton	0	1	$1/2$	2
slepton	0	1	0	1

MSSM lagrangian: a reminder

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- contains Yang-Mills field strength

$$F^a_{\mu\nu} = \partial_{\mu} A^a_{\nu} - \partial_{\nu} A^a_{\mu} - g f^{abc} A^b_{\mu} A^c_{\nu} \qquad \left[T^a, T^b\right] = i f^{abc} T^c$$

leading to the kinetic terms of the gauge fields, trilinear interactions (AAA) and quartic interaction of the gauge bosons

- contains kinetic term for the gauginos

with covariant derivative $D_{\mu}\lambda^a = \partial_{\mu}\lambda^a - g f^{abc} A^b_{\mu}\lambda^c$

leading to trilinear interactions of gauginos $(A\lambda\lambda)$

MSSM lagrangian: a reminder

- in unbroken supersymmetry (SUSY) superpartners are expected to be degenerate in mass with the SM particles of the same supermultiplet and massless

Non observation of superpartners requires SUSY to be broken

to avoid quadratic divergences in the radiative corrections of scalar particles masses (i.e. avoid the "return" of the hierarchy problem of the SM)

SUSY breaking must take a specific form i.e. **soft supersymmetry breaking**

→ **most general terms to be added to the lagrangian**

$$L_{soft} = \sum_{\tilde{q},\tilde{l},H_{u,d}} m_{o,i}^2 |\phi_i|^2 - \frac{1}{2} m_{1/2,a} \lambda_a \lambda_a - A_{0,i} W_{3,i} - B\mu H_u H_d$$

the parameters of soft SUSY breaking have positive mass dimension

$m_{o,i}$	mass parameter of the scalar (in principle a matrix in generation space)
$m_{1/2,a}$	mass parameter the gauginos
$A_{0,i}, B$	parameter of dimension of mass (A is called trilinear coupling)
$W_{3,i}$	trilinear terms of the superpotential

MSSM lagrangian: a reminder

- in AMSB in principle three inputs parameters :

$$m_{3/2},\ \tan\beta,\ sign(\mu)$$

- μ^2 and B are obtained by requiring correct electroweak symmetry breaking (as in the previous case)

- BUT anomaly mediation contribution to the slepton scalar masses squared is negative

- problem can be in principle handled by adding a positive non-anomaly mediated contribution to the soft masses i.e. a m_o^2 term at M_U as in mSUGRA thus one ends up with

$$m_{3/2},\ \tan\beta,\ sign(\mu),\ m_o$$

MSSM lagrangian: a reminder

More models of SUSY breaking :

- mixed dilaton/moduli SUSY breaking (heterotic superstrings inspired)

  soft terms dominated by loop contributions (group dependent and independent)

  possible non-universal soft masses

- Susy breaking from extra dimensions : Scherk Schwarz, Hosotani mechanism, Wilson lines, Orbifolding