String Phenomenology and Model Building

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Abstract

This is a set of 3 lectures (50 minutes each) prepared for the Natal School 2014. The aim of these lectures is to introduce the basic ideas of string theory and their applications to particle physics. These lectures hopefully serve to complement the lectures by Eduardo Ponton (BSM and Extra Dimensions), Marc Besanon (Phenomenology and experimental aspects of SUSY and searches for extra dimensions at the LHC), Dmitry Melnikov (Holography and Hadron Physics), and Jorge Noronha (AdS/CFT and applications to particle physics). These lectures are an update of my lectures at the TASI 2008 Summer School and the Trieste 2010 Spring School – both serve as useful references for this course.

1 Introduction

This is the first of 3 lectures on "String Phenomenology and Model Building". My plan for these lectures is roughly as follows:

- Lecture I: String Phenomenology, with Branes
- Lecture II: D-brane Model Building
- Lecture III: Randall-Sundrum Scenario in String Theory: Warped Throats & their Field Theory Duals

Basic Theme:

The basic theme of these lectures is to introduce some modern string theory tools useful for building models and scenarios of particle physics beyond the Standard Model. Of course, particle physics is not all what string phenomenology is about. String theory, being a quantum theory of gravity, is also a natural arena to address questions about early universe cosmology. However, string cosmology is a topic that warrants a set of lectures on its own. Instead of thinning ourselves too much, these lectures will focus on the particle physics aspects of string theory. Fortunately, the tools that we introduce in these lectures like D-branes, warping, and holography also found applications in cosmology model building, especially in realizing inflation in string theory. So, these lectures will hopefully give you a good start to explore these other string phenomenology topics as well.

As Eduardo discussed in his lectures, a major driving force behind physics beyond the Standard Model (and also the LHC) is the hierarchy problem. Many scenarios in solving this problem have been proposed, but it is fair to say that the main contenders are:

- Supersymmetry
- Extra Dimensions
- Strong dynamics (technicolor, composite Higgs etc)

So, what does string theory have to do with these BSM physics? The answer is - a whole lot. Supersymmetry and extra dimensions are common ingredients in string compactifications. In fact, supersymmetry was discovered to some extent first in the context of string theory, as a way to introduce fermions and to remove the unwanted tachyons. The idea of extra dimensions has been around for over a hundred years so string theory cannot claim credit for inventing this idea. However, theories with extra dimensions are not renormalizable. To make sense of these theories, we need to complete them in the UV. String theory provides such a UV completion. Moreover, as we will see in these lectures, there are stringy constraints on extra dimensional physics that are not apparent from a low-energy bottom-up approach. So, one might hope that string theory can shed light on what kinds of extra dimensional scenarios are more likely to be realized in nature. Finally, models that involve strong coupling dynamics are difficult to analyze as perturbative techniques break down. However, the gauge/gravity correspondence (aka holography) can provide a dual weakly coupled description of such theories. As Eduardo discussed, this duality has offered insights into technicolor model building.

So, given that the the hierarchy problem is intrinsically about the existence of a high cutoff scale and string theory is our best developed framework for describing physics at such high energies, it is worthwhile to explore BSM physics in the context of string theory. With these motivations in mind, we will discuss in the first two lectures how models of particle physics can be constructed from string theory. In the last two lectures, we will discuss how string theory provides us a tool to analyze strong coupling dynamics in BSM scenarios.

2 String Phenomenology, with Branes

Before we talk about D-brane model building, let us go back in time to the mid 80s (when most of you were not yet born) to see how life was like without branes. It was known at that time that there are five consistent string theories, all formulated in ten dimensions: Type I theory with gauge symmetry $SO(32)$ which involves both open and closed strings, Type IIA and IIB which are theories of closed strings, and two closed heterotic strings with gauge symmetries $E_8 \times E_8$ and $SO(32)$ respectively:

- Among these formulations, three of them already contain gauge bosons in ten dimensions. They seem more promising as a starting point to construct the Standard Model and chiral fermions upon dimensional reduction.

- For this reason, Type II theories didn't look too interesting. There was even a no-go theorem which forbids them to produce the Standard Model upon compactification `[Dixon, Kaplunovsky, Vafa (1987)]`.

- The heterotic $E_8 \times E_8$ attracted much of the attention as it had many of the features one would like in a BSM scenario: upon compactification to 4D, it can give rise to chiral $\mathcal{N} = 1$ SUSY models with familiar gauge and matter content. The observable sector comes from the first E_8 which contains the Standard Model gauge symmetry

$$E_8 \supset SU(3) \times SU(2) \times U(1) \tag{1}$$

and replication of matter families. The second E_8 gives rise to a hidden sector which can be responsible for SUSY breaking. Gravity is often assumed to be the messenger of SUSY breaking to the observable sector.

This picture changed dramatically in the mid 1990s. The discovery of string dualities suggested that the five consistent string theories together with 11D SUGRA are just different limits of the same underlying theory. There is no fundamental reason why we have to start from any particular description. The techniques of constructing realistic models have also been much extended. We can now start from any of these descriptions and obtain models with features of the Standard Model:

1. *11D SUGRA:* There are two broad ways in which chiral fermions can arise from 11D SUGRA. One is purely geometrical. Just as the requirement of $\mathcal{N} = 1$ supersymmetry in 4D singles out Calabi-Yau compactifications of string theory, G_2 manifolds provide a purely geometric construction if our starting point is 11D. Smooth G_2 manifolds do not support chiral fermions, a necessary feature

of the Standard Model. Nevertheless, chiral fermions can arise from co-dimension 7 singularities in a G_2 manifold while gauge fields are supported on a codimension 4 subspace.

Another way in which chiral fermions can arise from 11D is the Horava-Witten construction. In this construction, the 11-th dimension is an interval. One can think of the two E_8 as living at the endpoints of the interval which are 10D surfaces. Further compactification on Calabi-Yau manifolds give rise to interesting chiral models. Incidentally, an additional dimension accessible only to gravity helps solve the discrepancy between the unification scale and the Planck scale.

2. *Type I, IIA, IIB Strings:* We can also construct chiral models in Type II string theories with D-branes. This will be the subject of the first two lectures. Jumping a bit ahead of ourselves, there are also several ways in which chiral fermions can arise: branes at singularities, intersecting branes, and so on.

   These D-brane constructions can be generalized to include the backreaction of branes in F-theory (proposed by Vafa). In Type IIB string theory, the two scalar fields of the 10D theory, the dilaton and axion are combined into one complex field $S = a + i\phi$, which realizes the S-duality symmetry $SL(2, \mathbf{Z})$:

$$S \to \frac{aS+b}{cS+d} \qquad a,b,c,d \in \mathbf{Z} \text{ with } ad-bc=1 \tag{2}$$

which is similar to how the modular parameter of a torus transforms. Therefore, one can "geometrize" this varying axio-dilaton background as compactification on an elliptically fibered CY 4-fold which is locally a product of this torus with a six-dimensional (non Calabi-Yau) base B_3. These compactifications naturally incorporate D7-branes, which are given by points in the base[1] where the elliptic fibration degenerates.

What do these constructions have in common? They involve one way or the other the notion of branes – either a D-brane or the end-of-the-world brane in the Horava-Witten construction, or a singularity in a G_2 manifold, or a degeneration of the fibration in an elliptically fibered Calabi-Yau four-fold. This brane world idea has triggered a foray of phenomenological studies as explained in Eduardo's lectures. Here we will discuss their string theory realizations. In particular, we will focus on D-brane models both because we don't need to introduce very complicated mathematical machineries to describe them and that they are closely accessible by holographic tools through the AdS/CFT correspondence.

One of the interesting features of scenarios with branes is that the fundamental scale of nature can be much lower than the Planck scale and therefore closer to the energies accessible by experiments. To see this, consider the Standard Model gauge fields to be localized on the worldvolume of D_p branes,

[1] More precisely, points in P_1.

the low energy effective action in 4D takes the form:

$$S_{4d} = \int d^4x\sqrt{-g}\left(M_s^8 V_6 e^{-2\phi}\mathcal{R}_4 + \frac{1}{4}M_s^{p-3}V_{p-3}e^{-\phi}F_{\mu\nu}^2 + \ldots\right) \quad (3)$$

There are two crucial differences in comparison to the heterotic case where all the low energy degrees of freedom come from closed strings. Firstly, the power of the dilaton is different for the gravity and the gauge kinetic terms. Secondly, the total volume V_6 enters on the gravity part but only the volume of the cycle of the internal manifold that the p-branes wrap around appears in the gauge part of the action. In particular for a $D3$-brane, there is no volume factor contribution to the gauge coupling. The gravitational and gauge couplings are then related to the fundamental string scale as follows:

$$M_P^2 = \frac{M_s^8}{(2\pi)^7}\frac{V_6}{g_s^2}\,, \qquad \alpha_{YM}^{-1} = 4\pi\frac{M_s^{p-3}V_{p-3}}{g_s} \quad (4)$$

where we have restored factors of 2 and π. We can easily see that if the Standard Model fits inside a D3-brane, for instance, we may have M_s substantially smaller than M_P as long as the volume of the extra dimensions are large enough, without affecting the gauge couplings. More generally, the above relations illustrate the “brane world” effect where the dimensions transverse to the branes (which are not necessarily D3-branes) if large can lower the fundamental string scale. In the heterotic case, setting the volume very large would make the gauge couplings extremely small and furthermore the KK modes are light, which are unrealistic.

The possibility of having large dimensions add a “geometrical” view of the hierarchy problem, though we still need to explain why the size of the dimensions are stabilized to a large value. Several scenarios have been proposed:

1. $M_s \sim M_P$. This is analogous to the heterotic scenario and similar comments apply.
2. $M_s \sim M_{GUT} \sim 10^{16}$ GeV. This corresponds to a compactification scale $r \equiv V_6^{1/6} \sim 10^{-30}$cm or equivalently $M_c \sim 10^{14}$ GeV. This is analogous to the Horava-Witten construction and allow the possibility of the unification of gauge *and* gravitational couplings (if the Standard Model is realized on the same set of branes).
3. $M_s \sim M_I \sim 10^{10-12}$ GeV $\sim \sqrt{M_{EW} M_P}$. If the Standard Model is realized on a set of D3-branes, this corresponds to a compactification scale $r \sim 10^{-23}$ cm. This proposal was based on the special role played by the intermediate scale M_I in different issues beyond the Standard Model. Examples include the scale of SUSY breaking in graviy mediated SUSY breaking scenario and the scale of the axion field introduced to solve the strong CP problem.
4. $M_s \sim M_{EW} \sim$ TeV. This is the string theory realization of the large extra dimensions scenario a la ADD. Note that the hierarchy problem is not totaly solved. We still need to explain why the compactification size is so large.

D-branes thus offer many more scenarios, in contrast to the heterotic string where the fundamental string scale is rigidly fixed: $M_s \sim g_{YM} M_P \sim 10^{18-19}$ GeV. Without going into details of the constructions to which we turn next, a few observations can already be made:

- Gauge unification is not necessarily realized in D-brane models. The tree-level gauge coupling depends on the volume of the cycles the branes wrap around and can be different for different gauge factors of the Standard Model.

- We may or may not consider SUSY in D-brane constructions because the fundamental string scale can be much lower than the Planck scale. However, stable D-brane configurations realizing non-SUSY models are harder to construct since they usually come with (i) closed string tachyons, and (ii) runaway potentials for moduli (dilaton, volume, etc). So, for the purpose of the first two lectures, we wil focus on $\mathcal{N} = 1$ SUSY examples. We will discuss metastable SUSY breaking in Lectures III and IV.

- D-brane models can be easily combined with other ingredients such as sources of moduli stabilization and supersymetry breaking e.g. background fluxes. We will discuss compactifications with fluxes in Lecture III.

- Although we will focus on D-brane models, much of our discussions can be generalized to other brane constructions in M/F theory models.

- Besides motivating new BSM scenarios, D-brane models also realize more traditional scenarios such as hidden sector (useful for dark matter scenarios) and gravity mediated SUSY breaking, but now in a more geometrical and stringy way.

3 D-branes and Chirality

We now turn to the main point of these lectures. D-branes contain non-Abelian gauge fields so they are a good starting point. But in addition to the $SU(3) \times SU(2) \times U(1)$ gauge structure, we need another fundamental property of the SM, i.e., 4D chiral fermions.

The worldvolume theory on a stack of N D-branes in flat space is an $\mathcal{N} = 4$ $SU(N)$ SYM. Type IIB string theory in flat space has maximal SUSY and the D3-branes are 1/2 BPS. The extended SUSY forbids chiral fermions. One might think that choosing a compactification which preserves fewer supersymmetries like a CY space or turning on fluxes can immediately change the situation, but it does not because this is a local issue at the location of the branes.

To make this point more precise, consider a D3-brane sitting at a point P in the extra dimensional space $\mathcal{M}_6$, with background fluxes. The background fluxes can in principle break the supersymmetry to $\mathcal{N} = 1$ or $\mathcal{N} = 0$ so chirality is possible. However, one can continuously deform $\mathcal{M}_6$ and dilute the fluxes to reach again flat space. Along this deformation, the gauge group does not change, so gauge protected quantities, like the number of chiral families should not change. Therefore, the spectrum in the original configuration is non-chiral.

In general, an open string with both ends on a stack of N D-branes tansforms in the adjoint representation of $U(N)$, hence the theory is non-chiral. Let us consider instead a stack of N_{D3} D3-branes and N_{D7} D7-branes. If we place them on top of each other, we have

$$U(N_{D3}) \times U(N_{D7}) \tag{5}$$

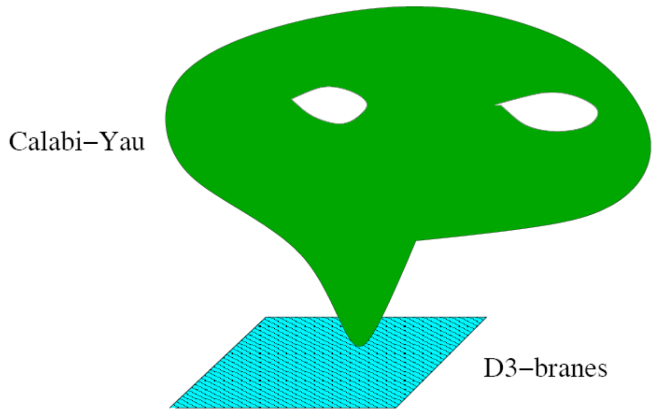
gauge groups and matter in the representations:

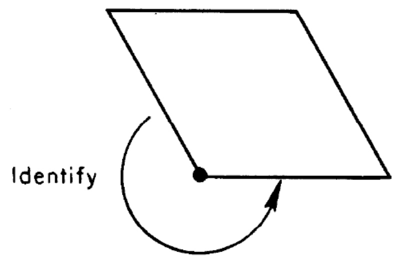
$$m(N_{D3}, \overline{N}_{D7}) + m'(\overline{N}_{D3}, N_{D7}) \tag{6}$$

where m, m' are multiplicities. But since we can separate them by a distance, the strings between D3 and D7 can gain a mass. Hence $m = m'$ and the theory is non-chiral.

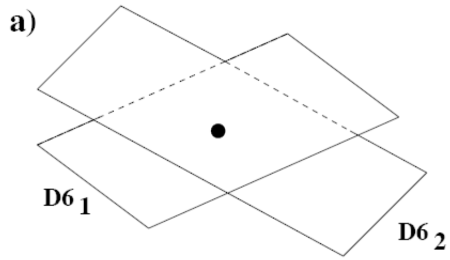
How could one obtain chirality? Four dimensional chirality is a violation of parity. In string theory, 4D chirality is correlated with the chirality in the 6 extra dimensions. Hence to achieve 4D chirality, we should consider D-brane configurations that somehow introduce a preferred orientation in 6D. There are several closely related ways to achieve this:

1. **D-branes at singularities:** We can consider placing D-branes in spaces that are not smooth. Chirality can arise if the D-branes are sitting at a singularity. A simple example is to consider a stack of D3-branes sitting at an orbifold singularity. An orbifold is a discrete identification of space and this defines a preferred orientation.

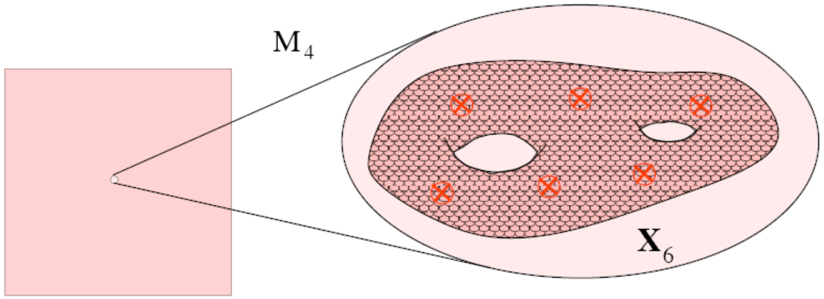




2. **Intersecting branes:** We can also consider pairs of D-branes that cannot be separated from one another. Intersecting D-branes lead to chiral fermions in the sector of open strings stretched between different kinds of D-branes. (Again, the angle between one stack of branes with respect to another defines a preferred orientation).



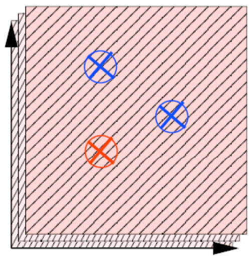
3. **Magnetized D-branes:** Finally, chirality also arises when we turn on a non-trivial field strength background for the worldvolume $U(1)$ gauge fields. The magnetic fields introduce a preferred orientation in the internal dimensions through the wedge product $F \wedge F \wedge F$ as the volume form.



An important point to make here is that these constructions are "modular" in the sense that we can locally obtain the Standard Model witout having to know all the details of the compactification. This is of great importance because we can follow a bottom-up approach instead of looking at random compactifications hoping that they could give rise to the Standard Model.

We will present this bottom-up approach for building realistic model. Most of the important details of the model, such as the gauge group, the number of chiral families, etc, will depend only on the local properties of the geometry felt by the branes. Therefore we can keep the main properties of the model intact regardless of whether we are talking about a complicated Calabi-Yau space or a simple toroidal orbifold compactification. This makes the models constructed more robust. This is the main practical advantage of D-brane model building over the heterotic string.

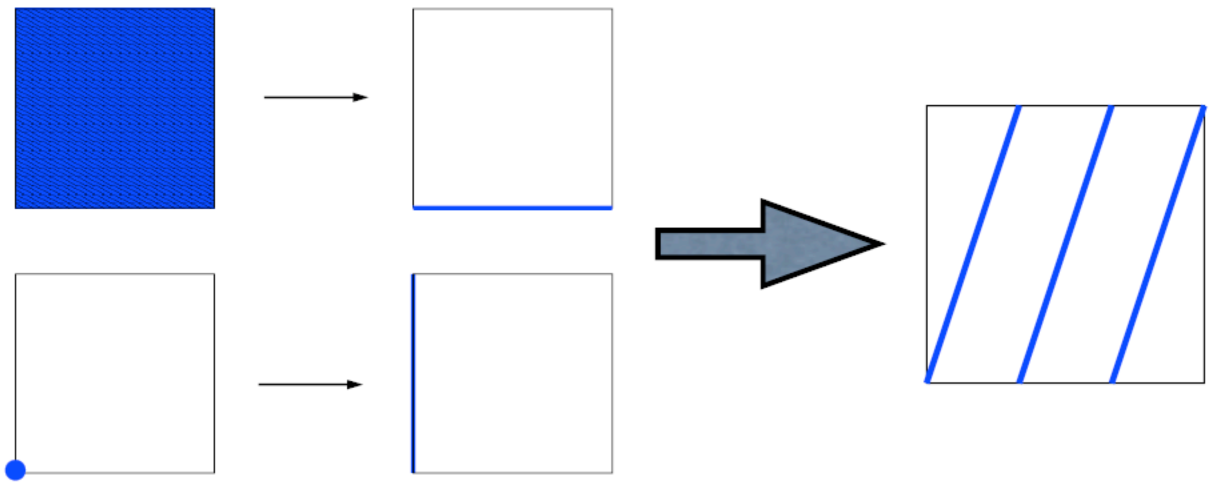
Before we go on, let me point out that these methods of obtaining chirality are in fact related by dualities. The simplest way to see this is to note that turning on a gauge bundle on the worldvolume of D-branes induces lower-dimensional D-brane charges. For example, one can think of a D_p brane with n units of magnetic flux on a torus:



as a bound state of a D_p brane and n D_{p-2} brane. Under T-duality:

$$R \to \alpha'/R \tag{7}$$

the closed string spectrum remains the same but the dimension of D-branes changes. A D_p brane with its worldvolume extended along the T-dual direction will become a D_{p-1} brane whereas a D_p brane with its worldvolume transverse to the T-dual direction will become a D_{p+1} brane. Now let's T-dualize along one direction:



The D_p and the D_{p-2} branes both turn into a D_{p-1} brane but they orient along different directions.

Thus branes with magnetic fluxes become branes at angles. Likewise, we will see later that the D3-branes at singularities which give rise to chiral fermions, known as fractional branes, can be thought of higher dimensional branes wrapping around a collapsed cycle with some gauge bundle on them.

So, why do we discuss these approaches separately if the setups are dual to one another? It turns out that in some situations, one side of the duality is simplier than the other. Furthermore, we will soon introduce background fluxes. Under duality, the background flux can turn into a non-trivial metric background and the two descriptions are no longer equivalent at least in this simple form. So it is useful to have an intuition about each of these approaches independently.

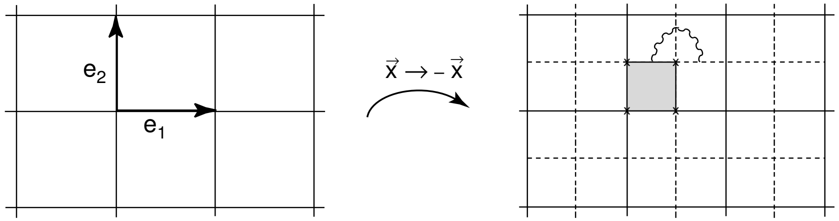
4 D-branes at Singularities

First, let us discuss how one can construct chiral models from D-branes at singularities. The simplest type of singularities is perhaps orbifold singularities, and we will begin with that. We will review what we need to know about orbifolds as we go along.

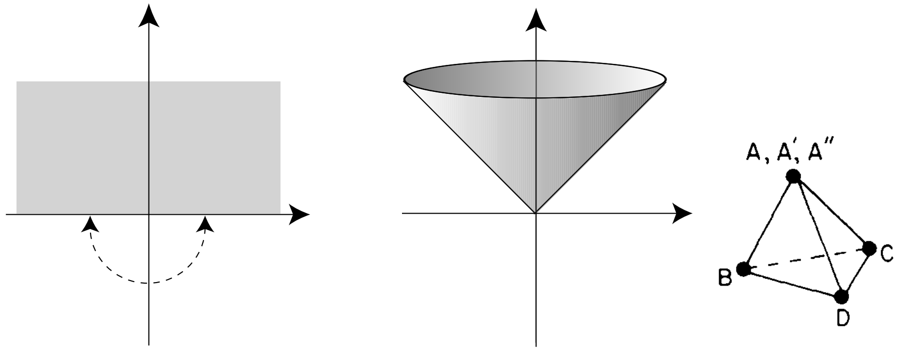
- A manifold $\mathcal{M}_6$ is by definition locally like $\mathbb{R}^6$, but string theory is defined even on manifolds that are not smooth. The extra dimensions can contain singularities, like orbifold or conifold singularities.
- Orbifolds are spaces that locally look like $\mathbb{R}^6$ or $\mathbb{R}^6/\Gamma$ where Γ is a discrete subgroup of $SO(6)$, the rotation group of the 6 extra dimensional space.
- Although the gravity background is singular, strings are well-behaved at orbifold singularities. This has been shown in the classic papers of `(Dixon, Harvey, Vafa, Witten, 1985)` for closed strings and `(Douglas and Moore, 1996)` for open strings.
- What is an orbifold? Consider a simple case $T^2/\mathbb{Z}_2$. The $\mathbb{Z}_2$ orbifold symmetry acts on the two-dimensional torus as follows:

$$\theta : (x_1, x_2) \to (-x_1, -x_2) \tag{8}$$

There are 4 fixed points: $(0,0),(1/2,0),(0,1/2),(1/2,1/2)$ (if we normalize the radii of T^2 to 1).



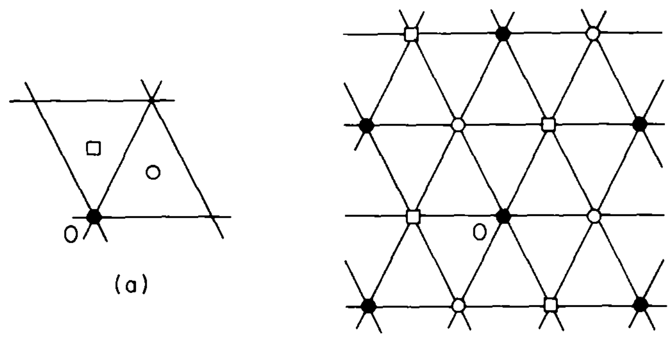
Locally, the singularity looks like a cone and globally the $T^2/\mathbb{Z}_2$ orbifold is a tetrahedral (or ravioli):



- Let's look at a slightly more non-trivial example: $T^2/\mathbb{Z}_3$. The discrete $\mathbb{Z}_3$ orbifold symmetry acts on the complex coordinates $z = x_1 + ix_2$ of the torus as follows:

$$\theta : z \to \alpha z \qquad \text{where} \qquad \alpha = e^{2\pi i/3} \tag{9}$$

There are three fixed points as shown in the figure:



(a) (b)

Note that vectors undergo a non-trivial rotation when transported along a closed curve around the singularity (localized holonomy).

- In general, we can consider a local singularity of the form $\mathbb{C}^3/\Gamma$ where $\Gamma = \mathbb{Z}_N$. The discrete symmetry $\mathbb{Z}_N$ acts on the complex coordinates of $\mathbb{C}^3$ as follows:

$$\theta : (z_1, z_2, z_3) \to (\alpha^{\ell_1} z_1, \alpha^{\ell_2} z_2, \alpha^{\ell_3} z_3) \tag{10}$$

where $\alpha = e^{2\pi i/N}$, $\ell_i \in \mathbb{Z}$ such that $\theta^N = 1$.

- One can check that if θ is a matrix of determinant 1:

$$\theta \in SU(3) \Leftrightarrow \ell_1 \pm \ell_2 \pm \ell_3 = 0 \quad (\text{mod } N) \tag{11}$$

for some choices of sign, then

$$\Gamma \subset SU(3) \subset SO(6) \ , \qquad \text{SUSY is preserved} \tag{12}$$

If this condition is not satisfied, then we find closed tachyons in the closed string spectrum. For concreteness, we will take the choice of signs, $\ell_1 + \ell_2 + \ell_3 = 0 \pmod N$.

- Furthermore, only if we require:

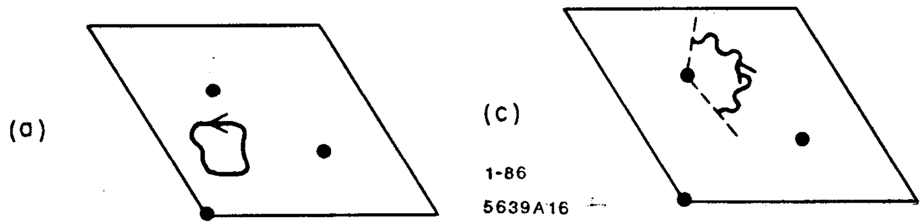
$$\sum_i \ell_i = \text{even} \tag{13}$$

do we have fermions in the spectrum. In more technical terms, the orbifold is called a spin manifold.

- The orbifold action has the following effects on closed strings. It projects out states that are not invariant under the twist, reducing the number of states in the spectrum. It also increases the number of states in another way since it now includes the so called "twisted sector". An open string around a fixed point with its two endpoints lying at points which are identified under the orbifold is not included in the spectrum of states in the unorbifolded space but it is a valid closed string in the orbifold. In other words, there are two sectors:

  – Untwisted sector: strings closed on $\mathbb{C}^3$

– Twisted sector: strings not closed on $\mathbb{C}^3$ but closed on $\mathbb{C}^3/\Gamma$ (confined to the singularity)



- Closed strings are not charged under the D-brane gauge group and so they will not give us the Standard Model particles. We will not analyze their spectrum. However, the closed string spectrum determines the types and number of moduli fields we have. They will be important later on when we discussed moduli stabilization.

- The open string spectrum is our main concern for particle physics model building. We know how to quantize open strings exactly to all orders in α' in these backgrounds. We will however discuss only the massless spectrum.

- If we are away from the singularity, things work as in flat space. The branes are arranged in a $\mathbb{Z}_N$ invariant fashion. They are identified and the open string spectrum is that of $U(N)$ $\mathcal{N} = 4$ SYM (plus α' corrections).

- If the branes sit on the singularity, we have an "orbifolded" gauge theory. The spectrum is given by the open strings that are well defined at the singularity. This means open strings invariant under the action of θ.

- The orbifold twist θ has two actions
  - $\theta \in SO(6)$: R-symmetry group of $D = 4$, $\mathcal{N} = 4$ SYM.
  - $\theta \in U(N)$: "permutes" the D-brane positions.

  The action of θ on the $U(N)$ gauge degrees of freedom is then given by an $N \times N$ matrix Γ_θ. We will elaborate on this shortly.

4.1 Explicit Examples

We now see explicitly how the spectrum of D-branes at a $\mathbb{Z}_N$ singularity, like the fixed points of orbifolds, become chiral. As discussed before, the spectrum is determined by the local properties so for simplicity, let us consider a D3-brane in flat 10D space with the six extra dimensions modded out by a $\mathbb{Z}_N$ twist θ. (There are too many N in this business! Here, we refer to N for the order of the twist, n for the number of overlapping D-branes, and $\mathcal{N}$ for the number of supersymmetries).

If we have a stack of n D-branes, the original gauge group is $U(n)$. The gauge degrees of freedom are represented by the Chan-Paton matrices λ^i_j, $i, j = 1, \ldots, n$, associated to the endpoints of the open strings and which belongs to the adjoint of $U(n)$. The action of the orbifold twist on the gauge degrees of freedom are given by:

$$\lambda \rightarrow \Gamma_\theta \lambda \Gamma_\theta^{-1} \tag{14}$$

where Γ_θ is of order N and can be diagonalized to take the simple form:

$$\Gamma_\theta = \begin{pmatrix} I_{n_0} & & & & & \\ & \alpha I_{n_1} & & & & \\ & & \alpha^2 I_{n_2} & & & \\ & & & . & & \\ & & & & . & \\ & & & & & \alpha^{N-1} I_{n_{N-1}} \end{pmatrix} \tag{15}$$

Here $\alpha = \exp(2\pi i/N)$, I_{n_k} is the identity matrix in n_k dimensions and the integers n_k satisfy the constraint $\sum_k n_k = n$. (*i.e., n D-branes are split into groups of* n_k*'s*).

Let us now see how the original $\mathcal{N} = 4$ vector multiplet transforms under the action of the twist defined by θ and Γ_θ. We can write the $\mathcal{N} = 4$ multiplet in terms of the $\mathcal{N} = 1$ multiplets:

- Vector Multiplet: $V \equiv (A_\mu, \lambda)$

- Chiral Multiplets: $\Phi_a \equiv (\phi_a, \psi_a)$, $a = 1, 2, 3$ (a labels the 3 complex extra dimensions)

Therefore Φ_a which has an a index feels both the action of θ and Γ_θ where V feels only the action through Γ_θ.

Remember that we have to keep only the states that are invariant under the twist. This means that $V = \Gamma_\theta V \Gamma_\theta^{-1}$. This breaks the gauge group to:

$$U(n) \to U(n_0) \times U(n_1) \times \cdots \times U(n_{N-1}) \tag{16}$$

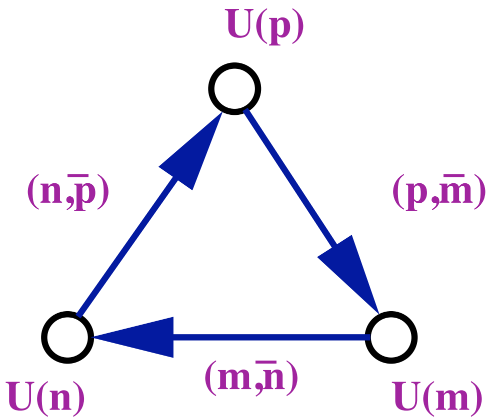
with the number of factors equal to the order of the twist N. This means that if we want three gauge factors, we should have a $\mathbb{Z}_3$ twist and so on.

The surviving chiral superfields satisfy $\Phi_a = \alpha^{\ell_a} \Gamma_\theta \Phi_a \Gamma_\theta^{-1}$. The first factor being the action of θ. Therefore remembering that λ carries adjoint indices (which are composed of fundamentals and anti-fundamentals) we can easily see that the remaining matter fields transform as:

$$\sum_{a=1}^{3} \sum_{i=0}^{N-1} (\mathbf{n}_i, \overline{\mathbf{n}}_{i+\ell_a}) \tag{17}$$

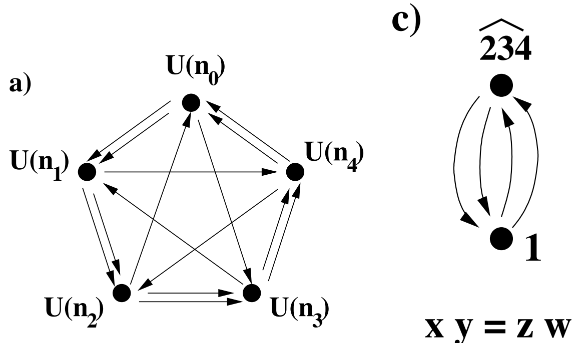
Here the sum over i is understood to be mod N, and n_i means the fundamental of $U(n_i)$.

This is a typical spectrum in this class of models. The matter fields tend to come in bi-fundamentals of the product of gauge groups. These can be arranged into "quiver" diagrams (see figure). These diagrams are made out of one node per group factor, i.e., the i-th node corresponding to the gauge group $U(n_i)$. There are also arrows joining the nodes. An arrow going from the i-th to the j-node correspond to a chiral field in the representation $(n_i, \overline{n}_j)$ (note the orientation). A closed triangle of arrow would indicate the existence of a gauge invariant cubic superpotetial for those fields.



A D3-brane in a quiver node is called a “fractional” D3-brane, and it cannot be taken away from the singularity. This is why two fractional D3-branes in different nodes can host a chiral fermion.

Other examples of local singularities and their quivers include $\mathbb{C}^3/\mathbb{Z}_5$ and the conifold. We will discuss D-branes at conifold singularities in Lecture III.



Back to our example. From the generic chiral spectrum in eqn. (17), we can extract a very simple but powerful conclusion: Only for $\mathbb{Z}_3$ will we get the chiral matter spectrum in three identical copies or families. The reason is that only for that case we have $\ell_1 = \ell_2 = \ell_3$ mod N, since $\ell_1 = \ell_2 = 1$ and $\ell_3 = -2 = 1$ mod 3. Other twists given by $(1/N, 1/N, -2/N)$ will give rise to two families. Therefore three is not only the maximum number of families for this class of models but is obtained only for one twist, the $\mathbb{Z}_3$ twist.

If we want to have the Standard Model we can consider $n_0 = 3$, $n_1 = 2$, $n_2 = 1$ to get the gauge group $U(3) \times U(2) \times U(1)$. The spectrum will then be:

$$3 \times \left[(\mathbf{3}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}) + (\overline{\mathbf{3}}, \mathbf{1}) \right] \tag{18}$$

where we have suppressed the $U(1)$ quantum numbers. This gives the 3 families of left-handed quarks, right-handed up quarks, and leptons, just as in the Standard Model. However, we can easily see that we are missing at least the right-handed down quarks. Actually, the spectrum as it is, is anomalous. What happens from the string theory point of view is that there are uncancelled tadpoles for twisted sector fields. We will take care of this shortly and construct models that are fully consistent, but until then we can explore some general properties of the model as it stands now.

First, there are actually three $U(1)$'s. Only one combination of them is anomaly free and it is defined in general (for any N) by:

$$Q_Y = -\left(\frac{1}{3}Q_3 + \frac{1}{2}Q_2 + \sum_{s=1}^{N-2} Q_i^{(s)}\right) \tag{19}$$

In a general orbifold all other $N-1$ additional $U(1)$ factors are anomalous and therefore massive due to a version of the Green-Schwarz mechanism. The Green-Schwarz mechanism is the cancellation of one-loop diagrams by tree-level diagrams due to the exchange of p-forms. We will discuss the Green-Schwarz mechanism in more detail later. For now, we can quickly check that this $U(1)$ does correspond to the hypercharge, as expected since hypercharge is essentially the only non-anomalous $U(1)$ (other than $B-L$) with the spectrum of the Standard Model. For instance, fields transforming in the $(\mathbf{3}, \mathbf{2})$ representation have Q_Y charge $-\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$ as correspond to left-handed quarks. Fields transforming in the $(\overline{\mathbf{3}}, \mathbf{1})$ (which necessarily have charge $+1$ under one of the $Q_1^{(s)}$ generators) have a Q_Y charge $+\frac{1}{3} - 1 = -\frac{2}{3}$, as corresponds to right-handed U quarks, etc.

It is worth noticing that the normalization of this hypercharge $U(1)$ depends on the order of the twist N. In fact, by normalizing $U(n)$ generators such that $\mathrm{Tr}T_a^2 = \frac{1}{2}$, the normalization of the Y generator is fixed to be

$$k_1 = 5/3 + 2(N-2) \tag{20}$$

This amounts to a dependence on N in the Weinberg angle, namely

$$\sin^2\theta_W = \frac{g_1^2}{g_1^2+g_2^2} = \frac{1}{k_1+1} = \frac{3}{6N-4} \tag{21}$$

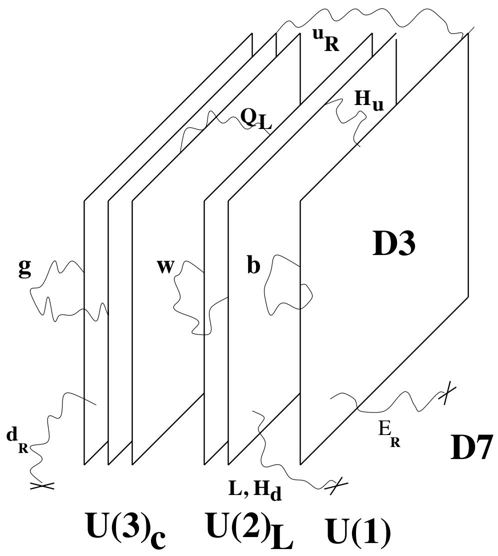
Thus the weak angle decreases as N increases. Notice that the $SU(5)$ result $3/8$ is only obtained for a $\mathbb{Z}_2$ singularity. However in that case the D3-brane spectrum is necessarily vector-like and hence one cannot reproduce the Standard Model spectrum. For the interesting case for us, $N=3$, we find $\sin^2\theta_W = 3/14$.

Now, back to the uncancelled tadpoles. To cancel these tadpoles at the singlarity it is necessary to have not only the D3-branes but also D7-branes. There are three types of D7-branes that can be introduced depending on which 2 of the 3 complex dimensions they contain. The consistency condition (tadpole cancellation) can be written as

$$\mathrm{Tr}\Gamma_{\theta,7_3} - \mathrm{Tr}\Gamma_{\theta,7_1} - \mathrm{Tr}\Gamma_{\theta,7_2} + 3\mathrm{Tr}\Gamma_{\theta,3} = 0 \tag{22}$$

This condition can be obtained by analyzing one-loop open string diagrams with boundaries on various combinations of D3 and D7-branes. More intuitively, we can understand this condition as equivalent to non-Abelian anomaly cancellation in the effective field theory (since $3-7$ strings introduce chiral matter fields charged under the D3-brane gauge groups). Notice that without the D7-branes we could not have the Standard Model on the D3-brane. (e.g., a solution for the 7-brane CP factors is $u_0^r = 0$, $u_1^r = 1$, $u_2^r = 2$ for $r = 1, 2, 3$). Nevertheless, the choice $n_0 = n_1 = n_2 = 3$ gives an anomaly free spectrum even without introducing D7-branes, and so the trinification model with gauge group $U(3)^3$ can be realized.

The D7-branes will have extra gauge groups and matter fields living on the D7-brane which can be obtained in a similar way, with a matrix $\Gamma_{\theta,7_i}$ (with $i = 1, 2, 3$ labeling different D7-branes) acting on the gauge degrees of freedom of the D7-branes. There are also massless matter fields living at the intersection of the D7 and D3-branes, corresponding to open strings with one endpoint on the D3-branes and the other on the D7-branes. This will complete the spectrum of the Standard Model and render the model anomaly free at the singularity. The D7-brane gauge couplings depend on the volumes of the wrapped cycles. If the volumes are large, the D7 gauge groups act essentially as global symmetries. We can picture the Standard Model realized on the D3-D7 system as follows:



As an illustration, a particular example of configuration of D3 and D7 branes and the resulting spectrum is given in Table 1.

A similar model can be constructed choosing $n_0 = 3$, $n_1 = 2$, and $n_2 = 2$ with

$$\Gamma_\theta = \mathrm{diag}\left(I_3, \alpha I_2, \alpha^2 I_2\right) \tag{23}$$

giving rise to a left-right symmetric model with gauge group:

$$U(3) \times U(2)_L \times U(2)_R \tag{24}$$

and three families of chiral matter:

$$3 \times \left[(\mathbf{3}, \mathbf{2}, \mathbf{1}) + (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{2})\right] \tag{25}$$

Matter fields	Q_3	Q_2	Q_1	$Q_{u_1^r}$	$Q_{u_2^r}$	Y
33 sector						
$3(3,2)$	1	-1	0	0	0	1/6
$3(\bar{3},1)$	-1	0	1	0	0	-2/3
$3(1,2)$	0	1	-1	0	0	1/2
$\mathbf{37}_r$ sector						
$(3,1)$	1	0	0	-1	0	-1/3
$(\bar{3},1;2')$	-1	0	0	0	1	1/3
$(1,2;2')$	0	1	0	0	-1	-1/2
$(1,1;1')$	0	0	-1	1	0	1
$\mathbf{7}_r\mathbf{7}_r$ sector						
$3(1;2)'$	0	0	0	1	-1	0

Table 1: Spectrum of $SU(3) \times SU(2) \times U(1)$ model. We present the quantum numbers under the $U(1)^9$ groups. The first three $U(1)$'s come from the D3-brane sector. The next two come from the D7_r-brane sectors, written as a single column with the understanding that e.g. fields in the $\mathbf{37}_r$ sector are charged under the $U(1)$ in the $\mathbf{7_r 7_r}$ sector.

where we suppressed the $U(1)$ charges above. Again, $B-L$ is the only anomaly free $U(1)$, the other $U(1)$'s acquire a mass by the Green-Schwarz mechanism.

It is important to emphasize that this is not the full story for these models. Remember that we are building them step by step from a bottom-up approach. There are further issues involved in constructing compact models. So far we have concentrated only on a singularity in flat space modded out by the action of $\mathbb{Z}_N$. If we compactify the extra dimensions, the total RR charge of the D7-branes has to cancel, since there is no place for the RR flux to escape in the compact space. This will force us to add objects with negative RR charges like anti D7-branes or orientifold planes. For stability reasons, the anti D7-branes have to be separated from the D7-branes or else they will annihilate. They can be placed at different orbifold fixed points, for instance.

An anti D7-brane breaks supersymmetry since it preserves the half of the supersymmetry that the D-brane breaks. Therefore if both are present, the full supersymmetry is broken. If the anti-branes are trapped at different fixed points, then only bulk fields can mediate the breaking of supersymmetry to the observable brane. This is a realization of the gravity mediated SUSY breaking scenario. In order to obtain a realistic spectrum of supersmmetric particles, the scale of SUSY breaking (string scale in this model) is typically the intermediate scale $M_I \sim 10^{11}$ GeV. Therefore, one might realize gauge unification at the string scale and low energy supersymmetry breaking solving the hierarchy problem. (The intermediate scale M_I is also motivated from axion physics).

Finally, let us emphasize again the flexibility of this bottom-up approach in model building. Our discussions can be generalized to F-theory since the D3-brane gauge and matter content depends only on the local geometry. However, F-theory allows for more general D7-brane configurations, e.g., it allows D7-branes carrying both "electric" and "magnetic" charges to coexist in the model (more precisely, they are the so called (p,q) 7-branes). Furthermore, singularities beyond the simplest $\mathbb{Z}_N$ singularities discussed here have been considered in `[Aldazabal, Ibanez, Quevedo, Uranga (2000)]`. These include non-Abelian twists, orientifold singularities, and conifold singularities. For the case of non-Abelian singularities, an interesting model was proposed in `[Berenstein, Jejjala, Leigh (2001); see also Grana (2002)]`. There, a singularity of the type $\mathbb{C}^3/G$ with $G = \Delta_{27}$ was considered. The group $G = \Delta_{27}$ is one of the non-Abelian discrete subgroups of $SU(3)$ and thus preserves SUSY on the D3-brane, like in the $\mathbb{Z}_N$ cases. The Δ_{27} group is actually one of the Δ_{3n^2} series whose action on $\mathbb{C}^3$ is given by:

$$\begin{aligned} e_1 : (z_1, z_2, z_3) &\rightarrow (\omega_n z_1, \omega_n^{-1} z_2, z_3) \\ e_2 : (z_1, z_2, z_3) &\rightarrow (z_1, \omega_n z_2, \omega_n^{-1} z_3) \\ e_3 : (z_1, z_2, z_3) &\rightarrow (z_3, z_1, z_2) \end{aligned} \tag{26}$$

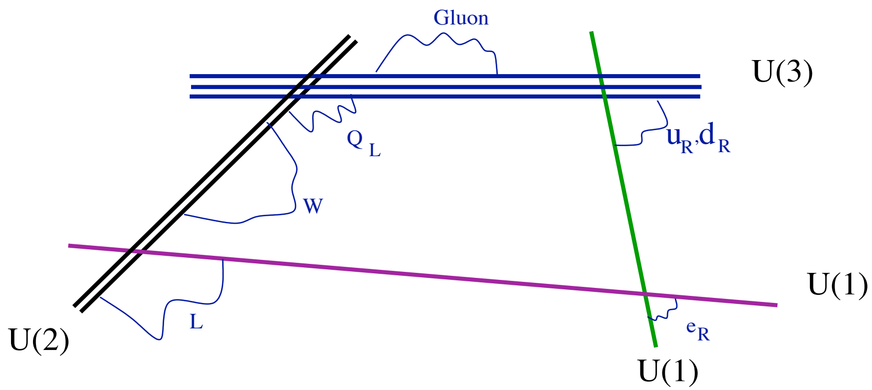
One of the interesting properties of this model is that there is no need to introduce D7-branes to cancel the local tadpoles. The gauge group on the D3-branes is $U(3)^2 \times U(1)^9$ which can further be broken to the Standard Model with three families (due partly to the $\mathbb{Z}_3$ subgroup of Δ_{27}).

Lecture II: D-brane Model Building

5 Intersecting Branes

Another mechanism to obtain $D = 4$ chiral fermions is to consider intersection of branes. We will discuss these models in the Type IIA context, although the issues we discuss (e.g., how chiral fermions are localized at brane intersections) will analogously arise in Type IIB and F theory models as well. Before we discuss all the details and subtleties, let's begin with the overall picture. We know by now that an open string carries indices in the adjoint representation of $U(n)$. The adjoint can be seen as the product of fundamental and anti-fundamental representation, therefore one end of the open string transform as the fundamental and the other as the anti-fundamental. When the two endpoints of the open string lie on the same stack of branes, we have particles like the gauge bosons in the adjoint. But when they lie on different stacks of branes it gives rise to bi-fundamentals. This is what happens at the intersections of two branes. The states corresponding to open strings ending on each of the two branes correspond to bi-fundamentals that can naturally lead to a chiral spectrum.

A way to obtain the Standard Model group and spectrum is to intersect several stacks of branes. One stack of three correspond to the strong interactions, it can intersect with a stack of two D-branes corresponding to $SU(2)_L$. At the intersection, we have then the quark doublets. At a different intersection point the stack of two D-branes will intersect with one brane carrying $U(1)$ and the leptons will be at the intersection and so on. As it turns out, we need to introduce minimally four stacks of branes to fully account for the quantum number of all the Standard Model particles. Pictorially, the building block looks something like this:

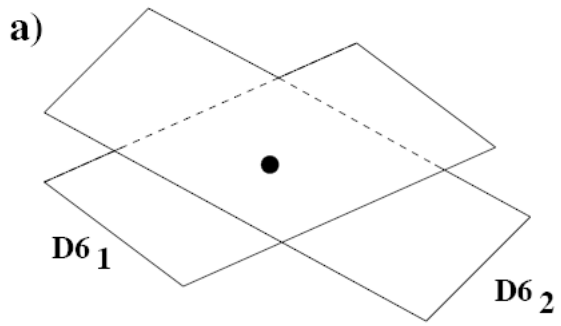


The four stacks of branes are named the baryonic branes, the left brane, the right brane, and the leptonic brane for obvious reasons.

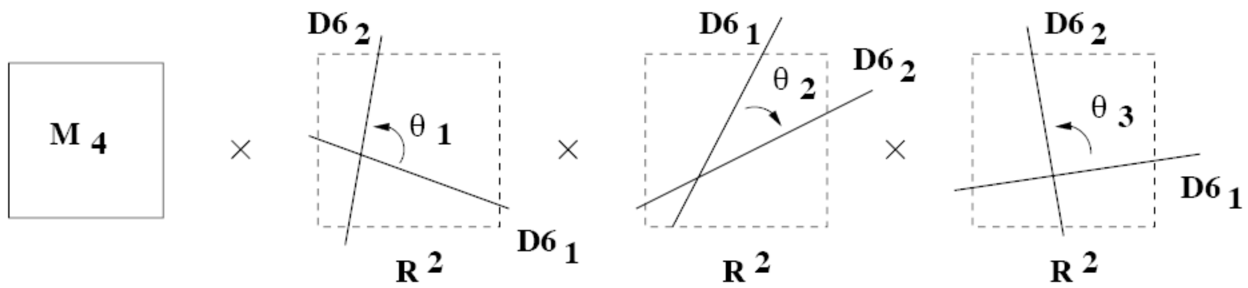
If this is all it takes to construct the Standard Model from intersecting branes, life will be simple. As we now discuss, there are further string theory constraints both in constructing a "local model" and in embedding this setup in a compact setting.

5.1 Local Geometry and Spectrum

The basic configuration of intersecting D-brane models leading to 4D chiral fermions involve two stacks of D6-branes, each spanning our 4D space and three additional real dimensions.



The local geometry is fully specified by the angles of rotations between the branes, which can be depicted as follows:



As we discussed, chiral fermions in bi-fundamental representations are localized at the intersection of the brane worldvolumes, which is our usual 4D space. The appearance of chirality can be understood from the fact that the geometry of the two D-branes introduces a preferred orientation in the 6D space. We can see this by considering the relative rotation of the second D6-brane with respect to the first. This also explains why we consider configuration of D6-branes (and not other types of branes, like D4 and D5, etc). D6-branes are the only type of branes that intersect at a point (rather than a line or a surface) and so their intersection defines a particular orientation in 6D, giving rise to chiral fermions.

The open string spectrum can also be obtained easily. In fact, one can quantize open strings in this intersecting brane background and obtain the full string spectrum and not only the massless states but we will skip over these details. As far as massless states go, the open strings ending on the same stack of D-branes provide the $U(N)$ gauge bosons, three real adjoint scalars and their superpartners propagating over the 7D worldvolume of the D6-branes. The open strings stretching between different kinds of branes lead to a 4D chiral fermion transforming in the bi-fundamental representation and localized at the intersection. The chirality is encoded in the orientation defined by the intersection.

This last point requires some elaboration. Notice that:

- Two intersecting D6-branes define 3 angles:

$$\vec{\theta}^{ab} = (\theta_1^{ab}, \theta_2^{ab}, \theta_3^{ab}) \tag{27}$$

This is because each 3-plane has an orientation, which differentiates between a D6-brane and an $\overline{D6}$-brane. Under a π rotation of any of these angles, a D6-brane becomes an $\overline{D6}$-brane:

$$\begin{array}{ccccc} (\theta_1^a, \theta_2^a, \theta_3^a) & \to & (\theta_1 + \pi, \theta_2^a, \theta_3^a) & \to & (\theta_1^a + \pi, \theta_2^a + \pi, \theta_3) \\ D6 & & \overline{D6} & & D6 \end{array} \tag{28}$$

- We can always choose $-\pi \leq \theta_i^{ab} \leq \pi$. Then if $\theta_i^{ab} \neq 0, \pm\pi$,

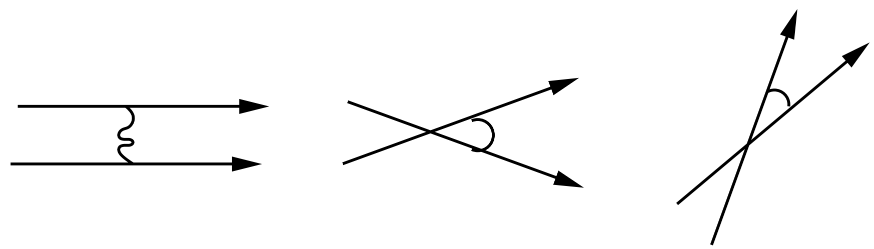
$$\vec{\theta}^{ab} = -\vec{\theta}^{ba} \tag{29}$$

and so

$$\epsilon^{ab} \equiv \mathrm{sign}(\theta_1^{ab}\theta_2^{ab}\theta_3^{ab}) = -\epsilon^{ba} \tag{30}$$

is a well defined quantity.

- If some $\theta_i^{ab} = 0$ or π, then ϵ^{ab} is not well defined, but the system is non-chiral since one can separate the branes.



- If all $\theta_i^{ab} \neq 0, \pi$, then the intersection cannot be removed by deforming the D6-branes. Therefore, there could be a chiral fermion at the intersection.

- It was shown by `[Berkooz, Douglas, Leigh (hep-th/9606139)]` that there is indeed an $D = 4$ chiral fermion, of chirality ϵ^{ab}, at the intersection, as well as some light scalars (also in bi-fundamental reps.) whose masses depend on the relative angles:

$$\begin{array}{ll} \frac{1}{2\pi}(\theta_1 + \theta_2 + \theta_3) & \frac{1}{2\pi}(-\theta_1 + \theta_2 - \theta_3) \\ \frac{1}{2\pi}(\theta_1 - \theta_2 - \theta_3) & 1 - \frac{1}{2\pi}(-\theta_1 - \theta_2 + \theta_3) \end{array} \tag{31}$$

  These light scalars can be massless, massive or tachyonic depending on the relative angles. This point will become clear as we analyze the SUSY preserved by the branes.

- Notice that:

  – An open string from a to b has quantum number $(N_a, \overline{N}_b)$ and chirality ϵ^{ab}.

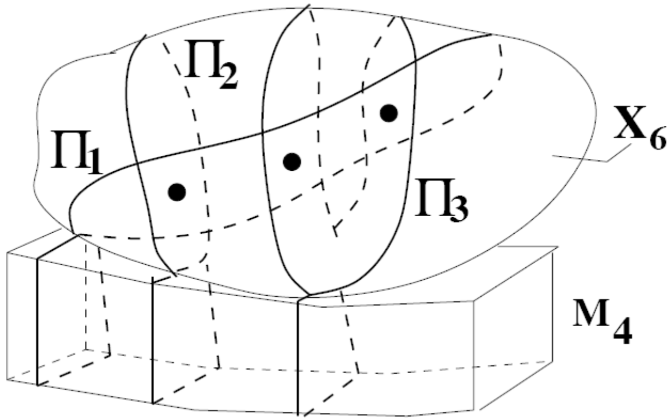
– An open string from b to a has quantum number $(\overline{N}_a, N_b)$ and chirality $-\epsilon^{ab}$.

They are anti-particles of one another and together gives the two fermionic degrees of freedom corresponding to one chiral Weyl fermion from a 4D spacetime point of view.

5.2 Compactification

Although intersecting D6-branes provide a mechanism to obtain 4D chiral fermions, the gauge bosons can propagate in the entire worldvolume of the D6-branes and so the gauge interactions remain 7D. Likewise, the gravitational interactions remain 10D before compactification. So, let us introduce the intersecting D-branes in a compact setting.

The general kind of configurations we will consider is string theory on a spacetime of the form $M_4 \times X_6$ where X_6 is compact.



The D6-branes are space-filling and wrap 3-cycles of the compact space. The new feature is that two 3-cycles in the compact space intersect several times, leading to replicated families of chiral fermions.

- Consider N_a D6-branes on $\mathcal{M}_4 \times \Pi_3^a$ and N_b D6-branes on $\mathcal{M}_4 \times \Pi_3^b$, we have
  - $U(N_a) \times U(N_b)$ gauge group
  - One chiral fermion in $(N_a, \overline{N}_b)$ representation at each intersection

  since we have locally the setup of flat space.

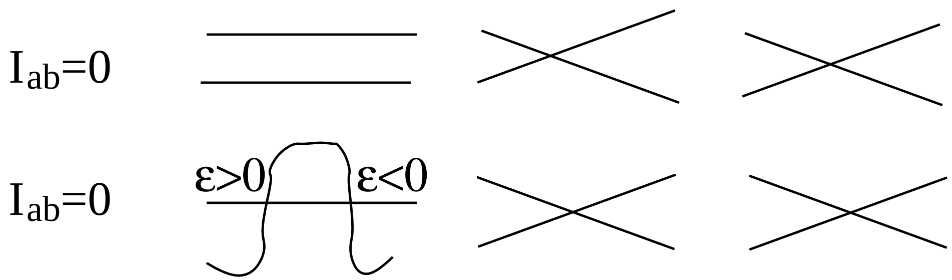
- Now, different intersections may have different ϵ^{ab}'s, so different chiralities. The gauge protected quantity is the net number of chiral fermions (say left-handed):

$$(\#\text{Intersections with } \epsilon^{ab} > 0) - (\#\text{Intersections with } \epsilon^{ab} < 0) \tag{32}$$

- The above is a topological quantity, known as the intersection number of two 3-cycles:

$$I_{ab} = [\Pi_a] \cdot [\Pi_b] \tag{33}$$

- I_{ab} is topological because it does not depend on the specific embedding of Π_a, only on the topology (more precisely, the homology class $[\Pi_a]$). It doesn't change as we deform the background geometry or the D-branes:



- The spectrum is thus:
  - Gauge Group: $\Pi_a U(N_a)$
  - Left chiral Fermions: $\sum_{a,b} I_{ab}(N_a, \overline{N}_b)$

where in our convention, $I_{ab} < 0$ means right-handed chiral fermions. The possibility of $I_{ab} \neq 0, 1$ gives rise to an interesting mechanism of family replication.

5.3 Supersymmetry for intersecting branes

Thus, we have seen intersecting D6-branes realize a particular brane world scenario where:

- Gravity propagates in 10D
- Gauge bosons propagate in 7D
- Chiral matter propagates in 4D

So, in principle, we can consider both high and low string scale scenarios. In the former case, SUSY at the string scale helps to protect the Higgs mass from large radiative corrections. In the latter, one can consider the possibility of breaking SUSY at the string scale. We will focus here on $\mathcal{N} = 1$ models whose effective theory is better understood. SUSY models have the advantage that they are free of tachyons and the brane configurations are stable. We will consider metastable SUSY breaking in next lecture.

So, let us analyze the conditions for the D6-brane configuration to preserve SUSY. A D-brane preserves 1/2 of the supersymmetries but two D-branes can a priori preserve a different SUSY. The general condition for the D6-branes to preserve SUSY is that the three-cycles on which they wrap are special Lagrangian (sLag) defined as follows.

On a Calabi-Yau manifold there exist a covariantly constant holomorphic three-form, Ω_3, and a Kähler 2-form J. Locally, the holomorphic 3-form Ω_3 and the Kähler form J can be defined by

$$\Omega_3 = dz_1 \wedge dz_2 \wedge dz_3, \quad J = i \sum_{i=1}^{3} dz_i \wedge d\bar{z}_i. \tag{34}$$

A three-cycle π_a is called Lagrangian if the restriction of the Kähler form on the cycle vanishes

$$J|_{\pi_a} = 0. \tag{35}$$

If the three-cycle in addition is volume minimizing, which can be expressed as the property that the imaginary part of the three-form Ω_3 vanishes when restricted to the cycle,

$$\Im(e^{i\varphi_a}\,\Omega_3)|_{\pi_a} = 0, \tag{36}$$

then the three-cycle is called a sLag cycle. The parameter φ_a is a constant over the 3-cycle and determines which $\mathcal{N} = 1$ SUSY is preserved by the brane. Thus, different branes with different values for φ_a preserve different $\mathcal{N}=1$ SUSY. One can show that (36) implies that the volume of the three-cycle is given by

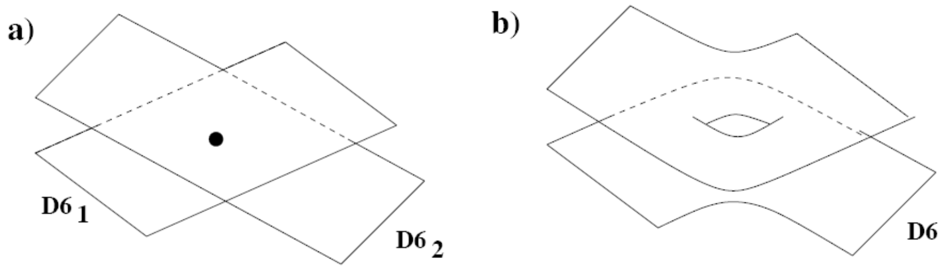
$$\mathrm{Vol}(\pi_a) = \left| \int_{\pi_a} \Re(e^{i\varphi_a}\,\Omega_3). \right| \tag{37}$$

A shift of $\varphi_a \to \varphi_a + \pi$ corresponds to exchanging a D-brane by its anti-D-brane, where the D-brane really satisfies (37) without taking the absolute value. Therefore a supersymmetric cycle π_a is calibrated with respect to $\Re(e^{i\varphi_a}\Omega_3)$. To obtain a globally $\mathcal{N}=1$ SUSY intersecting D-brane model all D6-branes have to wrap sLag three-cycles which are calibrated with respect to the same three-form.

Exercise: Show that for a torus, the condition that π_a is sLag reduces to $\theta_1+\theta_2+\theta_3=0$.

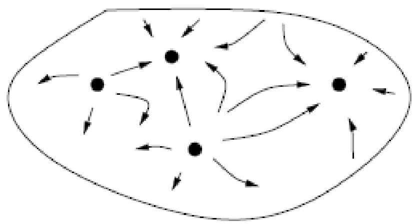
As mentioned, the light scalars at the intersection can be massless, massive or tachyonic:

- The massless case corresponds to a situation with some unbroken supersymmetry. The massless scalar is a modulus whose vacuum expectation value parametrizes the possibility of recombining the two intersecting D-branes into a single smooth one.



- The configuration with tachyonic scalars corresponds to situations where this recombination is triggered dynamically, as a result of tachyon condensation at the intersection.
- The configuration where all light scalars have positive squared mass corresponds to a non-supersymmetric configuration, which is nevertheless dynamically stable against recombination. Namely, the recombined 3-cycle has a volume larger than the sum of the volumes of the intersecting 3-cycles.

5.4 RR tadpole Cancellation



However, compactification also leads to new subtleties. This is because D-branes act as a source for RR fields via the disk coupling:

$$\int_{W^{p+1}} C_{p+1} \tag{38}$$

In a compact space, the total RR charge must vanish as required by Gauss's law.

The RR charges are characterized by the 3-cycles on which the branes wrap. Hence, the sum of the homological cycles wrapped by all the branes must be topologically trivial.

$$[\pi_{total}] = \sum_a N_a [\pi_a] = 0 \tag{39}$$

Equivalently, one can understand this constraint as the consistency requirement of the equations of motion for the RR fields. Keeping only terms in the spacetime action which depend on the RR 7-form C_7, we have:

$$\begin{aligned} S_{C7} &= \int_{M_4 \times X_6} H_8 \wedge *H_8 + \sum_a N_a \int_{M_4 \times \pi_a} C_7 \\ &= -\int_{M_4 \times X_6} C_7 \times dH_2 + \sum_a N_a \int_{M_4 \times X_6} C_7 \wedge \delta(\pi_a) \end{aligned} \tag{40}$$

where H_8 is the 8-form field strength, H_2 its Hodge dual, and $\delta(\pi_a)$ is a bump 3-form localized on the 3-cycle. The equation of motion reads:

$$dH_2 = \sum_a N_a \delta(\pi_a) \tag{41}$$

Integrating this equation gives the tadpole constraint Eq. (39) on the homology classes.

5.5 Anomaly Cancellation

Cancellation of RR tadpoles in the underlying string theory is important because it implies the cancellation of chiral anomalies in the four dimensional effective field theory. These anomalies include the cubic non-Abelian anomalies, and mixed $U(1)$ non-Abelian anomalies, and mixed gravitational anomalies. Let's discuss them one by one.

1. **Cubic non-Abelian anomalies**

   The $SU(N_a)^3$ anomaly is proportional to the number of fundamental minus anti-fundamental representations of $SU(N)$. Since the matter fields transform as bifundamentals, the cubic anomaly is proportional to the sum of the intersection number times the number of branes:

   $$A_a = \sum_b I_{ab} N_b \tag{42}$$

   It is easy to check that this vanishes due to RR tadpole cancellation. By taking the intersection of the tadpole condition with any 3-cycle, we find

   $$0 = [\pi_a] \circ \sum_b N_b [\pi_b] = \sum_b N_b I_{ab} \tag{43}$$

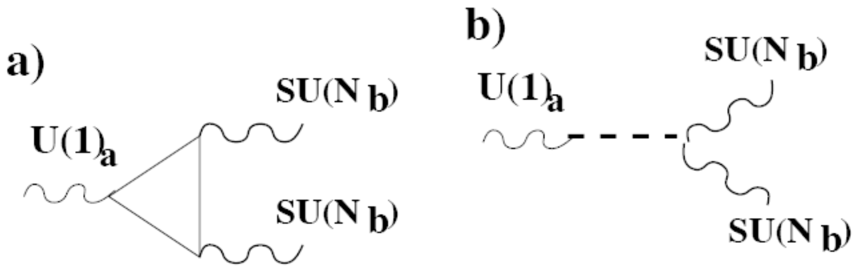
as claimed. Note that the tadpole condition is slightly stronger than the absence of cubic anomalies because it must hold even for $N = 1, 2$ where no cubic anomaly exists. This observation will turn out important in phenomenological model building.

2. **Mixed anomalies**

What about mixed $U(1)_a - SU(N_b)^2$ anomalies? The usual field theory triangle diagram gives a non-zero contribution, even after using the RR tadpole conditions:

$$A_{ab} \simeq N_a I_{ab} \tag{44}$$

However, in string theory there is an extra diagram known as the Green-Schwarz diagram, where the $U(1)$ gauge boson mixes with a 2-form which subsequently couples to two gauge bosons of $SU(N)$:



Exercise: Show that the following couplings arising in the KK reduction of the D6-brane worldvolume action

$$N_a \int_{D6_a} C_5 \wedge trF_a, \qquad \int_{D6_b} C_3 \wedge trF_b^2 \tag{45}$$

give rise to the Green-Schwarz term which leads to a cancellation of the residual field theory triangle anomalies.

Solutions: Introducing a basis of 3-cycles $[\Lambda_k]$ and its dual $[\Lambda_{\tilde{\ell}}]$, we can define the KK reduced 4d fields

$$(B_2)_k = \int_{[\Lambda_k]} C_5, \qquad \phi_{\tilde{\ell}} = \int_{[\Lambda_{\tilde{\ell}}]} C_3 = -\delta_{k\tilde{\ell}} *_{4d} (B_2)_k \tag{46}$$

The KK reduced 4d couplings read

$$N_a q_{ak} \int_{4d} (B_2)_k trF_a, \qquad q_{b\tilde{\ell}} \int_{4d} \phi_{\bar{\ell}} trF_b^2 \tag{47}$$

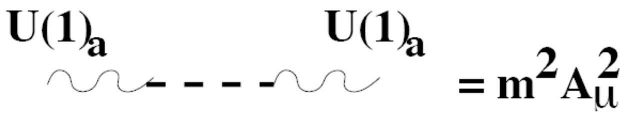
with $q_{ak} = [\pi_a] \circ [\Lambda_k]$, and similarly for $q_{b\tilde{\ell}}$. The total amplitude is proportional to

$$A_{ab}^{GS} = -N_a \sum_k q_{ak} q_{b\bar{\ell}} \delta_{k\tilde{\ell}} = \ldots - N_a I_{ab} \tag{48}$$

leading to cancellation between both kinds of contributions.

3. **Mixed gravitational anomalies:** It is left as an exercise to show that for toroidal models, the mixed gravitational anomalies cancel automatically, without the Green-Schwarz contribution. **(Exercise)** This is no longer true for orbifolds/orientifolds.

An important observation is that any $U(1)$ gauge boson with $B \wedge F$ coupling gets massive, with mass roughly of the order of the string scale (more precisely, the mass M_A of the $U(1)$ is g_s suppressed, i.e., $M_A \sim g_s M_s$).



There can be several of them, and they are not necessarily anomalous. Such $U(1)$'s disappear as gauge symmetries from the low energy effective theory, but remains as global symmetries, unbroken in perturbation theory. In constructing D-brane Standard Model, we need to make sure that the candidate for the hypercharge is not one of these massive $U(1)$.

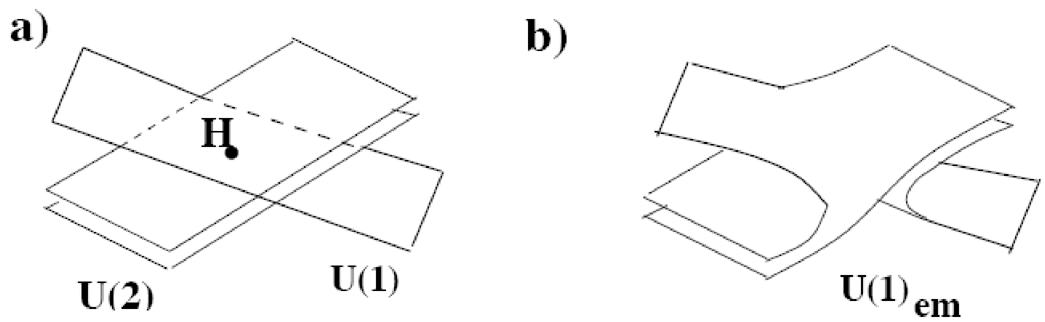
5.6 Phenomenological features

Before we construct more realistic examples, let us briefly mention some phenomenological features we can extract from the examples studied so far. These features are natural in the general setup of intersecting brane models and are not restricted only to toroidal models.

- **Proton Stability:** First of all, in these models, the proton is perturbatively stable. This is because the $U(1)$ within the $U(3)$ plays the role of baryon number, and is preserved as a global symmetry, unbroken in perturbation theory though it can be broken by non-perturbative instanton effects. In some cases, the instanton effects breaking the Standard Model baryon number is calculable, e.g., as a Euclidean D2-brane wrapped on 3-cycles.

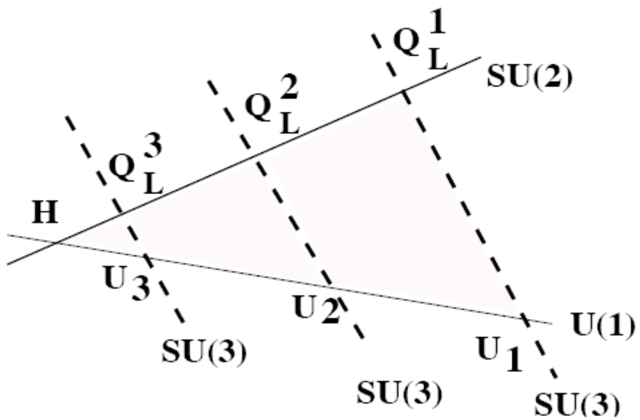
- **Gauge Unification:** These models do not have a natural gauge coupling unification, even at the string scale. Each gauge factor has a gauge coupling controlled by the volume of the wrapped 3-cycle. Gauge couplings are related to geometric volumes, hence the moduli controlling the sizes of these volumes are constrained by experiments.

- **Electroweak Symmetry Breaking:** These exists a geometric interpretation for the spontaneous electroweak symmetry breaking. In explicit models, the Higgs particle arises from the light scalar at the intersections, whose vev parametrizes the possibility of recombining two intersecting branes into a single smooth one.



In this process, the gauge symmetry is reduced, corresponding to a Higgs mechanism in the effective field theory.

- **Yukawa Couplings:** There is a natural way to generate exponential hierarchy. Yukawa couplings among the scalar Higgs and chiral fermions arise at tree level in the string coupling from open string worldsheet instantons, namely from string worldsheet spanning the triangle with vertices at the intersections and sides on the D-branes.



Their values are exponentially suppressed by the area of the string worldsheet in string units. Different families are located at different intersections, leading to an exponential hierarchy in the Yukawa couplings between different families.

5.7 Orientifolding

It turns out what we have introduced so far are not yet sufficient to construct a realistic model. We need an extra element, namely, the orientifold planes.

So, why orientifolds? We have already seen that the total RR charge in a compact space must vanish[2]. So, we need objects with negative charge. In order to preserve SUSY, these objects must also

[2]The discrete K-theory charges must also cancel but we will ignore this subtlety for now.

carry negative tension. An orientifold plane has precisely these properties. Another reason to consider orientifolds is that as it turns out, intersecting D-brane models without orientifold planes are bounded to contain chiral exotics.

An orientifold projection flips the worldsheet orientation and reflects the spacetime coordinates. An O-plane corresponds to the invariant subspace. More precisely, an O6-plane can be defined by the quotient $\Omega\overline{\sigma}(-1)^{F_L}$ where $\overline{\sigma}$ is an antiholomorphic $\mathbf{Z}_2$ involution which acts on the Kahler class J and the holomorphic 3-form Ω_3 as:

$$\overline{\sigma}J = -J \qquad \overline{\sigma}\Omega_3 = \pm\overline{\Omega}_3 \tag{49}$$

The O6-plane is localized at the fixed point of $\overline{\sigma}$ which is a three-cycle Π_{O6}.

The presence of the O-planes introduces the following changes to the gauge and matter content of the model:

- For each D6-brane on Π_a, we need to add the orientifold image wrapping $\overline{\sigma}(\Pi_a)$. There are two possibilities:
  - $\overline{\sigma}(\Pi_a) \neq \Pi_a$, then the gauge groups $U(N_a)$ of both D6-branes are identified.
  - $\overline{\sigma}(\Pi_a) = \Pi_a$, then the gauge group is broken to $SO(N_a)$ or $USp(N_a)$ depending on some subtle choices of sign.

- The matter spectrum also changes since there are chiral multiplets at the intersection of Π_a with $\overline{\sigma}(\Pi_b)$; Π_a with $\overline{\sigma}(\Pi_a)$ transforming in (N_a, N_b), symmetric/antisymmetric representations respectively.

Skipping some details, this is the chiral spectrum of the orientifold model.

Representation	Multiplicity
$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a$	$\frac{1}{2}\left(\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a\right)$
$\Box\Box_a$	$\frac{1}{2}\left(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a\right)$
$(\overline{\Box}_a, \Box_b)$	$\pi_a \circ \pi_b$
$(\Box_a, \Box_b)$	$\pi'_a \circ \pi_b$

5.8 Getting just the Standard Model

In addition to their importance in canceling the D-brane charges, there is in fact a general argument which shows that in the absence of O-planes, D-brane models necessarily contain $SU(2)$ chiral exotics. To see this, first notice that without the O-planes, the electroweak $SU(2)$ must belong to a $U(2)$ gauge factor. An important point emphasized earlier is that the tadpole condition implies that the number of fundamentals and anti-fundamentals must be equal, even for $U(2)$ (where the 2 and the $\overline{2}$

are distinguished by their $U(1)$ charge). Now, since there are 3 families and the left-handed quarks transform as $(3, \overline{2})$, they contribute altogether 9 anti-fundamentals of $SU(2)$. The complete spectrum must necessarily contain 9 fundamentals, three of which may be interpreted as left-handed leptons; the remaining six doublets are however exotic chiral fermions, beyond the spectrum of the SM. There are two ways to avoid these exotics, both involving orientifolds.

5.8.1 The U(2) case

One possibility is to exploit the fact there are two kinds of bi-fundamental fields in an orientifold model, namely, $(N_a, \overline{N}_b)$ and (N_a, N_b). Consider realizing the three families of left-handed quarks as a combination of $2(3, \overline{2}) + (3, 2)$. The number of $SU(2)$ doublets needed to cancel the tadpoles is 3 which is precisely the number of left-handed leptons.

Exercise: Consider four stacks of branes denoted a, b, c, d (and their images), giving rise to a gauge group $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$. If the intersection numbers between the corresponding 3-cycles are given by:

$$
\begin{aligned}
&I_{ab} = 1 \qquad I_{ab'} = 2 \qquad I_{ac} = -3 \qquad I_{ac'} = -3 \\
&I_{bd} = 0 \qquad I_{bd'} = -3 \qquad I_{cd} = -3 \qquad I_{cd'} = 3
\end{aligned}
\tag{50}
$$

Show that the chiral spectrum has the non-Abelian quantum numbers of the Standard Model (plus three right-handed neutrinos). In order to reproduce exactly the Standard Model one also needs to require that the following linear combination of $U(1)$'s to be massless:

$$Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c + \frac{1}{2}Q_d \tag{51}$$

However, several comments are in order. It is important to emphasize that at this level, we have not constructed any explicit model. In particular, we have only presented a set of intersection numbers. We haven't shown that there exist cycles on which the D-branes wrapped which lead to the given intersection numbers. Moreover, even if we manage to do so, the intersection numbers only define a local model. We still need to introduce additional hidden sector branes to cancel the tadpoles in constructing compact models. Despite these caveats we made, it was shown that such topological data can indeed arise in some Calabi-Yau orientifolds and Gepner constructions.

5.8.2 The USp(2) Case

Another way to avoid the $SU(2)$ exotics is to make use of the fact that $D6$-branes in the presence of orientifold planes can give rise to symplectic groups. For symplectic groups, all representations are real, and RR tadpole conditions do not impose any constraint on the number of doublets. Since

$USp(2) \equiv SU(2)$, it is possible to realize the electroweak $SU(2)$ as a symplectic group, and thus circumvent the constraints on the number of doublets.

Again, as an exercise, you can show that the following set of branes with gauge groups $U(3)_a \times USp(2)_b \times U(1)_c \times U(1)_d$ and intersection numbers

$$
\begin{array}{cccccc}
 & & I_{ab} = 3 & I_{ab'} = 3 & I_{ac} = -3 & I_{ac'} = -3 \\
I_{db} = 3 & I_{db'} = 3 & I_{dc} = -3 & I_{dc'} = 3 & I_{bc} = -1 & I_{bc'} = 1
\end{array}
\tag{52}
$$

give rise to the chiral spectrum of the Standard Model. The $U(1)$ that needs to be massless in order to reproduce the Standard Model hypercharge is

$$
Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d \tag{53}
$$

In fact, explicit models with D6-branes on 3-cycles and these intersection numbers have been constructed. See Marchesano and Shiu, `hep-th/0408059, hep-th/0409132`. **Exercise:** Construct other compact $\mathcal{N} = 1$ examples with the gauge and matter content of the (supersymmetric) Standard Model.

5.9 M Theory Lift

The only branes we introduced in these models are D6-branes and O6-planes. They are special because they correspond to pure geometry at strong coupling (unlike other branes which carry additional sources, e.g., M-branes or G-flux). Therefore from the number of supercharges the background preserves, the $\mathcal{N} = 1$ SUSY intersecting brane models are expected to lift up to M theory compactifications on singular G_2 manifolds. This is an interesting subject which I have no time to discuss. For a brief discussion, see my review.

5.10 Summary

I hope these examples suffice to illustrate the concepts we have learned. Much of our discussions so far have centered on the open string sector since we are interested in getting the gauge and chiral spectrum of the Standard Model from string theory. The closed string sector is however also relevant for low energy dynamics, and leads to an additional set of important questions. This is the topic to which we now turn.

Lecture III: Warped Throats & their Field Theory Duals

6 Warped Compactifications

The main such question that we have not yet addressed is the existence of moduli, massless fields with flat potential and hence undetermined vevs. Stabilization of these moduli (namely, providing them with a potential which fixes their vevs and give them masses) is an important question, of immediate relevance to phenomenology and cosmology. We will begin by discussing how turning on fluxes can stabilize moduli. Because the background flux provides a source term to the Einstein equations, its backreaction warps the geometry of spacetime. This allows us to roughly realize the Randall-Sundrum scenario in the context of string theory. We will view these warped backgrounds in two ways: as string compactifications[3] with a stabilized hierarchy, and as holographic duals of strongly coupled field theories. The latter view gives us a tool to study BSM scenarios that involve otherwise uncalculable strong dynamics. To illustrate this point, we will find a gravity dual of metastable SUSY breaking and compute holograhically the visible sector soft terms in a SUSY mediation scenario where the messengers are strongly coupled. If there is time, we will briefly describe a gravity dual of a technicolor-like model.

[3]The effective action for warped compactifications is highly subtle see, e.g., `Shiu, Torroba, Underwood, Douglas, 0803.3068`. In lack of time, we will not discuss this interesting topic.

6.1 Moduli Stabilization

Flux compactification has developed into a very broad subject. Here, we will focus on aspects which are relevant for us. General discussions can be found in some recent reviews (e.g., [Douglas and Kachru, arXiv:hep-th/0610102]). To be concrete, let us consider Type IIB string theory in the presence of NSNS and RR 3-form fluxes H_3 and F_3. These 3-form fluxes must satisfy the Bianchi identity

$$dF_3 = 0 \qquad dH_3 = 0 \tag{54}$$

and they should be properly quantized, namely, for any 3-cycle $\Sigma \subset X_6$

$$\frac{1}{(2\pi)^2\alpha'}\int_\Sigma F_3 \in \mathbb{Z} \qquad \frac{1}{(2\pi)^2\alpha'}\int_\Sigma H_3 \in \mathbb{Z} \tag{55}$$

An immediate consequence is that these background fluxes induce an effective D3 charge through the Chern-Simons terms in the Type IIB effective action:

$$\int_{M_4\times X_6} H_3 \wedge F_3 \wedge C_4 \tag{56}$$

where C_4 is the IIB self-dual 4-form gauge potential. This coupling implies that the flux background contributes to a tadpole for C_4, and hence the 3-form fluxes carry D3-brane charges. With proper normalization factor restored, the flux-induced D3 charge is:

$$N_{flux} = \frac{1}{(4\pi^2\alpha')^2}\int_{X_6} H_3 \wedge F_3 = \frac{i}{(4\pi^2\alpha')^2}\int_{X_6}\frac{G_3\wedge\overline{G}_3}{2\mathrm{Im}\tau} \tag{57}$$

where $\tau = a + i/g_2$ is the IIB complex dilaton, and we complexify the 3-form fluxes

$$G_3 = F_3 - \tau H_3 \tag{58}$$

for convenience. The kinetic term for G_3 is

$$V = \frac{1}{4\kappa_{10}^2\mathrm{Im}\tau}\int_{X_6} d^6y G_3 \wedge *_6\overline{G}_3 \tag{59}$$

which induces a scalar potential. After some algebra, it can be written as:

$$V = \frac{1}{2\kappa_{10}^2\mathrm{Im}\tau}\int_{X_6} d^6y\ G_3^- \wedge *_6\overline{G}_3^- - \frac{i}{4\kappa_{10}^2\mathrm{Im}\tau}\int_{X_6} d^6y\ G_3\wedge\overline{G}_3 \tag{60}$$

Here $G_3^{\pm}$ is the imaginary self-dual/anti-self-dual (ISD/IASD) part of G_3, i.e.,

$$*_6 G_3^{\pm} = \pm i G_3^{\pm} \tag{61}$$

The second term in the potential is a topological term proportional to N_{flux}. They will be canceled by other sources of RR charges in a compact model. The first term is positive definite and precisely vanishes if the flux is imaginary self dual. Thus, the equation of motion imposes self-duality of the

3-form flux. The ISD condition should not be regarded as an additional constraint on the flux but rather a condition on the moduli. For fixed flux quanta, only when the ISD condition is satisfied would the moduli minimize the potential.

In 4D EFT language, the potential V above can be derived as an F-term from the Gukov-Vafa-Witten superpotential:

$$W = \int_{X_6} G_3 \wedge \Omega \tag{62}$$

which depends only on the complex structure (shape) moduli and the dilaton and vanishes if these moduli are chosen such that G_3 is ISD. Thus all such moduli are generically stabilized. Ω here is the holomorphic 3-form. We can decompose the 3-form flux G_3 in terms of the Hodge cohomology according to the complex structure of the Calabi-Yau. The ISD condition implies that G_3 consists of only $(2,1)$ and $(0,3)$ forms. The $(2,1)$ component of the flux preserves $\mathcal{N} = 1$ SUSY whereas the $(0,3)$ component breaks SUSY while preserving the no-scale structure. We can see this from the F-term equation for the Kahler modulus ρ

$$0 = D_\rho W \sim W = \int G_3 \wedge \Omega \Rightarrow G = G^{(2,1)} \tag{63}$$

Indeed, $G^{(0,3)}$ induces a gravitino mass[4] of the order:

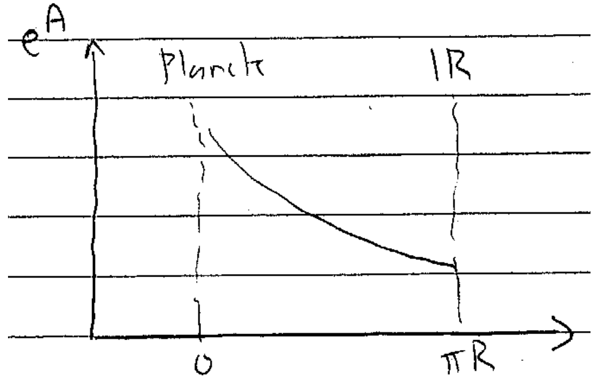
$$m_{3/2}^2 \sim \frac{|G_3 \wedge \Omega|^2}{Im\tau Vol(M_6)^2} \tag{64}$$

6.2 Warped Throats

Besides stabilizing moduli, another appealing feature of fluxes is that they backreact on the metric leading to a non-trivial warp factor, and strongly warped throats can be generated. These warped geometries appear not only in the Randall-Sundrum scenario, they are heavily used in the construction of string inflationary scenarios and in finding holographic duals of strongly coupled gauge theories.

With these motivations in mind, we now discuss how this picture:

[4] The factors in the denominator come from $e^{\mathcal{K}}$ where $\mathcal{K}$ is the Kahler potential.



can be roughly realized in string theory. Unlike the Randall-Sundrum scenario, the bulk will be SUSY. We will, however, put SUSY breaking on the IR brane eventually.

6.3 A Confining Gravity Dual

To realize this picture, we need a background with both an IR cutoff and a UV cutoff. As we have seen in Eduardo's lectures, the energy scale redshifts as the warp factor along the holographic direction. An IR cutoff on the the warp factor implies a minimal energy scale. The interpretation is that the field theory dual develops a mass gap at low energies. Likewise, a UV cutoff implies a maximal energy scale, beyond which the field theory is coupled to strings and quantum gravity. The "UV brane" can therefore be thought of as summarizing the rest of the compact geometry.

6.3.1 AdS/CFT at the Conifold

We already know, by considering N D3-branes at a smooth part, that $\mathcal{N} = 4$ SYM is dual to string theory on $AdS_5 \times S_5$. The $\mathcal{N} = 4$ SYM can be seen from the open string perspective as the low energy degrees of freedom living on the branes, while $AdS_5 \times S_5$ is the near horizon limit of the closed string background.

How do we get other examples? Any smooth point looks the same in the near horizon limit. So, we must place D3-branes at a singular point!

Perhaps the simplest next case is to consider D3-branes at orbifold singularities (like what we did in the first lecture), but more useful for us will be the conifold which can be defined as a submanifold of $\mathbb{C}^4$:

$$\sum_{i=1}^{4} z_i^2 = 0 \tag{65}$$

The conifold singularity arises in many compact Calabi-Yau spaces. We can see that it is singular at $(z_1, z_2, z_3, z_4) = 0$ because the normal space is ill defined.[5] We can picture the conifold singularity as a cone over $S^3 \times S^2$. The fact that it is a cone is obvious because if z_i satisfy the above equation, so does λz_i for any complex constant λ. To identify the base of the cone, we can intersect it with a

[5]In general, a variety defined by $f(z_i) = 0$ in $\mathbb{C}^4$ is singular at $f = df = 0$ since the normal space (associated with df) is ill defined.

seven-sphere in $\mathbb{R}^8$: $\sum_i |z_i|^2 = r^2$. At the singular point, both the S^3 and S^2 shrink to zero size.

To make the translation to the field theory dual more direct, let us do a linear change of variables. We can describe the conifold as:

$$z_1 z_2 - z_3 z_4 = 0 \tag{66}$$

This equation can be "solved" by writing:

$$z_1 = A_1 B_1 \ , \quad a_2 = A_2 B_2 \ , \quad z_3 = A_1 B_2 \ , \quad z_4 = A_2 B_1 \tag{67}$$

But we get the same z_i if we act by:

$$A_k \to \lambda A_k \ , \quad B_\ell \to \lambda^{-1} B_\ell \ , \quad k, \ell = 1, 2 \tag{68}$$

where $\lambda \in \mathbb{C}^*$. The $SO(4) = SU(2) \times SU(2)$ symmetry of the conifold is easy to describe in this formulation: one acts on the A_k, and one on the B_ℓ. If we write,

$$\lambda = s e^{i\alpha} \ , \quad s \in \mathbb{R}^+, \ \alpha \in \mathbb{R} \tag{69}$$

then away from the singular point $z_i = 0$, s can be selected to set

$$|A_1|^2 + |A_2|^2 = |B_1|^2 + |B_2|^2 \tag{70}$$

To get the conifold, we must divide by the $U(1)$:

$$A_k \to e^{i\alpha} A_k \ , \quad B_\ell \to e^{-i\alpha} B_\ell \ , \tag{71}$$

since it keeps the coordinates z_i unchanged.

6.3.2 Dual CFT to N D3 at Conifold (Klebanov-Witten)

We are now ready to find the field theory dual corresponding to placing N D3-branes at the conifold singularities (`Klebanov, Witten, hep-th/9807080`). Consider the $\mathcal{N} = 1$ SUSY gauge theory with $U(1)$ gauge group and the following matter content:

Field	**Charge**
$A_{1,2}$	+1
$B_{1,2}$	-1

The D-term equation for this theory is:

$$D = |A_1|^2 + |A_2|^2 - |B_1|^2 - |B_2|^2 = 0 \tag{72}$$

Then the moduli space is given by setting $D = 0$ and dividing by the $U(1)$:

$$\mathcal{M}_{\text{vacua}} = \{D = 0\}/U(1) \tag{73}$$

So this gauge theory gives the confiold as its moduli space of vacua.

However, this cannot be the whole story. This $U(1)$ is Higgsed, but a D3-brane should have an unbroken $U(1)$ on its worldvolume. We really should introduce another $U(1)$ All chiral multiplets

Field	$U(1)$	$U(1)$
$A_{1,2}$	$+1$	-1
$B_{1,2}$	-1	$+1$

are neutral under the diagonal $U(1)$, which thus decouples and is the expected unbroken $U(1)$ on the 3-brane worldvolume. The other $U(1)$ is the same as before. Thus this SUSY gauge theory describes a single D3-brane placed at the conifold.

What about the theory corresponding to N D3-branes? The natural guess is that the gauge group should be $U(N) \times U(N)$ with chiral fields:

Field	**U(N)**	**U(N)**
$A_{1,2}$	$\mathbf{N}$	$\overline{\mathbf{N}}$
$B_{1,2}$	$\overline{\mathbf{N}}$	$\mathbf{N}$

Here, the $SU(2) \times SU(2)$ and $U(1)_R$ quantum numbers of these fields are suppressed. Naively, a renormalizable superpotential is not possible, so as a first guess we suppose $W = 0$. We can think of A_k and B_ℓ as $N \times N$ matrices and in some basis, they are diagonal with distinct eigenvalues, corresponding to a family of vacua parametrized by the positions of N D3-branes at distinct points on the conifold. The gauge group is broken down to $U(1)^N$, one factor of $U(1)$ for each 3-brane. But this cannot be the whole story, since there exists massless charged chirals which should not be present for N D3-branes at generic smooth points. So we need a superpotential!

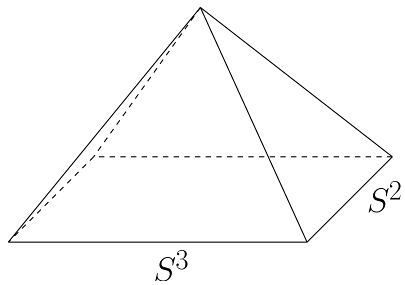
The lowest order single trace $SU(2) \times SU(2)$ invariant is:

$$W = \frac{\lambda}{2}\epsilon^{ij}\epsilon^{k\ell}\mathrm{Tr}A_i B_k A_j B_\ell \tag{74}$$

This does the job. The A and B both have R-charge 1/2 and so the superpotential has R-charge 2 as desired. The CFT is strongly coupled. The conformal dimension of A and B are 3/4 and not 1 as they receive large anomalous dimension.

6.3.3 Perturbing to get a confining theory (Klebanov-Strassler)

Since the conifold is a cone over $S^3 \times S^2$, we can consider in addition to N D3s, having M D5s on S^2.



The resulting gauge theory is:
with the same superpotential as before. What about its dynamics?

QFT side: Cascade of Seiberg dualities

Field	SU(N+M)	SU(N)
$A_{1,2}$	**N+M**	$\overline{\mathbf{N}}$
$B_{1,2}$	$\overline{\mathbf{N+M}}$	**N**

Recall that Seiberg duality is the statement that an $SU(N_c)$ theory with N_f flavors at strong coupling can be transformed to a weakly coupled dual $SU(N_f - N_c)$ theory with N_f flavors plus some mesons:

$$\begin{array}{ccc} SU(N_c) & \longrightarrow & SU(N_f - N_c) + \text{“mesons”} \\ N_f \text{ flavors} & \text{strong coupling} & N_f \text{ flavors} \end{array} \tag{75}$$

Here the gauge factor that runs to strong coupling is the $SU(N+M)$ (since it has relatively less N_f than N_c compared to the other factor):

$$\text{“}N_c\text{”} = N + M \qquad \text{“}N_f\text{”} = 2N \tag{76}$$

The dual gauge group is then $SU(2N - (N + M)) = SU(N - M)$. Therefore, under Seiberg duality, the roles of the gauge groups are interchanged:

$$SU(N + M) \times SU(N) \to SU(N) \times SU(N - M) \tag{77}$$

We can check that the field content is also self-similar, as is the superpotential W (the “mesons” are massive and can be integrated out) at each step. Therefore, the product gauge groups undergo a duality cascade with $N \to N - M$ at each step.

What is the end of the cascade? Let's consider $N = KM$. Then eventually, we reach a step:

$$SU(2M) \times SU(M) \to SU(M) \tag{78}$$

The end of the cascade is a pure gauge theory!

This $\mathcal{N} = 1$ pure SU(M) gauge theory has the following well known properties:

- A non-perturbative superpotential $W = \Lambda^3_{SU(M)}$.
- It has M vacua (Witten index is M), rotated by phase rotations of Λ.

Gravity Side:

We now turn to the gravity side to understand this cascade. Klebanov Strassler worked out the supergravity background (we will see more explicitly later) but the essential physics is as follows.

The modification from KW to KS is that the conifold is deformed:

$$\sum_{i=1}^{4} z_i^2 = \epsilon^2 \tag{79}$$

i.e., the S^3 has a finite size. We call this 3-cycle A and its dual (non-compact) 3-cycle B. The D3 and D5 branes are replaced by fluxes:

$$\frac{1}{2\pi\alpha'}\int_A F_3 = 2\pi M \qquad \frac{1}{2\pi\alpha'}\int_B H_3 = -2\pi K \tag{80}$$

and so $N = KM$. We see that the D3 and D5 charges match (if $N \neq KM$ for $K \in \mathbb{Z}^+$, there are probe D3-branes left).

What do these fluxes do to the moduli? A superpotential is induced:

$$W = \int_{\mathcal{M}} G_3 \wedge \Omega = \int_{\mathcal{M}} (F_3 - \tau H_3) \wedge \Omega = \int_A F_3 \int_B \Omega + \tau \int_B H_3 \int_A \Omega \tag{81}$$

where the sign flip in the last term is due to the ordering of A and B. The integrals appearing here are the *periods* defining the complex structure of the conifold. In particular, the complex coordinate for the collapsing A cycle is defined by

$$z = \int_A \Omega \tag{82}$$

The integral over the B-cycle can be determined from the monodromy around $z = 0$ where the A-cycle shrinks (as $z \to e^{2\pi i} z$, $A \to A$ but $B \to B + A$). The result is:

$$\int_B \Omega = \mathcal{G}(z) = \frac{z}{2\pi i} \ln z + \text{holomorphic} \tag{83}$$

Putting things together, the superpotential is then

$$W = (2\pi)^2\alpha' \left(M\mathcal{G}(z) - K\tau z\right) \tag{84}$$

Consider the F-term equation

$$0 = D_z W \propto M\partial_z\mathcal{G} - K\tau + \partial_z\mathcal{K}(M\mathcal{G} - K\tau z) \tag{85}$$

where $\mathcal{K}$ is the Kahler potential. Here $z \sim \epsilon^{\#}$; A is a vanishing cycle as one approaches conifold point $\epsilon \to 0$, $z \to 0$.

$$D_z W \propto \frac{M}{2\pi i}\ln z - i\frac{K}{g_s} + \mathcal{O}(1) \tag{86}$$

It follows that for $K/Mg_s >> 1$, z is indeed exponentially small,

$$z \sim \exp(-2\pi K/Mg_s) \tag{87}$$

Thus, we obtain a hierarchy of scales if, for example, $M = 1$ and K/g_s is of order 5. Actually, there are M vacua related by the phase of z (This is because of the monodromy $B \to B + A$, $\int_B F_3$ is defined mod M). Note also that $z = A$-cycle volume is near conifold in moduli space (But the warp factor keeps the SUGRA approximation valid).

To determine the actual warp factor requires solving the supergravity equation of motion, but one can estimate it as follows. The warp metric for a D3-brane is:

$$ds^2 = \left(\frac{r}{R}\right)^2 ds_4^2 + \left(\frac{R}{r}\right)^2 \left(dr^2 + r^2 d\Omega_5^2\right) \qquad \text{where} \quad R^4 = 4\pi g_s N(\alpha')^2 \tag{88}$$

where r is the distance (in unwarped metric) from the D3-branes located at $r = 0$. The deformation parameter sets a minimum for r and hence the warp factor $e^{A_{min}}$:

$$e^{A_{min}} \simeq r_{min} \simeq z^{1/3} \simeq \exp(-2\pi K/3Mg_s) \tag{89}$$

Thus the warped IR scale is the scale of gluino condensate in the $\mathcal{N} = 1$ pure YM at the end of the cascade!

To summarize, the background fluxes generate and stabilize an exponent hierarchy. The role of the IR and UV branes in RS are played by the S^3 of the deformed conifold and the bulk geometry.

6.3.4 The IR Geometry

So, how does the SUGRA solution look like in the IR? More will come in the next lecture, here we sketch the result.

The metric on the conifold is deformed by the fluxes:

$$ds^2 = a_0^2 dx_\mu dx^\mu + g_s M b_0^2 \left(dr^2 + d\Omega_3^2 + r^2 d\Omega_2^2\right) \tag{90}$$

where

$$a_0^2 \sim \frac{\epsilon^{4/3}}{g_s M} , \qquad \epsilon \sim e^{-\pi K/g_s M} ; \qquad b_0 \sim \mathcal{O}(1) \tag{91}$$

The S^3 has a finite size while the S^2 collapses at the tip. The 3-form RR flux F_3 is non-vanishing on S^3:

$$F_3 \sim f \epsilon_{ijk} \qquad f \sim \frac{2}{\sqrt{g_s^3 M} b_0} \tag{92}$$

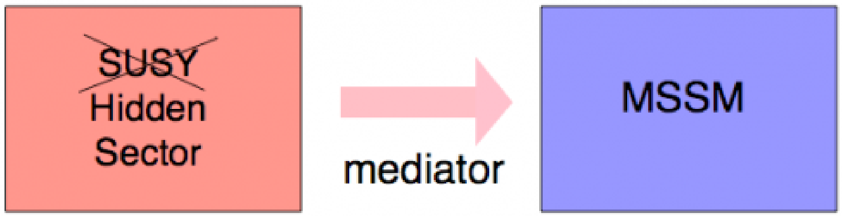
We will discuss in the next lecture how to introduce SUSY breaking states in this geometry, in both open and closed string descriptions.

Lecture IV: Holographic Applications in String Model Building

In this lecture, we will discuss how holography can be used to study BSM scenarios which involve strong coupling dynamics. We will illustrate this approach by calculating soft terms in a SUSY breaking scenario where the messengers are strongly coupled. If there is time, we will describe how one might construct gravity duals of technicolor-like theories.

6.4 Holographic SUSY Breaking

SUSY is arguably the leading candidate for physics BSM. Among its many appeals is its calculability as an effective theory. Although strong coupling physics is often involved in SUSY breaking, its influence on the visible sector can be parametrized by a set of soft terms in a weakly coupled theory. The perturbativity of these weakly coupled models make them more amendable to quantitative studies. Even in specific SUSY mediation scenarios where we specify how the effects of strong coupling physics get transmitted to the visible sector,



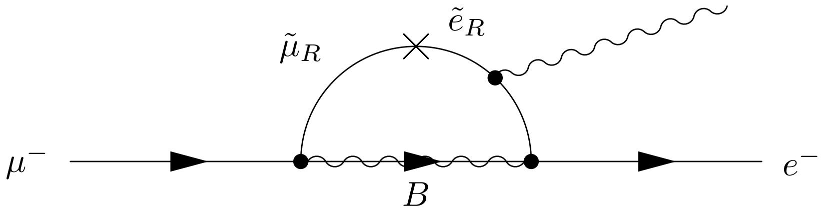
we don't necessarily need to know all the details of its dynamics. For example, in gravity mediation, the effect of SUSY breaking is summarized by a spurion field:

$$< X >= M + \theta^2 F \tag{93}$$

The messengers, which feel directly the effects of SUSY breaking, couple to the visible sector gravitationally:

$$m_0 \sim m_{1/2} \sim m_{3/2} \sim \frac{F}{M_P} \tag{94}$$

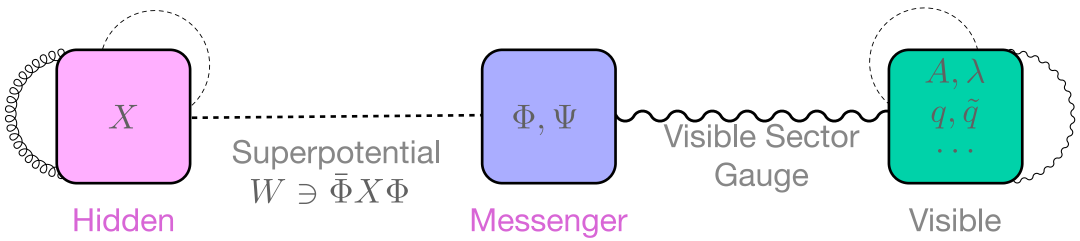
As far as the visible sector is concerned, the physics is weakly coupled. This calculability as an EFT however does not mean that finding a UV complete model is easy. This is because the soft terms in gravity mediation are all generated by Planck suppressed operators. Any Planck suppressed corrections to the effective action, including those of the Kahler potential which is not protected by holomorphy can contribute. Other than the UV sensitivity, these Planck suppressed operators generically give unacceptable flavor mixing terms:



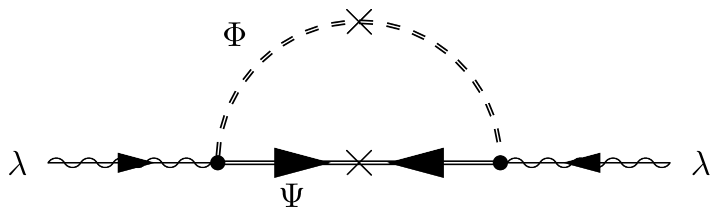
This is where gauge mediation (**`Dine, Fischler, Nappi, Ovrut, Alvarez-Gaume, Claudson, Wise,...`**) comes in. The flavor problem is naturally solved in gauge mediation because the messengers couple to the visible sector through the usual Standard Model gauge interactions which are flavor-blind.

Minimal Gauge Mediation (Dine, Nelson, Nir, Shirman)

In the simplest model of gauge mediation, known as minimal gauge mediation, the messengers are *assumed* to be neutral under the hidden sector gauge group. They talk to the hidden sector only through superpotential couplings.



Our ignorance of the strongly coupled hidden sector is again parametrized by the vacuum expectation of some spurion field X. Because the messengers are not charged under the hidden sector gauge group, they are weakly coupled and their effects are easy to compute:



This gives a rather simple prediction of the gaugino masses

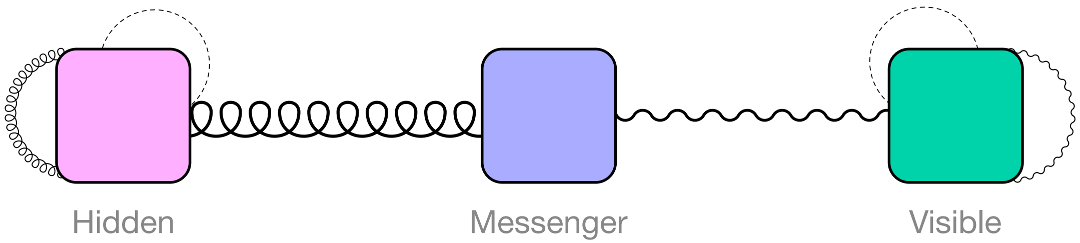
$$m_{\lambda_r} \sim \alpha_r \frac{F}{M} \ , \qquad r = 1, 2, 3 \tag{95}$$

Direct Gauge Mediation

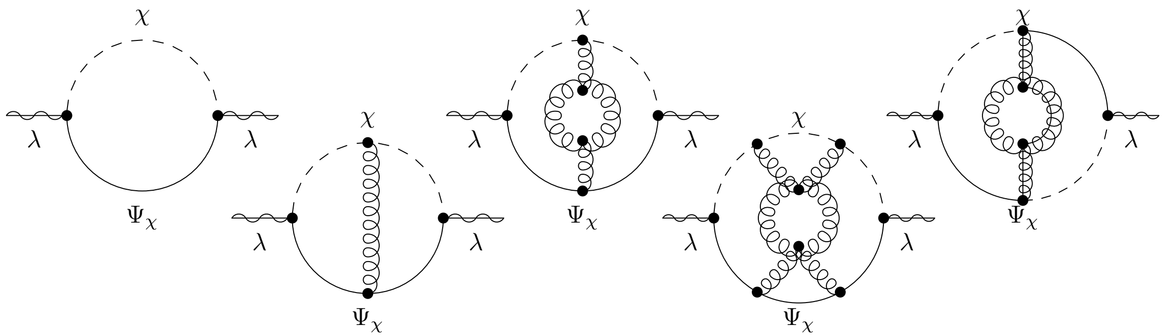
However, this is not the only type of gauge mediation. In fact, the original idea of gauge mediation is direct mediation. In direct gauge mediation, the messengers are charged under the hidden sector and they participate in SUSY breaking. Not only does the distinction between messengers and hidden sector become less clear, in order to get this scenario to work, one often needs to come up with rather complicated models. More importantly, the messengers are strongly coupled and their effects on the visible sector are difficult to compute.

Semi-direct Gauge Mediation (Seiberg, Volansky, Wecht)

More recently, a variation of this old idea was proposed. It was named semi-direct gauge mediation by its inventors because it is a compromise between the minimal and direct models.



The messengers are charged under the hidden sector (just like direct mediation), but they do not participate in SUSY breaking. This gives more flexibility in model building, though the computation of soft terms still involves strong dynamics. For example, in semi-direct gauge mediation, one would expect the visible sector soft terms to receive corrections at all orders of hidden sector loops.



Computing these contributions by brute force is clearly a formidable task. To study scenarios with strongly coupled messengers, we need a better tool than conventional perturbative techniques.

Holographic Gauge Mediation

One of these tools is holography. The idea is to find a gravity dual of the strongly coupled hidden sector and compute holographically the soft terms. But first, what kind of gravity dual would have potentially a state that breaks SUSY at exponentially small energies?

The natural idea is to start from the most explicit known gravity dual of a confining gauge theory, i.e., the warped deformed conifold. Given the tiny warped scale at the IR, one might expect to be able to locally break SUSY there to generate an exponentially small SUSY breaking scale. One way of breaking SUSY locally is to introduce an $\overline{D3}$ brane.

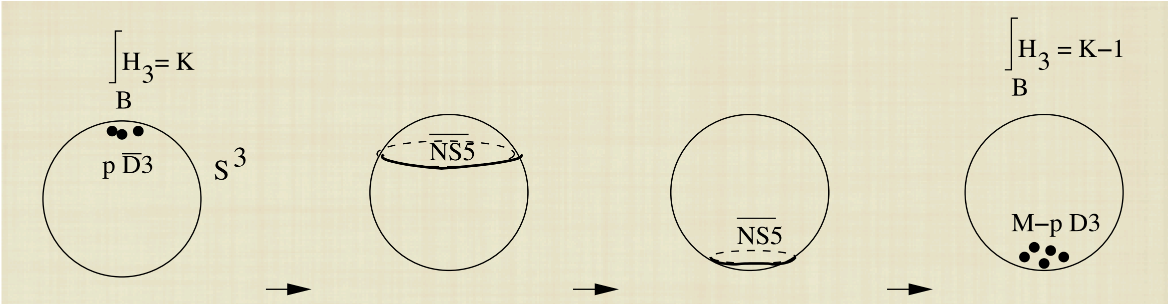
Brane-Flux Annihilation

But the KS background carries D3-charge, would the $\overline{D3}$ be annihilated? With the explicit KS solution, we see that the D3-charge is radially dependent and near the tip:

$$dF_5 = H_3 \wedge F_3 \,, \qquad \int F_5 = g_s M^2 \tau^2 + \ldots \tag{96}$$

At the tip, there are no D3-branes and so putting p $\overline{D3}$ there is *perturbatively* stable.

But this is too quick. There is another state, visible in the same gravity theory, with the same charges: KS solution with $\tilde{N} = (K-1)M$, $\tilde{M} = M$ and $M - p$ D3-branes. Although there is no D3-charge locally to annihilate the $\overline{D3}$ branes, there exists a brane/flux annihilation process (`Kachru, Pearson, Verlinde`) that allows the SUSY breaking state to decay to the SUSY state described schematically by this picture:



The 3-form flux polarizes the anti D3-branes into an NS5-brane. The SUSY breaking state can therefore decay via nucleation of a bubble of supersymmetric vacuum surrounded by a spherical NS 5-brane domain wall. This picture is reliable only if $p << M$; otherwise, the blob of the expanded $\overline{D3}$ is bigger than the space at the tip! For $p << M$, the tunneling rate, determined by this instanton, is exponentially small giving evidence that the state is metastable.

Back-reacted Solution

To study holographically this metastable SUSY breaking state, we should find the back-reacted solution of $p\ \overline{D3}$ branes in the KS background. Furthermore, according to the usual gauge/gravity dictionary, if the SUGRA fields sourced by the $\overline{D3}$-branes are normalizable modes, then the field theory dual is a SUSY breaking state. If the SUGRA fields are not normalizable, then the field theory dual corresponds to a different field theory. Since $p << M$, the backreaction can be treated as a perturbation. Here are some upshots if one goes about and solve for the back-reacted solution:

- Solution in the entire conifold is difficult to find, but solution in the UV (`DeWolfe, Kachru, Mulligan`) and IR regions (`McGuirk, Shiu, Sumitomo`) have been obtained.

- In the UV region, the perturbations were found to fall off faster than the unperturbed fields, e.g., $\delta h/h_{KS} \sim r^{-4}$, etc.

- There has been a recent contradicting claim (`Bena, Grana, Halmagyi`) that the perturbations are non-normalizable if some IR boundary conditions (vanishing of H_3) are imposed. However, the b.c. turn out to be too restrictive[6].

- In all these analysis, an ansatz respecting the isometry of KS was chosen, e.g.:

$$ds_{10}^2 = h^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}\left(p(\tau)d\tau^2 + u(\tau)g_5^2 + q(\tau)\left(g_3^2 + g_4^2\right) + s(\tau)\left(g_1^2 + g_2^2\right)\right) \qquad (97)$$

  (similar ansatz for fluxes, dilaton). Here, g_i are some angular 1-forms. To preserve the isometry, the $\overline{D3}$ are smeared over the S^3.

- To linear order in perturbations, the solution in the IR is:

$$\delta h \sim \delta q \sim \delta u \sim \delta s \sim \frac{\mathcal{S}}{\tau} + \mathcal{O}(\tau)\ , \qquad \mathcal{S} \sim \frac{pT_{D3}}{(g_s M)^2} \qquad (98)$$

[6]In fact, our IR analysis showed that a non-vanishing H_3 follows from the equations of motion.

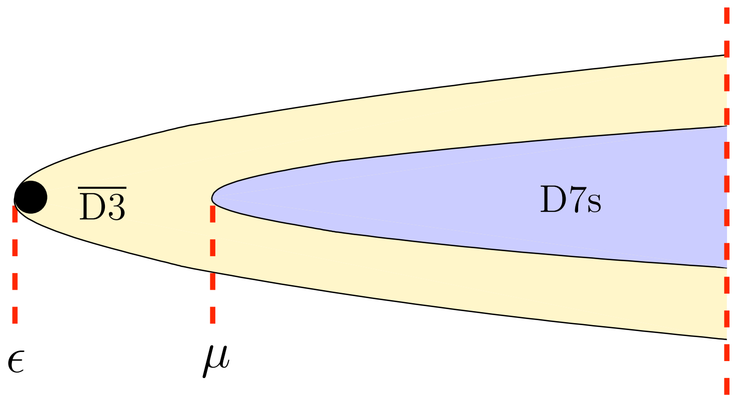
(similarly perturbations for fluxes and dilaton). The singular behavior accounts for the localized source. Detailed expressions can be found in `McGuirk, Shiu, Sumitomo`. To summarize the main features:

  - The angular geometry is squashed, internal metric is not conformally Calabi-Yau.
  - 3-form flux is non-ISD and has all SUSY breaking components: $(1, 2)$, $(0, 3)$, $(3, 0)$.
  - The dilaton has a non-trivial profile, but vanishes in the IR: $\delta\Phi \sim \mathcal{S}\tau$.

- Similar behavior in the UV (`DeWolfe, Kachru, Mulligan`), though with simpler squashing and only (2,1) and (1,2) fluxes are sourced.

Visible Sector

Now, let us introduce the visible sector and the messengers. An $SU(K)$ flavor group of the hidden sector can be introduced by adding K D7-branes to the SUSY breaking background we just found:



Although the coupling of the $SU(K)$ flavor group is zero when the 7-branes are non-compact, the flavor symmetry is weakly gauged once the theory is compactified. The visible sector is a subgroup of this weakly gauged $SU(K)$ symmetry[7].

A D7-brane has codimension 2 and can be defined by a complex equation. We will consider a simple

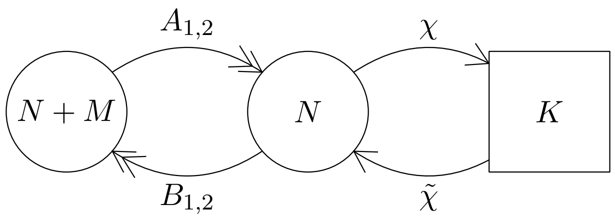
[7]We will work in the probe approximation, i.e., $K << M$ to neglect the D7-brane backreaction.

choice of D7-brane embedding given by this holomorphic equation:

$$z_4 = \mu \qquad \text{"Kuperstein embedding"} \tag{99}$$

Other embeddings (e.g., `Ouyang, Karch-Katz`) defined by other holomorphic equations generally require worldvolume flux to minimize the brane tension (`Chen, Ouyang, Shiu`). Holomophicity is not a sufficient condition because of the non-vanishing B_2 in the background.

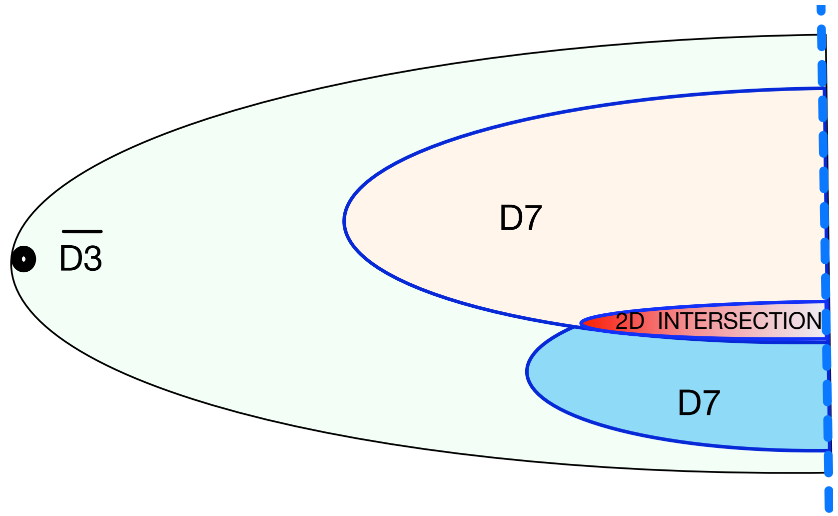
Besides the visible sector fields, adding D7-branes introduces messenger quarks whose masses are set by the scale μ.



The messenger quarks are bi-fundamental fields denoted by χ and $\tilde{\chi}$ here. They are charged under both the visible and hidden sectors, but they do not participate in creating the SUSY breaking state. This setup is therefore a gravity dual of semi-direct gauge mediation.

Standard Model Matter

As discussed in Lecture II, the chiral matter of the Standard Model lives at the intersection of branes (and a worldvolume field strength supported on the two-cycle at which they intersect is also needed to obtain chiral fermions):



The degree of compositeness depends on the location of the intersection. If the branes intersect in the UV, the corresponding matter fields are "elementary". If they intersect in the IR, the corresponding matter fields are composites in the field theory dual.

Model Building Features

Before we compute holographically the soft terms, let us contrast what we have found with a 5D phenomenological approach, and extract some novel features for model building.

- The first new feature is that unlike the warped models considered in the phenomenological literature, the gauge fields and the gauginos do not live in the entire bulk. They end where the D7-branes end. Therefore, the gaugino mass is more tunable, given a fixed IR SUSY breaking scale. We will consider two cases: (i) $m_\chi >> \Lambda_\epsilon$, (ii) $m_\chi \sim \Lambda_\epsilon$.

- Another feature is that the matter fields are not bulk fields. They are not even fields living in the bulk of the D7-branes. The matter fields reside at the intersection of D7-branes, and are expected to be localized at the IR end of the intersection. This gives us rather rich model building possibilities as the Standard Model can be part composite and part elementary. For simplicity, however, we will assume here that all the Standard Model matter fields are elementary, i.e. they live in the UV.

- One more feature to point out is that the SM and the SUSY-breaking branes can be separated by a large physical distance even for a small coordinate separation in the holographic direction:
$$\frac{1}{g_s M} << \tau < 1 \tag{100}$$
This is because the size of the S^3 at the tip $R^2_{S^3} \sim g_s M$ is large in string units. The $D7$-branes can dip down to very small τ within the validity of SUGRA.

Calculating Soft Terms

Let us finally turn to the computation of soft terms. On the gravity side, the gaugino masses can be calculated by analyzing how the worldvolume fields on the D7-branes react to the SUSY breaking background. Our final answer would correspond to summing all-loop planar diagrams on the gauge theory side.

In this gravity dual, the gaugino masses are the most straightforward to compute. The squarks and the sleptons are localized at the brane intersection and so their masses are harder to obtain from a worldvolume analysis. Nevertheless, if we assume the visible sector sftermions to be living in the UV, their dominant mass contributions should come from gaugino mediation which can be computed in field theory. We will consider the two cases (i) $m_\chi >> \Lambda_\epsilon$, (ii) $m_\chi \sim \Lambda_\epsilon$ in turn.

Case 1: Large Messenger Mass, i.e., $m_\chi >> \Lambda_\epsilon$

On the field theory side, the gaugino mass comes from loops. Schematically:

$$m_{1/2} = \int_0^{\Lambda_\epsilon} d^4p + \int_{\Lambda_\epsilon}^{m_\chi} d^4p + \int_{m_\chi}^{\Lambda_{UV}} d^4p \qquad (101)$$

Below the hidden sector confinement scale, the hidden sector and messenger fields are decoupled and do not contribute to the gaugino mass. Between the confinement scale and the messenger mass, the effective degrees of freedom are the messenger mesons. They are bound states of the messenger quarks and transform in the adjoint representation of the flavor group. If the hidden sector group has a large rank, these mesons are weakly coupled (`'t Hooft; Witten`). Therefore, the usual perturbative expression for gauge mediation applies:

$$m_{1/2} = \frac{g_{vis}^2 K}{16\pi^2} \sum_n \frac{F_n}{M_n} \tag{102}$$

There is nothing holographic about this gaugino mass except that the masses of the messenger mesons M_n and their F-terms F_n which are often just parametrized in gauge mediation can be computed holographically (see Benini et al for detailed expressions).

Above the messenger mass, the messenger quarks can no longer be integrated out. These messenger quarks are charged under the hidden sector and are therefore strongly coupled. They cannot be analyzed using standard perturbative techniques. However, their effect on the gaugino masses can be computed on the gravity side from the pullback of the SUSY breaking background on the D7-brane worldvolume. A direct calculation shows that this contribution vanishes. Why?

Field Theory Explanation: For $\mu >> \epsilon$, the 7-branes are in the UV region of the KS geometry (i.e., `Klebanov-Tseytlin`) where the isometry group is $SO(4) \times \mathbb{Z}_{2M}$. The $U(1)$ isometry is broken to $\mathbb{Z}_{2M}$ in the UV by the RR flux, and to $\mathbb{Z}_2$ in the IR by the deformation parameter. On the field theory side, this surviving $\mathbb{Z}_{2M}$ R-symmetry forbids a non-vanishing gaugino mass. One may say that the messenger mass already breaks R-symmetry completely. However, for energies above the messenger mass, this effect can be neglected and the surviving R-symmetry should largely be determined by the isometry of the background.

Case 1: Small Messenger Mass, i.e., $m_\chi \approx \Lambda_\epsilon$

Now consider the case where the messenger mass is comparable to the hidden sector confinement scale. This corresponds to having the D7-branes come closer to the $\overline{D3}$ branes. To calculate the soft terms, we make use of the supergravity solution in the IR region we just found. This is also a potentially more interesting parameter regime because the R-symmetry is reduced to Z_2 which allows for a gaugino mass.

We will also make use of some results on the fermionic D-brane action obtained earlier (for details, see `Marchesano, McGuirk, Shiu`). We can divide the holographic computation of the gaugino mass into 5 smaller steps. The complete calculation is rather involved, so we will sketch only part of the steps here.

- **Step 1: Getting the unperturbed gaugino wavefunction:**

  We first solve the gaugino equation of motion in the unpertubed SUSY background. Since we are working to first order in perturbations, we can calculate the gaugino mass to this order by inserting the unperturbed wavefunction into the perturbed D7-brane action. The perturbation of the gaugino wavefunction contributes in higher order.

The light fermionic degrees of freedom on D7-branes can be represented by a doublet of 10D Majorana-Weyl spinors:

$$\Theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \tag{103}$$

The advantage of doubling the fermionic degrees of freedom is that a symmetry of the fermionic action, known as κ symmetry, becomes more manifested. In terms of the bispinors, the fermionic action takes the form (`Martucci, Rosseel, Van der Bleeken, Van Proeyen`):

$$S_{\rm D7}^{\rm fer} = \tau_{\rm D7} \int \mathrm{d}^8\xi \, \mathrm{e}^{-\Phi} \sqrt{|\det\left(P[G] + \mathcal{F}\right)|} \, \bar{\Theta} P_-^{\rm D7} \big(\Gamma^\alpha \mathcal{D}_\alpha - \frac{1}{2}\mathcal{O}\big) \, \Theta \tag{104}$$

The κ symmetry acts as $\Theta \to \Theta + P_-^{\rm D7}\kappa$. We can use this redundancy to set one of the components of the bispinor to zero, e.g., $\theta_1 = \theta$, $\theta_2 = 0$. The SUGRA flux dependence is hidden in $\mathcal{D}_\alpha$ and $\mathcal{O}$ which are operators appearing in the gravitino and dilatino variation respectively. The projector $P_-^{\rm D7}$ depends on $\mathcal{F} = B_2 + F_2$.

It was noted in `Marchesano, McGuirk, Shiu` that in the presence of warping, the 5-form flux is sourced: $F_5^{\rm int} = \hat{*}_6 d(\Delta e^{\Phi})$. This 5-form flux couples to D7 fermions and affects the warping dependence of the gaugino wavefunction and we found that

$$\theta(x^\mu, x^a) = h^{3/8}\lambda(x^\mu) \otimes \eta(x^a) \tag{105}$$

where η is a covariantly constant spinor with respect to the unwarped metric. This is in contrast to $\theta(x^\mu, x^a) = h^{1/8}\lambda(x^\mu)\otimes\eta(x^a)$ found in `Acharya, Benini, Valandro` where the 5-form coupling is neglected. Moreover, the warping dependence is the same regardless of the worldvolume flux on the D7-branes.

- **Step 2: Perturbing the D7-brane action:**

  The gaugino bilinear terms come from the covariant derivative in the Dirac-like action, e.g., terms of this form which depend on the 3-form flux:

$$\int d^8x \sqrt{-\det(g_{\alpha\beta} + B_{\alpha\beta})}\,{\rm tr}\{\overline{\Theta} P_-^{\rm D7} \mathcal{G}_3^+ \Theta\} \tag{106}$$

  We perturb to linear order all the SUGRA fields appearing in the action including the metric, the 5-form, etc

- **Step 3: Dimensionally reducing the action:**

The wavefunction is peaked in the IR (from step 1). The main contribution comes from $\tau_{min} = 2\cosh^{-1}(\mu/\epsilon)$, and we get

$$-\tau_7 g_s M \epsilon^{2/3} \tau_{min}^4 \mathcal{S} \int_{\mathbb{R}^{3,1}} d^4x \ \mathrm{tr}(\lambda^2) \tag{107}$$

- **Step 4: Canonical Normalization:**

The kinetic term of the gauginos do not come out canonically normalized. Rescaling the gaugino kinetic term so that it becomes canonical, the bilinear calculated in step 3 gives a contribution to the gaugino mass that goes parametrically as follows:

$$\delta m_{1/2} \sim g_s M \epsilon^{2/3} \tau_{\mathrm{min}}^4 \ g_{\mathrm{vis}} \ \mathcal{S} \tag{108}$$

- **Step 5: Translating to 4D Gauge Theory:**

Instead of microscopic quantities like the flux quanta and the deformation parameter of the conifold, it is useful to express this gaugino mass in terms of field theory quantities.

At the bottom of the cascade, the hidden sector gauge group is simply $SU(M)$, and so the 't Hooft coupling in the far IR is

$$\lambda = g_s M \tag{109}$$

Using the holographic dictionary for cascading gauge theories (`Aharony, Buchel, Yarom`), the expansion parameter $\mathcal{S}$ in the gravity solution was found to be related to the vacuum energy of the dual 4D gauge theory (**DeWolfe, Kachru, Mulligan**):

$$\Lambda_{\mathcal{S}} \sim S^{1/4}\Lambda_\epsilon \tag{110}$$

Finally, the messenger mass is determined by the distance from the tip:

$$m_\chi^{3/2} \approx \Lambda_\epsilon^{3/2}\left(1+\frac{1}{8}\tau_{\min}^2\right) \tag{111}$$

Putting these together:

$$\delta m_{1/2} \sim g_{\text{vis}}\lambda\frac{\Lambda_{\mathcal{S}}^4}{\Lambda_\epsilon^3}\left(\left(\frac{m_\chi}{\lambda_\epsilon}\right)^{3/2}-1\right)^2 \tag{112}$$

A priori, there are potential contributions from the 5-form flux or from the perturbed spin connection. But a detailed analysis shows that these terms vanish.

The end result, to leading order, is:

$$\delta m_{1/2} \sim g_{\text{vis}}\lambda\frac{\Lambda_{\mathcal{S}}^4}{m_\chi^3}\left(\left(\frac{m_\chi}{\lambda_\epsilon}\right)^{3/2}-1\right)^2 \tag{113}$$

This suggests identifying an effective F-term (the non-messenger dependence):

$$F = \sqrt{\lambda}\Lambda_{\mathcal{S}}^2 \tag{114}$$

Therefore, we found after a long calculation that:

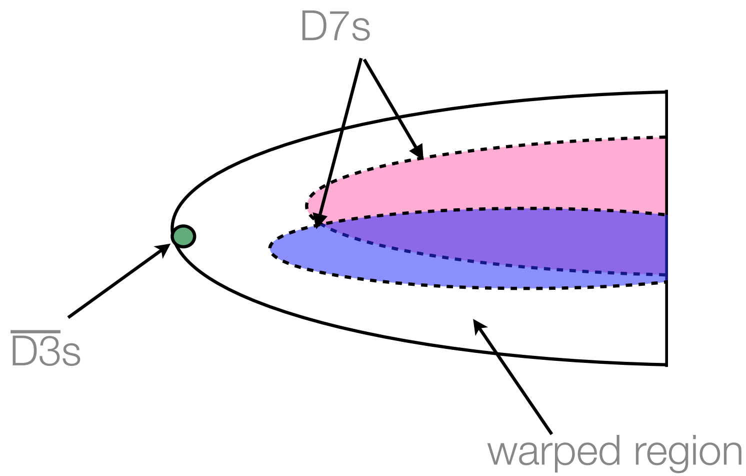
– Unlike the weakly coupled messenger case where $m_{1/2} \sim F/M$, the gaugino mass vanishes to leading order in F, which seems generic in semi-direct gauge mediation (`Seiberg, Volansky, Wecht`).

– All SUSY breaking components of the 3-form flux contribute to the gaugino mass. This is in contrast to an earlier model independent study (`Camara, Ibanez, Uranga`) where only the $(0, 3)$ flux was found to contribute. This is because a Kahler metric was assumed in their analysis. However, in the presence of a SUSY breaking source like an $\overline{D3}$ brane, the backreacted metric is not necessarily Kahler as our explicit solution shows. The non-Kahlerity of the metric can contribute in the same order.

– A deceasing gaugino mass as $m_\chi \to \Lambda_\epsilon$ is reflective of the duality cascade. The effective 't Hooft coupling decreases with scale and so $m_{1/2}$ becomes smaller.

Sfermion Masses

As for the visible sector matter, we assumed for simplicity that they live in the UV and hence the sfermion masses follow from gaugino mediation (`Kaplan, Kribs, Schmaltz; Chacko, Luty, Nelson, Ponton`). In a more complete model, the visible sector could be realized holographically on intersecting D7-branes with worldvolume flux. The computation of these soft masses requires more sophisticated tools.



Summary of this section

So, to summarize, holography provides a powerful tool to explore strong coupling extensions of the Standard Model. This example is a proof of concept that such scenarios can be analyzed even though conventional perturbative techniques break down.

6.5 Warped Throats and the Standard Model

Now, one might wonder if holographical techniques can be applied to BSM scenarios where the visible sector is the low energy limit of a strongly coupled gauge theory (i.e., technicolor-like). A natural question is whether the singularity which support the chiral spectrum of the Standard Model discussed earlier in Section 4 can arise at the tip of some warped throats. Unfortunately, the KS throat is too simple to generate chiral physics in the infrared. On the gravity side, the geometry is smooth after the deformation, while on the field theory side, the light degrees of freedom are simply the glueballs of the confining theory. Hence the KS throat does not lead to the SM degrees of freedom by putting D3-branes at the tip.

The simplest possibility is to consider throats which contain a singularity at the tip, e.g., a $\mathbb{Z}_3$ orbifold singularity. The first thing one might try is to construct the quotient of the deformed conifold by a $\mathbb{Z}_3$ action with isolated fixed points. Unfortunately, the deformed conifold does not admit such symmetries. For instance, we can change variables and describe the conifold as

$$xy - uv = z \tag{115}$$

There is a $\mathbb{Z}_3$ symmetry: $(x, u) \to e^{2\pi i/3}(x, u)$ and $(y, v) \to e^{-2\pi i/3}(y, v)$, which unfortunately is freely-acting[8]. Other possible $\mathbb{Z}_3$, such as $x \to e^{2\pi i/3}x$, $y \to e^{-2\pi i/3}y$ while leaving u, v invariant, have a whole complex curve of fixed points[9] so locally the singularity is $\mathbb{C} \times \mathbb{C}^2/\mathbb{Z}_3$. This singularity leads to $\mathcal{N} = 2$ worldvolume theories which are non-chiral.

Progress in understanding warped throats for other geometries, generalizing the conifold, as well as their interpretation in terms of duality cascades and infrared confinement provides useful techniques to implement these ideas. Skipping the details, it suffices for our purposes to consider a particular example. Here, we follow the discussion of `Cascales, Saad, Uranga`. Consider the so called suspended pinch point (SPP) singularity, which can be described as a hypersurface in $\mathbb{C}^4$ given by

$$xy - zw^2 = 0 \tag{116}$$

This geometry admits a complex deformation to the smooth geometry

$$xy - zw^2 = \epsilon w \tag{117}$$

which contains a finite size[10] S^3.

Moreover, the deformed geometry is invariant under a $\mathbb{Z}_3$ acting as

$$x \to \alpha x \qquad y \to \alpha y \qquad z \to \alpha z \qquad w \to \alpha^2 w \tag{118}$$

[8] The fixed point $x = y = z = w = 0$ does not solve the deformed conifold equation.

[9] defined by $uv = z$

[10] To see this, it is most convenient to change coordinates: $\rho = x/w$. The deformed SPP can now be written as $\rho y - zw = \epsilon$.

with $\alpha = e^{2\pi i/3}$. Notice that the $\mathbb{Z}_3$ action leaves invariant the holomorphic 3-form $\Omega = \frac{dxdydz}{zw}$ of the SPP, guaranteeing that the quotient is a new CY singularity. Furthermore, the $\mathbb{Z}_3$ action has the origin as the unique fixed point, and hence there is a left-over $\mathbb{Z}_3$ singularity after the deformation.

On general grounds, it is expected that turning on M units of RR flux on the finite size 3-cycle (as well as a suitable NSNS flux on its dual (non-compact) 3-cycle) leads to a warped throat. At its bottom the throat is cutoff by the finite size 3-cycle, leaving behind a $\mathbb{C}^3/\mathbb{Z}_3$ singularity. A chiral gauge theory is obtained by introducing a small set of D3-branes at the singularity. The latter are probes, and do not modify the structure of the throat significantly.

In fact, one can embed the local D-brane model described in Section 4 within this throat. The D7-branes can wrap the 4-cycle defined by $w = 0$ which pass through the D3-branes located at $x = y = z = w = 0$.

To summarize, we have succeeded in finding a throat with a semi-realistic D-brane sector at its tip. The warped geometry also admits a tractable holographic dual. It would be interesting to construct an explicit metric[11] for these types of warped throats since they allow us to study properties of the strongly coupled field theory dual. For example, the KK spectrum in such warped throat background

[11]The SPP singularity arises as a particular case of a cone over the $L^{a,b,c}$ families of Einstein-Sasaki metrics constructed in recent years. The warped deformed versions of such metrics have not been worked out yet.

tells us about the glueball masses of the dual field theory.

7 Final Thoughts

I hope these lectures have given you some flavor of string phenomenology, with an emphasis on D-brane models of particle physics. As you have seen, string theory has been a seed of many new ideas for physics beyond the Standard Model. It has also given us a powerful tool to explore scenarios that are otherwise not calculable with conventional perturbative techniques. The big question is whether any of these BSM ideas, string theoretical or not, are realized in Nature. For that, we have to wait for the LHC to find out ...