Point Charge as a Soliton in Truncated Quantum Electrodynamics

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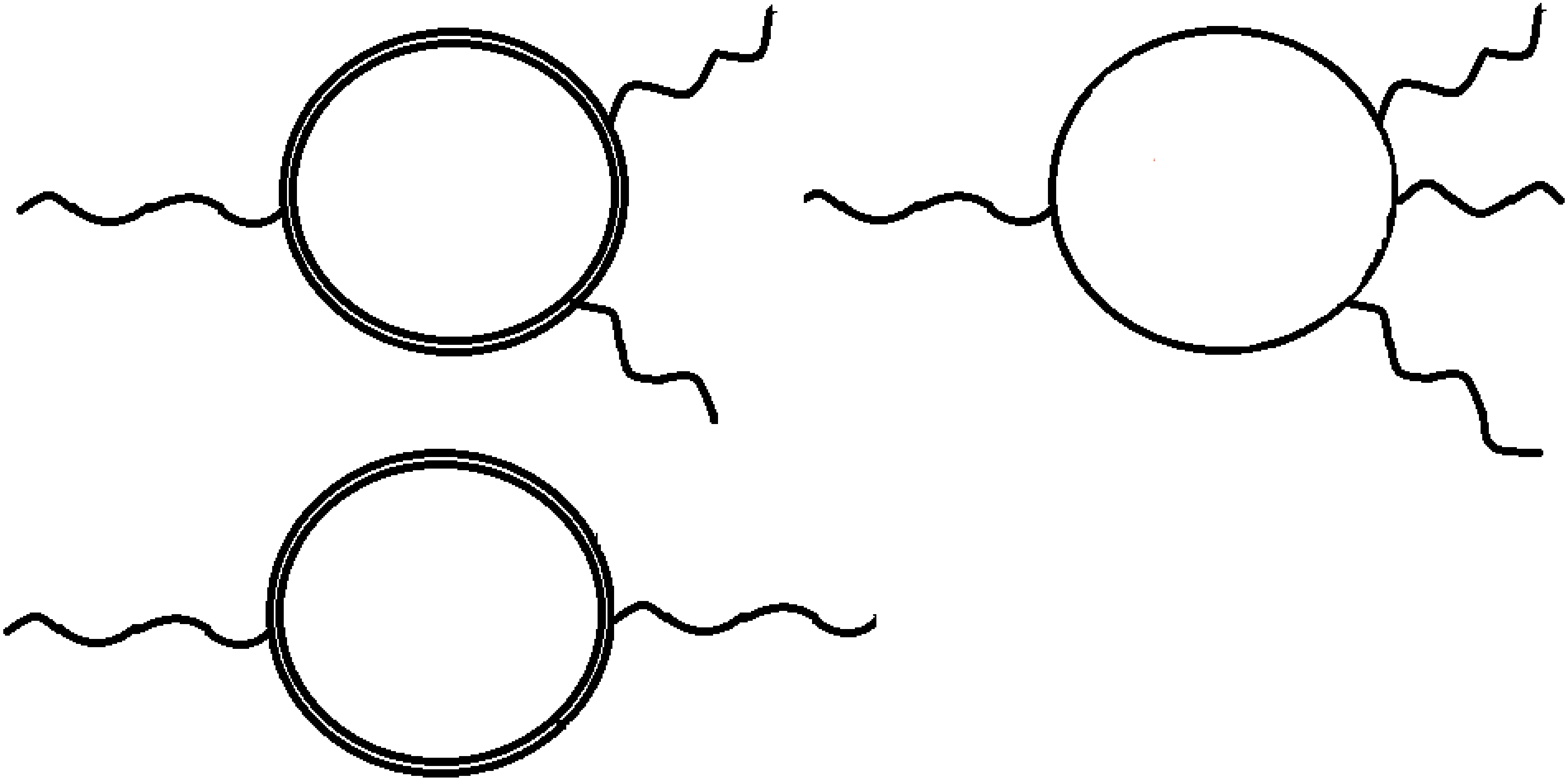


How strong should the fields be?

Electric field E_0
performs over electron charge e
along its Compton length 1/m
the work, equal to its rest energy m

$$E_0 = m^2/e = 1.3 \cdot 10^{16} \text{ V/cm}$$

$$B_0 = \mathbf{m}^2/e = 4.4 \cdot 10^{13} \text{ G}$$

Field at the edge of the neutron due to its magnetic moment is $3 \cdot 10^{16}$ G

Optically as dense as:

Air ----- 10^{13}G

Water ----- 10^{16} G

Diamond ------ 27 $\cdot 10^{16}$ G

$$B_{\mathrm{lim}} = B_0 \exp\{0.79 + 3\pi/\alpha\} = 10^{576}\mathrm{G}$$

– zero-charge instability limit

$B_{\mathrm{Pl}} = 5 \cdot 10^{57}\mathrm{G}$ – Planck scale

$B_{\mathrm{KawatiKokado}} = 1.3 \cdot 10^{47}$ – de Sitter stage

$$B_{\mathrm{max}} =$$

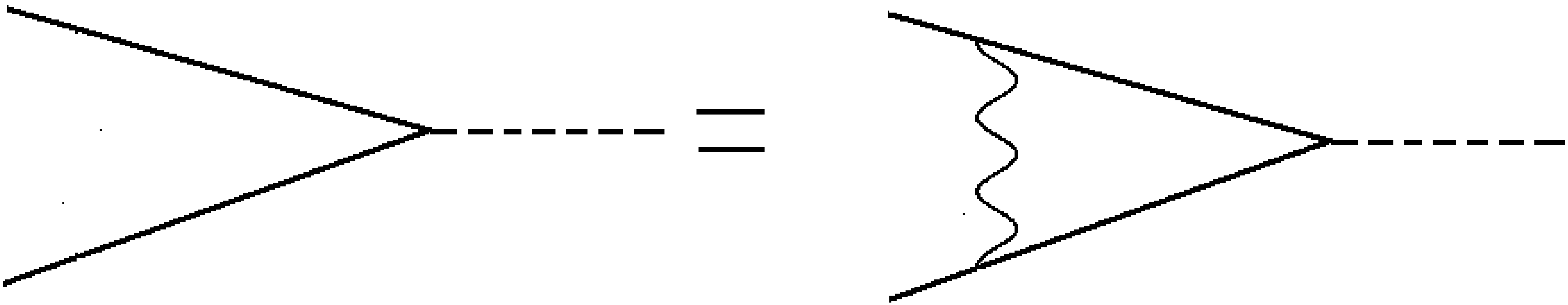
$$= \frac{1}{4} B_0 \exp\{\pi^{3/2}/\alpha^{1/2} + C_{\mathrm{E}}\} = 7 \cdot 10^{41}\mathrm{G}$$

– positronium collapse

Positronium collapse

and maximum magnetic field

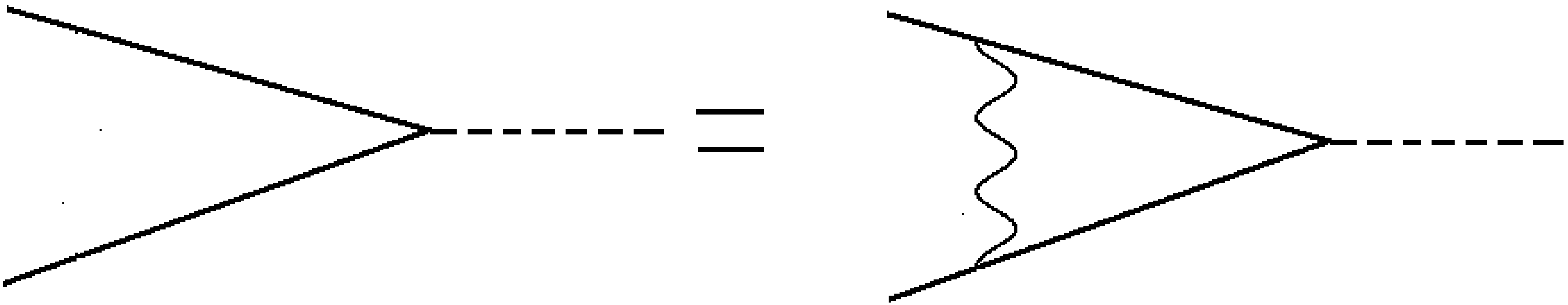
Bethe-Salpeter equation



$$D(x,y) = \frac{1}{(x-y)^2}$$

Positronium collapse and maximum magnetic field

Bethe-Salpeter equation



$$D(x,y) = \frac{1}{(x-y)^2}$$

$$B_{\max} = \frac{m^2}{4e} \exp\left\{\frac{\pi^{3/2}}{\sqrt{\alpha}} + 2C_{\mathrm{E}}\right\} \simeq 1.6 \times 10^{28} B_0,$$

1. Linear response of (the vacuum with) the background B – anisotropic medium

i) photon-positronium mixed state and photon capture

ii) modification of the Coulomb field and its one-dimensioning

iii) point charge as a sort of magnetic monopole in parallel electric and magnetic fields

- 2. Quadratic response of the strong magnetic field to an electric field: static charge is a magnetic dipole
- 3. Cubic response of the blank vacuum to electric field

i) self-coupling of a point charge and finiteness of its field energy

ii) self-coupling of a magnetic dipole and of an electric dipole

Higher-rank polarization tensors

$$\Pi_{\mu\tau}(x,x') = \left.\frac{\delta^2\Gamma}{\delta A^\mu(x)\,\delta A^\tau(x')}\right|_{A=\mathcal{A}^{\rm ext}},$$

$$\Pi_{\mu\tau\sigma}(x,x',x'') = \left.\frac{\delta^3\Gamma}{\delta A^\mu(x)\,\delta A^\tau(x')\,\delta A^\sigma(x'')}\right|_{A=\mathcal{A}^{\rm ext}},$$

$$\Pi_{\mu\tau\sigma\rho}(x,x',x'',x''') = \left.\frac{\delta^4\Gamma}{\delta A^\mu(x)\,\delta A^\tau(x')\,\delta A^\sigma(x'')\,\delta A^\rho(x''')}\right|_{A=\mathcal{A}^{\rm ext}}$$

$\Gamma[A] = \int \mathcal{L}(x)\mathrm{d}^4x$ is the effective action functional

Nonlinear Maxwell equations

truncated at the fourth power of the field $a^{\tau}(x)$

$$j_{\mu}(x) = [\Box\eta_{\mu\tau} - \partial_{\mu}\partial_{\tau}]\, a^{\tau}(x) + \int \mathrm{d}^4y\, \Pi_{\mu\tau}(x,y)\, a^{\tau}(y)$$
$$+\frac{1}{2}\int \mathrm{d}^4y\mathrm{d}^4u\, \Pi_{\mu\tau\sigma}(x,y,u)\, a^{\tau}(y)\, a^{\sigma}(u)$$
$$+\frac{1}{6}\int \mathrm{d}^4y\mathrm{d}^4u\mathrm{d}^4v\, \Pi_{\mu\tau\sigma\rho}(x,y,u,v)\, a^{\tau}(y)\, a^{\sigma}(u)\, a^{\rho}(v)$$

$$A_{\beta}(x) = \mathcal{A}_{\beta}^{\mathrm{ext}}(x) + a_{\beta}(x)$$

Polarization tensor is diagonal

$$\Pi_{\mu\tau}(k) = \sum_{a=1}^{3} \varkappa_a(k) \frac{\flat_\mu^{(a)} \flat_\tau^{(a)}}{(\flat^{(a)})^2}$$

$$\Pi_\mu{}^\tau \flat_\tau^{(a)} = \varkappa_a(k) \flat_\mu^{(a)}$$

Eigenvectors

$$\flat_\mu^{(1)} = (F^2k)_\mu k^2 - k_\mu(kF^2k),$$

$$\flat_\mu^{(2)} = (\tilde{F}k)_\mu, \ \flat_\mu^{(3)} = (Fk)_\mu, \ \flat_\mu^{(4)} = k_\mu$$

where $(\tilde{F}k)_\mu \equiv \tilde{F}_{\mu\tau}k^\tau$, $(Fk)_\mu \equiv F_{\mu\tau}k^\tau$, $(F^2k)_\mu \equiv F^2_{\mu\tau}k^\tau$, $kF^2k \equiv k_\mu F^2_{\mu\tau}k^\tau$

Photon propagator

$$D_{\mu\nu}(k) = \sum_{a=1}^{3} \frac{\flat_\mu^{(a)} \flat_\nu^{(a)}}{(\flat^{(a)})^2} \frac{1}{(k^2 - \varkappa_a(k))}$$

Polarization tensor is diagonal

$$\Pi_{\mu\tau}(k) = \sum_{a=1}^{3} \varkappa_a(k) \frac{\flat_\mu^{(a)} \flat_\tau^{(a)}}{(\flat^{(a)})^2}$$

$$\Pi_\mu^{\ \tau} \flat_\tau^{(a)} = \varkappa_a(k) \flat_\mu^{(a)}$$

Eigenvectors

$$\flat_\mu^{(1)} = (F^2k)_\mu k^2 - k_\mu(kF^2k),$$
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where $(\tilde{F}k)_\mu \equiv \tilde{F}_{\mu\tau}k^\tau, \ (Fk)_\mu \equiv F_{\mu\tau}k^\tau,$
$(F^2k)_\mu \equiv F^2_{\mu\tau}k^\tau, \ kF^2k \equiv k_\mu F^2_{\mu\tau}k^\tau$

Photon propagator

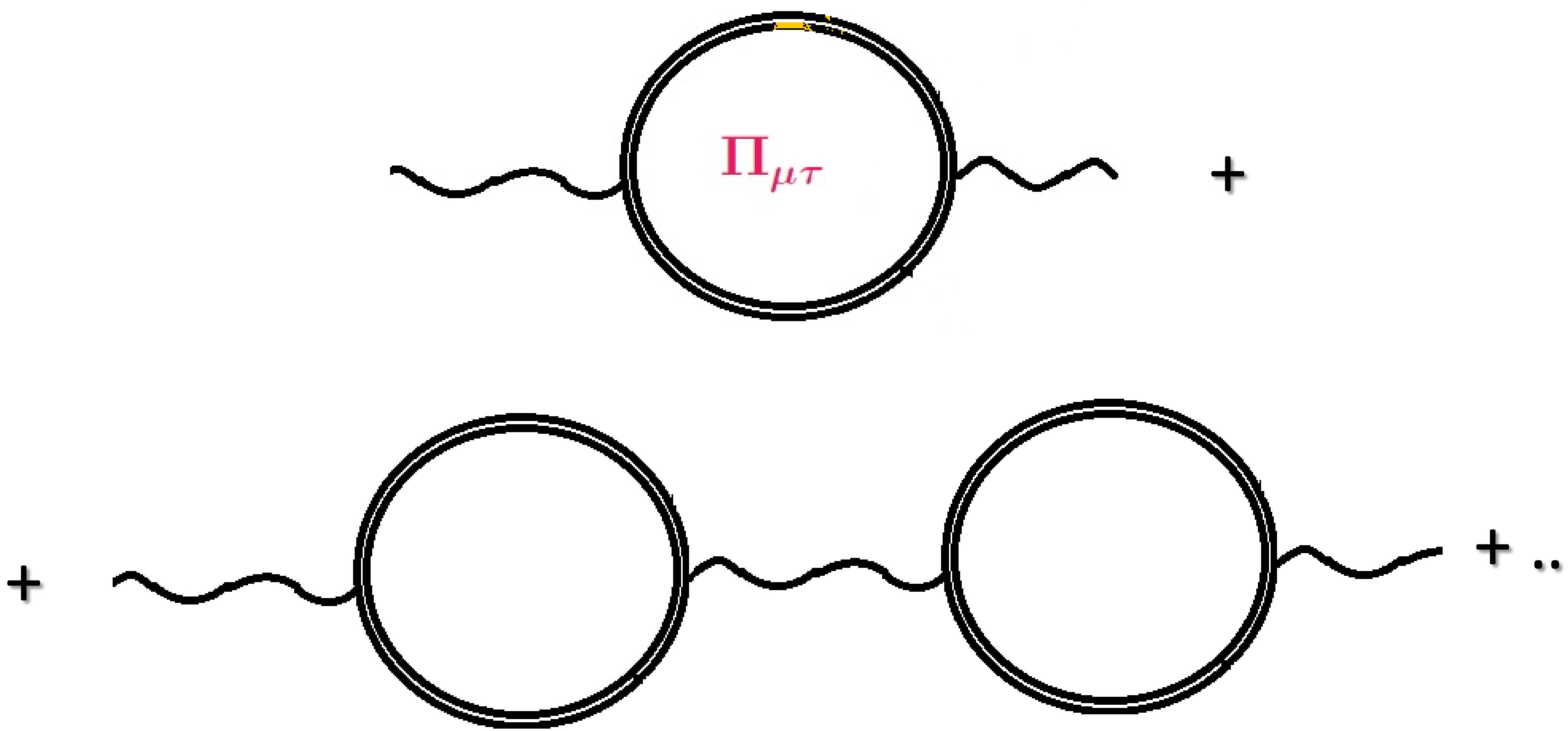
$$D_{\mu\nu}(k) = \sum_{a=1}^{3} \frac{\flat_\mu^{(a)} \flat_\nu^{(a)}}{(\flat^{(a)})^2} \frac{1}{(k^2 - \varkappa_a(k))}$$

Dispersion equations

$$\varkappa_a(k\tilde{F}^2k, kF^2k, \mathfrak{F}, \mathfrak{G}^2) = k^2$$

$$\mathfrak{F}(x) = \frac{1}{2}\left(B^2 - E^2\right) \qquad \mathfrak{G}(x) = (\mathbf{B} \cdot \mathbf{E})$$

One-loop approximation of QED



b=eB/m

$$\mathbf{D}_{\mu\nu}(\mathbf{k}) = \sum_{\mathbf{a}=1}^{4} \mathcal{D}_{\mathbf{a}}(\mathbf{k}) \frac{\flat_{\mu}^{(a)} \ \flat_{\nu}^{(a)}}{(\flat^{(a)})^2}$$

$$\mathcal{D}_{\mathbf{a}}(\mathbf{k}) = -\frac{1}{(k^2 + \kappa_a(k))}, \qquad \mathbf{a} = 1, 2, 3$$

Linear growth with magnetic field

$$\kappa_2 = -\frac{2\alpha b m^2}{\pi} \exp\left(-\frac{k_\perp^2}{2m^2 b}\right) T\left(\frac{k_3^2 - k_0^2}{4m^2}\right)$$

$$T(0) = 0, \ T(\infty) = 1$$

Static potential of a point charge

$$A_0(\mathbf{x}) = \frac{q}{(2\pi)^3} \int \frac{\mathrm{e}^{-\mathrm{i}\mathbf{k}\mathbf{x}} \mathrm{d}^3 k}{\mathbf{k}^2 - \kappa_2(0, k_3^2, k_\perp^2)}, \quad A_{1,2,3}(\mathbf{x}) = 0.$$

Characteristic field

$$B_0 = \frac{m^2}{e} = 4.4 \times 10^{13} \textbf{ Gauss}$$

Magnetic field measured by $b \equiv \frac{B}{B_0}$

Electron Compton half-length

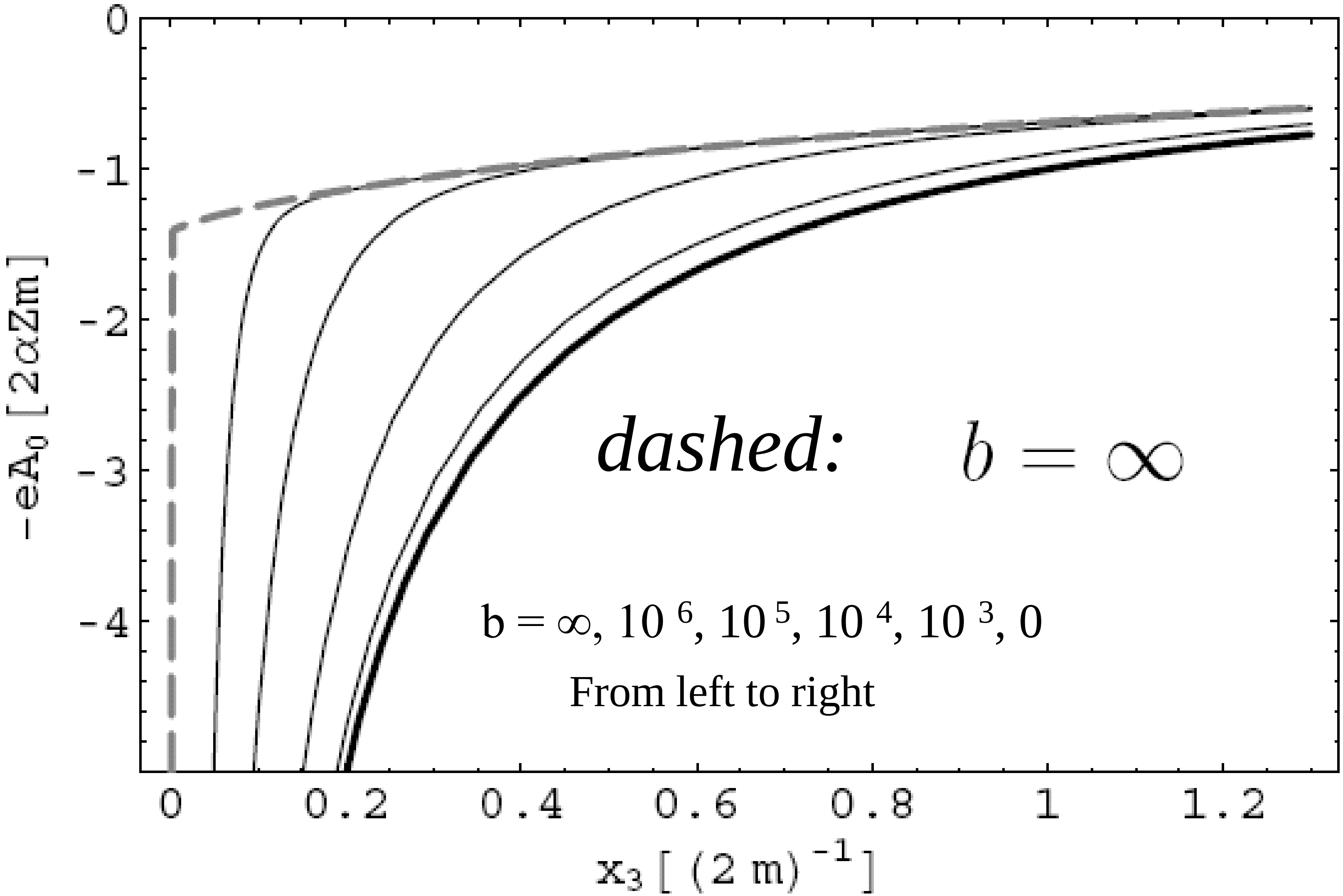
$$\lambda_C/2 = \frac{1}{2m}$$

Larmour length $L_B = (eB)^{-1/2} = \frac{1}{m\sqrt{b}}$

The hierarchy of two scales: $L_B \ll \lambda_C$

Electron potential energy plotted against longitudinal distance at $X_\perp = 0$

$q = eZ$



Modification of Coulomb field in strong magnetic field

$$A_0(\mathbf{x}) = A_{\mathrm{s.r.}}(\mathbf{x}) + A_{\mathrm{l.r.}}(\mathbf{x})$$

Yukawa law for shorter distances: $L_B < |\mathbf{x}| \ll \lambda_C$

$$A_{\mathrm{s.r.}}(\mathbf{x}) \simeq \frac{q}{4\pi} \frac{\exp\{-\left(\frac{2\alpha b}{\pi}\right)^{\frac{1}{2}} m|\mathbf{x}|\}}{|\mathbf{x}|}$$

"Debye" mass squared

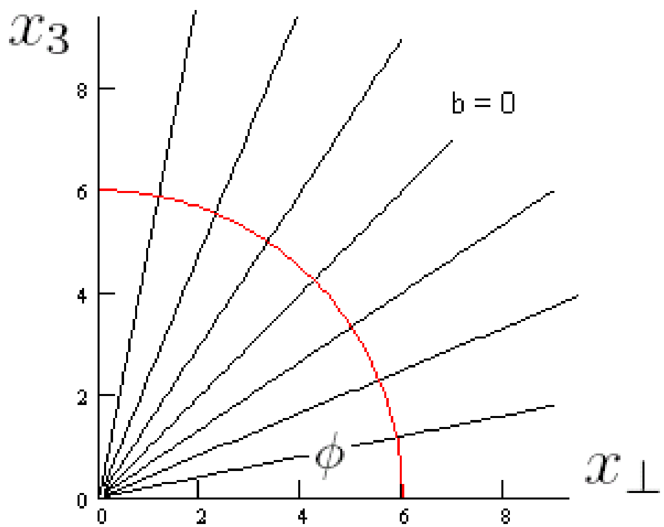
$$\frac{2\alpha b}{\pi} m^2$$

For larger distances $|\mathbf{x}| > \lambda_C$

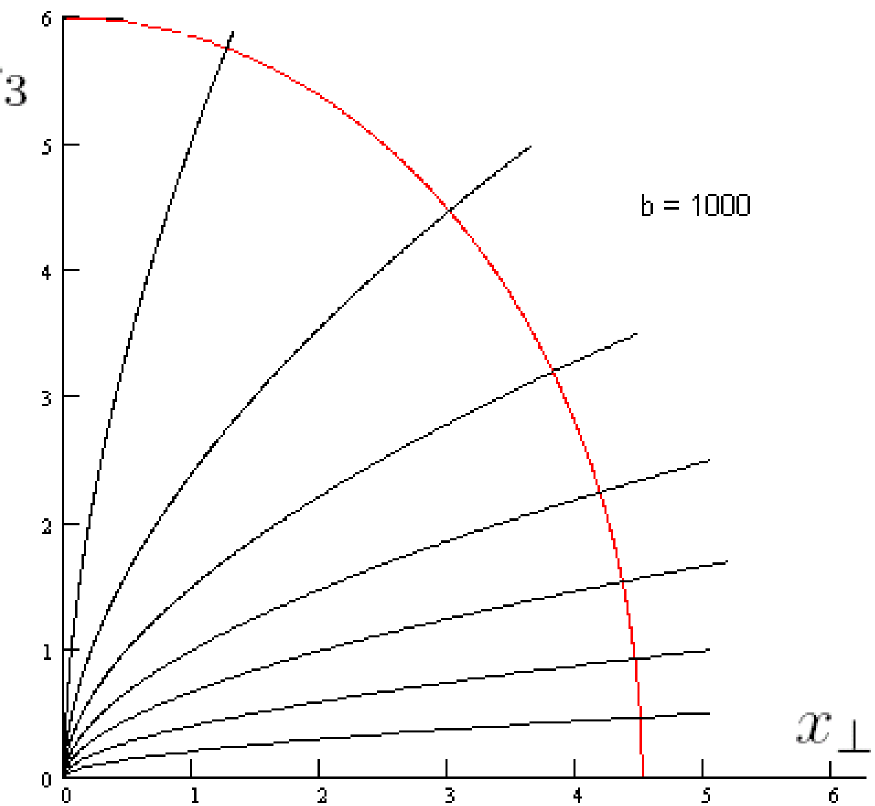
Anisotropic Coulomb law

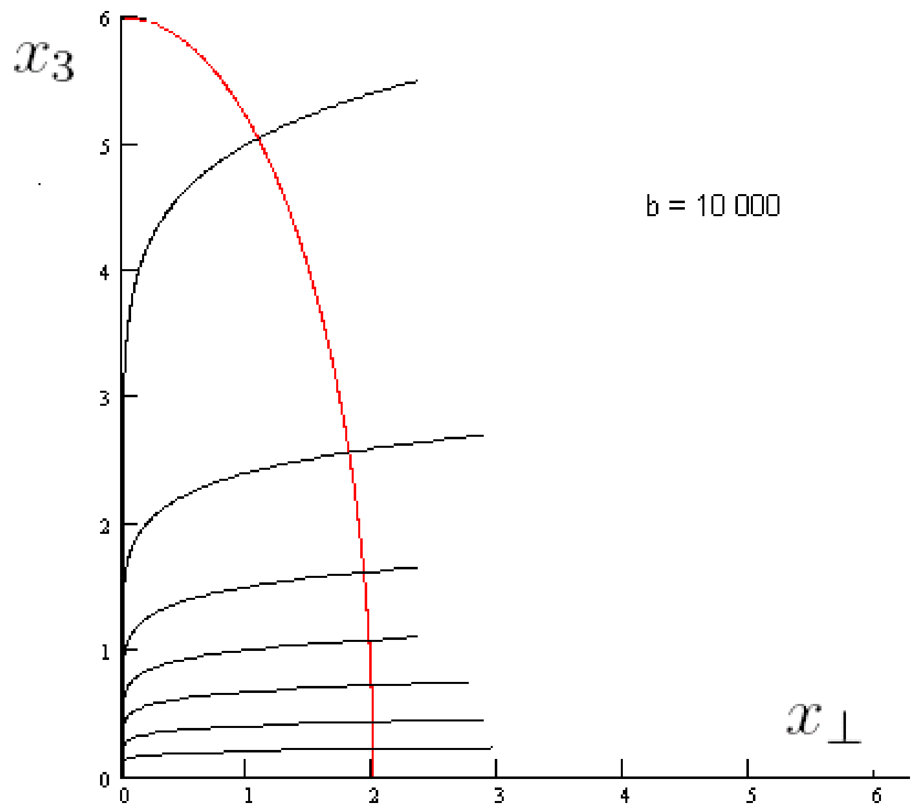
$$A_0(x_3, x_\perp) \simeq \frac{1}{4\pi} \frac{q}{\sqrt{(x'_\perp)^2 + x_3^2}},$$

$$x'_\perp = x_\perp \left(1 + \frac{\alpha b}{3\pi}\right)^{1/2}, \quad x'_\perp > x_\perp$$



$$x_3 = \tan\phi \, x_\perp^{\left(\frac{1}{1+\frac{\alpha b}{3\pi}}\right)}$$





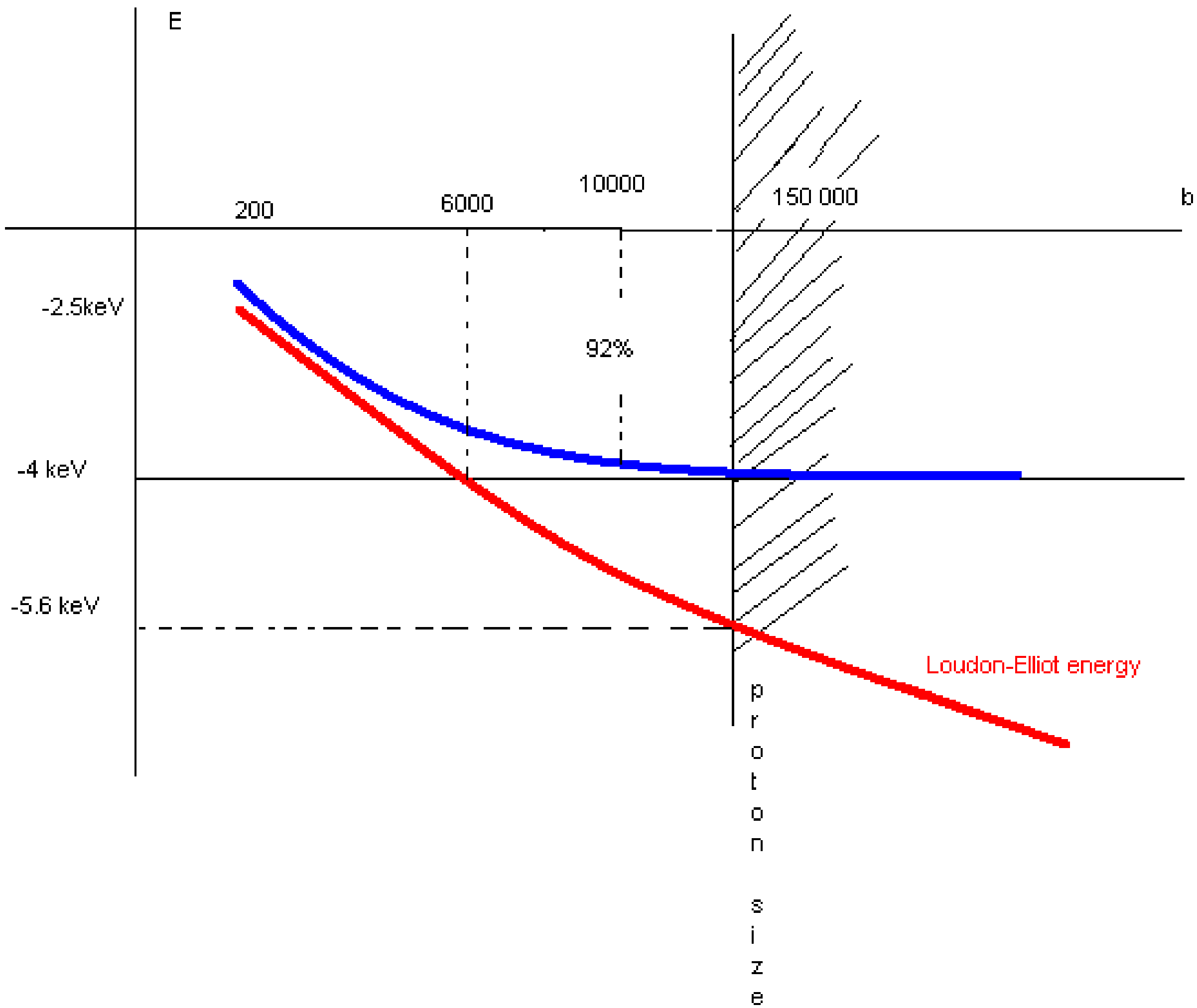
Potential on the string
(infinite magnetic field)

Long-range part on the axis $X_\perp = 0$
near the charge $2x_3 m \ll 1$

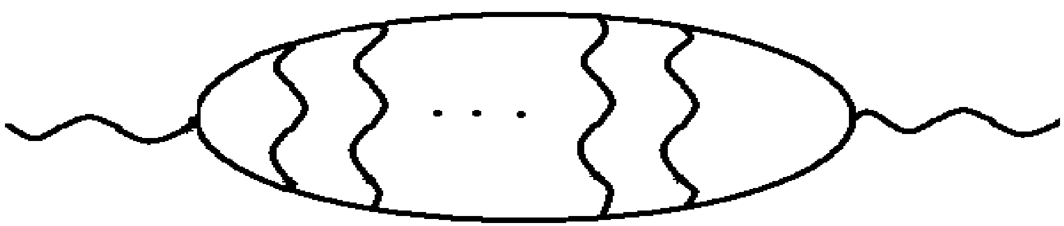
$$A_{\mathrm{l.r.}}(x_3,0)\big|_{b=\infty} = \frac{qm}{2\pi}[1.4152 + 0.495 \cdot 2mx_3(\ln(2mx_3) - 0.77)]$$

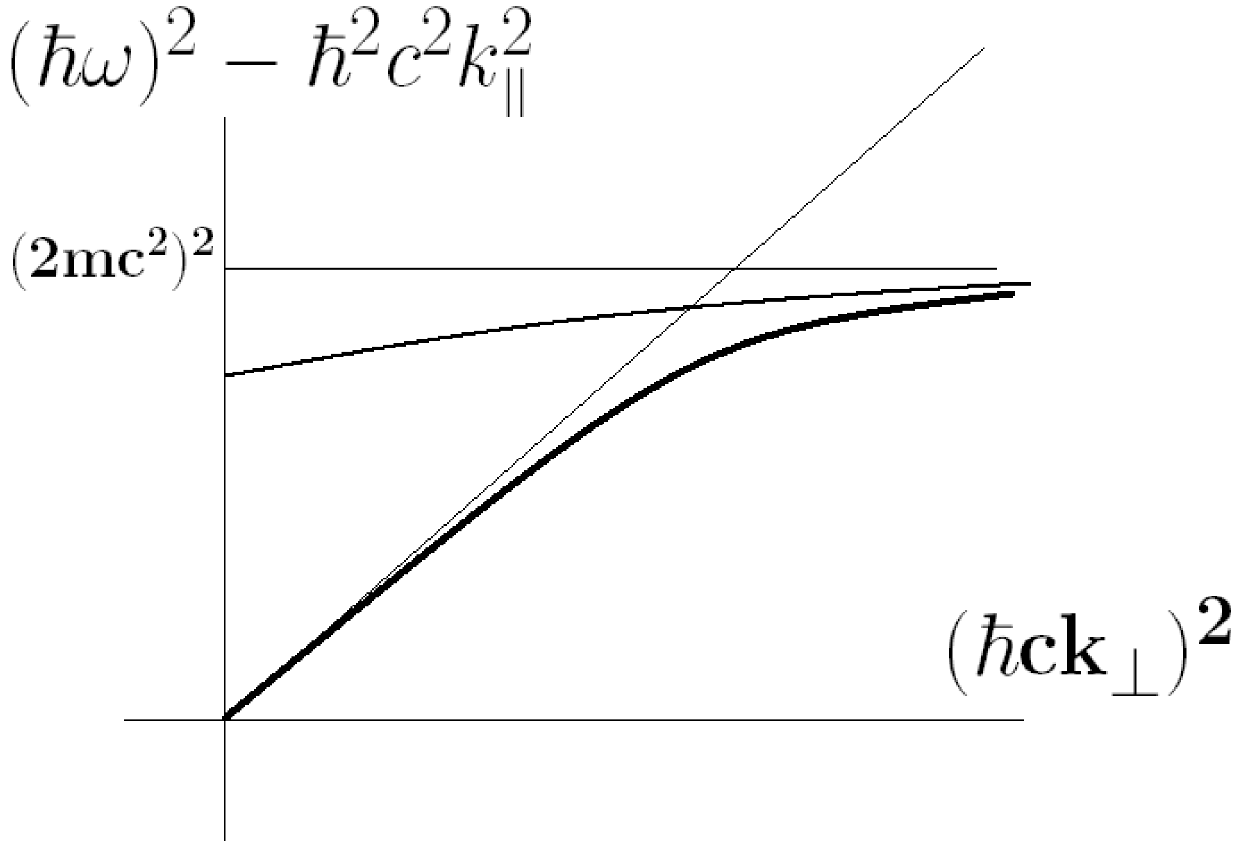
Short-range part on the axis $X_\perp$

$$A_{s.r.}(x_3,0)\big|_{b=\infty} = 2.180\frac{q}{2\pi}\delta(x_3)$$


E
200
6000
10000
150 000
b
-2.5keV
92%
-4 keV
-5.6 keV
Loudon-Elliot energy
proton size

Including positronium state into second-rank polarization tensor

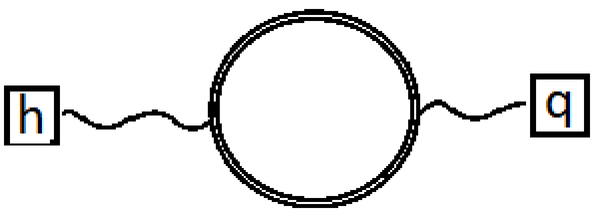


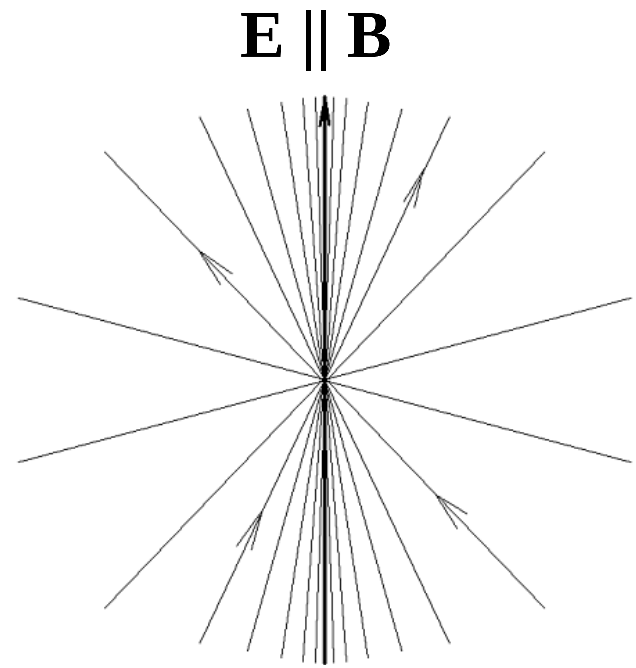


Photon capture by strong magnetic field:

$$\mathbf{v}_{\mathrm{gr}}^{\perp} = \frac{\mathrm{d}\omega}{\mathrm{d}\mathbf{k}_{\perp}} \to \mathbf{0}$$

Simplest magneto-electric effect: point-like electric charge at rest in parallel constant electric and magnetic fields





$$\mathbf{h} = \frac{\mathbf{x}}{|\mathbf{x}|}\frac{q\mathfrak{M}}{8\pi}\frac{1}{|\mathbf{x}|^2}\frac{x_{\parallel}}{|\mathbf{x}_{\perp}|}$$

$$\mathfrak{M} = \mathfrak{L}_{\mathfrak{F}\mathfrak{G}}(\mathrm{B}^2 + \mathrm{E}^2) - (\mathfrak{L}_{\mathfrak{G}\mathfrak{G}} + \mathfrak{L}_{\mathfrak{F}\mathfrak{F}})\mathfrak{G}$$

Furry-like theorem for static case:
number of electric and magnetic legs are both even (space parity conservation).

Why?
Because *B* is a pseudovector, hence it must enter even number of times, while the total number of legs is even from charge conjugation invariance.

Basis for magneto-electric phenomena

Nonlinear Maxwell equations

truncated at the fourth power of the field $a^{\tau}(x)$

$$j_{\mu}(x) = [\Box \eta_{\mu\tau} - \partial_{\mu}\partial_{\tau}]\, a^{\tau}(x) + \int \mathrm{d}^4 y\, \Pi_{\mu\tau}(x,y)\, a^{\tau}(y)$$
$$+\frac{1}{2}\int \mathrm{d}^4 y \mathrm{d}^4 u\, \Pi_{\mu\tau\sigma}(x,y,u)\, a^{\tau}(y)\, a^{\sigma}(u)$$
$$+\frac{1}{6}\int \mathrm{d}^4 y \mathrm{d}^4 u \mathrm{d}^4 v\, \Pi_{\mu\tau\sigma\rho}(x,y,u,v)\, a^{\tau}(y)\, a^{\sigma}(u)\, a^{\rho}(v)$$

$$A_{\beta}(x) = \mathcal{A}_{\beta}^{\mathrm{ext}}(x) + a_{\beta}(x)$$

Infrared approximation

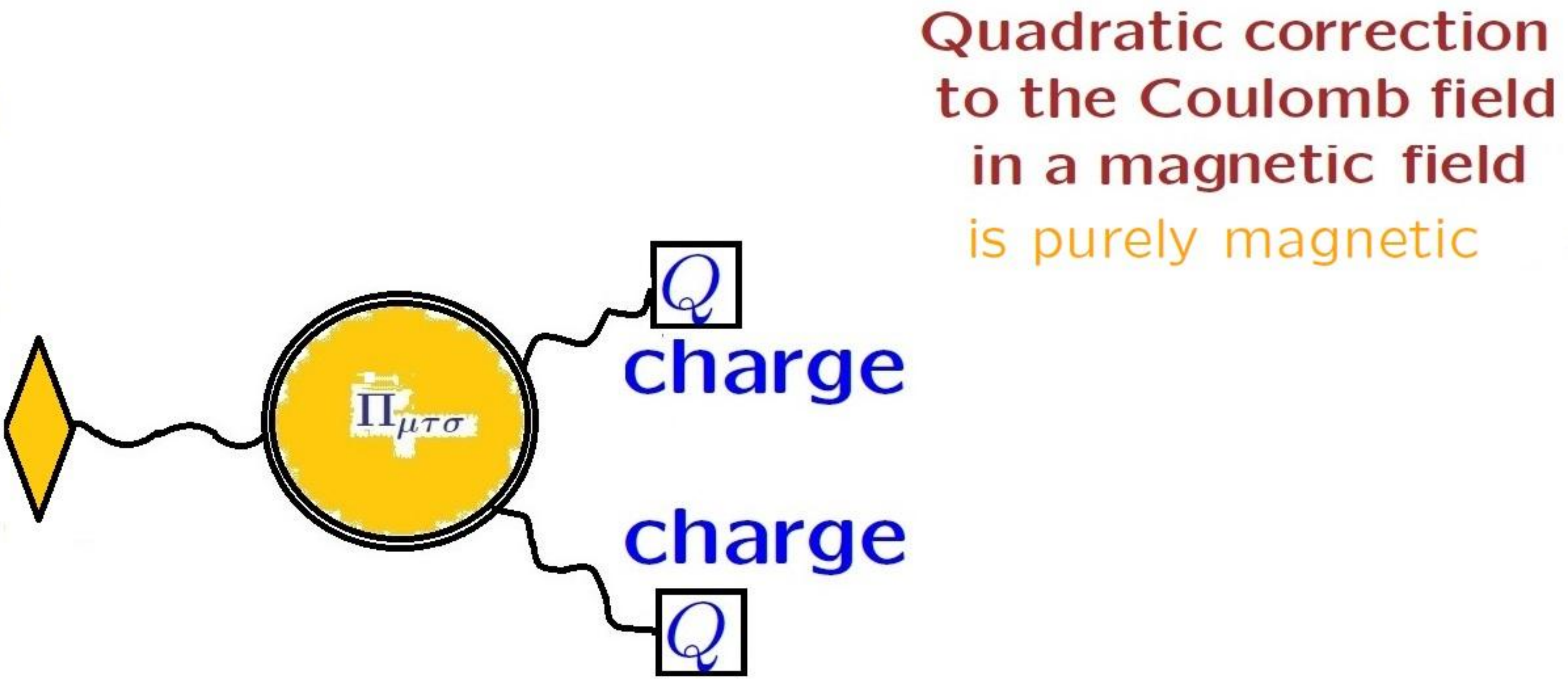
$$\Gamma\left[A\right] = \int \mathcal{L}\left(z\right) d^4 z = \Gamma\left[\mathfrak{F}, \mathfrak{G}, \cancel{\partial_x F_{\mu\nu}, \ldots}\right]$$

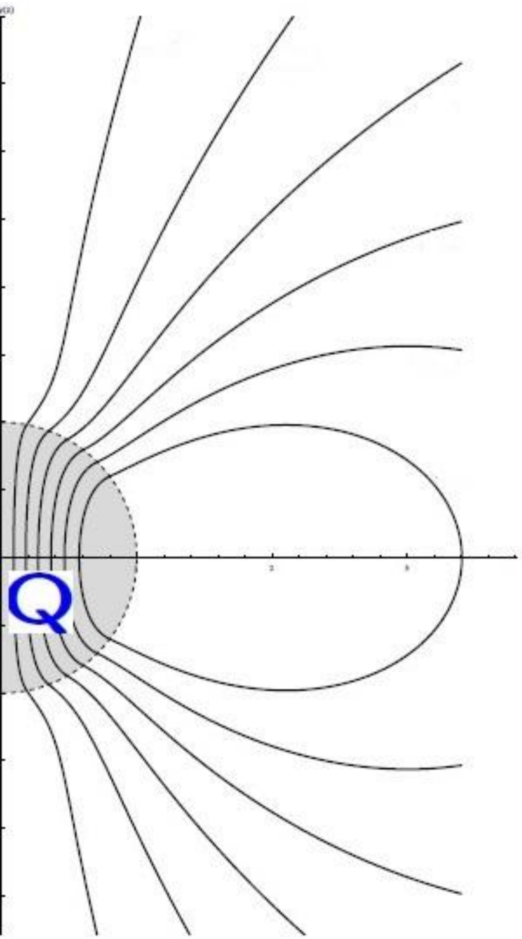
$$\mathfrak{F}\left(z\right) = \frac{1}{4} F^{\mu\nu} F_{\mu\nu}\,, \quad \mathfrak{G} = \frac{1}{4} F^{\rho\sigma} \tilde{F}_{\rho\sigma}$$

$$F^{\mu\nu} = \partial_\mu A^\nu(x) - \partial_\nu A^\mu(x),$$

$$\tilde{F}_{\rho\sigma} = \frac{1}{2} \epsilon_{\rho\sigma\lambda\kappa} F^{\lambda\kappa}$$

- 3. Quadratic response of the strong magnetic field to an electric field: static charge is a magnetic dipole





Quadratic correction to the Coulomb field in a magnetic field is purely magnetic

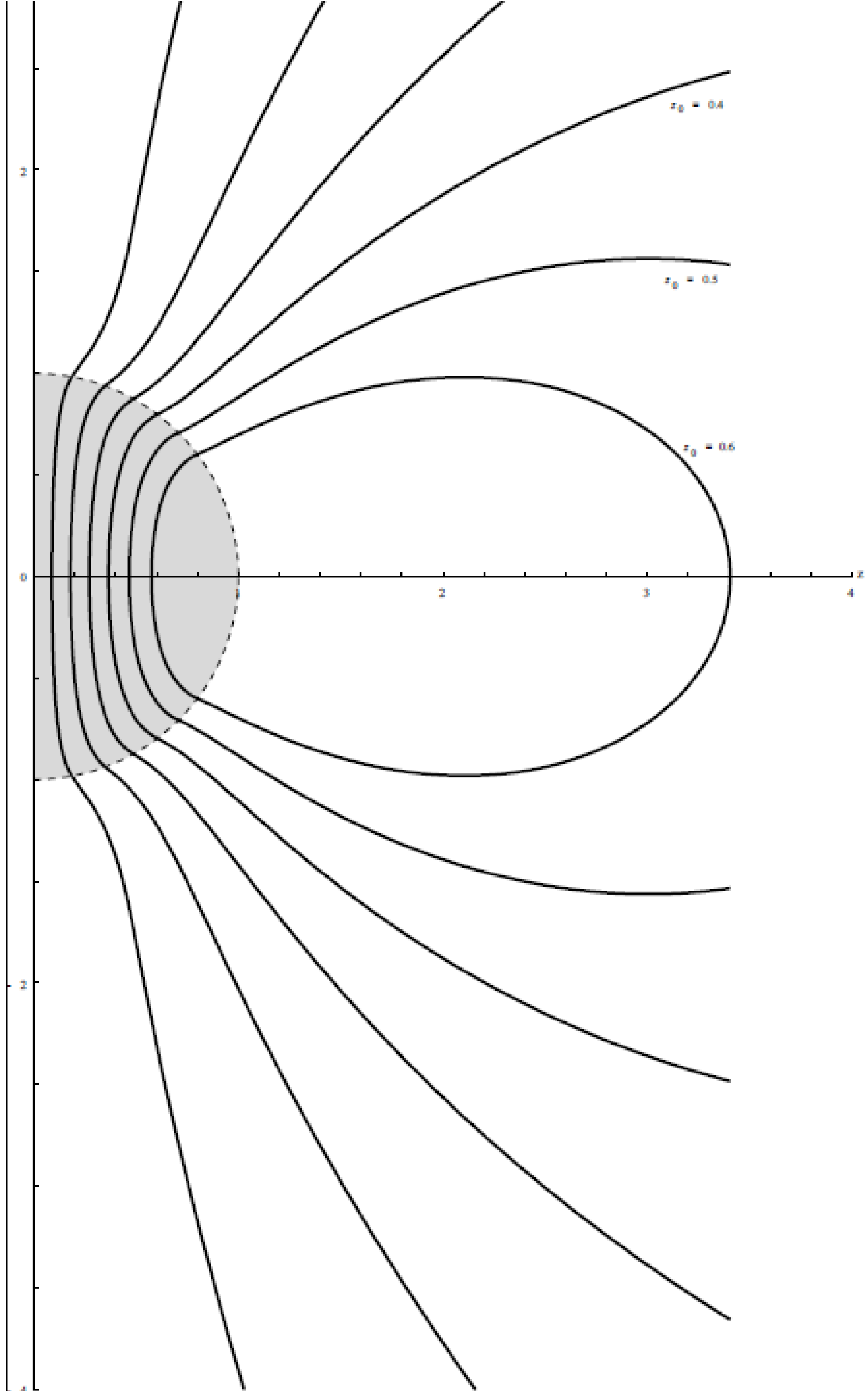
Charge

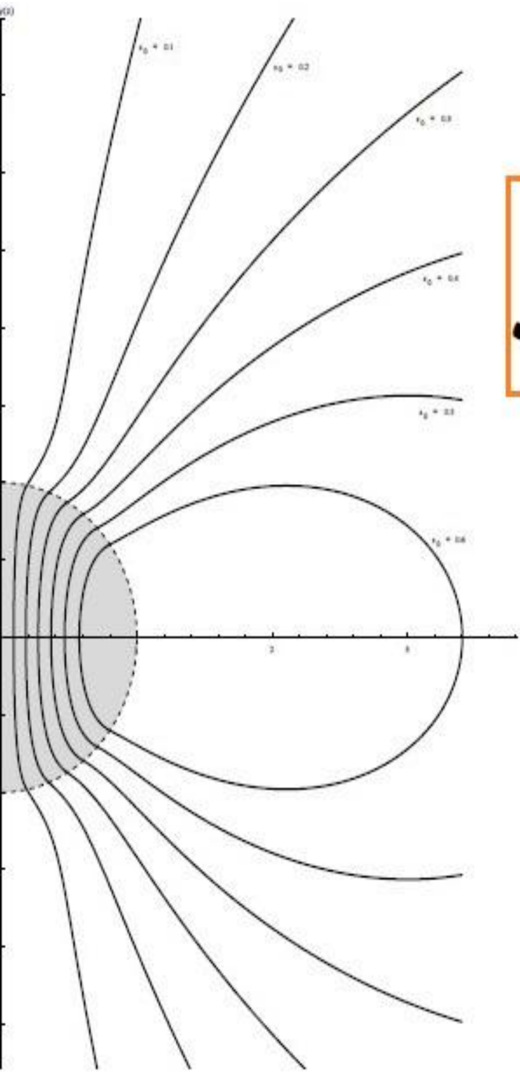
Q

homogeneously distributed

over a sphere

Magnetic lines of force emitted by charged sphere





Magnetic moment

$$\boldsymbol{\mathcal{M}} = \frac{Q^2}{5a}\left(3\mathcal{L}_{\mathfrak{F}\mathfrak{F}} - 2\mathcal{L}_{\mathfrak{G}\mathfrak{G}} - B^2\mathcal{L}_{\mathfrak{F}\mathfrak{G}\mathfrak{G}}\right)\boldsymbol{B}$$

of charge Q

distributed over sphere of radius a

in magnetic field $\boldsymbol{B}$

in terms of derivatives of effective Lagrangian over invariants

$$\mathfrak{F} = \frac{B^2}{2},\ \mathfrak{G} = 0$$

Magnetic moment of a charge plotted against background magnetic field

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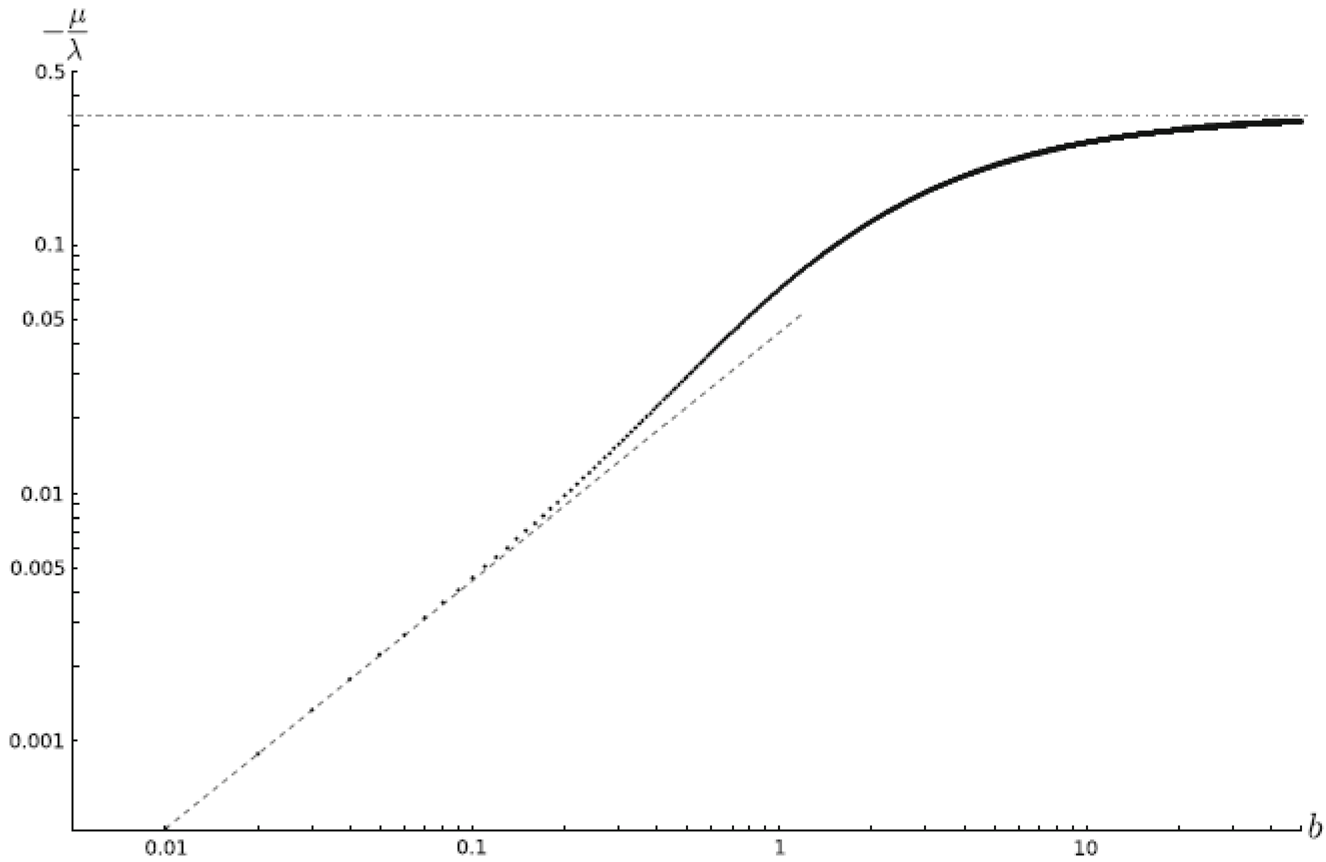


Fig. 1 The magnetic moment (69) of a charge plotted in logarithmic scale against the magnetic field $b = B/B_{\mathrm{Sch}}$ in the range $10^{-2} < b < 50$. The scaling parameter is $\lambda = (Ze/4\pi)^2(\alpha/5\pi R B_{\mathrm{Sch}})$. The *dot-dashed line* corresponds to the large-field constant asymptotic value, while the *dashed line* is the small-field linear asymptotic behavior (73). The step in b is 10^{-2}. The *leftmost value* for the magnetic moment is approximately $-4.4 \times 10^{-3}\lambda$

- 4. Cubic response of the blank vacuum to electric field

i) self-coupling of a point charge and finiteness of its field energy (solitonic configuration)

ii) self-coupling of a magnetic dipole and of an electric dipole

Nonlinear Maxwell equations

truncated at the fourth power of the field $a^{\tau}(x)$

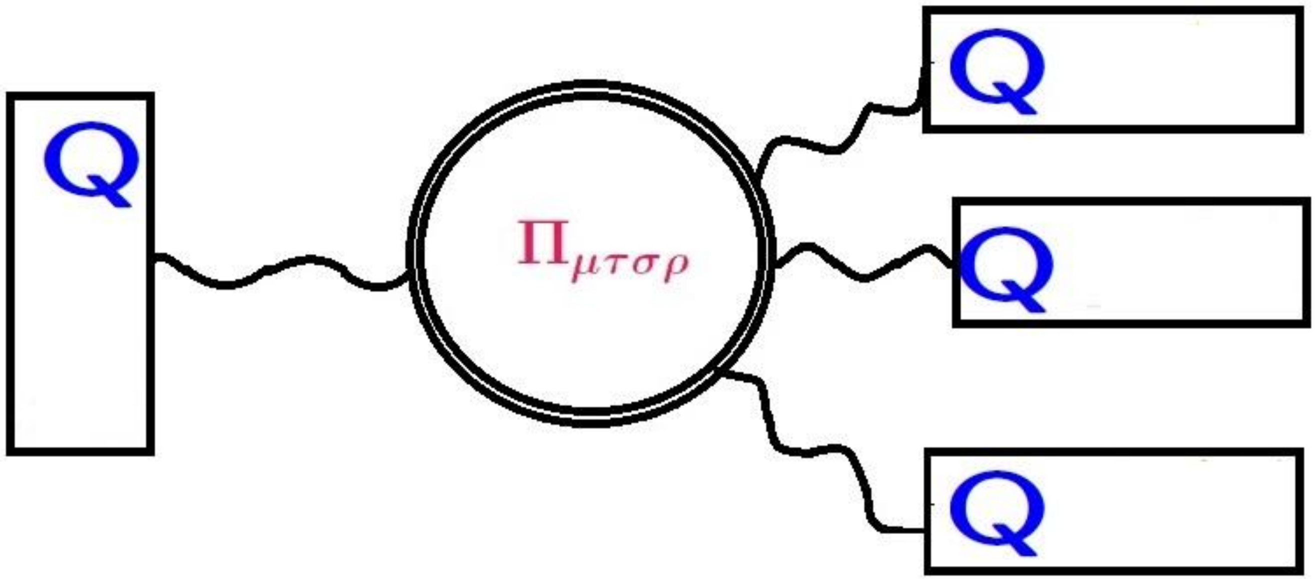
$$j_{\mu}(x) = \left[\Box\eta_{\mu\tau} - \partial_{\mu}\partial_{\tau}\right] a^{\tau}(x) + \int \mathrm{d}^4y\, \Pi_{\mu\tau}(x,y)\, a^{\tau}(y)$$

$$+\frac{1}{2}\int \mathrm{d}^4y \mathrm{d}^4u\, \Pi_{\mu\tau\sigma}(x,y,u)\, a^{\tau}(y)\, a^{\sigma}(u)$$

$$+\frac{1}{6}\int \mathrm{d}^4y \mathrm{d}^4u \mathrm{d}^4v\, \Pi_{\mu\tau\sigma\rho}(x,y,u,v)\, a^{\tau}(y)\, a^{\sigma}(u)\, a^{\rho}(v)$$

$$A_{\beta}(x) = \mathcal{A}_{\beta}^{\mathrm{ext}}(x) + a_{\beta}(x)$$

Cubic corrections to the field of charge



Quartic model

$$L(x) = -\mathfrak{F}(x) + \frac{\gamma}{2}(\mathfrak{F}(x))^2$$

Electromagnetic field invariant

$$\mathfrak{F}(x) = \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \frac{1}{2}\left(B^2 - E^2\right)$$

In truncated QED: $\gamma \equiv \mathfrak{L}_{\mathfrak{F}\mathfrak{F}}$

Nonlinear Maxwell equations

$$\partial_\mu \left[\left(1 - \gamma \mathfrak{F}\left(x\right)\right) F^{\mu\nu} \right] = 0$$

Electrostatic case

$$\nabla \left[\left(1 + \frac{\gamma}{2} E^2 \right) \mathbf{E} \right] = 0$$

Spherical-symmetric solution for point charge Q

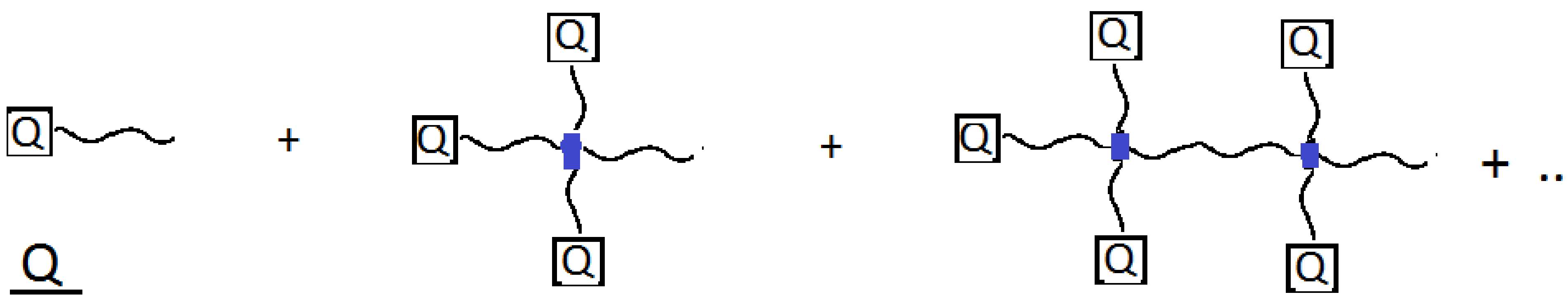
$$\left(1 + \frac{\gamma}{2} E^2(r) \right) \mathbf{E}\left(\mathbf{x}\right) = \frac{Q}{4\pi r^2} \frac{\mathbf{x}}{r}$$

$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi r^2}\left(1 - \frac{2\alpha}{45\pi}\left(\frac{e\,Q}{4\pi r^2\, m^2}\right)^2\right)$$

Wichmann and Kroll 1954

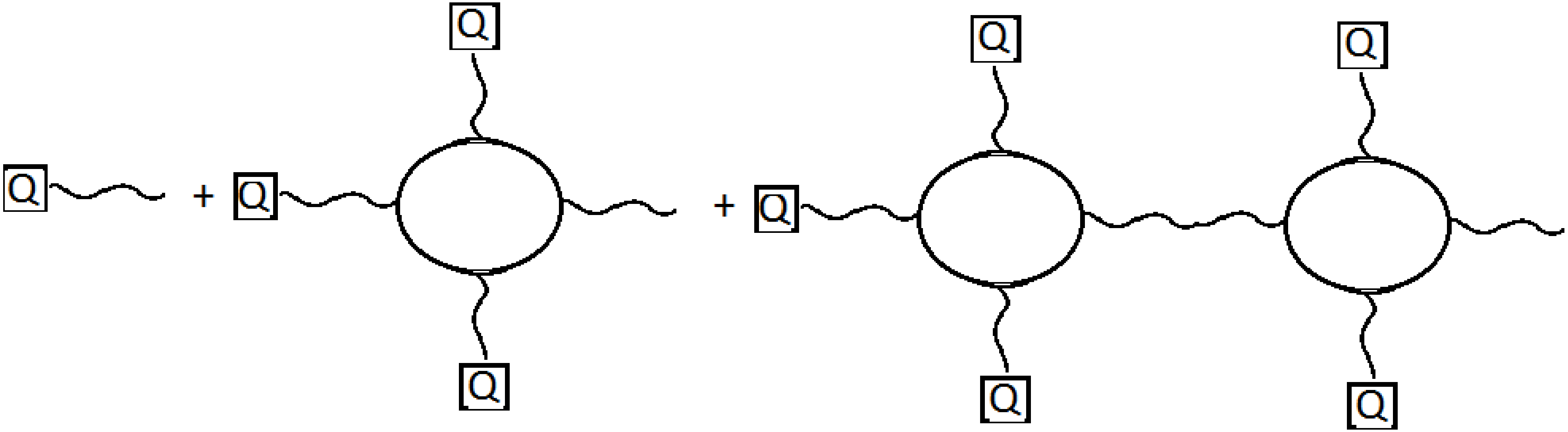
$$\gamma \equiv \mathfrak{L}_{\mathfrak{F}\mathfrak{F}} \equiv \left.\frac{\mathrm{d}^2\mathfrak{L}^{\mathrm{EH}}(\mathfrak{F},0)}{\mathrm{d}\mathfrak{F}^2}\right|_{\mathfrak{F}=0} = \frac{e^4}{45\pi^2 m^4}$$

Iterations of the field equation



$\frac{Q}{r^2}$ $\quad$ $\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}\frac{Q^3}{r^6}$ $\quad$ $\mathfrak{L}^2_{\mathfrak{F}\mathfrak{F}}\frac{Q^5}{r^{10}}$

One-photon reducible chain



Electrostatic energy of electron charge *Q* = *e*

$$\Theta^{00} = \frac{E^2}{2} + \frac{3\mathcal{L}_{\mathfrak{F}\mathfrak{F}} E^4}{8} \qquad \int \Theta^{00} d^3x = 2.09m$$

Soliton field configuration

$$E = \sqrt[3]{\sqrt{\left(\frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^2 + \left(\frac{2}{3\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^3} + \frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}}$$

$$- \sqrt[3]{\sqrt{\left(\frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^2 + \left(\frac{2}{3\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^3} - \frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}}$$

$$E_{\text{lin}}(r) = \frac{e}{4\pi r^2}$$

Electrostatic energy of electron charge $Q = e$

$$\Theta^{00} = \frac{E^2}{2} + \frac{3\mathcal{L}_{\mathfrak{F}\mathfrak{F}} E^4}{8} \qquad \int \Theta^{00} d^3x = 2.09m$$

Soliton field configuration

$$E = \sqrt[3]{\sqrt{\left(\frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^2 + \left(\frac{2}{3\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^3} + \frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}}$$

$$- \sqrt[3]{\sqrt{\left(\frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^2 + \left(\frac{2}{3\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^3} - \frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}}$$

$$E \sim r^{-2/3}$$

$$E_{\text{lin}}(r) = \frac{e}{4\pi r^2}$$

General local regular monotonous Lagrangian

The field energy of a point charge *e*

$$4\pi\int_0^\infty \Theta^{00} r^2 dr =$$

$$\frac{e^{\frac{3}{2}}}{4\pi^{\frac{1}{2}}}\int_0^\infty \left(-\mathfrak{F}+2\mathfrak{F}\mathfrak{L}_{\mathfrak{F}}-\mathfrak{L}\right)\frac{\left[\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}E^2+1-\mathfrak{L}_{\mathfrak{F}}\right]\mathrm{d}E}{\left(\left(1-\mathfrak{L}_{\mathfrak{F}}\right)E\right)^{\frac{5}{2}}}$$

converges if

$$\mathfrak{L}(\mathfrak{F}) \sim (-\mathfrak{F})^w, \quad w > \frac{3}{2}$$

or if

$$\mathfrak{L}(\mathfrak{F}) \sim (-\mathfrak{F})^{\frac{3}{2}} \ln^u(-\mathfrak{F}), \quad u > 2$$

General local regular monotonous Lagrangian

The field energy of a point charge *e*

$$4\pi \int_0^\infty \Theta^{00} r^2 dr =$$

$$\frac{e^{\frac{3}{2}}}{4\pi^{\frac{1}{2}}} \int_0^\infty \left(-\mathfrak{F} + 2\mathfrak{F}\mathfrak{L}_{\mathfrak{F}} - \mathfrak{L}\right) \frac{\left[\mathfrak{L}_{\mathfrak{F}\mathfrak{F}} E^2 + 1 - \mathfrak{L}_{\mathfrak{F}}\right] \mathrm{d}E}{\left(\left(1 - \mathfrak{L}_{\mathfrak{F}}\right) E\right)^{\frac{5}{2}}}$$

converges if

$$\mathfrak{L}(\mathfrak{F}) \sim (-\mathfrak{F})^w, \quad w > \frac{3}{2}$$

or if

$$\mathfrak{L}(\mathfrak{F}) \sim (-\mathfrak{F})^{\frac{3}{2}} \ln^u(-\mathfrak{F}), \quad u > 2$$

Is obeyed

by truncated QED: polynomial Lagrangian

$\mathfrak{L} \sim \mathfrak{F}^n, \ n \geq 2$

Cubic corrections to magnetic dipole field

current, producing dipole field $\boldsymbol{j}(\boldsymbol{r}) = 3\frac{\boldsymbol{\mathcal{M}}^{\text{lin}} \times \mathbf{r}}{r^4}\delta(r-a)$

Dipole field reproduces itself in the remote region

$$\boldsymbol{\mathcal{M}} = \boldsymbol{\mathcal{M}}^{\text{lin}} - \frac{7}{5}\mathcal{L}_{\mathfrak{F}\mathfrak{F}}\boldsymbol{\mathcal{M}}\left(\frac{\mathcal{M}}{a^3}\right)^2$$

Cubic corrections to electric dipole field

current, producing dipole field $j_0(\mathbf{r}) = 3\frac{\mathbf{r}\cdot\mathbf{p}^{\text{lin}}}{r^4}\delta(r-a)$

Dipole field reproduces itself in the remote region

$$\mathbf{p} = \mathbf{p}^{\text{lin}} - \frac{\mathbf{p}}{10}\mathcal{L}_{\mathfrak{F}\mathfrak{F}}\left(\frac{p}{a^3}\right)^2$$

Summary: List of nonlinear effects in QED

1. *Linearization above a strong background field*

*Photon capture by magnetic field with formation of positronium
*Screening of the Coulomb field by a strong magnetic field
*Finiteness of the ground state of Hydrogen when $B \longrightarrow \infty$
*Magnetic monopole-like perturbation of parallel electric and magnetic fields

Summary: List of nonlinear effects in QED

2. Quadratic effects above a strong background field:

*Static charge in a magnetic field is a magnetic dipole

3. Cubic effects without any background field

*Solitonic solution for the field of a charge

*Nonlinear renormalization of electric and magnetic dipole moments

Thank you for attention.


Obrigado pela vossa atenção

Characteristic field

$$B_0 = \frac{m^2}{e} = 4.4 \times 10^{13} \text{ Gauss}$$

Magnetic field measured by $b \equiv \frac{B}{B_0}$

Electron Compton half-length

$$\lambda_C/2 = \frac{1}{2m}$$

Larmour length $L_B = (eB)^{-1/2} = \frac{1}{m\sqrt{b}}$

The hierarchy of two scales: $L_B \ll \lambda_C$

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Effective action

$$\Gamma[A] = \int \mathfrak{L}(x) d^4x$$

Field equations

$$[\Box \eta_{\mu\tau} - \partial_\mu \partial_\tau] \; A^\tau(x) + \frac{\delta \Gamma[A]}{\delta A^\mu(x)}$$

$$= j_\mu(x) + \mathcal{J}_\mu(x)$$

$$A^\tau(x) = a^\tau(x) + \boldsymbol{\mathcal{A}}^\tau(x)$$

Background field equations

$$[\Box u\tau - \partial_\mu \partial_\tau] \; \boldsymbol{\mathcal{A}}^\tau(x) + \left. \frac{\delta \Gamma[A]}{\delta A^\mu(x)} \right|_{A=\boldsymbol{\mathcal{A}}} = \mathcal{J}_\mu(x)$$

Designations

$$\mathfrak{L}_{\mathfrak{F}} = \left.\frac{\mathrm{d}\mathfrak{L}(\mathfrak{F},0))}{\mathrm{d}\mathfrak{F}}\right|_{F=\mathcal{F}} ; \mathfrak{L}_{\mathfrak{F}\mathfrak{F}\mathfrak{F}} = \left.\frac{\mathrm{d}^3\mathfrak{L}(\mathfrak{F},0)}{\mathrm{d}\mathfrak{F}^3}\right|_{F=\mathcal{F}}$$

$$\mathfrak{L}_{\mathfrak{F}\mathfrak{F}} = \left.\frac{\mathrm{d}^2\mathfrak{L}(\mathfrak{F},0)}{\mathrm{d}\mathfrak{F}^2}\right|_{F=\mathcal{F}},$$

$$\mathfrak{L}_{\mathfrak{G}\mathfrak{G}} = \left.\frac{\partial^2\mathfrak{L}(\mathfrak{F},\mathfrak{G})}{\partial\mathfrak{G}^2}\right|_{F=\mathcal{F},\mathfrak{G}=0},$$

$$\mathfrak{L}_{\mathfrak{F}\mathfrak{G}\mathfrak{G}} = \frac{\mathrm{d}}{\mathrm{d}\mathfrak{F}}\left.\frac{\partial^2\mathfrak{L}(\mathfrak{F},\mathfrak{G})}{\partial\mathfrak{G}^2}\right|_{F=\mathcal{F},\mathfrak{G}=0}$$