Light-by-light scattering at the LHC from new physics

Sylvain Fichet
International Institute of Physics, Natal

27/10/14

based on 1311.6815 (with G. von Gersdorff)
1312.5153, 1XXX.XXXX
(with G. von Gersdorff, O. Kepka, B. Lenzi, C. Royon, M. Saimpert)

Outline

1. Introduction: precision physics
2. Four-photon interactions at the LHC
3. New physics candidates for 4γ

Introduction

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- No direct observations so far ! Up to now the LHC just found a Higgs.

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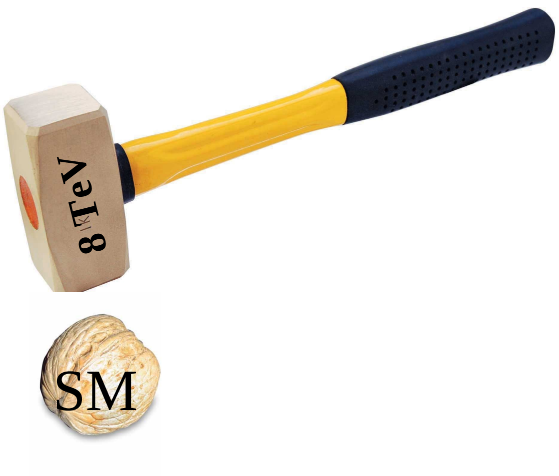

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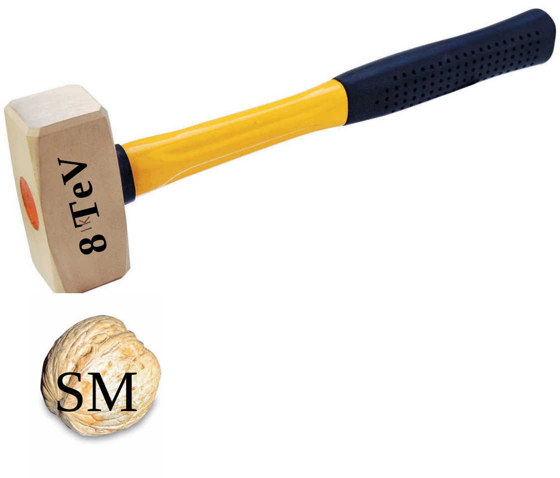

The effective Lagrangian

- Low-energy effects of any heavy new physics are captured by higher-dimensional operators,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{\alpha_i}{\Lambda^2}\mathcal{O}_i + \text{higher order terms} \qquad E < \Lambda$$

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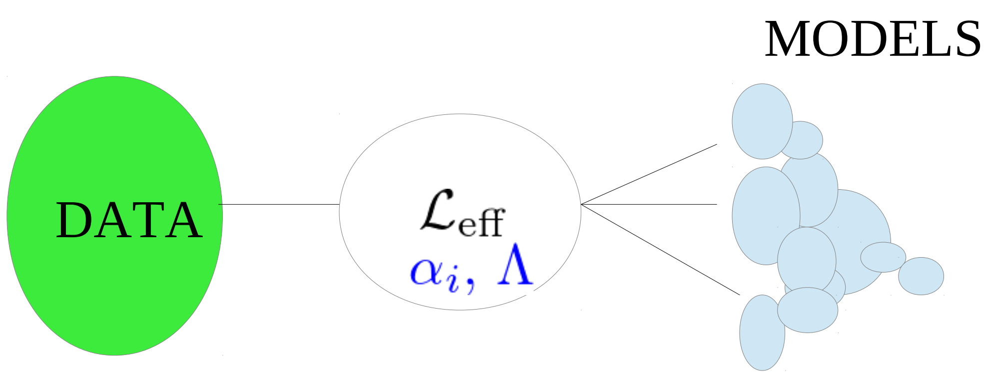
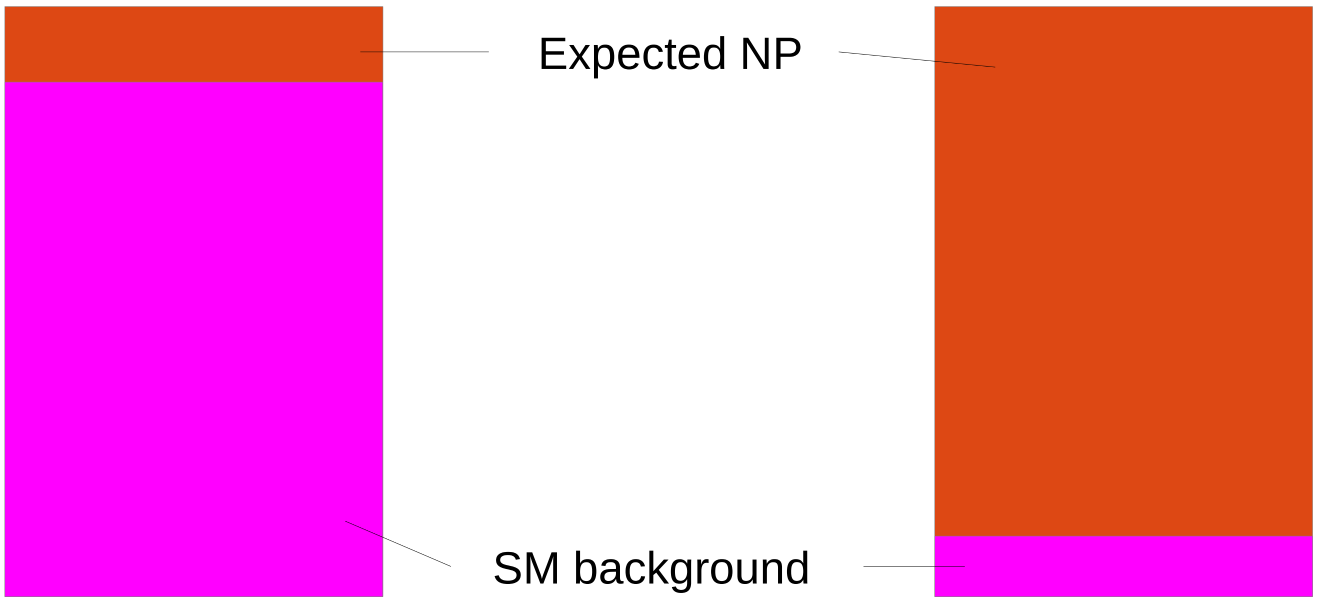
- Precision physics consists in getting information on α_i , Λ



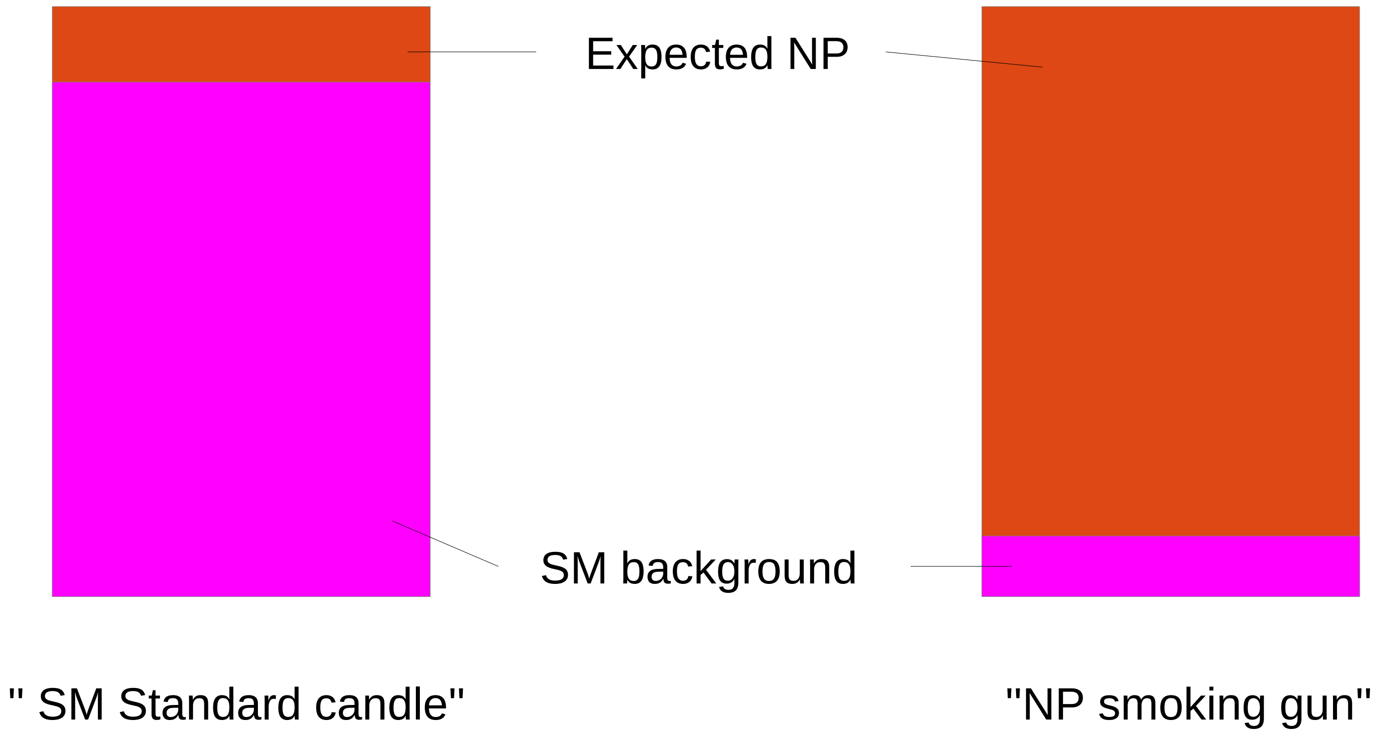
Data for precision physics



" SM Standard candle"

"NP smoking gun"

Data for precision physics



- This depends on the amount of data, as

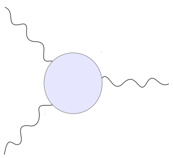
Discovery significance = $\dfrac{n_{\rm obs} - n_{bg}}{\sqrt{n_{bg}}}$ (for large background)

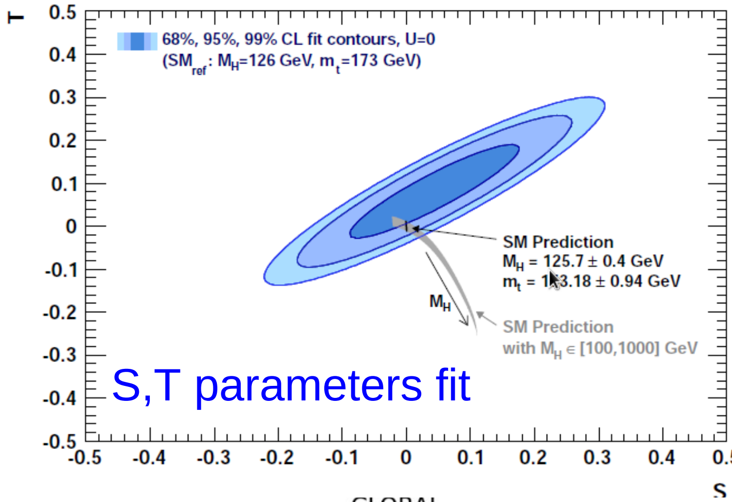
Data for precision physics

- The LEP legacy:

	Best fit	68%	95%
$\delta\kappa_\gamma$	-0.016	$[-0.036, +0.02]$	$[-0.054, 0.021]$
δg_1^Z	-0.018	$[-0.060, +0.024]$	$[-0.099, 0.066]$
λ	-0.022	$[-0.041, -0.003]$	$[-0.059, 0.017]$

Triple gauge couplings fit
(see also last LH report)



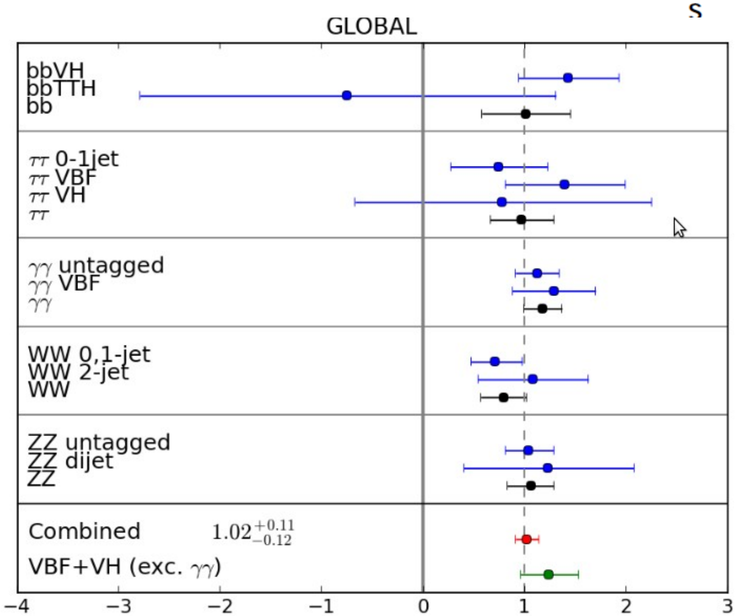


S,T parameters fit

- From LHC and Tevatron:

$$\mu_{XY} = \frac{N_{evts}}{\sigma(X \to h)\mathcal{B}(h \to Y)\epsilon_{XY} \times \mathcal{L}}$$

Higgs couplings to the SM



Data for precision physics

- Fitting Higgs data is a big subject. Today we will just need

$$\mathcal{L}_h^{tree} \supset a_W \frac{h}{v} 2m_W^2 W_\mu^+ W_\mu^- + a_Z \frac{h}{v} m_Z^2 (Z_\mu)^2$$

$0.96 < a_V < 1.21$ 95% BCI

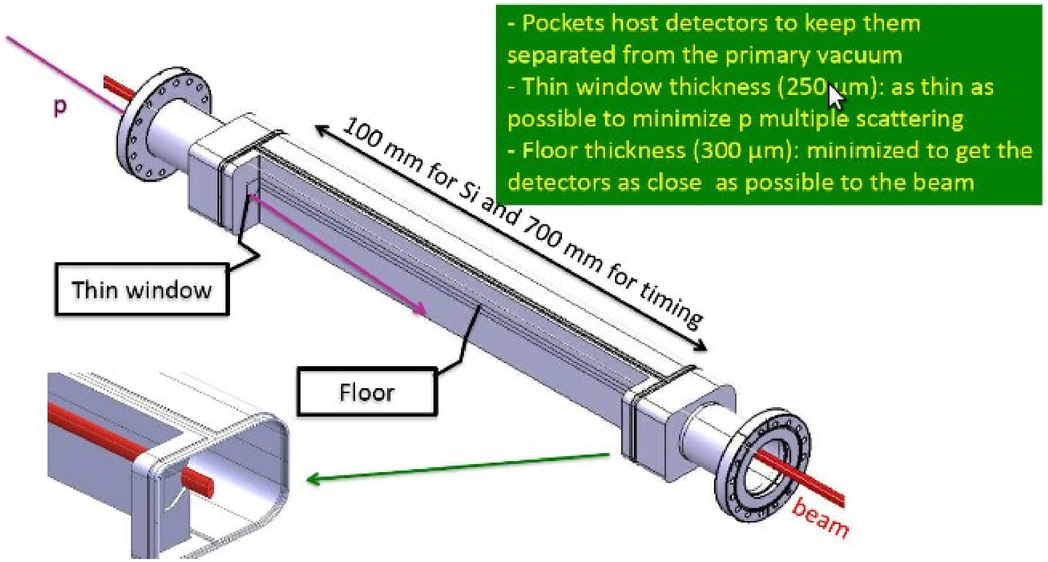
(Assuming $a_W \approx a_Z$)

- Obained through a Bayesian fit of the dimension-6 effective Lagrangian [Dumont/SF/Gersdorff '13]

Four-photon interactions at the LHC

Forward proton detectors

- New detectors scheduled at CMS (CT-PPS) and ATLAS (AFP) to detect intact protons from proton diffraction at small angles



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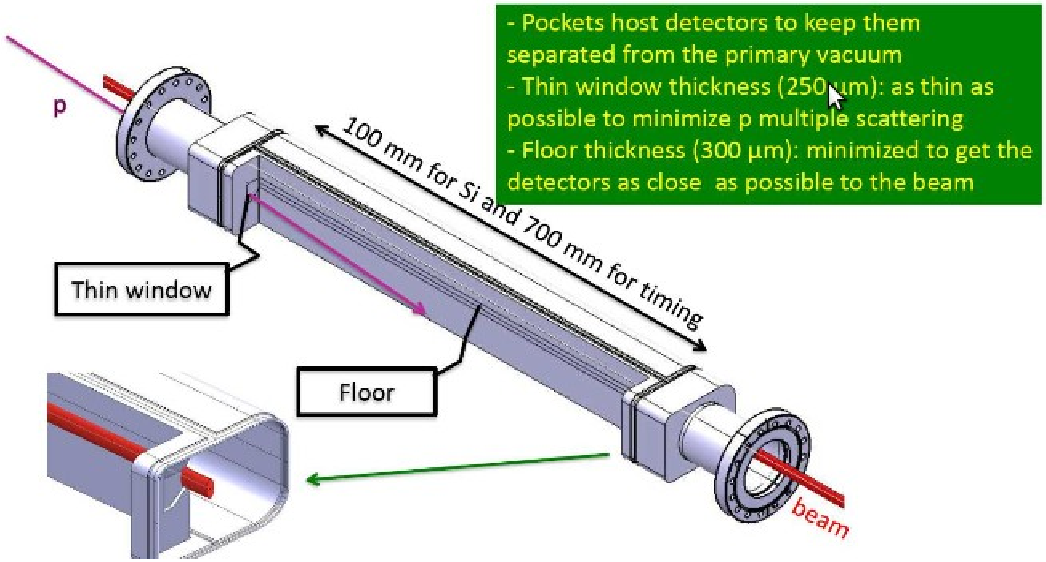
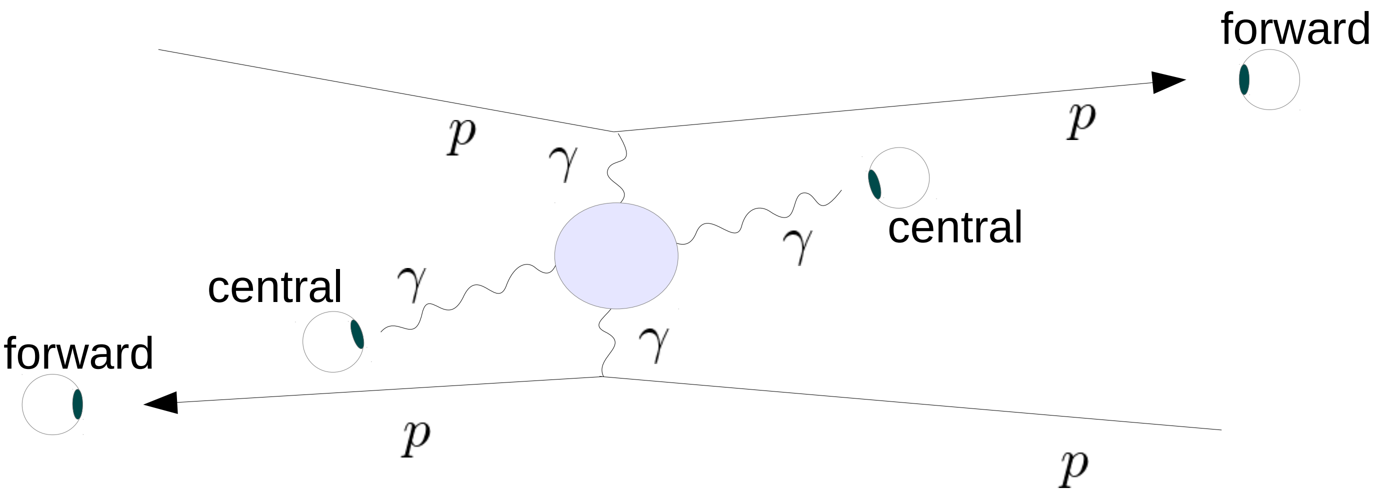


- Intact protons: all kinematics is known !

- More details on the experimental setup : see Matthias' talk

Forward proton detectors

- Proton diffraction can be studied with a good sensitivity.
  Among the processes: light-by-light scattering



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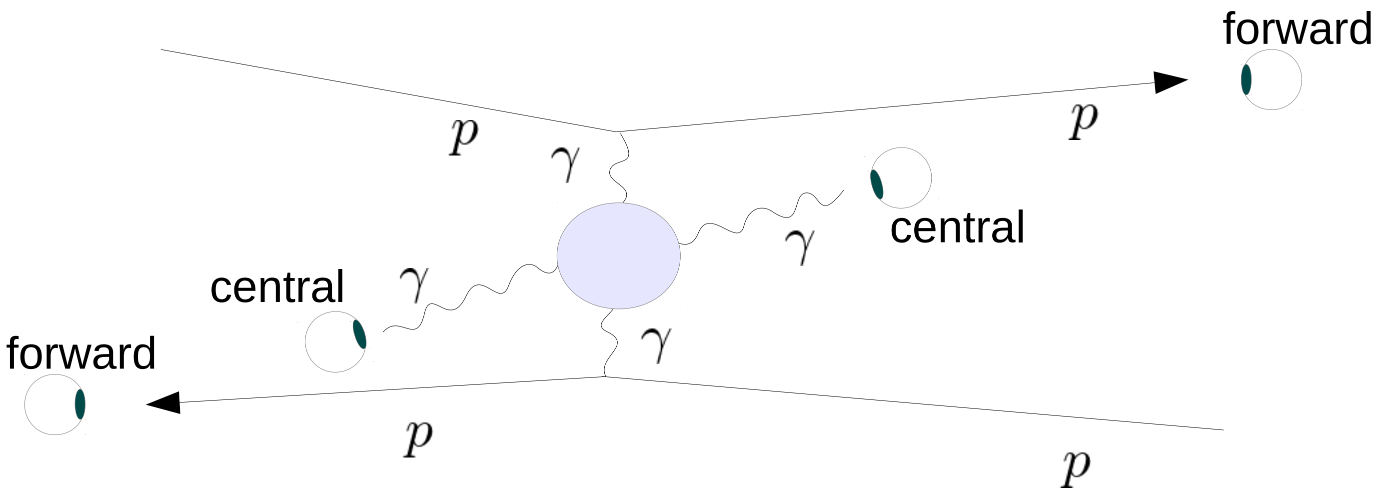
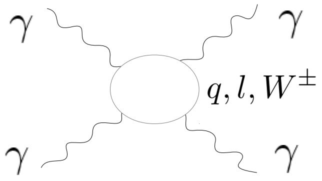


- Can the SM process be observed ?



- Potential probe for new physics in 4γ vertices ?

Can the SM process be observed at the LHC ?

Can the SM process be observed at the LHC ?

- # NO

- Long answer: see Matthias' talk

Potential for new physics discovery: EFT regime

- In the limit M>>E, new physics effects are enclosed in effective operators. The study is model-independent in this limit.

- What are the effective operators ?

The dimension 8 effective Lagrangian

- For quartic gauge couplings, one needs to go up ot dimension 8 operators

$$\mathcal{L}_{(8)} = \frac{\alpha_1}{\Lambda^4} D^\mu H^\dagger D_\mu H D^\nu H^\dagger D_\nu H + \frac{\alpha_2}{\Lambda^4} D^\mu H^\dagger D_\nu H D^\nu H^\dagger D_\mu H$$
$$+ \frac{\alpha_3}{\Lambda^4} D^\rho H^\dagger D_\rho H B_{\mu\nu} B^{\mu\nu} + \frac{\alpha_4}{\Lambda^4} D^\rho H^\dagger D_\rho H W^I_{\mu\nu} W^{I\mu\nu} + \frac{\alpha_5}{\Lambda^4} D^\rho H^\dagger \sigma^I D_\rho H B_{\mu\nu} W^{I\mu\nu}$$
$$+ \frac{\alpha_6}{\Lambda^4} D^\nu H^\dagger D_\rho H B_{\mu\nu} B^{\mu\rho} + \frac{\alpha_7}{\Lambda^4} D^\nu H^\dagger D_\rho H W^I_{\mu\nu} W^{I\mu\rho}$$

DH^2 F^2 : comes with EW breaking

$$+ \frac{\alpha_8}{\Lambda^4} B^{\mu\nu} B_{\mu\nu} B^{\rho\sigma} B_{\rho\sigma} + \frac{\alpha_9}{\Lambda^4} W^{I\mu\nu} W^I_{\mu\nu} W^{J\rho\sigma} W^J_{\rho\sigma} + \frac{\alpha_{10}}{\Lambda^4} W^{I\mu\nu} W^J_{\mu\nu} W^{I\rho\sigma} W^J_{\rho\sigma}$$
$$+ \frac{\alpha_{11}}{\Lambda^4} B^{\mu\nu} B_{\mu\nu} W^{I\rho\sigma} W^I_{\rho\sigma} + \frac{\alpha_{12}}{\Lambda^4} B^{\mu\nu} W^I_{\mu\nu} B^{\rho\sigma} W^I_{\rho\sigma}$$
$$+ \frac{\alpha_{13}}{\Lambda^4} B^{\mu\nu} B_{\nu\rho} B^{\rho\sigma} B_{\sigma\mu} + \frac{\alpha_{14}}{\Lambda^4} W^{I\mu\nu} W^I_{\nu\rho} W^{J\rho\sigma} W^J_{\sigma\mu} + \frac{\alpha_{15}}{\Lambda^4} W^{I\mu\nu} W^J_{\nu\rho} W^{I\rho\sigma} W^J_{\sigma\mu}$$
$$+ \frac{\alpha_{16}}{\Lambda^4} B^{\mu\nu} B_{\nu\rho} W^{I\rho\sigma} W^I_{\sigma\mu} + \frac{\alpha_{17}}{\Lambda^4} B^{\mu\nu} W^I_{\nu\rho} B^{\rho\sigma} W^I_{\sigma\mu} .$$

F^4 : pure field strengh

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- What are the effective operators ?
  In the broken phase, 2 operators parameterize any new physics contribution :

$$\mathcal{L}_{4\gamma} = \zeta_1^\gamma F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + \zeta_2^\gamma F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu}$$

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- The amplitudes are just :

$$\begin{aligned} \alpha_{em}^2 \mathcal{M}_{++++} &= -\frac{1}{4} \left(4\zeta_1 + 3\zeta_2\right) s^2 \\ \alpha_{em}^2 \mathcal{M}_{++--} &= -\frac{1}{4} \left(4\zeta_1 + \zeta_2\right) \left(s^2 + t^2 + u^2\right) \\ \alpha_{em}^2 \mathcal{M}_{+++-} &= 0 \,. \end{aligned}$$

Potential for new physics discovery: EFT regime

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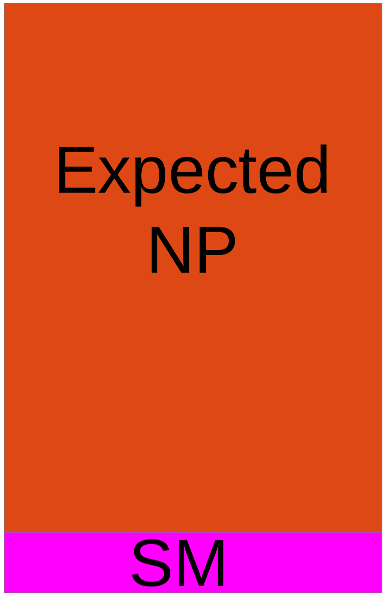
$4\gamma =$



Potential for new physics discovery: EFT regime

- Simulation for 300 fb-1 , mu= 50 (medium luminosity) and 6000 fb-1, mu=200 (high luminosity)

medium

$$\zeta_1^\gamma < 9 \cdot 10^{-15}\ \mathrm{GeV}^{-4}$$

$$\zeta_2^\gamma < 2 \cdot 10^{-14}\ \mathrm{GeV}^{-4}$$

high

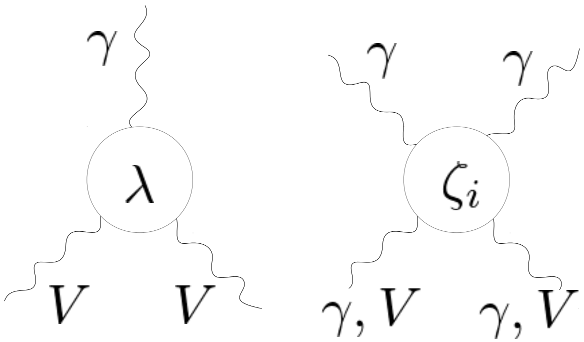
$$\zeta_1^\gamma < 7 \cdot 10^{-15}\ \mathrm{GeV}^{-4}$$

$$\zeta_2^\gamma < 1.5 \cdot 10^{-14}\ \mathrm{GeV}^{-4}$$

- Results do not improve much at high luminosity because pile-up is higher

New physics candidates for 4γ

New charged particles



Spin 0,1/2,1 contributions to F^3 , F^4

[SF/Gersdorff '13]

- All vertices are fixed by SU(2)xU(1) gauge invariance.

  So contributions from new particles caracterized only by masses and quantum numbers. So one can derive the contributions for arbitrary new physics. We use the background field method to compute the loops.

- Because it is SU(2)xU(1), traces $Tr(t^a t^b t^c)$, $Tr(t^a t^b t^c t^d)$ can be computed generally as a function of hypercharge Y and dimension of the SU(2) irrep d . For a given model, the only remaining work is to decompose $t^a_{S,f,X} \to \bigoplus d_Y$!

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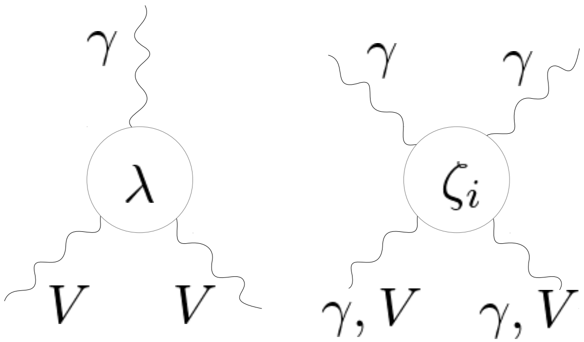

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- Apply this to the 4γ vertices : $\mathcal{L}_{4\gamma} = \zeta_1^\gamma F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + \zeta_2^\gamma F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}$

$$\zeta_i^\gamma = \alpha_{\rm em}^2 Q^4 m^{-4} N\, c_{i,s} \quad c_{1,s} = \begin{cases} \frac{1}{288} & s=0 \\ -\frac{1}{36} & s=\frac{1}{2} \\ -\frac{5}{32} & s=1 \end{cases}, \quad c_{2,s} = \begin{cases} \frac{1}{360} & s=0 \\ \frac{7}{90} & s=\frac{1}{2} \\ \frac{27}{40} & s=1 \end{cases}$$

Scalar loops are smaller !

Spin 0,1/2,1 charged particles in the EFT limit

- In realistic models, electrically charged particles come with some multiplicity with respect to e.m. For example they have multiplicity if they are colored. In the loop formula this can be summarized by replacing Q by

$$Q^4 \to Q_{\rm eff}^4 = {\rm tr}\, Q^4$$

- Model-independent results, depending only on $m, Q_{\rm eff}, S$

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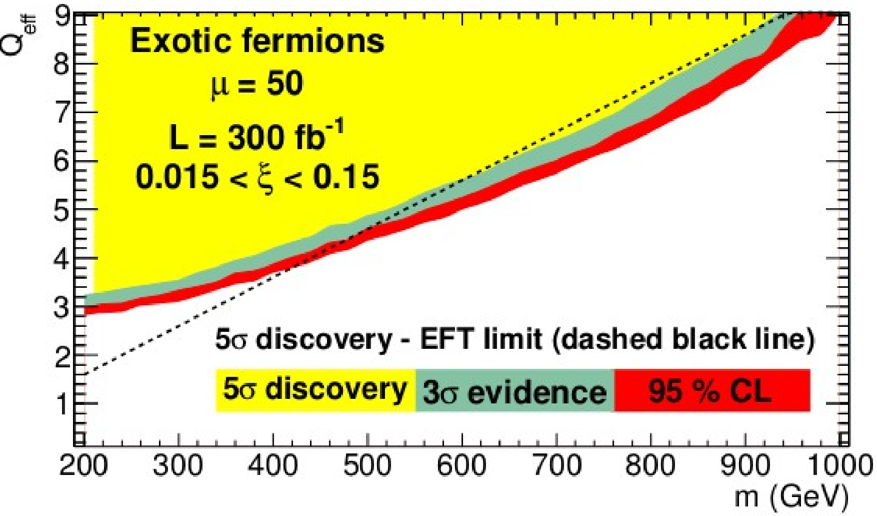
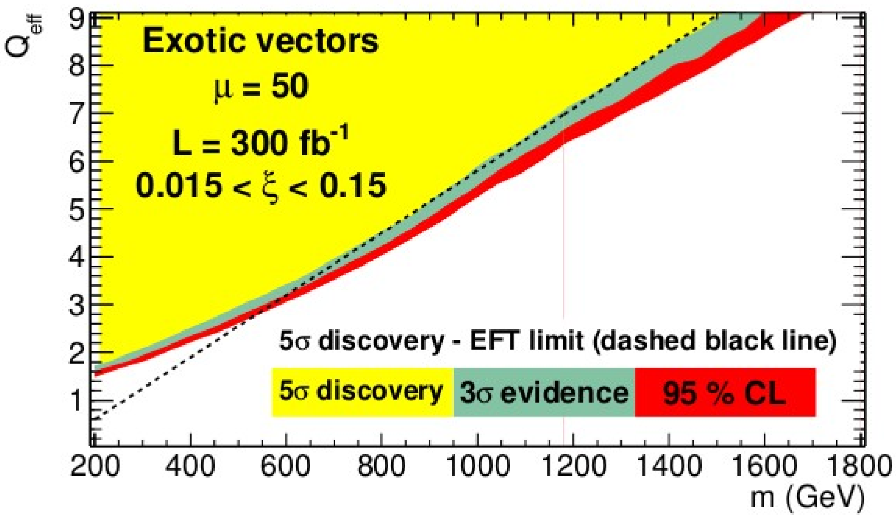
- Model-independent results, depending only on $m, Q_{\rm eff}, S$
- In familiar BSM theories (at least their minimal form), $Q_{\rm eff}$ hardly exceeds 3 or 4. For these values, masses up to 800 GeV (500 GeV) for S=1 (S=1/2) can be reached.
- But EFT is not valid anymore at these low masses

Spin 1/2,1 charged particles for any mass

- We implemented the full amplitudes in FPMC for S=1/2, 1.





- Result valid for any mass, model-independent, no ad-hoc form factors

Warped KK gravitons

Warped extra dimensions

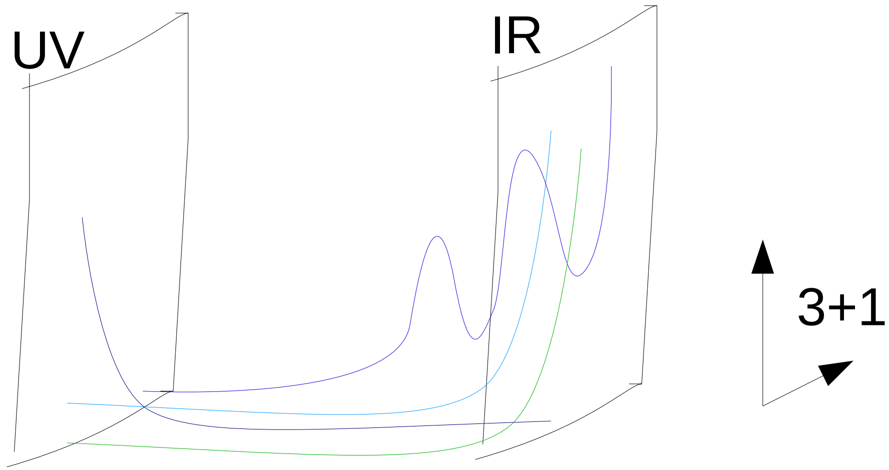
$$ds^2 = a(z)^{-2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

$$a(z) = kz \quad \text{(pure AdS)}$$

$$z \in [z_0, z_1] \quad z_0/z_1 = \epsilon$$

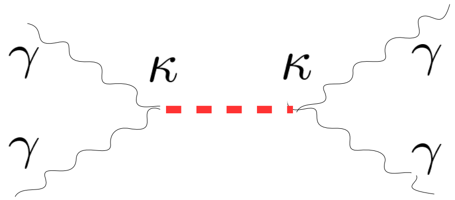
$$V = |\log \epsilon| \approx 37$$

$$\tilde{k} = \epsilon k$$

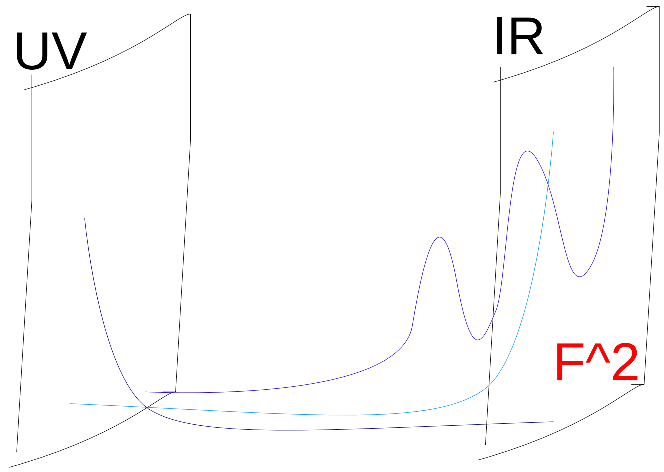


- KK gravitons, radion, Higgs near the IR brane.

- KK gravitons couple strongly to the photon through the 5d stress-energy tensor with warped gravity strength $\kappa = \tilde{k}/M_{Pl}$



WED : IR brane gauge scenario



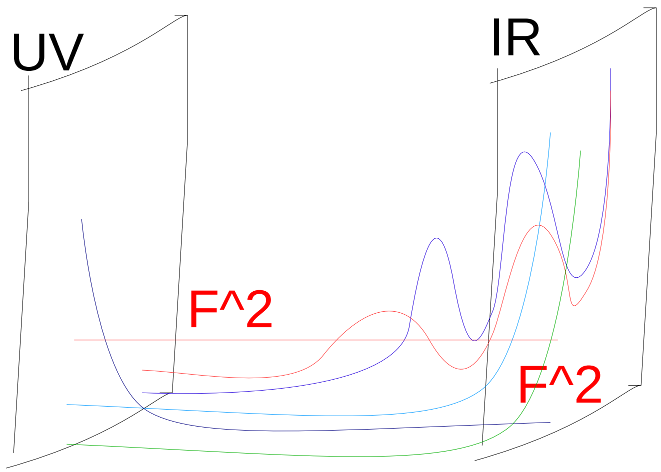
- Sizeable coupling to KK gravitons

$$\zeta_1^\gamma = -\frac{\kappa^2}{64\tilde{k}^4} \qquad \zeta_2^\gamma = \frac{\kappa^2}{16\tilde{k}^4} \qquad\qquad m_2 = 3.83\,\tilde{k}$$

- Using our estimated sensitivities on zeta's and $\kappa = 2$, KK gravitons can be detected up to

$$m_{\mathbf{2}} = 6.5\,\mathrm{TeV}$$

WED : bulk gauge scenario



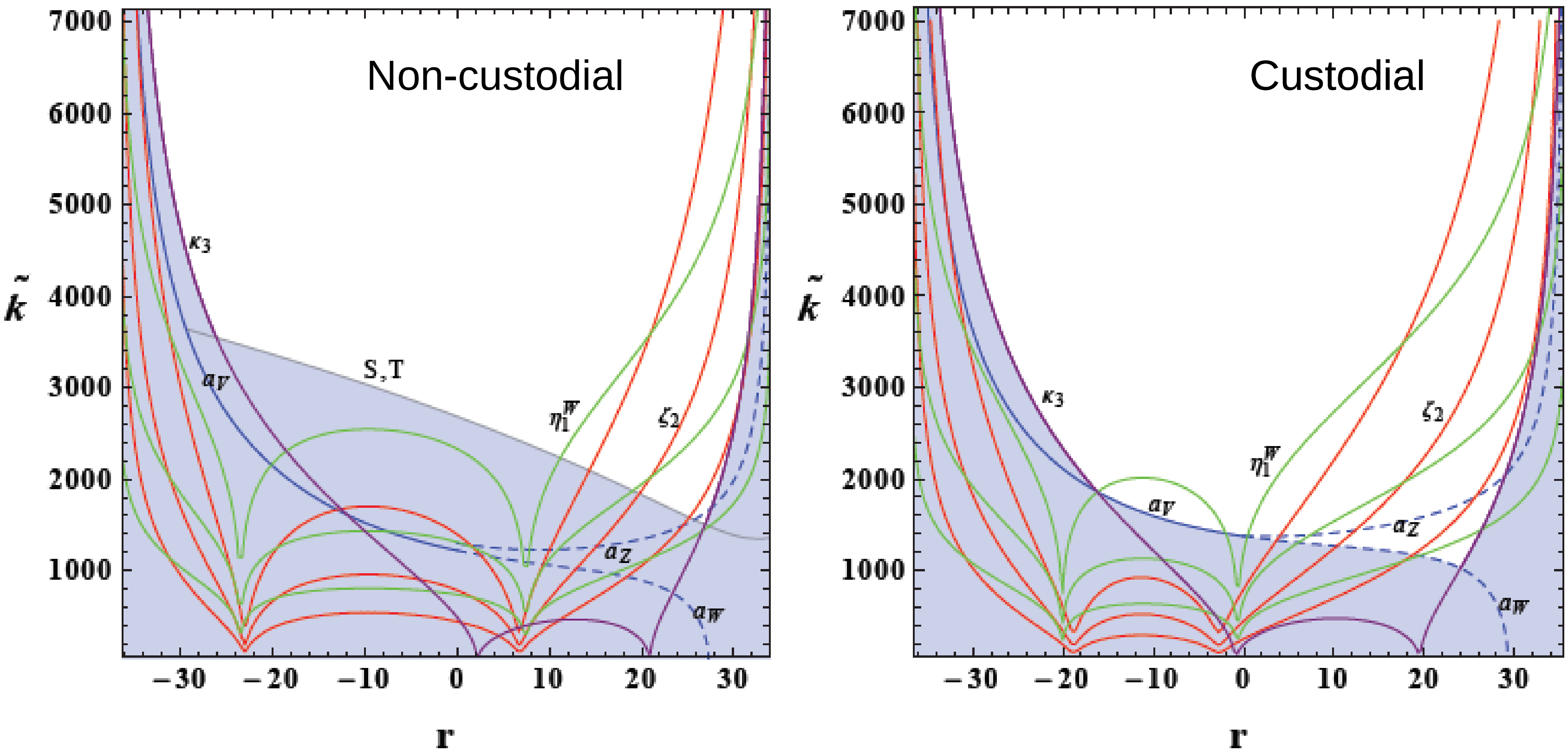
- We now have KK gauge fields, which contribute to S,T, Higgs, triple gauge couplings

- IR brane EW kinetic terms are crucial

$$\mathcal{L}_{IR} \supset \frac{r}{4}(W^a_{\mu\nu})^2 + \frac{r'}{4}(B^a_{\mu\nu})^2$$

$$SU(2)_L \qquad U(1)_Y$$

WED : bulk gauge scenario



- In red, $\zeta_2^\gamma = (10^{-13},\ 10^{-14},\ 10^{-15}\ \mathrm{GeV}^{-4})$ from KK gravitons. ($m_2 = 3.83\,\tilde{k}$)
- Using our EFT sensitivity, KK gravitons can be detected in the multi-TeV range

The SIHD

Strongly-interacting heavy dilaton

- BSM theories often feature a new strongly-coupled sector (e.g CH models). If conformal in the UV, conformality is broken in the IR (at least by EWSB and QCD).
- The spectrum then features a neutral scalar, the dilaton. Unless the theory is fine-tuned, its mass is of order of the conformal breaking scale. In absence of fine-tuning, the dilaton couplings are unsuppressed with respect to this scale. We call this the Strongly-Interacting Heavy Dilaton (SIHD)

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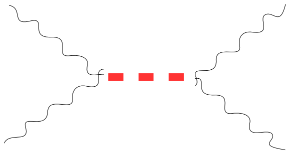
- The SIHD couples to the trace of the SE tensor ϕT^{μ}_{μ} . The SE tensor contains $(F^{\mu\nu})^2,\ (Z^{\mu\nu})^2,\ (W^{\mu\nu})^2$, thus the tree-level dilaton exchange generates $\zeta_1^{\gamma,Z,W}$

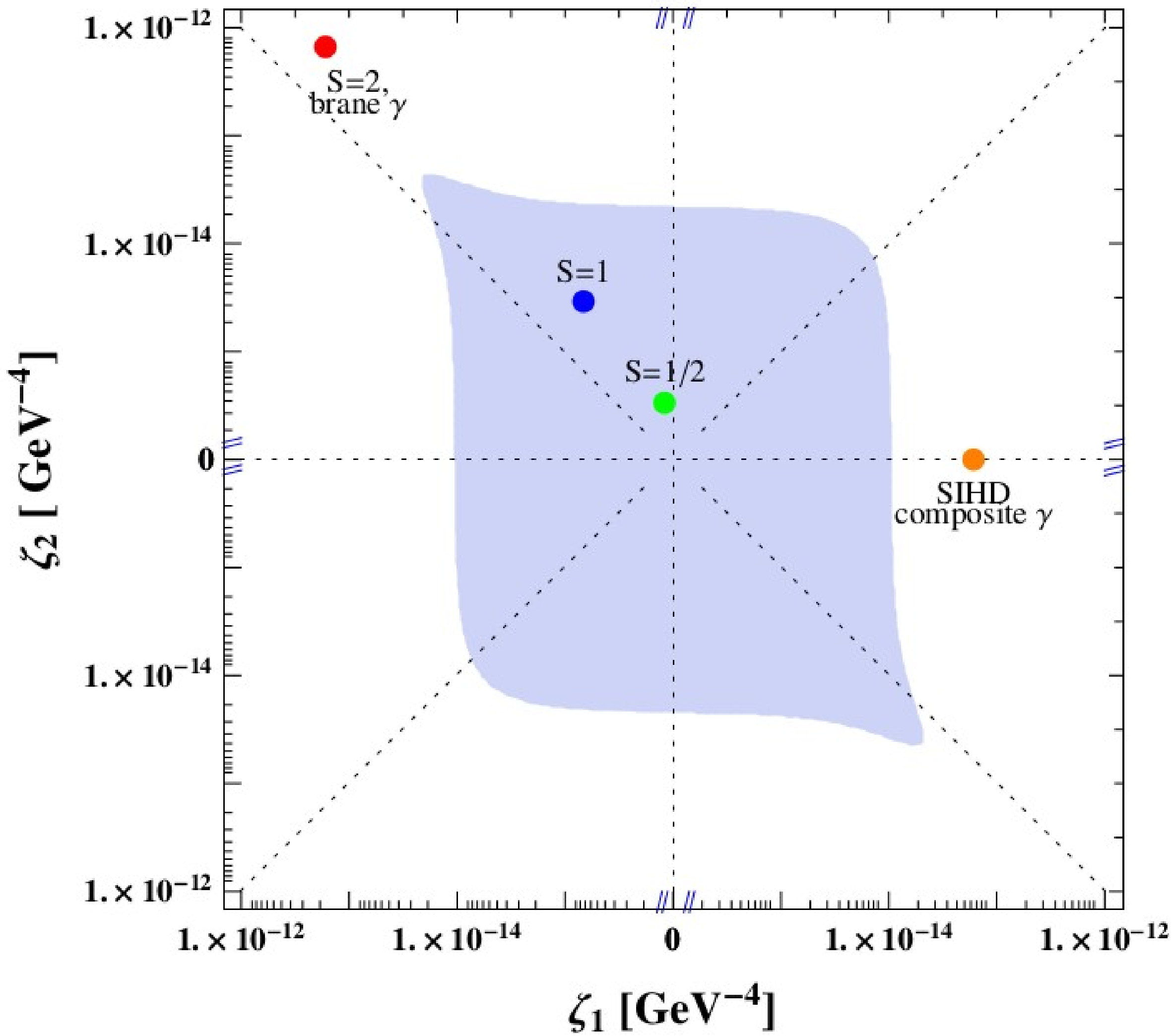
$$\mathcal{L}_{4\gamma} = \zeta_1^{\gamma} F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}$$



- The contribution is large if one has a partially composite photon. For a pure composite photon,

$$\zeta_1^{\gamma} \sim \frac{\pi^2}{2\, m_\phi^4} \longrightarrow m_\phi = 4.8\,\mathrm{TeV}$$

EFT summary plot



Magnetic monopoles

Open problem: Magnetic monopoles

- [Ginzburg/Panfil 82']: Assume a heavy point-like monopole. Its Lagrangian is unknown, but one can use electromagnetic duality to deduce its coupling to the photon.

$$B \to E \qquad F_{\mu\nu} \to \tilde{F}_{\mu\nu}$$
$$E \to -B \qquad \tilde{F}_{\mu\nu} \to -F_{\mu\nu}$$

$$g = \frac{2\pi n}{e} \quad n \in \mathbf{N}$$

$$\zeta_i^\gamma = \alpha_{\rm em}^2 Q^4 \, m^{-4} \, N \, c_{i,s} \qquad \zeta_{i,s} \to \frac{g^4}{e^4}\zeta_{i,s} = \left(\frac{n}{2\alpha_e}\right)^4 \zeta_{i,s}$$

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- Very nice reasoning... but what about higher loops ?

  - As far as I understand, in the GP paper higher-loops are assumed to be absorbed by renormalization. In reality this does not happen.

  - The formal computation they provide goes through the background field method. This computation provides only the one-loop result and neglects higher loops.

→ Open problem !
(let me know if you have any idea)

Look at the EFT results for charged particles.

The zeta's grow fast with the spin...

$$c_{1,S} = \begin{cases} \frac{1}{288} & S = 0 \\ -\frac{1}{36} & S = \frac{1}{2} \\ -\frac{5}{32} & S = 1 \end{cases}$$

⋮

?

Higher-spin particles

EFT for higher-spin objects

- Any strongly-interacting extension of the SM potentially features higher-spin composites in its spectrum (analogous to higher-spin hadrons in QCD)

- In low-energy strings scenarios, strings feature higher-spin excited modes

→ HS objects are well motivated

EFT for higher-spin objects

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- In low-energy strings scenarios, strings feature higher-spin excited modes

→ HS objects are well motivated

- Assuming the size of the high-spin object is small, it appears to be pointlike at low-energy.

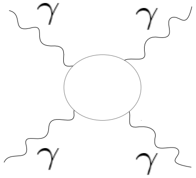
→ EFT Lagrangian for higher-spin particles

Virtual effects of HS particles

- HS couplings to the SM have to be bilinear, ie $\mathcal{L} \supset \mathcal{O}\phi_{(s)}\phi^*_{(s)}$

  → HS particles can be spotted in loops.

- A very naive generalization of the background field computation gives



$$\propto S^5$$

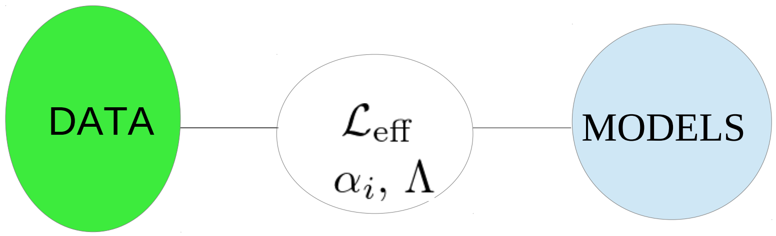
  → Light-by-light scattering may be a good place to look for HS particles

- HS QFT computations: never done and challenging

Summary

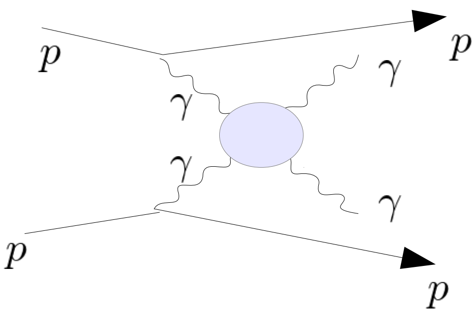
Summary

- Precision physics + EFT are powerful
  (but be careful with inference on Λ)



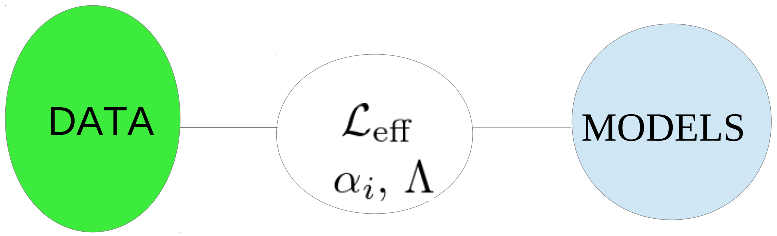
- Forward proton detectors at the LHC will provide new precision measurements

- We find from simulation that light-by-light scattering from NP can potentially be observed at the LHC14



Summary

- Precision physics + EFT are powerful (but be careful with inference on Λ)



- Forward proton detectors at the LHC will provide new precision measurements

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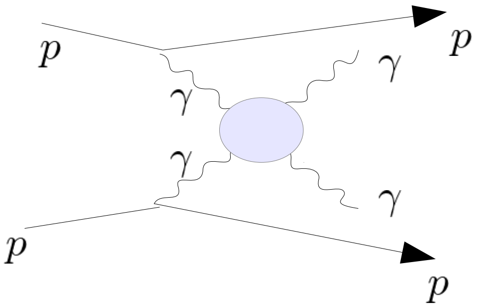

- Model-independent bounds on massive charged particles.

- Warped KK gravitons and the SIHD can be detected in the multi-TeV range

- Charged higher-spin particles might be detected through LbL scattering. But first the QFT tools need to be set up → upcoming work, stay tuned !

More

The dimension 6 effective Lagrangian

- Piece of the basis relevant for the available data:

Potentially Tree-level

$$\mathcal{O}_{D^2} = |H|^2 |D_\mu H|^2 , \qquad \mathcal{O}'_{D^2} = |H^\dagger D_\mu H|^2 \qquad \mathcal{O}_f = 2 y_{ij} f |H|^2 H \bar{\Psi}_i \Psi_j$$

$$\mathcal{O}_D = J^a_{H\,\mu} J^a_{f\,\mu} , \qquad \mathcal{O}'_D = J^Y_{H\,\mu} J^Y_{f\,\mu} , \quad \mathcal{O}_{4f} = J^a_{f\,\mu} J^a_{f\,\mu} , \qquad \mathcal{O}'_{4f} = J^Y_{f\,\mu} J^Y_{f\,\mu} ,$$

$$\mathcal{O}_{WW} = H^\dagger H \, (W^a_{\mu\nu})^2 , \qquad \mathcal{O}_{BB} = H^\dagger H \, (B_{\mu\nu})^2 , \qquad \mathcal{O}_{WB} = H^\dagger \, W_{\mu\nu} H \, B_{\mu\nu} ,$$

$$\mathcal{O}_{W^3} = g \varepsilon^{IJK} W^I_{\mu\nu} W^J_{\nu\rho} W^K_{\rho\mu} .$$

Loop-level

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Loop-level

- Contribute to triple gauge couplings $\kappa_1, \kappa_2, \kappa_3$

$$\mathcal{L}^{\mathrm{SM},v}_{\mathrm{CGC}} = -ie(1+\kappa_1) F_{\mu\nu} W^-_\mu W^+_\nu - i g_Z (1+\kappa_2) Z_{\mu\nu} W^-_\mu W^+_\nu - i g_Z (1+\kappa_3) \left[\hat{W}^+_{\mu\nu} W^-_\nu - \hat{W}^-_{\mu\nu} W^+_\nu \right] Z_\mu$$

$$\kappa_\gamma = 1 + \kappa_1 \qquad \kappa_Z = 1 + \kappa_2 \qquad g_1^Z = 1 + \kappa_3$$

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Loop-level

- Contribute to electroweak precision parameters S, T (and even more at loop level)

The dimension 6 effective Lagrangian

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Loop-level

- Contribute to Higgs couplings $a_{W,Z}, c_i$ and ζ_i

$$\mathcal{L}_h^{tree} = a_W \frac{h}{v} 2 m_W^2 W^+_\mu W^-_\mu + a_Z \frac{h}{v} m_Z^2 (Z_\mu)^2 - \sum_i c_i \frac{h}{v} m_f \bar{\Psi}_L \Psi_R - h.c.$$

$$+ \zeta_\gamma h (F_{\mu\nu})^2 + \zeta_g h (G_{\mu\nu})^2 + \zeta_Z h (Z_{\mu\nu})^2 + + \zeta_W h W^+_{\mu\nu} W^-_{\mu\nu} + \zeta_{Z\gamma} h Z_{\mu\nu} F_{\mu\nu}$$

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$$\mathcal{O}_{W^3} = g \varepsilon^{IJK} W^I_{\mu\nu} W^J_{\nu\rho} W^K_{\rho\mu} \,.$$

Loop-level

- We performed a (Bayesian) global fit of these operators [Dumont/SF/Gersdorff '13]

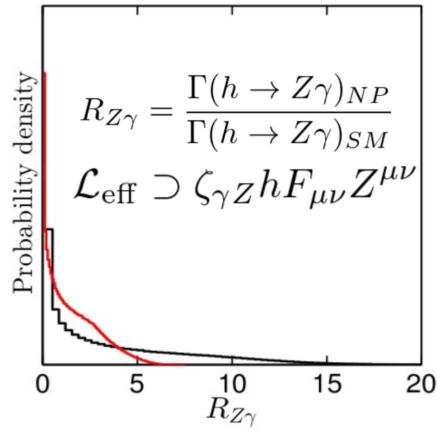
- Result to keep for today :

$0.96 < a_V < 1.21$ 95% BCI

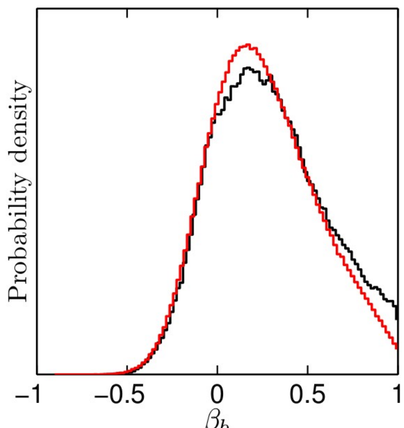
(Assuming $a_W \approx a_Z$)

Some results...

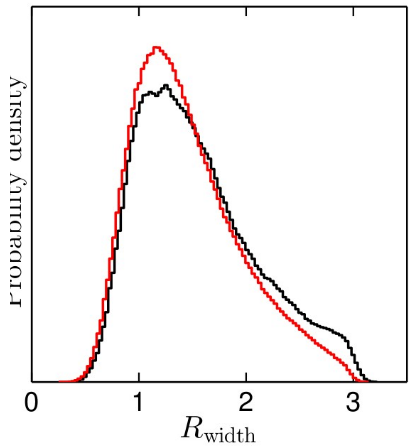
- $h \to Z\gamma$ can be enhanced up to a factor 12 (95% BC) for democratic HDOs and up to a factor 4 (95% BC) for loop-suppressed $\mathcal{O}_{FF}$'s



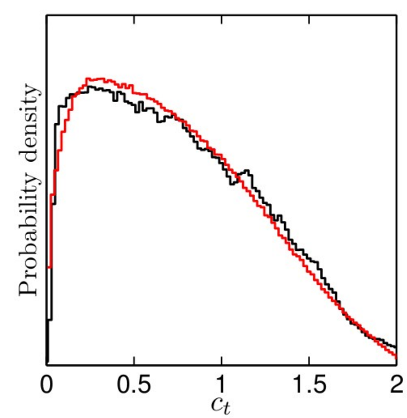
- Bottom and total width can be enhanced together



$$c_b \approx 1 + \beta_b = 1 + \alpha_b \frac{v^2}{\Lambda^2}$$



- The top coupling is suppressed



The anomalous Lagrangian in the broken phase

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm CGC} + \mathcal{L}_{\rm NGC}$$

$$\mathcal{L}_{\rm NGC} = \mathcal{L}^{v}_{\rm NGC} + \mathcal{L}^{\partial}_{\rm NGC}$$

$$\mathcal{L}_{\rm CGC} = \mathcal{L}^{{\rm SM},v}_{\rm CGC} + \mathcal{L}^{\partial}_{\rm CGC}$$

- The $\mathcal{L}^{v}$ piece comes with EWSB. It dominates for $E < v$
  The $\mathcal{L}^{\partial}$ piece has only field strengths. It dominates for $v < E < \Lambda$ (if no GBs on legs)

$$\mathcal{L}^{{\rm SM},v}_{\rm CGC} = -ie(1+\kappa_1)F_{\mu\nu}W^-_\mu W^+_\nu - ig_Z(1+\kappa_2)Z_{\mu\nu}W^-_\mu W^+_\nu$$
$$- ig_Z(1+\kappa_3)\left[\hat{W}^+_{\mu\nu}W^-_\nu - \hat{W}^-_{\mu\nu}W^+_\nu\right]Z_\mu - g_Z^2(1+\kappa_4)^2\left(|W^+_\mu|^2|Z_\nu|^2 - |Z_\mu W^+_\mu|^2\right)$$
$$+\frac{1}{4}g_W^2(1+\kappa_5)^2(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)^2 + \eta_1^W F^{\mu\nu}F_{\mu\nu}W^{+\rho}W^-_\rho + \eta_2^W F^{\mu\nu}F_{\mu\sigma}W^{+\nu}W^-_\sigma\,,$$

Assuming broken SU(2)xU(1), one has $\kappa_2 = \kappa_3 - \kappa_1\dfrac{s_w^2}{c_w^2}$

$$\kappa_\gamma = 1+\kappa_1$$
$$\kappa_Z = 1+\kappa_2$$
$$g_1^Z = 1+\kappa_3$$

- $\mathcal{L}^{v}_{\rm NGC} = \eta_1^Z F^{\mu\nu}F_{\mu\nu}Z_\rho Z_\rho + \eta_2^Z F^{\mu\nu}F_{\mu\sigma}Z_\nu Z_\sigma$

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$$\mathcal{L}_{\rm CGC} = \mathcal{L}^{{\rm SM},v}_{\rm CGC} + \mathcal{L}^{\partial}_{\rm CGC}$$

- $$\begin{aligned}\mathcal{L}^{\partial}_{\rm CGC} =& \lambda^Z \left[i g_Z Z_{\mu\nu} (\hat{W}^-_{\nu\rho} \hat{W}^+_{\rho\mu} - \hat{W}^+_{\nu\rho} \hat{W}^-_{\rho\mu}) \right] + \lambda^\gamma \left[ie F_{\mu\nu} (\hat{W}^-_{\nu\rho} \hat{W}^+_{\rho\mu} - \hat{W}^+_{\nu\rho} \hat{W}^-_{\rho\mu}) \right] \\ &+ \zeta^W_1 F^{\mu\nu} F_{\mu\nu} W^{+\rho\sigma} W^-_{\rho\sigma} + \zeta^W_2 F^{\mu\nu} F_{\nu\rho} W^{+\rho\sigma} W^-_{\sigma\mu} \\ &+ \zeta^W_3 F^{\mu\nu} W^+_{\mu\nu} F^{\rho\sigma} W^-_{\rho\sigma} + \zeta^W_4 F^{\mu\nu} W^+_{\nu\rho} F^{\rho\sigma} W^-_{\sigma\mu} \,.\end{aligned}$$

Keep only $\geq 2\,\gamma$ operators

- $$\begin{aligned}\mathcal{L}^{\partial}_{\rm NGC} =& \zeta^\gamma_1 F^{\mu\nu} F_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} + \zeta^\gamma_2 F^{\mu\nu} F_{\nu\rho} F^{\rho\sigma} F_{\sigma\mu} \\ &+ \zeta^{\gamma Z}_1 F^{\mu\nu} F_{\mu\nu} F^{\rho\sigma} Z_{\rho\sigma} + \zeta^{\gamma Z}_2 F^{\mu\nu} F_{\nu\rho} F^{\rho\sigma} Z_{\sigma\mu} \\ &+ \zeta^Z_1 F^{\mu\nu} F_{\mu\nu} Z^{\rho\sigma} Z_{\rho\sigma} + \zeta^Z_2 F^{\mu\nu} F_{\nu\rho} Z^{\rho\sigma} Z_{\sigma\mu} \\ &+ \zeta^Z_3 F^{\mu\nu} Z_{\mu\nu} F^{\rho\sigma} Z_{\rho\sigma} + \zeta^Z_4 F^{\mu\nu} Z_{\nu\rho} F^{\rho\sigma} Z_{\sigma\mu} \,,\end{aligned}$$

The background field method

- Decompose gauge fields into background and fluctuation $A^\mu \to A^\mu + \mathcal{A}^\mu$. Plug into the generating functional, integrate over fluctuations :

$$S_{\rm eff}^S = +\frac{i}{2}\log\left(-D^2 - m_S^2\right)$$
$$S_{\rm eff}^f = -\frac{i}{2}\log\left(-D^2 - m_f^2 + g\, S^{\mu\nu}\, V_{\mu\nu}^a t_f^a\right)$$
$$S_{\rm eff}^X = \frac{i}{2}\log\left(-[D^2 + m_X^2]\eta_{\mu\nu} + 2ig\, V_{\mu\nu}^a t_X^a\right) + \frac{i}{2}(1-2)\log\left(-D^2 - m_X^2\right)$$

- The one-loop effective action remains covariant in background fields. To obtain the 3 and 4-point functions one expands at 3rd and 4th order. Coefficients (Gilkey-De Witt) of the expansion are computed in [Gilkey '75, Tseytlin et al '83, 88'].

- $\mathcal{L}^{(6)} \supset \frac{g^3}{16\pi^2}\left(-\frac{1}{144\, m_S^2} + \frac{1}{36\, m_f^2} - \frac{1}{48\, m_X^2}\right)\frac{(d^2-1)d}{24}\mathcal{O}_{W^3}$

- $$\mathcal{L}^{(8)} \supset \frac{1}{16\pi^2 m_S^4}\left\{\frac{1}{576}\mathcal{A}+\frac{1}{720}\mathcal{B}+\frac{1}{420}\mathcal{C}+\frac{2}{35}\mathcal{D}\right\} + \frac{1}{16\pi^2 m_f^4}\left\{-\frac{1}{36}\mathcal{A}+\frac{7}{90}\mathcal{B}-\frac{64}{105}\mathcal{C}+\frac{104}{35}\mathcal{D}\right\} + \frac{1}{16\pi^2 m_X^4}\left\{-\frac{5}{64}\mathcal{A}+\frac{27}{80}\mathcal{B}-\frac{111}{140}\mathcal{C}+\frac{342}{35}\mathcal{D}\right\},$$

- $$\mathcal{A} = g'^4\, d\, Y^4\, \mathcal{O}_8 + g^4\left(\frac{(d^4-1)d}{240}\mathcal{O}_9 + \frac{(d^2-1)(d^2-4)d}{120}\mathcal{O}_{10}\right) + g^2 g'^2\,\frac{(d^2-1)d}{6}Y^2\left(\mathcal{O}_{11}+2\mathcal{O}_{12}\right),$$

$$\mathcal{B} = g'^4\, d\, Y^4\, \mathcal{O}_{13} + g^4\left(\frac{(d^4-1)d}{120}\mathcal{O}_{14} + \frac{(d^2-9)(d^2-1)d}{240}\mathcal{O}_{15}\right) + g^2 g'^2\,\frac{(d^2-1)d}{6}Y^2\left(2\mathcal{O}_{16}+\mathcal{O}_{17}\right).$$

$$\mathcal{C} = g^4\,\frac{d(d^2-1)}{1152}\left(\mathcal{O}_{10}-\mathcal{O}_9\right), \qquad \mathcal{D} = g^4\,\frac{d(d^2-1)}{1152}\left(\mathcal{O}_{15}-\mathcal{O}_{14}\right),$$

Depend only on m, d, Y

Scalar loops are small

- Consider $G = SO(4+N)$, $H = SO(4)$, and embeddings into the fundamental $\mathbf{F}_{2/3}$ and the traceless symmetric $\mathbf{S}_{2/3}$

- Decomposition under $G \to G_{EW}$: $\mathbf{F}_{2/3} = \mathbf{2}_{1/6} \oplus \mathbf{2}_{7/6} \oplus N \times \mathbf{1}_{2/3}$

$$\mathbf{S}_{2/3} = \mathbf{3}_{5/3} \oplus \mathbf{3}_{2/3} \oplus \mathbf{3}_{-1/3} \oplus N \times \left(\mathbf{2}_{1/6} \oplus +\mathbf{2}_{7/6}\right) \oplus \frac{N(N+1)}{2} \times \mathbf{1}_{2/3}$$

- Patterns :

	$\mathbf{1}_{2/3}$	$\mathbf{2}_{1/6}$	$\mathbf{2}_{7/6}$	$\mathbf{3}_{5/3} \oplus \mathbf{3}_{2/3} \oplus \mathbf{3}_{-1/3}$
$\lambda^{Z,\gamma}$	0	1.7×10^{-4}	1.7×10^{-4}	0.002
ζ_1^{γ}	-1.0×10^{-6}	-1.1×10^{-6}	-4.1×10^{-5}	-0.5×10^{-4}
$\zeta_1^{\gamma Z}$	2.2×10^{-6}	-5.8×10^{-6}	-1.7×10^{-5}	-2.7×10^{-4}
ζ_1^{Z}	-0.6×10^{-6}	-4.3×10^{-6}	-1.3×10^{-5}	-2.1×10^{-4}
ζ_2^{γ}	2.9×10^{-6}	1.2×10^{-6}	11×10^{-5}	1.0×10^{-4}
$\zeta_2^{\gamma Z}$	-6.2×10^{-6}	3.0×10^{-6}	3.4×10^{-5}	4.6×10^{-4}
ζ_2^{Z}	3.4×10^{-6}	-2.6×10^{-6}	5.0×10^{-5}	5.7×10^{-4}
ζ_3^{Z}	-1.2×10^{-6}	-8.6×10^{-6}	-2.7×10^{-5}	-4.1×10^{-4}
ζ_4^{Z}	1.7×10^{-6}	-0.1×10^{-6}	2.5×10^{-5}	2.8×10^{-4}
ζ_1^{W}	0	-12×10^{-6}	-7.2×10^{-5}	-3.3×10^{-4}
ζ_2^{W}	0	38×10^{-6}	37×10^{-5}	11×10^{-4}
ζ_3^{W}	0	-2.5×10^{-6}	-12×10^{-5}	-3.9×10^{-4}
ζ_4^{W}	0	-28×10^{-6}	14×10^{-5}	1.7×10^{-4}

$$\times \frac{1}{m_\Psi^4}$$

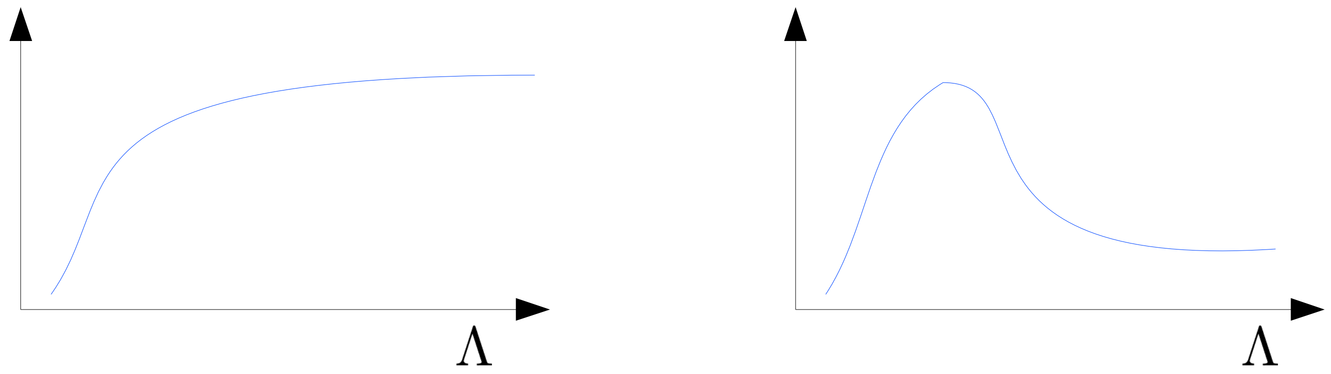
The electroweak and Higgs precision observables

- small
$$S = \left(2 s_w c_w \alpha_{WB} + s_w^2(\alpha_D - 2\alpha_{4f}) + c_w^2(\alpha'_D - 2\alpha'_{4f})\right)\frac{v^2}{\Lambda^2}$$
$$T = \left(-\frac{1}{2}\alpha'_{D^2} + \frac{1}{2}\alpha'_D - \frac{1}{2}\alpha'_{4f}\right)\frac{v^2}{\Lambda^2}$$
$$a_{Z,W} = 1 + \left(\frac{1}{2}\alpha_{D^2} - \frac{1}{4}(\alpha_D - \alpha_{4f}) \pm \frac{1}{4}\alpha'_{D^2}\right)\frac{v^2}{\Lambda^2}$$
- S,T with BKTs : $S = 2\pi\, f_2(\nu)\left(1 + (r + r')\,\frac{2+\nu}{3+\nu}\right)\frac{v^2}{\tilde{k}^2}$ exact !
$$T = \frac{\pi\, V}{2\, c_w^2}\, f_1(\nu)\left(1 + \frac{r'}{V}\right)\frac{v^2}{\tilde{k}^2}$$ (non-Custodial), $T = 0$ (Custodial)
- Higgs couplings : $a_Z \approx a_W \approx 1 - \frac{3g^2 + 3g'^2}{16} V f_1(\nu)\frac{v^2}{\tilde{k}^2}$ (Custodial)
$$a_W - a_Z \propto g'^2$$
- With BKTs: $a_W - a_Z$ strongly depends on r'

A statistical aside

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{\alpha_i}{\Lambda^2}\mathcal{O}_i + \text{higher order terms} \qquad E < \Lambda$$

- No one actually tried yet to get information on Λ . If you try, you get a a distribution like



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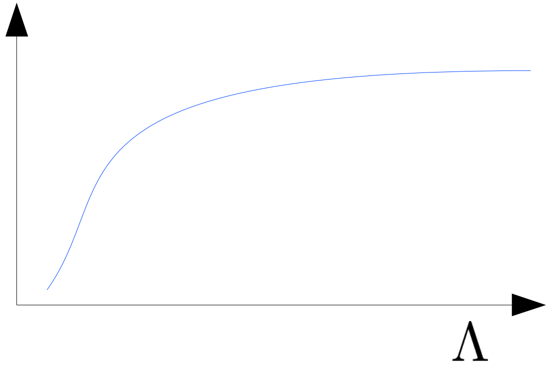
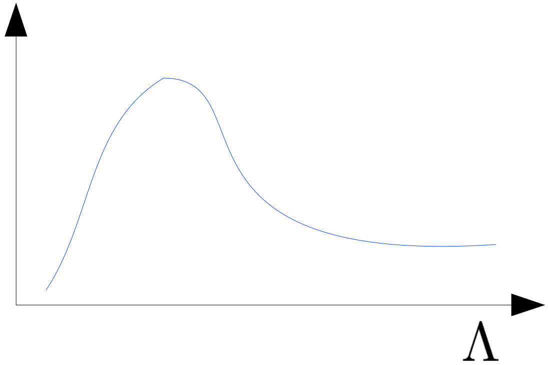
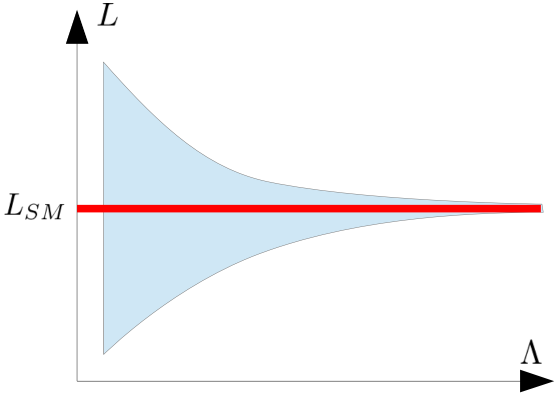



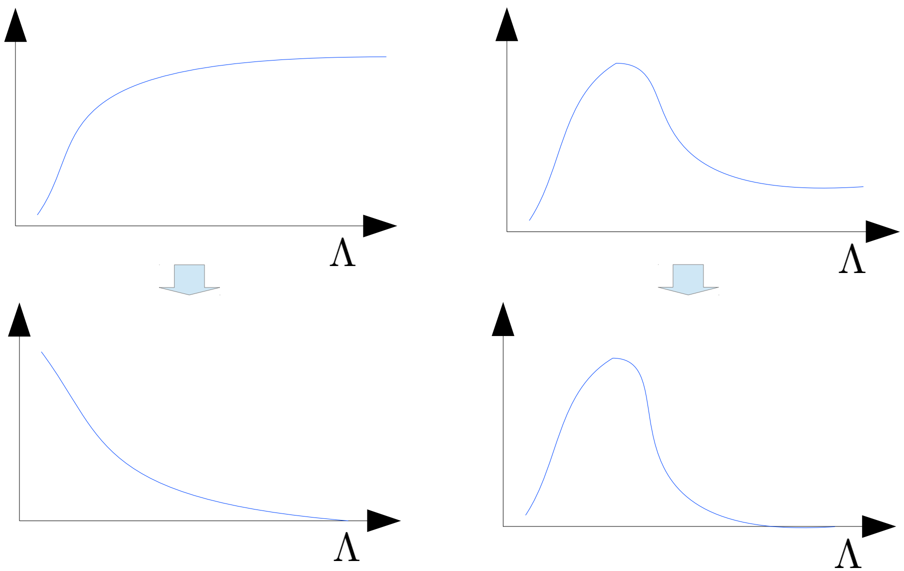
- These distributions are not normalized (``unproper''), so one cannot quantify properly the credibility intervals.

WHY ?

A statistical aside

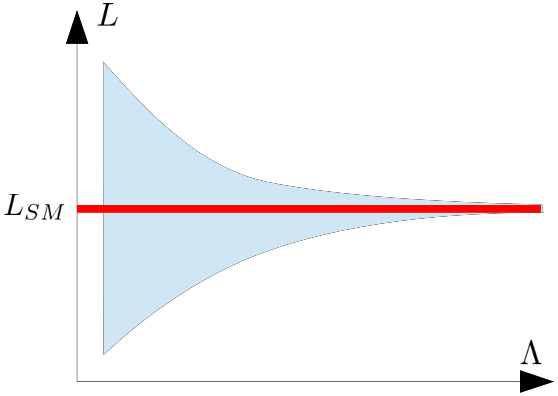
- This is because $p(\Lambda \to \infty | d) \to \frac{L_{SM}}{\Lambda}$
- However by definition $L = L_{SM}$ is not testable. So the slice of parameter space such that $L = L_{SM}$ should be excluded from the inference process.

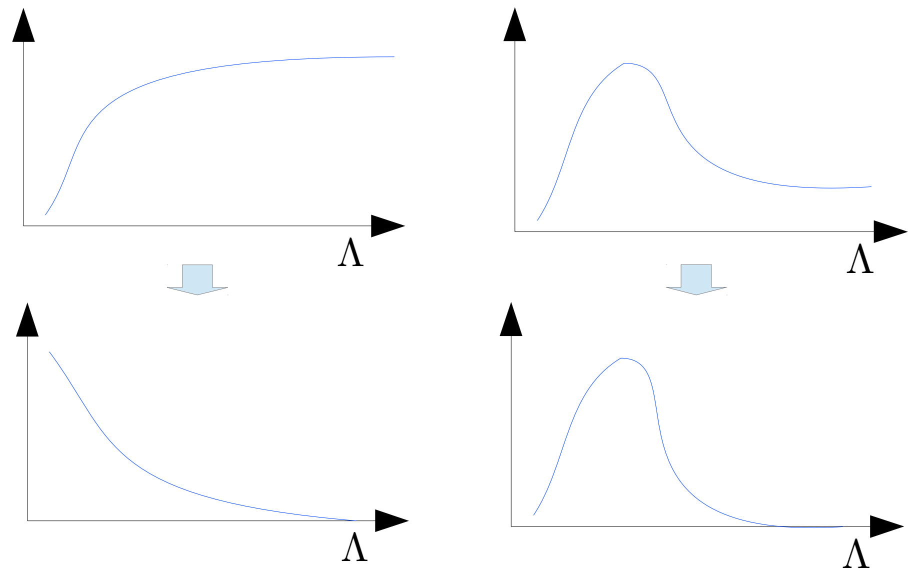




A statistical aside

- This is because $p(\Lambda \to \infty | d) \to \frac{L_{SM}}{\Lambda}$
- However by definition $L = L_{SM}$ is not testable. So the slice of parameter space such that $L = L_{SM}$ should be excluded from the inference process.



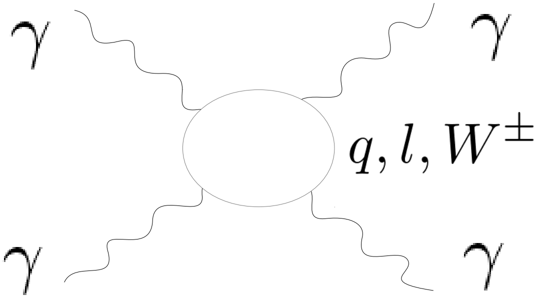


- Easy to implement using MCMCs. Convergence proof involves measure theory and Lesbegue integration.

Keep in mind if you want to evaluate the new physics scale from data !

SM four-photon processes

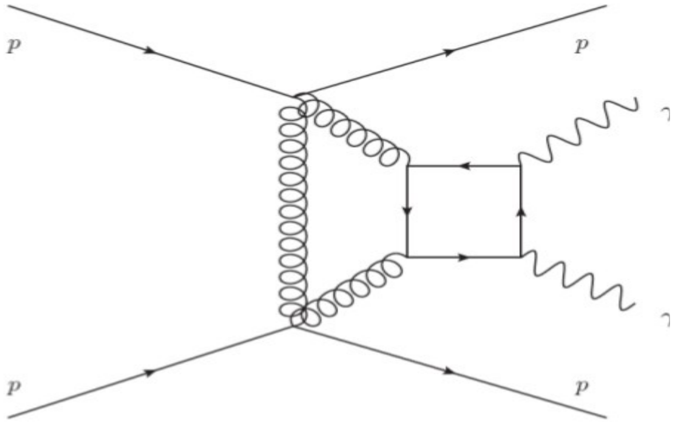
- Occurs through quark, lepton and W loops



- At high energy, fermion amplitudes are constant with respect to energy, while the W loop goes as $\log[s/m_W^2]^2$.

→ The W loop dominates at high energy.

- Exclusive two-photon production: there are also QCD graphs



Potential for new physics discovery: EFT regime

- The NP signal is controlled by

$$|\mathcal{M}|^2 - |\mathcal{M}_{SM}|^2 = |\mathcal{M}_{NP}|^2 + 2Re[\mathcal{M}_{SM}\mathcal{M}_{NP}]$$

Interference

- Not obvious what cut is optimal, as $\mathcal{M}_{SM}$ appears both in the background and the signal. We find it is the one cutting $\mathcal{M}_{SM}$ as much as possible.

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- With such cuts,

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2\, s}(s^2+t^2+st)^2\left[48(\zeta_1)^2 + 40\zeta_1\zeta_2 + 11(\zeta_2)^2\right]$$

- A single angle-independent combination appears !

QFT for higher spin

- Never done and challenging
- HS particle always feature higher-dimensional interactions in QED $\mathcal{M} \supset \frac{\Lambda^2}{m^2} \ldots$
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