A covariant model for the nucleon spin structure

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In collaboration with Franz Gross and M.T.Peña
PRD 85, 093005 (2012), **PRD 85, 093006 (2012)**

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Motivation

- Start with Constituint Quark Model for $\gamma^* N \to N^*$ reactions
  $\Longrightarrow$ Nucleon Deep Inelastic Scattering (DIS)
  P: nucleon momentum; q: momentum transfer
  $\frac{q \cdot P}{M}, \; -q^2 = Q^2 \to \infty \;\; x = \frac{Q^2}{2q \cdot P} \to$ const

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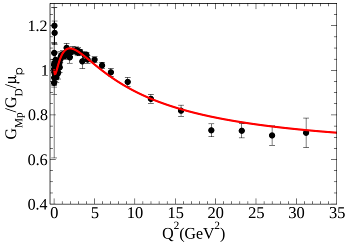
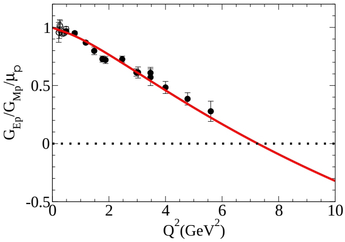
- Calculate Parton Distribution Functions: PDF(x)
- Include quarks with orbital angular momentum
- Check if the data can be explained only by valence quark effects
  [Reparametrization of the model]
- Interpret the effects of orbital angular momentum states

Model for the Nucleon form factors [PRC 77, 015202 (2008)]

Quark-Diquark model (S-state configuration: $L = 0$)





- Describes proton and neutron data
  Explains the G_{Ep}/G_{Mp} suppression (Jlab 2000)

  Jones, PRL 84, 1398 (2000); Gayou, PRL 88 092301 (2002); Puckett, PRL 104, 242301 (2010)

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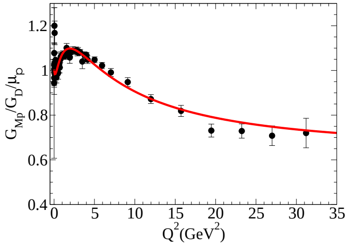

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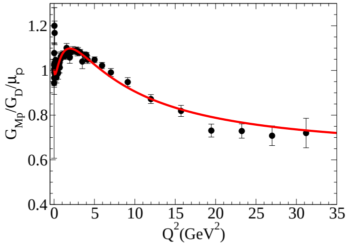

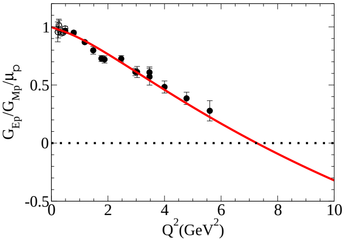

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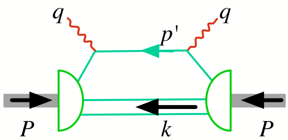
- Shape (wave functions) and Quark form factors (e.m. structure – gluon and $q\bar{q}$ effects) fitted to nucleon form factor data
- No pion cloud or sea quark effects included

DIS with a quark model



$$(J_{s_q,\lambda,\lambda_n})^\mu = -\bar{u}(p', s_q) j^\mu(q) (\Psi_N)_{\Lambda\lambda}(P,k)$$

λ: nucleon; Λ: quark-pair

Hadronic tensor:

$$W^{\mu\nu}(\lambda) = 3 \sum_{\Lambda, s_q} \iint_{p'k} (J^\dagger_{s_q,\Lambda,\lambda})^\mu (J_{s_q,\Lambda,\lambda})^\nu$$

$$\iint_{p'k} \equiv \iint \frac{d^3p'}{(2\pi)^3 2e_q} \frac{d^3k}{(2\pi)^3 2E_s} (2\pi)^4 \delta^4(p' + k - q - P)$$

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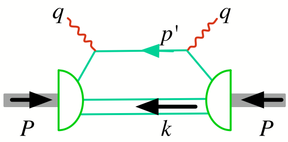

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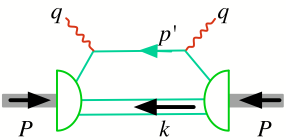

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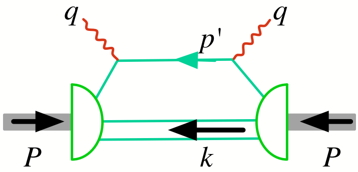
Input:

- Nucleon wave function $(\Psi_N)_{\Lambda\lambda}$
- Quark current: $\frac{\not{q} q^\mu}{q^2}$: include interaction currents behind impulse [Z. Batiz, F. Gross, PRC 58, 2963 (1998)]

$$j^\mu(q) = j_q(+\infty)\left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2}\right) + \mathcal{O}\left(\frac{i\sigma^{\mu\nu} q_\nu}{2M}\right)$$

DIS structure (structure functions)



$$W^{\mu\nu} = -2\pi \left\{ \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) W_1 - \left(P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left(P^\nu - \frac{P \cdot q}{q^2} q^\nu \right) \frac{W_2}{M^2} \right.$$
$$\left. + i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha S_\beta}{P \cdot q} (g_1 + g_2) - \frac{S \cdot q}{M(P \cdot q)} i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha P_\beta}{q \cdot P} g_2 \right\}$$

W_1, W_2: unpolarized PDFs

g_1, g_2: polarized PDFs (encode details of the spin structure)   Spin S^μ

S-state (previous work): W_1, W_2

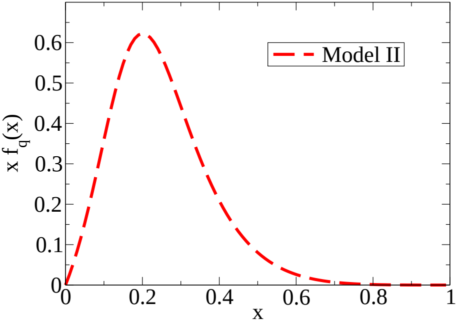
PRC 77,015202 (2008)

S-state approach

Qualitative description of DIS

Callen-Gross scaling $x = \frac{Q^2}{2M\nu}$

$$\nu W_2(x) = 2MxW_1(x)$$
$$= e_I^2 x f_q(x)$$



Quark distribution function (normalized to 1):

$$f_q(x) = \frac{\mathcal{N}}{4\pi} \int \frac{d^2 k_\perp}{(2\pi)^2 (1-x)} \psi_S^2(k_\perp; x).$$

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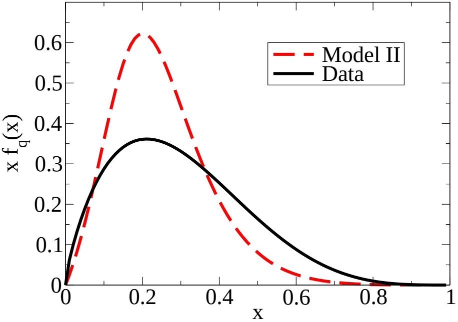

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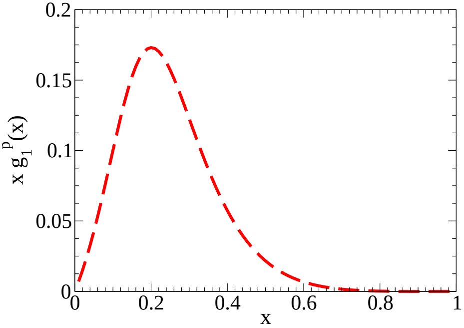
S-state (new result): g_1

Predicts $g_2 \equiv 0$, but

$$g_1(x) = \frac{5}{18} e_N f_q(x)$$

First moment (proton)

$$\Gamma_1 = \int_0^1 dx g_1(x) = \frac{5}{18} = 0.28$$
$$>> \Gamma_1^{exp} = 0.17.$$



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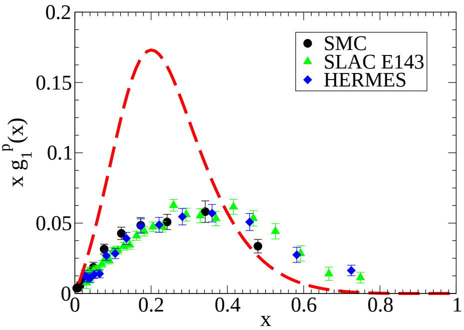

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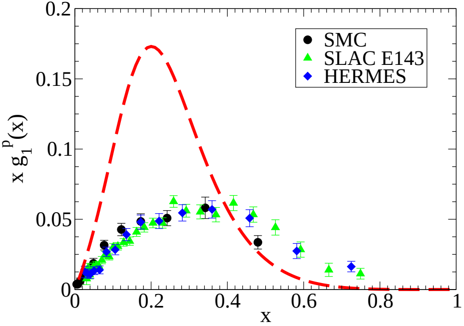
$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma}_{q \text{ spin}} + \underbrace{L_q}_{q \text{ OAM}} + \underbrace{J_g}_{\text{gluons}}$$



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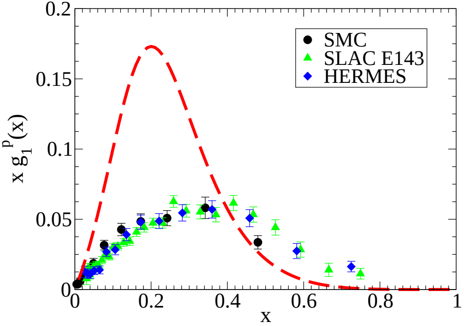

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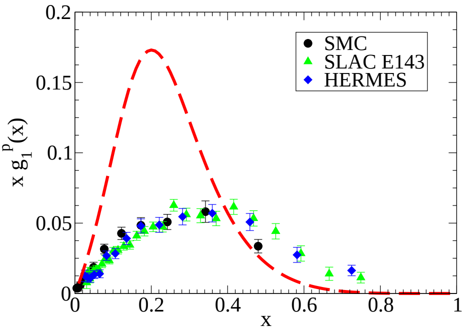


$$\Psi_N = \sum_q [n_S \Psi_q^S + n_P \Psi_q^P + n_D \Psi_q^D]$$

S-state quarks are insufficient !!

What about $P, D \oplus \Psi_u^L \neq \Psi_d^L$?

Covariant Spectator Theory †

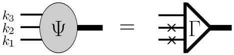
Covariant Spectator Theory ©, Franz Gross *et al.*, applied to:
[See A. Stadler and F. Gross, FBS 49, 91 (2011)]

- NN scattering, deuteron and three-nucleon bound states
- Deuteron and triton electromagnetic form factors
- πN scattering
- $q\bar{q}$ models of mesons

Covariant Spectator Quark Model (See arXiv:1008.0371 [hep-ph])
GR, F. Gross, M. T. Peña, K. Tsushima, ...

- Nucleon and Δ electromagnetic form factors
- Electromagnetic transition form factors $\gamma^* N \to N^*$
  $N^* = \Delta(1232), N^*(1440), N^*(1520), N^*(1535), \Delta(1600), N^*(1710), ...$
  ... [SQTM: PRD 90, 033010 (2014)]
- Octet baryon and decuplet baryon e.m. form factors:
  physical regime, nuclear medium and **extension to lattice QCD**
- $\Delta(1232)$ mass distribution for the Dalitz decay: $\Delta \to N e^+ e^-$ $(pp \to pp)$

Covariant Spectator Quark Model (1) †



Wave function Ψ with 2 on-mass-shell quarks

- On-shell integration $(k_1, k_2) \Rightarrow$ $k = k_1 + k_2$, $r = \frac{1}{2}(k_1 - k_2)$
  $\Rightarrow$ integration in $\mathbf{k}$ and $s = (k_1 + k_2)^2$
  F. Gross, GR and M. T. Peña: PRC 77, 015202 (2008); PRD 85, 093005 (2012)

$$\int \frac{d^3 k_1}{2E_{k_1}} \int \frac{d^3 k_2}{2E_{k_2}} = \frac{\pi}{4} \int d\Omega_{\hat{\mathbf{r}}} \int_{4m_q^2}^{+\infty} ds \sqrt{\frac{s - 4m_q^2}{s}} \int \frac{d^3 \mathbf{k}}{2\sqrt{s + \mathbf{k}^2}}$$

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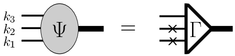

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Mean value theorem $\Rightarrow$ covariant int. in diquark **on-shell** momentum

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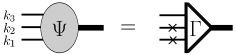

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- Integrating over the diquark internal degrees of freedom

$$\int_k \int_r \psi(\mathbf{k}, \mathbf{r})(\ldots) \to \int \frac{d^3 \mathbf{k}}{2E_k} \tilde{\psi}(\tilde{k}, \zeta^\nu, \epsilon_D^\nu)(\ldots)$$

ζ^ν P-state diquark; ϵ_D^ν D-state diquark

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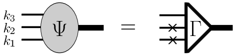

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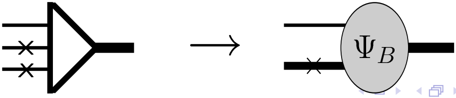

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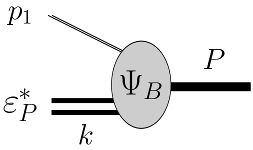

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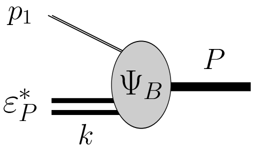

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  $\Rightarrow$ Covariant generalization using baryon proprieties

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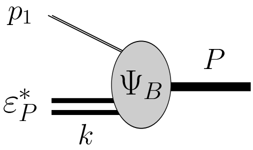

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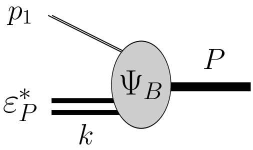

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- Wave function with manifest rotational invariance,
  well defined **angular momentum states** (important spin/DIS)
- Phenomenology in the **radial wf** (momentum scale parameters, ...)

$$\Psi^L_{\Lambda\lambda}(P,k) = \mathcal{O}^L_\Lambda\, u(P,\lambda)$$

Isospin: ϕ^0, ϕ^1

Covariant notation

$$\tilde{k}^\alpha = k^\alpha - \frac{P\cdot k}{M^2}P^\alpha$$

$$\tilde{\gamma}^\alpha = \gamma^\alpha - \frac{P^\alpha}{M^2}\not{P}$$

$$\tilde{g}^{\alpha\beta} = g^{\alpha\beta} - \frac{P^\alpha P^\beta}{M^2}$$

$$D^{\alpha\beta}(P,k) = \tilde{k}^\alpha\tilde{k}^\beta - \frac{1}{3}\tilde{k}^2\tilde{g}^{\alpha\beta}$$

$$G^{\alpha\beta}(\tilde{k},\zeta_\nu) = \tilde{k}^\alpha\zeta_\nu^\beta + \zeta_\nu^\alpha\tilde{k}^\beta - \frac{2}{3}(\tilde{k}\cdot\zeta_\nu)\tilde{g}^{\alpha\beta}$$

Replacements:

$\mathbf{k} \to -\tilde{k}$, $\mathbf{k}^2 \to -\tilde{k}^2$

$\delta_{\ell m} \to \tilde{g}^{\alpha\beta}$

- S-state

$$\mathcal{O}^{S,0}_\Lambda = \frac{1}{\sqrt{2}}\phi^0\psi_S(P,k)\mathbb{1}$$

$$\mathcal{O}^{S,1}_\Lambda = \frac{1}{\sqrt{2}}\phi^1\psi_S(P,k)(\varepsilon^*_\Lambda)_\alpha\gamma_5\tilde{\gamma}^\alpha$$

- P-state

$$\mathcal{O}^{P,0}_\Lambda = \frac{1}{\sqrt{2}}\phi^0\psi_P(P,k)\tilde{\not{k}}$$

$$\mathcal{O}^{P,1}_\Lambda = \frac{1}{\sqrt{2}}\phi^1\psi_P(P,k)\tilde{\not{k}}(\varepsilon^*_\Lambda)_\alpha\gamma_5\tilde{\gamma}^\alpha$$

- D-state

$$\mathcal{O}^{D,0}_\Lambda = \frac{\sqrt{3}}{\sqrt{2\sqrt{10}}}\phi^0|\tilde{k}|\psi_D(P,k)(\varepsilon^*_\Lambda)_\alpha \overbrace{G^{\alpha\beta}(\tilde{k},\zeta_\nu)}^{\ell=1}\gamma_5\tilde{\gamma}_\beta$$

$$\mathcal{O}^{D,1}_\Lambda = -\frac{1}{\sqrt{30}}\phi^1\tilde{k}^2\psi_D(P,k)\overbrace{(\epsilon^*_{D\Lambda})_\alpha}^{\ell=2}\gamma_5\tilde{\gamma}^\alpha$$

$$\mathcal{O}^{D,2}_\Lambda = \sqrt{\frac{3}{5}}\phi^1\psi_D(P,k)\overbrace{(\varepsilon^*_\Lambda)_\alpha D^{\alpha\beta}(P,k)}^{\ell=0}\gamma_5\tilde{\gamma}_\beta$$

Nucleon wave function (2) [PRD 85, 093005 (2012)]

- **Covariant** wave function consistent with **isospin** and **angular momentum**

$$\Psi_{\Lambda\lambda} = n_S \Psi^S_{\Lambda\lambda} + n_P \Psi^P_{\Lambda\lambda} + n_D \Psi^D_{\Lambda\lambda}$$

- L-states normalized

$$n_S^2 + n_P^2 + n_D^2 = 1$$

- We can also consider **Isospin breaking** $u \neq d$
- Radial wf $\psi_L(P,k)$?
  - **Determined by DIS phenomenology**

- S-state

$$\mathcal{O}^{S,0}_\Lambda = \frac{1}{\sqrt{2}} \phi^0 \psi_S(P,k) \mathbb{1}$$

$$\mathcal{O}^{S,1}_\Lambda = \frac{1}{\sqrt{2}} \phi^1 \psi_S(P,k) (\varepsilon^*_\Lambda)_\alpha \gamma_5 \tilde{\gamma}^\alpha$$

- P-state

$$\mathcal{O}^{P,0}_\Lambda = \frac{1}{\sqrt{2}} \phi^0 \psi_P(P,k) \tilde{\not{k}}$$

$$\mathcal{O}^{P,1}_\Lambda = \frac{1}{\sqrt{2}} \phi^1 \psi_P(P,k) \tilde{\not{k}} (\varepsilon^*_\Lambda)_\alpha \gamma_5 \tilde{\gamma}^\alpha$$

- D-state

$$\mathcal{O}^{D,0}_\Lambda = \frac{\sqrt{3}}{\sqrt{2\sqrt{10}}} \phi^0 |\tilde{k}| \psi_D(P,k) (\varepsilon^*_\Lambda)_\alpha \overbrace{G^{\alpha\beta}(\tilde{k},\zeta_\nu)}^{\ell=1} \gamma_5 \tilde{\gamma}_\beta$$

$$\mathcal{O}^{D,1}_\Lambda = -\frac{1}{\sqrt{30}} \phi^1 \tilde{k}^2 \psi_D(P,k) \overbrace{(\epsilon^*_{D\Lambda})_\alpha}^{\ell=2} \gamma_5 \tilde{\gamma}^\alpha$$

$$\mathcal{O}^{D,2}_\Lambda = \sqrt{\frac{3}{5}} \phi^1 \psi_D(P,k) \overbrace{(\varepsilon^*_\Lambda)_\alpha D^{\alpha\beta}(P,k)}^{\ell=0} \gamma_5 \tilde{\gamma}_\beta$$

Write wave function in terms of u and d isospin states

Proton: $\chi^{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = u$; Neutron: $\chi^{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = d$

ψ_L dependent of the flavor of the quark 3

Isospin-0 component:

$$\Phi_I^0 \psi_L = \phi^0 \chi^I \psi_L = \frac{1}{\sqrt{2}}(ud - du) \begin{pmatrix} u \\ d \end{pmatrix} \psi_L \longrightarrow \phi^0 \chi^I \psi_{u(d)}^L$$

Isospin-1 component:

$$\Phi_I^1 \psi_L = \underbrace{-\frac{1}{\sqrt{6}}(ud + du) \begin{pmatrix} u \\ d \end{pmatrix}}_{\ell=0} \psi_L + \underbrace{\sqrt{\frac{2}{3}} \begin{pmatrix} (uu)d \\ -(dd)u \end{pmatrix}}_{\ell=\mp 1} \psi_L$$

$$\to \left(\phi_{\ell=0}^1\right) \chi^I \psi_{u(d)}^L + \left(\phi_{\ell=\mp 1}^1\right) \chi^I \, \psi_{d(u)}^L$$

$\psi_L \to \psi_q^L$: different distributions for u and d

Normalization / quark charge

$$j_q(+\infty) = \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3}, \quad Q_d = -\frac{1}{3}$$

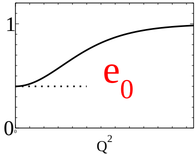
$$j_q(0) = e_0\left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3}e_0, \quad Q_d = -\frac{1}{3}e_0$$

Wave function normalized by the nucleon charge ($Q^2 = 0$); $P = (M, 0, 0, 0)$

Elastic form factors

$$J^0 = 3\sum_\Lambda \int_k \overline{\Psi}_{\Lambda\lambda}(P,k)\overbrace{j_q(0)}^{e_0}\gamma^0\Psi_{\Lambda\lambda}(P,k)$$

$$= \frac{1}{2}(1+\tau_3)e_0 \underbrace{\int_k \overline{|\psi_N|^2}}_{1}$$



Wave functions normalization $Q^2 = 0$: (rest frame $\tilde{k}^2 = -\mathbf{k}^2$)

$$e_0 \int_k |\psi_S|^2 = e_0 \int_k (-\tilde{k}^2)|\psi_P|^2 = e_0 \int_k \tilde{k}^4|\psi_D|^2 = 1$$

Normalization / quark charge

$$j_q(+\infty) = \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right): \quad Q_u = +\frac{2}{3}, \quad Q_d = -\frac{1}{3}$$

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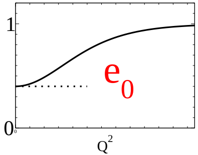


Wave functions normalization $Q^2 = 0$: (rest frame $\tilde{k}^2 = -\mathbf{k}^2$) $u \neq d$, $e_0 \to e_q^0$

$$e_q^0 \int_k |\psi_q^S|^2 = e_q^0 \int_k (-\tilde{k}^2)|\psi_q^P|^2 = e_q^0 \int_k \tilde{k}^4 |\psi_q^D|^2 = 1$$

Normalization (more details) ††

$$f_q^S(x) = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2$$

DIS normalization

$$\int_0^1 dx f_q^S(x) = 1,$$

WF normalization ($Q^2 = 0$)

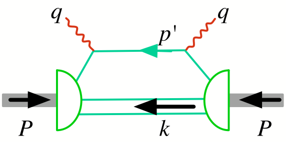
$$e_q^0 \int_k |\psi_q^S(\chi)|^2 \big|_{Q^2=0} = e_q^0 \underbrace{\int_{-\infty}^1 dx f_q^S(x)}_{N} = 1$$

How to fix e_q^0 ?

$$e_q^0 = \frac{1}{N} = \frac{\int_0^1 dx f_q^S(x)}{\int_{-\infty}^0 dx f_q^S(x) + \int_0^1 dx f_q^S(x)} < 1$$

- DIS define ψ_q^L for $0 < x < 1$
- f_q^S for $x < 0 \Rightarrow$ determine quark charge (e_q^0) at $Q^2 = 0$

DIS with a quark model (2) †



$$(J_{s_q,\lambda,\lambda_n})^\mu = -\bar{u}(p', s_q) j^\mu(q) \Psi_{\Lambda\lambda}(P,k)$$

Hadronic tensor: $\Psi_{\Lambda\lambda} = \mathcal{O}_\Lambda u(P,\lambda)$, S^μ spin operator

$$W^{\mu\nu}(\lambda) = 3 \sum_\Lambda \iint_{p'k} \frac{1}{2} \mathrm{tr} \left\{ \mathcal{O}^\dagger_\Lambda j^\mu(q) \underbrace{(m_q + \not{p}')}_{\text{quark p}'} j^\nu(q^2) \mathcal{O}_\Lambda \underbrace{(M + \not{P})(1 + \gamma_5 \not{S})}_{N \text{ spin-}S \text{ proj}} \right\}$$

Integration: $|\mathbf{k}|$, $z = \cos\theta = \frac{k_z}{|\mathbf{k}|}$, $\qquad p'^2 = m_q^2 \approx 0$

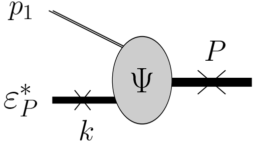
$|\mathbf{k}| = M\kappa$, $E_s = ME_\kappa = M\sqrt{\frac{m_s^2}{M^2} + \kappa^2}$

$$\iint_{p'k} = \int \frac{d^4k}{(2\pi)^2} \delta_+(m_q^2 - p'^2) \delta_+(m_s^2 - k^2) = \int \frac{d^3\mathbf{k}}{(2\pi)^2(2E_s)} \overbrace{\delta\left(\frac{Q^2}{Mx}\left[(1-x) - E_s + |\mathbf{k}|z\right]\right)}^{\text{DIS condition}}$$

$$= \frac{M^2 x}{Q^2} \int_0^{+\infty} \frac{\kappa d\kappa}{4\pi E_\kappa} \int_{-1}^{1} dz \delta(z - z_0) = \frac{M^2 x}{Q^2} \int_{\kappa_{\min}}^{+\infty} \frac{\kappa d\kappa}{4\pi E_\kappa} = \frac{\pi x}{Q^2} \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi$$

DIS: wave functions †

$\psi_L(P,k)$ can be represented using the covariant variable



$$\chi = \frac{(M-m_s)^2-(P-k)^2}{Mm_s} = 2\frac{P\cdot k}{Mm_s} - 2$$

because $P^2 = M^2$ and $k^2 = m_s^2$.

When $r = \frac{m_s}{M} \to 1$ (diquark mass = nucleon mass)

$$\iint_{p'k} \psi^2(\chi) = \frac{\pi x}{Q^2}\frac{Mm_s}{16\pi^2}\int_\xi^{+\infty} d\chi \psi^2(\chi), \qquad \xi = \frac{x^2}{1-x}$$

S-state PRC 77,015202 (2008): PRD 85, 093006 (2012)

$$f_q^S(x) = \frac{Mm_s}{16\pi^2}\int_\xi^{+\infty} d\chi [\psi_q^S(\chi)]^2, \qquad \frac{df_q^S}{dx} = -\frac{x(2-x)}{(1-x)^2}\frac{Mm_s}{16\pi^2}[\psi_q^S(\xi)]^2$$

DIS: gluon effects †

Gluon contributions are not considered in this model
Only valence quarks (no sea quarks):

$$\int_0^1 dx f_u(x) = \int_0^1 dx f_d(x) = 1,$$

$$\frac{1}{3}\int_0^1 dx\,[2f_u(x) + f_d(x)] = 1 \qquad \text{[proton charge]}$$

Proton momentum sum rule: N_g gluon contribution, $N_g \approx 0.5$

$$2\int_0^1 dx x f_u(x) + \int_0^1 dx x f_d(x) + N_g = 1.$$

S-state approximation $(n_P = n_D = 0)$ and $f_u = f_d \equiv f_q$:

$$\int_0^1 dx x f_q^S(x) = 0.167, \qquad e_q^0 \int_{-\infty}^1 dx f_q^S(x) = 1, \qquad \underbrace{e_q^0 = 0.41}_{f_q^S(x) \propto \delta(x - x_0)}$$

Structure functions (1) †

Use elementary structure functions $\int_\chi = \frac{M m_s}{16\pi^2} \int_\xi^{+\infty} d\chi$

$$f_q^L(x) = \int_\chi k^{2L} \left[\psi_q^L(\chi)\right]^2 \qquad L = 0, 1, 2 \quad (S, P, D)$$

$$g_q^L(x) = \int_\chi P_2(z_0) k^{2L} \left[\psi_q^L(\chi)\right]^2 \qquad L = 1, 2 \quad (P, D)$$

$$d_q(x) = \int_\chi P_2(z_0) k^2 \psi_q^S(\chi)\psi_q^D(\chi) \qquad (SD \text{ interference})$$

$$h_q^0(x) = \int_\chi z_0 k \, \psi_q^S(\chi)\psi_q^P(\chi) \qquad (SP \text{ interference})$$

$$h_q^2(x) = \int_\chi z_0 k^3 \, \psi_q^P(\chi)\psi_q^D(\chi) \qquad (PD \text{ interference})$$

$$h_q^1(x) = \int_\chi (1 - z_0^2) \frac{k^2}{4Mx} \, \psi_q^S(\chi)\psi_q^P(\chi) \qquad (SP \text{ interference})$$

$$h_q^3(x) = \int_\chi (1 - z_0^2) \frac{k^4}{4Mx} \, \psi_q^P(\chi)\psi_q^D(\chi) \qquad (PD \text{ interference})$$

Structure functions (2) [Using charge symmetry]

$f_u(x) = u$-distribution in the proton [d-distribution in the neutron] $a_{SD} = -3\sqrt{\frac{2}{5}} n_S n_D$

$f_d(x) = d$-distribution in the proton [u-distribution in the neutron] $a_{PD} = -3\sqrt{\frac{2}{5}} n_P n_D$

$g_i^u(x)$: u-contribution of g_i in the proton [d-contribution of g_i in the neutron]

$g_i^d(x)$: d-contribution of g_i in the proton [u-contribution of g_i in the neutron]

Proton:

$$f_p(x) = \sum_q e_q^2 f_q(x) \qquad g_i^p(x) = \frac{1}{2}\sum_q e_q^2 g_i^q(x)$$

Neutron: $e_u \leftrightarrow e_d$ $\qquad f_q = n_S^2 f_q^S + n_P^2 f_q^P + n_D^2 f_q^D - 2 n_S n_P h_q^0$

$$\begin{aligned}
g_1^u &= \frac{2}{3} f_u - n_D^2 f_u^D - \frac{8}{9} n_P^2 f_u^P + \frac{8}{9} n_P^2 g_u^P + \frac{29}{60} n_D^2 g_u^D - \frac{2}{9} a_{SD} d_u + \frac{2}{9} a_{PD} h_u^2 \\
g_1^d &= -\frac{1}{3} f_d + \frac{8}{15} n_D^2 g_d^D + \frac{4}{9} n_P^2 f_d^P - \frac{4}{9} n_P^2 g_d^P - \frac{8}{9} a_{SD} d_d + \frac{8}{9} a_{PD} h_d^2 \\
g_2^u &= -\frac{4}{3} n_P^2 g_u^P - \frac{29}{40} n_D^2 g_u^D + \frac{1}{3} a_{SD} d_u - \frac{4}{3} n_S n_P (h_u^1 - h_u^0) + \frac{2}{9} a_{PD} (h_u^3 - h_u^2) \\
g_2^d &= +\frac{2}{3} n_P^2 g_d^P - \frac{4}{5} n_D^2 g_d^D + \frac{4}{3} a_{SD} d_d + \frac{2}{3} n_S n_P (h_d^1 - h_d^0) + \frac{2}{9} a_{PD} (h_d^3 - h_d^2)
\end{aligned}$$

Functional form for the radial wave functions

Wave functions for $L = S, P, D$: $\theta \neq 0$

Apart kinematic factors:

$$\psi_q^L(\chi) \approx \frac{\beta \cos\theta + \chi \sin\theta}{\chi^{n_0}(\beta + \chi)^{n_1 - n_0}}$$

β is a (momentum range scale$/M)^2$
n_1 define the high Q^2 behaviour of the nucleon form factor (large x)
n_0 define $f_q(x)$, $x \to 0$

β, θ, n_0, n_1 depend of L and q

Data:

SMC, SLAC, HERMES, Jlab and COMPASS

obtained for several regions of Q^2 (not only large Q^2)

Fit to the data – parametrization for $Q^2 = 1$ GeV^2

- Unpolarized
$$\int_0^1 dx\, f_q^{\rm exp}(x) = 1$$

  Martin, Roberts, Stirling and Thorne, PLB 531, 216 (2002)-(MRST02)

$$x\, f_u^{\rm exp}(x) = \mathbf{0.130}\, x^{0.31}(1-x)^{3.50}(1 + 3.83\sqrt{x} + 37.65x)$$
$$x\, f_d^{\rm exp}(x) = \mathbf{0.061322}\, x^{0.35}(1-x)^{4.03}(1 + 49.05\sqrt{x} + 8.65x)$$

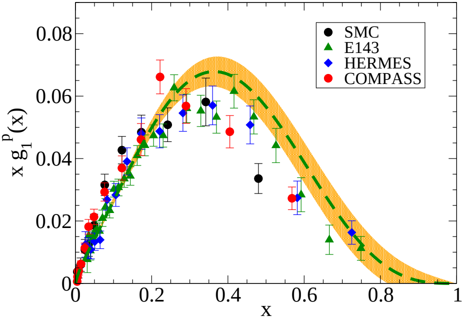
- Polarized: Leader, Sidorov and Stamenov (LSS10)
  PRD 82, 114018 (2010), $Q^2 = 1$ GeV^2

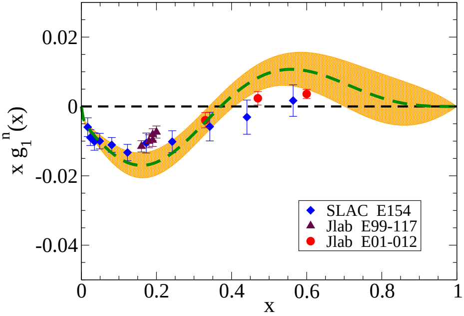
$$g_1^q(x) = \Delta q(x) + \frac{h^q(x)}{Q^2}$$

$$x\Delta u(x) = 0.548\, x^{0.782}(1-x)^{3.335}(1 - 1.779\sqrt{x} + 10.2\, x)$$
$$x\Delta d(x) = -0.394\, x^{0.547}(1-x)^{4.056}(1 + 6.758\, x)$$

Data (polarized)

$$g_1^q(x) = \Delta q(x) + \frac{h^q(x)[1 \pm \delta_q(x)]}{Q^2} \quad \leftarrow \text{high twist corrections}$$





$$\Gamma_1^p = \int_0^1 dx g_{1p}^{\text{exp}}(x) = 0.128 \pm 0.013 \qquad \Gamma_1^u = \tfrac{3}{5}(4\Gamma_1^p - \Gamma_1^n) = 0.333 \pm 0.039$$

$$\Gamma_1^n = \int_0^1 dx g_{1n}^{\text{exp}}(x) = -0.042 \pm 0.013 \qquad \Gamma_1^d = \tfrac{6}{5}(4\Gamma_1^n - \Gamma_1^p) = -0.355 \pm 0.080$$

Fit to the data: Structure functions (review)

Proton:

$$f_p(x) = \sum_q e_q^2 f_q(x) \qquad g_i^p(x) = \frac{1}{2}\sum_q e_q^2 g_i^q(x)$$

Neutron: $e_u \leftrightarrow e_d$

$$f_q = n_S^2 f_q^S + n_P^2 f_q^P + n_D^2 f_q^D - 2 n_S n_P h_q^0$$

$$\begin{aligned}
g_1^u &= \frac{2}{3} f_u - n_D^2 f_u^D - \frac{8}{9} n_P^2 f_d^P + \frac{8}{9} n_P^2 g_u^P + \frac{29}{60} n_D^2 g_u^D - \frac{2}{9} a_{SD} d_u + \frac{2}{9} a_{PD} h_u^2 \\
g_1^d &= -\frac{1}{3} f_d + \frac{8}{15} n_D^2 g_d^D + \frac{4}{9} n_P^2 f_d^P - \frac{4}{9} n_P^2 g_d^P - \frac{8}{9} a_{SD} d_d + \frac{8}{9} a_{PD} h_d^2 \\
g_2^u &= -\frac{4}{3} n_P^2 g_u^P - \frac{29}{40} n_D^2 g_u^D + \frac{1}{3} a_{SD} d_u - \frac{4}{3} n_S n_P (h_u^1 - h_u^0) + \frac{2}{9} a_{PD} (h_u^3 - h_u^2) \\
g_2^d &= +\frac{2}{3} n_P^2 g_d^P - \frac{4}{5} n_D^2 g_d^D + \frac{4}{3} a_{SD} d_d + \frac{2}{3} n_S n_P (h_d^1 - h_d^0) + \frac{2}{9} a_{PD} (h_d^3 - h_d^2)
\end{aligned}$$

Covariant Spectator QM: very rich structure in the DIS regime

Fit to the data

Covariant Spectator QM: very rich structure in the DIS regime

How can we study the effect of the individual L states ?
(and learn from the phenomenology)

Fit to the data

Covariant Spectator QM: very rich structure in the DIS regime

How can we study the effect of the individual L states ?
(and learn from the phenomenology)

Fitting process (MRST02 & LSS10 parametrizations):

- Step 1: S-state component - fitted to unpolarized PDFs
  (Adjust $\beta_{Sq}, \theta_{Sq}, n_{0Sq}$ and n_{1Sq}) $f_q = n_S^2 f_q^S + n_P^2 f_q^P + n_D^2 f_q^D - 2 n_S n_P h_q^0$

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- Step 2: Estimate the strength of the P and D states (n_P, n_D)
  Same ψ_q^L **for all** L**:** $\psi_q^L \approx \psi_q^S \Rightarrow$ fit moments Γ_1^u, Γ_1^d

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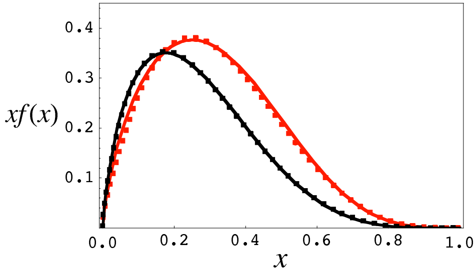
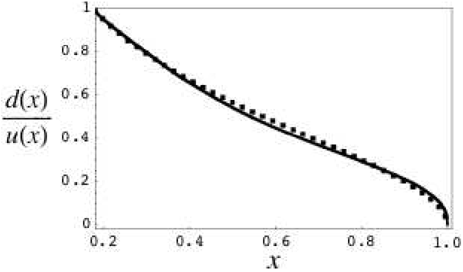
  Same ψ_q^L for all L: $\psi_q^L \approx \psi_q^S \Rightarrow$ fit moments Γ_1^u, Γ_1^d
- Step 3: Global fit
  Improves the description **breaking the symmetry** between S, P and D radial wave functions

Fits to the data: step 1 $q(x) \equiv f_q(x)$

	β_{Sq}	θ_{Sq}	n_{0Sq}	n_{1Sq}	C_q^S	e_q^0
u	**0.9**	**0.4**π	**0.51**	3	2.197	0.3545
d	**1.25**	$\frac{1}{4}\pi$	**0.49**	**3.2**	2.279	0.3940





f_u, f_d (Models) . . . . .

$$x f_u(x) \approx x^{0.20}(1-x)^{3.0} \qquad x f_u^{\rm exp}(x) \approx x^{0.31}(1-x)^{3.50}$$

$$x f_d(x) \approx x^{0.36}(1-x)^{3.4} \qquad x f_d^{\rm exp}(x) \approx x^{0.35}(1-x)^{4.03}$$

Fits to the data: step 2

Same functional form for all states: $\kappa^2 = \frac{1}{4}\chi(\chi+4) = \frac{\mathbf{k}^2}{M^2}$

$\psi_q^S = (M\kappa)\psi_q^P = (M\kappa)^2\psi_q^D$

Fit n_P and n_D to the moments Γ_1^q

$$\Gamma_1^u = 0.333 \pm 0.039, \qquad \Gamma_1^d = -0.355 \pm 0.080$$

solution	$n_P(u)$	$n_D(u)$	$n_P(d)$	$n_D(d)$
1	0.43	0.18	−0.43	−0.18
2	0.08	0.59	0.08	−0.59

Two possible solutions (equal quality):

- Solution 1: large P state; $n_P(d) < 0$, $n_D(d) < 0$
- Solution 2: large D state; $n_D(d) < 0$, $n_P(q) \approx 0$

Fits to the data: step 3

n_P, n_D **fixed for models 1 and 2**

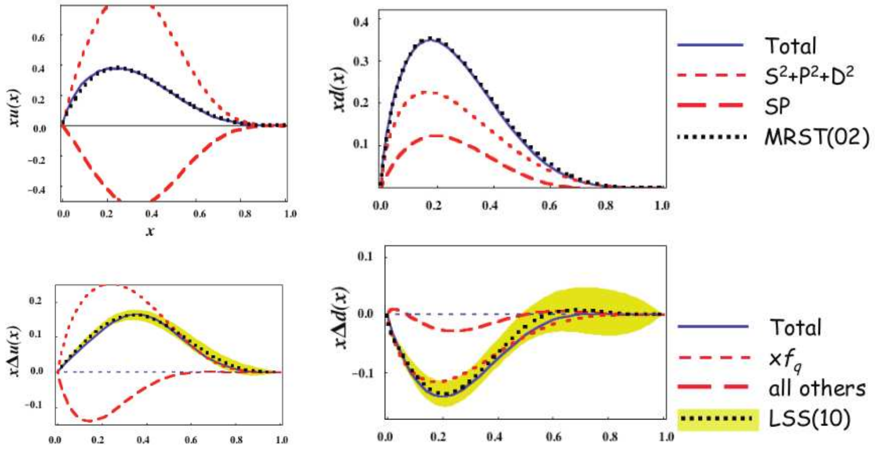
$$\psi_q^L(\chi) \approx \frac{\beta \cos\theta + \chi \sin\theta}{\chi^{n_0}(\beta+\chi)^{n_1-n_0}}$$

Refit wave functions

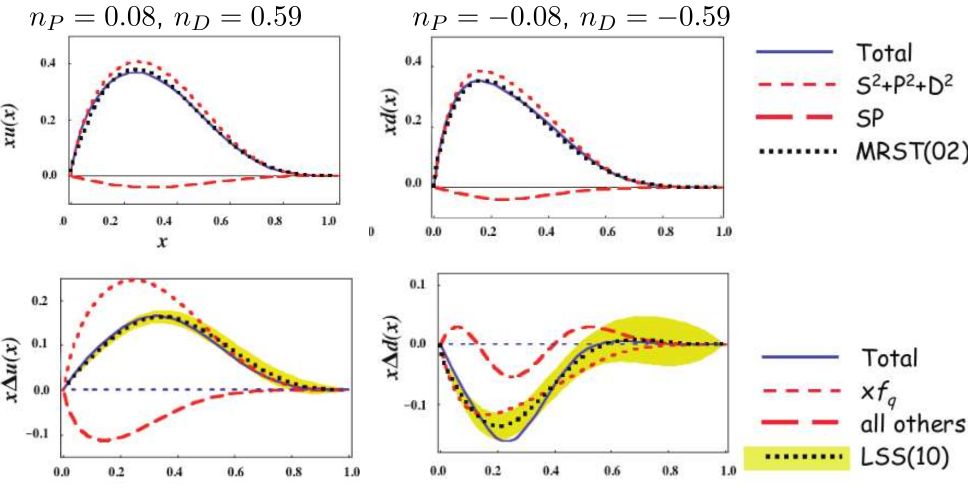
- β_{Lq}, n_{0Lq}, n_{1Lq} same for **all** L
- θ_{Lq} adjusted in same cases

Fits to the data: step 3 (model 1) [P: 18%, D: 3%]

$n_P = 0.43$, $n_D = 0.18$ $\qquad$ $n_P = -0.43$, $n_D = -0.18$

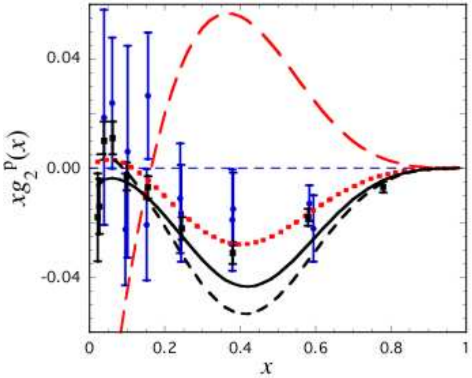


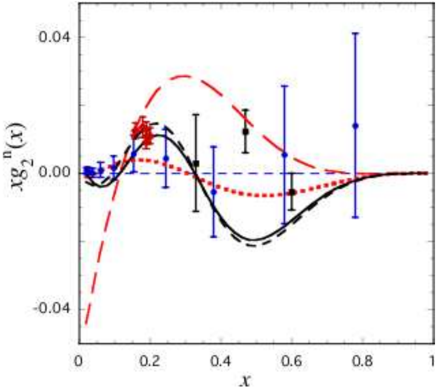
Fits to the data: step 3 (model 2) [P: 0.6%, D: 35%]



Predictions for g_2

— — Model 1; —- Model 2; · · · Model 1 (P=0); - - - Model 2 (P=0)

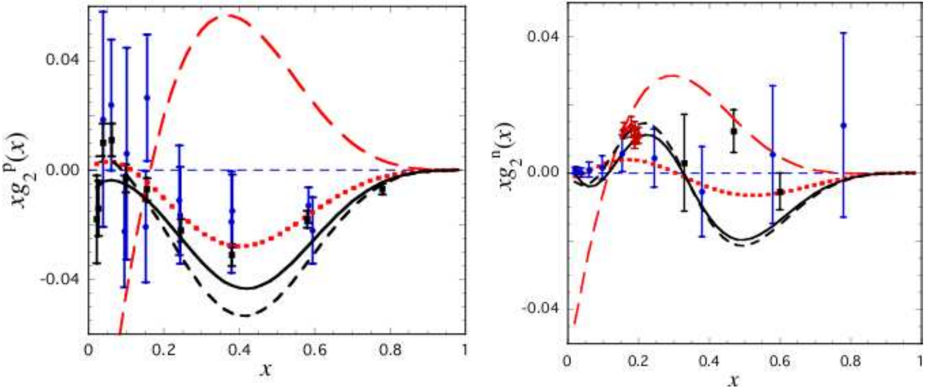




Data: g_2^p: SLAC-E143, SLAC-E155 g_2^n: SLAC-E155, Jlab-Kramer, Jlab-Hall A

Predictions for g_2

— — Model 1; —- Model 2; · · · Model 1 (P=0); - - - Model 2 (P=0)



Data: g_2^p: SLAC-E143, SLAC-E155 g_2^n: SLAC-E155, Jlab-Kramer, Jlab-Hall A

Only **Model 2** gives a good result (35% D-state)

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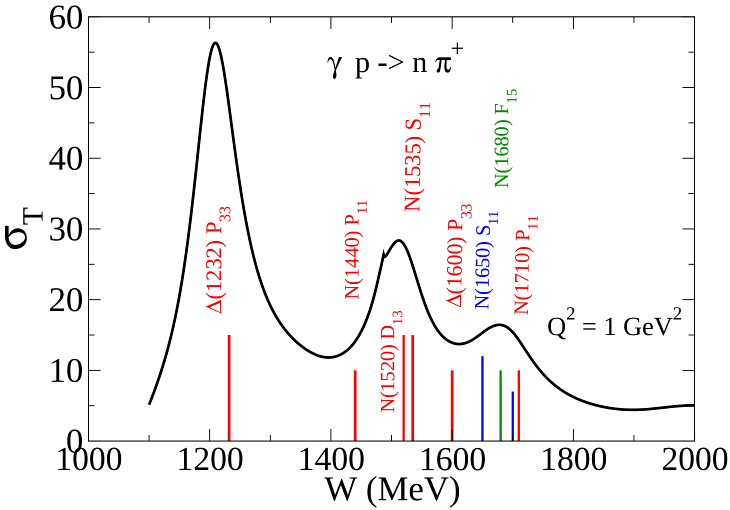
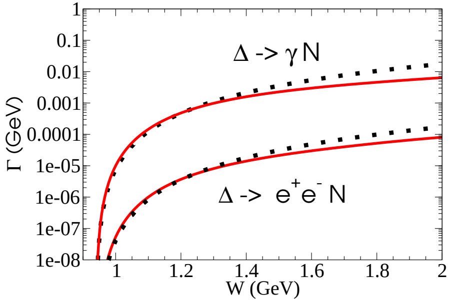
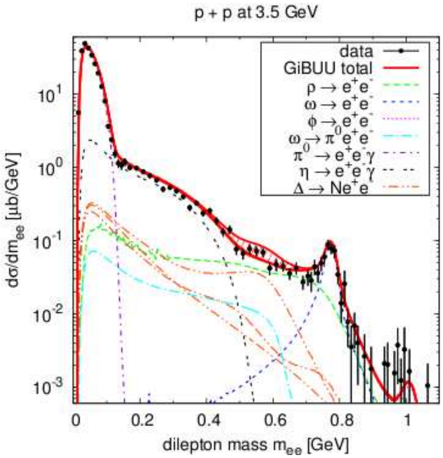
Nucleon Resonance Structure (e.m. structure)



$\gamma^* N \to \Delta$: Dalitz decay ($\Delta \to e^+ e^- N$)



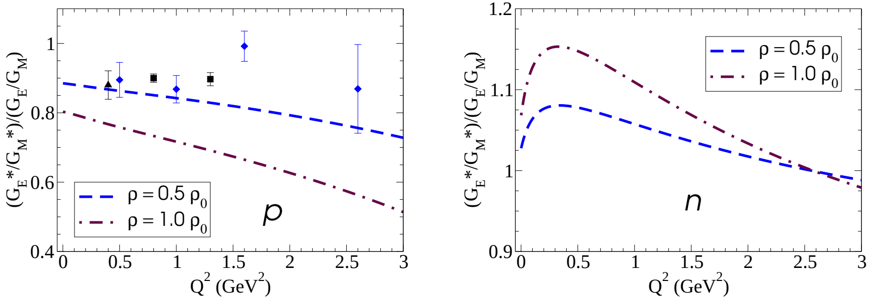


GR and MT Peña, PRD 85113014 (2012)

Cross-section: J. Weil, H. van Hees and U. Mosel, EPJA 48, 111 (2012)

Octet form factors in the nuclear medium

Proton and neutron double ratio: $\mathcal{R}_N = \frac{G_E^*/G_M^*}{G_E/G_M}$



GR, K Tsushima and AW Thomas, JPG 40 015102 (2013)

Proton data: S. Dieterich et al, PLB 500, 47 (2001); S. Strauch et al, PLB 91, 052301 (2003); M. Paolone et al, PLB 105, 072001 (2010)

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Thank you

Discussion

- **Where is the glue ?**
  - No need of gluon effects to explain nucleon spin (g_1^N data) (gluons included in constituent quark structure)
  - Maybe for g_2^N (more precise data needed)
  - ... gluon effects expected for larger Q^2 (QCD evolution equations)
- **Regime of application of the model ?**
  - Results derived in the large Q^2, ν limit; but no gluons included
  - To compare with the data we should use $Q_0^2 = 2 - 5$ GeV^2
  - For **very** large Q^2 use QCD evolution equations (DGLAP); gluon effects will emerge for larger Q^2 (even if there are only quarks at the low Q_0^2 regime)
- **D-state mixture**
  - What is the physical source of that effect (larger than other models) ?
  - Without a explicit interaction model it is not possible to explain the effect
  - We can however look for signs of that effect in other processes like the nucleon elastic form factors or $\gamma^* N \to \Delta$ reaction

Selected bibliography (part 1)

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  F. Gross, G. Ramalho and M. T. Pena, Phys. Rev. D **85**, 093006 (2012) [arXiv:1201.6337 [hep-ph]].
- **Covariant nucleon wave function with S, D, and P-state components**,
  F. Gross, G. Ramalho and M. T. Pena, Phys. Rev. D **85**, 093005 (2012) [arXiv:1201.6336 [hep-ph]].
- **A pure S-wave covariant model for the nucleon**,
  F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C **77**, 015202 (2008) [arXiv:nucl-th/0606029].
- **Fixed-axis polarization states: covariance and comparisons**,
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Selected bibliography (part 2)

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  Exclusive Reactions and High Momentum Transfer IV, 287 (2011)
  arXiv:1008.0371 [hep-ph].
- **Studies of Nucleon Resonance Structure in Exclusive Meson Electroproduction**
  I. G. Aznauryan et al, Int. J. Mod. Phys. E **22**, 1330015 (2013)
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- **A Covariant model for the nucleon and the Δ**,
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  [arXiv:0803.3034 [hep-ph]].
- **D-state effects in the electromagnetic $N\Delta$ transition**,
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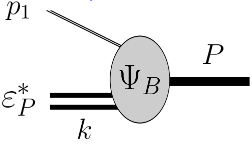
  [arXiv:0810.4126 [hep-ph]].

Backup slides

Nucleon wave function: S-state

S-state in quark-diquark [PRC 77,015202 (2008)]: k = diquark momentum



$$\Psi_S(P,k) = \frac{1}{\sqrt{2}}\left[\Phi_I^0\Phi_S^0 + \Phi_I^1\Phi_S^1\right]\psi_S(P,k)$$

$\Phi^0_{I,S}$: anti-symmetric in the exchange of quark states (12) - M_A
$\Phi^1_{I,S}$: symmetric in the exchange of quark states (12) - M_S

$$\Phi_I^0 \to \phi^0, \qquad \Phi_I^0 \to \phi^1$$
$$\Phi_S^0 \to u(P,\lambda), \qquad \Phi_S^1 \to -(\varepsilon^*_{\Lambda P})_\alpha U^\alpha(P,\lambda)$$

$\phi^{0,1}$ isospin operators acting in χ^I (nucleon isospin state)
ε_P^α diquark pol. vector: fixed-axis base [PRC 77, 015202 (2008)]
Vector spin 1/2 S-state $\left[1 \oplus \frac{1}{2} \to \frac{1}{2}\right]$: $U^\alpha(P,\lambda) = \frac{1}{\sqrt{3}}\gamma_5\left(\gamma^\alpha - \frac{P^\alpha}{M}\right)u(P,\lambda)$

Nucleon wave function: P-state and D-states

S-state: quark-diquark [PRC 77, 015202 (2008)]: (review)

$$\Psi_S(P,k) = \frac{1}{\sqrt{2}}\left[\phi^0 U(P,\lambda) - \phi^1 (\varepsilon^*_\Lambda)_\alpha U^\alpha(P,\lambda)\right]\psi_S(P,k)$$

P-state: quark-diquark [PRD 77, 093005 (2012)]: $\tilde{k} = k - \frac{P\cdot k}{M^2}P$

$$\Psi_P(P,k) = \frac{1}{\sqrt{2}}\tilde{\not{k}}\left[\phi^0 U(P,\lambda) - \phi^1 (\varepsilon^*_\Lambda)_\alpha U^\alpha(P,\lambda)\right]\psi_P(P,k)$$

D-state: quark-diquark [PRD 77, 093005 (2012)]:

$$\Psi_D(P,k) \approx \frac{1}{\sqrt{2}}\left[\phi^0 \psi^{Da}_{\Lambda\lambda} + \phi^1 \psi^{Ds}_{\Lambda\lambda}\right]\phi_D(\mathbf{k}_1,\mathbf{k}_2)$$

Integration in $\mathbf{r}$: $\iint \frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3} = \iint \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{d^3\mathbf{r}}{(2\pi)^3} \to \int \frac{d^3\mathbf{k}}{(2\pi)^3}\tilde{\phi}(\mathbf{k})$

Nucleon wave function: D-states (1)

D-state algebra (rest frame): $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$, $\mathbf{r} = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$

EPJA 36, 329 (2008); PRD 78, 114017 (2008)

$$U_m(\lambda) = \frac{1}{\sqrt{3}} \sigma_m | \tfrac{1}{2} \lambda \rangle \qquad D^{\ell m}(\mathbf{k}) = \mathbf{k}^\ell \mathbf{k}^m + \frac{1}{3} \mathbf{k}^2 \delta_{\ell m}$$

D-state function

$$\Theta^D_{\Lambda\lambda}(\mathbf{k}_i) = \frac{3}{\sqrt{2}} (\varepsilon^*_\Lambda)_\ell D^{\ell m}(\mathbf{k}_i) U_m(\lambda)$$

$$\begin{aligned} \psi^{Da}_{\Lambda\lambda} &= \frac{1}{\sqrt{2}} \left[\Theta^D_{\Lambda\lambda}(\mathbf{k}_1) - \Theta^D_{\Lambda\lambda}(\mathbf{k}_2) \right] \phi_D(\mathbf{k}^2, \mathbf{r}^2) \\ &= \frac{3}{2} (\varepsilon^*_\Lambda)_\ell G^{\ell m}(\mathbf{k}, \mathbf{r}) U_m(\lambda) \; \phi_D(\mathbf{k}^2, \mathbf{r}^2) \\ \psi^{Ds}_{\Lambda\lambda} &\simeq \left[\cos\phi \; \Theta^D_{\Lambda\lambda}(\mathbf{k}) + \sin\phi \; \Theta^D_{\Lambda\lambda}(\mathbf{r}) \right] \phi_D(\mathbf{k}^2, \mathbf{r}^2), \qquad \cos\phi = \frac{1}{5} \end{aligned}$$

Nucleon wave function: D-states (2)

$$G^{\ell m}(\mathbf{k},\mathbf{r}) = \mathbf{k}^\ell \mathbf{r}^m + \mathbf{r}^\ell \mathbf{k}^m + \frac{2}{3}(\mathbf{k}\cdot\mathbf{r})\delta_{\ell m}$$

$\mathbf{r}^\ell$: diquark with internal P-state; define spin-1 vector ζ_ν^ℓ

$\Theta_{\Lambda\lambda}^D(\mathbf{r})$: diquark with internal D-state; define spin-1 vector $\varepsilon_{D\Lambda}^m$

$$\psi_{\Lambda\lambda(\nu)}^{Da} \rightarrow \frac{3}{\sqrt{20}} \overbrace{(\varepsilon_\Lambda^*)_\ell G^{\ell m}(\mathbf{k},\zeta_\nu) U_m(\lambda)}^{\ell=1} \; \psi_D(P,k)$$

$$\psi_{\Lambda\lambda}^{Ds} \rightarrow \frac{3\sqrt{2}}{\sqrt{5}} \overbrace{(\varepsilon_\Lambda)_\ell D^{\ell m}(\mathbf{k}) U_m(\lambda)}^{\ell=0} \; \psi_D(P,k)$$

$$+\frac{1}{\sqrt{5}} \overbrace{\mathbf{k}^2 (\epsilon_{D\Lambda}^*)^m U_m(\lambda)}^{\ell=2} \; \psi_D(P,k)$$

Nucleon wave function: S-state; spin part

Example $|p\uparrow\rangle$: $\qquad \uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \chi_S = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Spin-0: $$\Phi_S^0 = \overbrace{\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)}^{\varepsilon^s}\uparrow \quad = \quad \varepsilon^s \chi_S$$

Spin-1: $$\Phi_S^1 = \frac{1}{\sqrt{6}}\left[2\uparrow\uparrow\downarrow - (\uparrow\downarrow + \downarrow\uparrow)\uparrow\right] \quad = \quad -\frac{1}{\sqrt{3}}(\sigma \cdot \varepsilon_P^*)\chi_S$$

Relativistic generalization:

$$\Phi_S^0 \to u(P,\uparrow) \qquad \Phi_S^1 \to -(\varepsilon_P^*)_\alpha U^\alpha(P,\uparrow)$$

- Dirac nucleon spinor $u(P,\uparrow)$; Diquark polarization vector: ε_P^α [**rest frame**-fixed-axis base PRC 77, 015202 (2008)]
- Vector spin 1/2 S-state $\left[1 \oplus \frac{1}{2} \to \frac{1}{2}\right]$:

$$U^\alpha(P,\lambda_n) = \frac{1}{\sqrt{3}}\gamma_5\left(\gamma^\alpha - \frac{P^\alpha}{M}\right)u(P,\lambda_n)$$