Fundamental principles of particle physics

Our description of the fundamental interactions and particles rests on two fundamental structures :

- Quantum Mechanics
- Symmetries

Symmetries

Central to our description of the fundamental forces :

Relativity - translations and Lorentz transformations

Lie symmetries - $SU(3) \otimes SU(2) \otimes U(1)$

Copernican principle : "Your system of co-ordinates and units is nothing special"

⇒ Physics independent of system choice

Fundamental principles of particle physics

Quantum Mechanics

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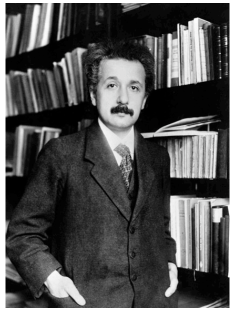
Relativity

} Quantum Field theory

Special relativity



- Space time point $a^{\mu} = (ct, x, y, z)$ not invariant under translations

- Space-time vector $(a+\Delta a)^{\mu} - a^{\mu} = \Delta a^{\mu} = (c\Delta t, \Delta x, \Delta y, \Delta z)$

  Invariant under translations …but not invariant under rotations or boosts

- Einstein postulate : the real invariant distance is

$$(\Delta a^0)^2 - (\Delta a^1)^2 - (\Delta a^2)^2 - (\Delta a^3)^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} \Delta a^{\mu} \Delta a^{\nu} = \Delta a^{\mu} \Delta a_{\mu} = (\Delta a)^2$$

$$g_{\mu\nu} = diag(+1, -1, -1, -1)$$

- Physics invariant under all transformations that leave all such distances invariant :

  Translations and Lorentz transformations

Lorentz transformations :

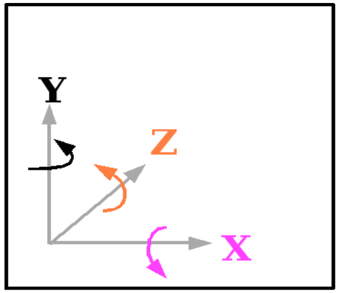
$g_{\mu\nu} = Diagonal(1,-1,-1,-1)$

$$x^{\mu} \rightarrow \Lambda^{\mu}_{\nu} x^{\nu} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} x^{\nu} = \tilde{x}^{\mu} \quad \Rightarrow \quad g_{\mu\nu} \tilde{x}^{\mu} \tilde{x}^{\nu} = g_{\mu\nu} x^{\mu} x^{\nu} \quad \Rightarrow \quad g_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} = g_{\alpha\beta}$$

(Summation assumed)

Solutions :

- 3 rotations R



$$R_z(\theta)\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ct \\ x\cos\theta + y\sin\theta \\ y\cos\theta - x\sin\theta \\ z \end{pmatrix}$$

Lorentz transformations :

$$x^\mu \to \Lambda^\mu_\nu x^\nu = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu = \tilde{x}^\mu \quad \Rightarrow \quad g_{\mu\nu} \tilde{x}^\mu \tilde{x}^\nu = g_{\mu\nu} x^\mu x^\nu \quad \Rightarrow \quad g_{\mu\nu} \Lambda^\mu_\alpha \Lambda^\nu_\beta = g_{\alpha\beta}$$

(Summation assumed)

Solutions :

- 3 rotations R

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- 3 boosts B

$$\begin{pmatrix} \cosh\alpha & \sinh\alpha & 0 & 0 \\ \sinh\alpha & \cosh\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Space reflection – parity P

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Time reflection, time reversal T

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- The Lorentz transformations form a group $G = SO(3,1) \quad (g_1 g_2 \in G \quad if \quad g_1, g_2 \in G)$

Rotations

$$R(\theta) = e^{-i\mathbf{J}.\theta/\hbar}, \quad J_z = i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)..$$

Angular momentum operator

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = R_z(\theta)\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$(c.f. \ \mathbf{J} = \mathbf{r} \times \mathbf{p})$

$$R_z(\theta) = e^{-iJ_z\theta/\hbar} = I - iJ_z\theta/\hbar + \frac{1}{2}(-iJ_z\theta/\hbar).(-iJ_z\theta/\hbar) + \ldots$$

$$-iJ_z/\hbar = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \left(-iJ_z/\hbar\right)^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R_z(\theta) = e^{-iJ_z\theta/\hbar} = \begin{pmatrix} 1 & & & \\ & 1+\frac{\theta^2}{2}+\ldots & \theta+\ldots & \\ & -\theta+\ldots & 1+\frac{\theta^2}{2}+\ldots & \\ & & & 1 \end{pmatrix}$$

Demonstration that $R_Z(\theta) = e^{-iJ_z\theta}$

$$R_Z(\theta)\psi(x,y) \equiv R_Z(\theta)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} \equiv \psi(x',y')$$

For small θ, $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x+\theta y \\ y-\theta x \end{pmatrix}$

$$R_Z(\theta)\psi(x,y) = \psi(x+\theta y, y-\theta x) \simeq \psi(x,y) + \theta(y\frac{\partial\psi}{\partial x} - x\frac{\partial\psi}{\partial y})$$
$$= (1 - i\theta(xp_y - yp_x))\psi(x,y)$$

$$i.e.\ R_Z(\theta) \simeq (1 - i\theta(xp_y - yp_x)) = 1 - i\theta J_z$$

For large θ

$$R_Z(\theta = n\varepsilon) = e^{-iJ_z\theta}$$

- **The Lorentz transformations form a group, G** $(g_1 g_2 \in G^{\dagger} \quad if \quad g_1, g_2 \in G)$

Rotations

$$R(\theta) = e^{-i\mathbf{J}.\theta/\hbar}, \quad J_z = i\hbar(x\tfrac{\partial}{\partial y} - y\tfrac{\partial}{\partial x})..$$

Angular momentum operator

$$(c.f.\ \mathbf{J} = \mathbf{r} \times \mathbf{p})$$

$$[J_i, J_j] = i\hbar \sum_{k=1}^{3} \varepsilon_{ijk} J_k$$

ε_{ijk} totally antisymmetric Levi-Civita symbol,

$$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = +1; \quad \varepsilon_{213} = \varepsilon_{132} = \varepsilon_{321} = -1$$

The J_i are the "generators" of the group.

Their commutation relations define a "Lie algebra"†.

Derivation of the commutation relations of SO(3) (SU(2))

$$R_x(\varepsilon)R_y(\eta)R_x(-\varepsilon)R_y(-\eta) \qquad \varepsilon, \eta \textit{ small} \text{ (infinitesimal)}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \varepsilon \\ 0 & -\varepsilon & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \eta \\ 0 & 1 & 0 \\ -\eta & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \eta \\ 0 & 1 & 0 \\ -\eta & 0 & 1 \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 & \varepsilon\eta & 0 \\ -\varepsilon\eta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = R_z(\varepsilon\eta) \simeq (1 - i\varepsilon\eta J_z)$$

$$R_x(\varepsilon)R_y(\eta)R_x(-\varepsilon)R_y(-\eta)$$

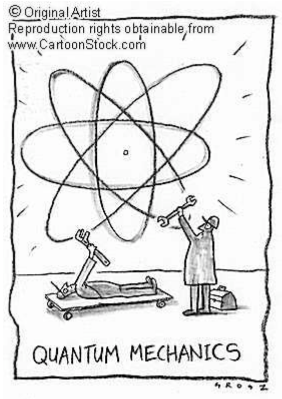
$$= (1 - i\varepsilon J_x)(1 - i\varepsilon J_y)(1 + i\varepsilon J_x)(1 + i\varepsilon J_y) = -\varepsilon\eta(J_x J_y - J_y J_x)$$

Equating the two equations implies

$$[J_x, J_y] = iJ_z$$

QED

Fundamental principles of particle physics



Quantum Mechanics

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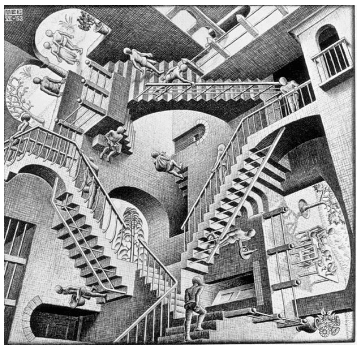
Relativity

} Quantum Field theory



Relativity, M.C.Esher 1953

Relativistic quantum field theory

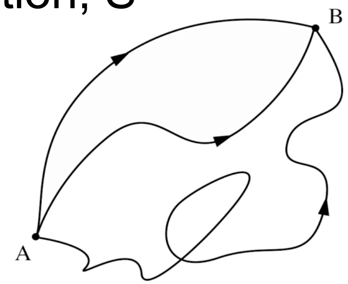
Fundamental division of physicist's world :

speed

Action (S)		slow	fast
	large	Classical Newton	Classical relativity
	small	Classical Quantum mechanics	Quantum Field theory

c →

Action, S



$$S_{classical} = \int_{t_A}^{t_B} (K.E. - P.E.)dt$$

Action $\quad S = \int_{t_1}^{t_2} L \, dt$

Lagrangian $\quad L = T - V \quad$ (Nonrelativistic mechanics)

- Quantum mechanics … sum over all paths with amplitude $\propto e^{iS/\hbar}$

- Classical path … minimises action

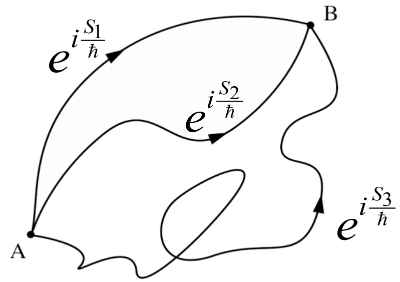
"Principle of Least Action"
Feynman Lectures in Physics
Vol II Chapter 19

(Lagrangian invariant under all the symmetries of nature

-makes it easy to construct viable theories)

Relativistic quantum field theory

Fundamental division of physicist's world :



speed

		slow	fast
Action (S)	large	Classical Newton	Classical relativity
	small	Classical Quantum mechanics	Quantum Field theory

$\hbar$ ↓ $(QM \text{ amplitude} \propto e^{i(\frac{S}{\hbar})})$

→ c

Quantum Mechanics : Quantization of dynamical system of particles

Quantum Field Theory : Application of QM to dynamical system of fields

Why fields?

- No right to assume that any relativistic process can be explained by single particle since $E=mc^2$ allows pair creation

- (Relativistic) QM has physical problems. For example it violates causality

Amplitude for free propagation from x_0 to x

$$U(t) = \langle x | e^{-i(p^2/2m)t} | x_0 \rangle = \int d^3p \langle x | e^{-i(p^2/2m)t} | p \rangle \langle p | x_0 \rangle$$

$$= \frac{1}{2\pi^3} \int d^3p \, e^{-i(p^2/2m)t} \, e^{ip(x-x_0)}$$

$$= \left(\frac{m}{2\pi i t}\right)^{3/2} e^{im(x-x_0)^2/2t} \quad \text{... nonzero for all } x, \ t$$

Relativistic case : $U(t) \propto e^{-m\sqrt{x^2-t^2}}$ ... nonzero for all $x, \ t$