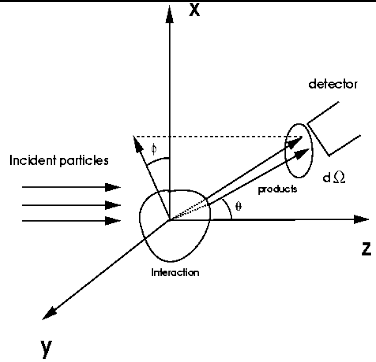
Theory confronts experiment - Cross sections and decay rates



Scattering in Quantum Mechanics

- Prepare state at $t = -\infty$ $\quad |\psi_{in}(t=-\infty)> = |i>$
- Time evolution (possibly scattering) $\quad |\psi_{in}(t=+\infty)> = S\,|\psi_{in}(t=-\infty)>$
- Observe resulting system in state $\quad |\psi_{out}(t=+\infty)> = |f>$

QM : probability amplitude :

$$<\psi_{out}(t=+\infty)\,|\,\psi_{in}(t=+\infty)> = <\psi_{out}(t=+\infty)\,|\,S\,|\,\psi_{in}(t=-\infty)>$$

$$= <f\,|\,S\,|\,i> = S_{fi}$$

$$S_{fi} = \delta_{fi} + iT_{fi}$$

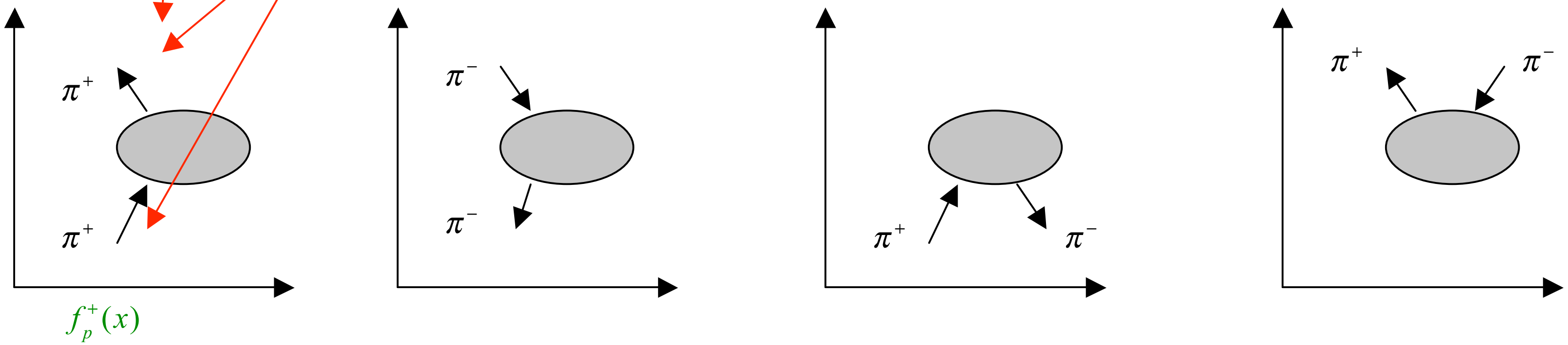
S matrix for Klein Gordon scattering

$$S_{\mathbf{p}'+,\mathbf{p}+} = <\psi_{out}(t=+\infty)\,|\,\psi_{in}(t=+\infty)> \;= \lim_{t\to\infty}\int d^3x\; f^{+*}_{p'+}\, i\partial_0 \psi(x)$$

$$\psi(x) = \phi(x) - \int d^4x'\,\Delta_F(x'-x)V(x')\phi(x') + \ldots$$

$$\Delta_F(x'-x) = -i\int d^3p\; f^{+}_{p}(x')f^{+*}_{p}(x)\theta(t'-t) - i\int d^3p\; f^{-}_{p}(x')f^{-*}_{p}(x)\theta(t-t')$$

$$S_{\mathbf{p}'+,\mathbf{p}+} = \delta^3(\mathbf{p}'_{+} - \mathbf{p}_{+}) - i\int d^4x'\; f^{+*}_{p+}(x')\,V(x')\,f^{+}_{p+}(x') + \ldots$$



Feynman rules

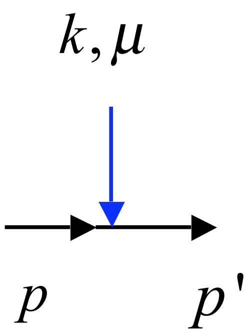
$$iT_{fi} = -i\int d^4y\; f_{p+}^{+*}(y)\, V(y)\, f_{p+}^{+}(y) = i\int d^4y\; f_{p+}^{+*}(y)\; ie(A^\mu\partial_\mu + \partial_\mu A^\mu)\; f_{p+}^{+}(y)$$

$$f_p^{\pm} = e^{\mp ip.x}\frac{1}{\sqrt{2p^0V}}$$

$$= -i\int d^4y\; j_\mu^{fi} A^\mu$$

$$j_\mu^{fi} = -ie\left(f_{p+}^{+*}(y)\left[\partial_\mu f_{p+}^{+}(y)\right] - \left[\partial_\mu f_{p+}^{*+}(y)\right] f_{p+}^{+}(y)\right)$$

$$= -e(p_f + p_i)_\mu e^{i(p_f - p_i).y}$$



$$-ie(p_\mu + p'_\mu)$$

Feynman rule associated with Feynman diagram

Fundamental experimental objects

$\Gamma(a_1 \to b_1 b_2 \ldots b_n)$ **Decay width = 1/lifetime** (Dimension 1/T=M)

$\sigma(a_1 a_2 \to b_1 b_2 \ldots b_n)$ **Cross section** (Dimension $L^2=M^{-2}$)

Units "barn"

$$
\begin{aligned}
1\,barn &= 10^{2}\ fm^{2} \\
1\,mb &= 10^{-1}\ fm^{2} \quad \text{"}milli\text{"} \\
1\,\mu b &= 10^{-4}\ fm^{2} \quad \text{"}micro\text{"} \\
1\,nb &= 10^{-7}\ fm^{2} \quad \text{"}nano\text{"} \\
1\,pb &= 10^{-10}\ fm^{2} \quad \text{"}pico\text{"} \\
1\,fb &= 10^{-13}\ fm^{2} \quad \text{"}fempto\text{"}
\end{aligned}
$$

(Natural Units $1 GeV^{-2} = 0.39 mb$)

Fundamental experimental objects

$\Gamma(a_1 \to b_1 b_2 \ldots b_n)$ **Decay width = 1/lifetime** (Dimension 1/T=M)

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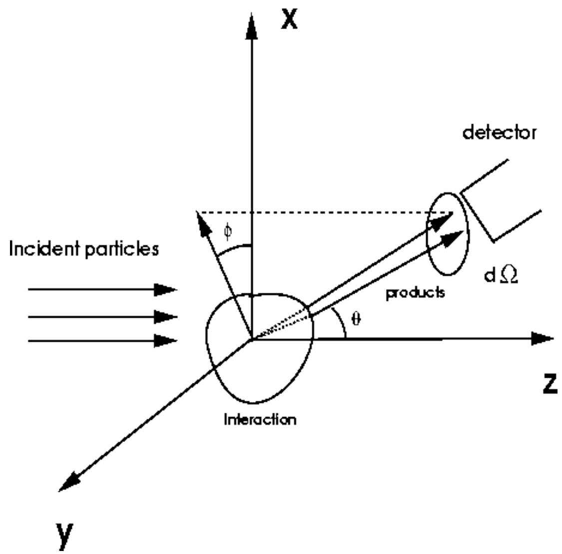


$$\text{Cross section} = \frac{\text{Transition rate x Number of final states}}{\text{Initial flux}}$$

Fundamental experimental objects

$\Gamma(a_1 \to b_1 b_2 \ldots b_n)$ **Decay width = 1/lifetime** (Dimension 1/T=M)

$\sigma(a_1 a_2 \to b_1 b_2 \ldots b_n)$ **Cross section** (Dimension $L^2=M^{-2}$)

Momenta of final state forms phase space

Cross section = $\dfrac{\text{Transition rate x Number of final states}}{\text{Initial flux}}$

For a single particle the number of final states in volume V with momenta

in element d^3p is $\frac{Vd^3p}{(2\pi)^3}$ $\Rightarrow$ $\Pi_{i=1}^{n} \frac{Vd^3p}{(2\pi)^3}$

Fundamental experimental objects

$\Gamma(a_1 \to b_1 b_2 \ldots b_n)$ **Decay width = 1/lifetime**

$\sigma(a_1 a_2 \to b_1 b_2 \ldots b_n)$ **Cross section**

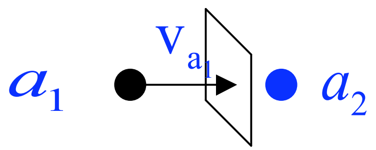
$$\prod_{i=1}^{n} \frac{V d^3 p}{(2\pi)^3}$$

$$\text{Cross section} = \frac{\text{Transition rate x Number of final states}}{\text{Initial flux}}$$

$$\frac{|v_{a_1}|}{V} \times \frac{1}{V}$$

particles passing through unit area in unit time

target particles per unit volume



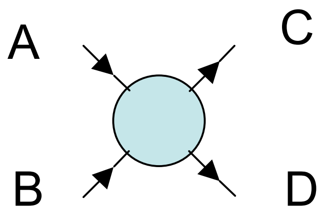
(Lab frame)

The transition rate

$$T_{fi} = -\int d^4x\, \phi_f^*(x) V(x) \phi_i(x) + \ldots$$

$$\phi_{i,f} \rightarrow f_p^{\pm} = e^{\mp ip.x} \frac{1}{\sqrt{2p^0 V}} \equiv \frac{N}{\sqrt{V}} e^{\mp ip.x}$$

e.g.



Transition rate per unit volume

$$W_{fi} = \frac{|T_{fi}|^2}{TV}$$

$$\phi_{f,i} = e^{\mp ip.x}$$

$$T_{fi} = -\frac{N_A N_B N_C N_D}{V^2} (2\pi)^4 \delta^4(p_C + p_D - p_A - p_B) \mathfrak{M}_{fi}$$

$$W_{fi} = (2\pi)^4 \frac{\delta^4(p_C + p_D - p_A - p_B) |\mathfrak{M}|^2}{V^4} \left(\frac{1}{2E_A}\right)\left(\frac{1}{2E_B}\right)\left(\frac{1}{2E_C}\right)\left(\frac{1}{2E_D}\right)$$

The cross section

$$\text{Cross section} = \frac{\text{Transition rate} \times \text{Number of final states}}{\text{Initial flux}}$$

$$d\sigma = \frac{V^2}{|\mathbf{v_A}|2E_A 2E_B}\frac{1}{V^4}|\mathfrak{M}|^2\frac{(2\pi)^4}{(2\pi)^6}\delta^4(p_C + p_D - p_A - p_B)\frac{d^3p_C}{2E_C}\frac{d^3p_D}{2E_D}V^2$$

$$d\sigma = \frac{|\mathfrak{M}|^2}{F}dQ$$

$$dQ = (2\pi)^4\delta^4(p_C + p_D - p_A - p_B)\frac{d^3p_C}{(2\pi)^3 2E_C}\frac{d^3p_D}{(2\pi)^3 2E_D}$$

Lorentz Invariant Phase space

$$F = |\mathbf{v_A}|2E_A 2E_B$$
$$= 4((p_A.p_B)^2 - m_A^2 m_B^2)^{1/2}$$

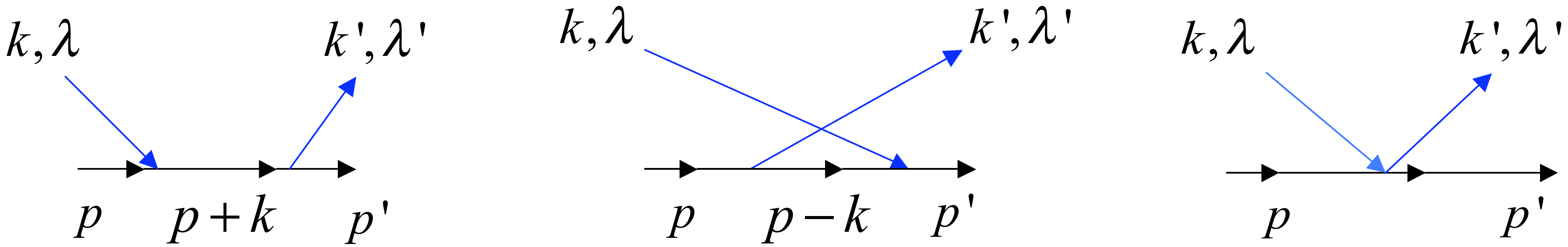
The decay rate

$$d\Gamma = \frac{1}{2E_A} |\mathfrak{M}|^2 \, dQ$$

$$dQ = (2\pi)^4 \delta^4 (p_A - p_{B_1} \ldots - p_{B_n}) \frac{d^3 p_{B_1}}{(2\pi)^3 2E_{B_1}} \cdots \frac{d^3 p_{B_n}}{(2\pi)^3 2E_{B_n}}$$

Compton scattering of a π meson

$$\gamma\pi \rightarrow \gamma\pi$$

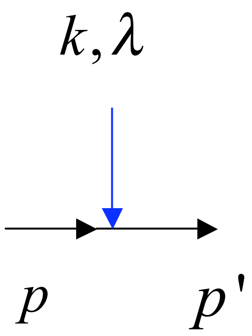


Feynman rules

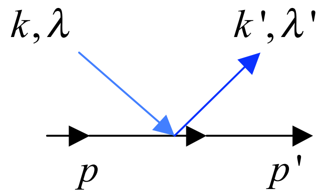
Klein Gordon

$$(\partial_\mu \partial^\mu + m^2)\psi = -V\psi$$

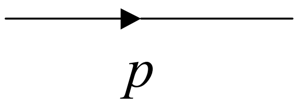
$$V = -ie(\partial_\mu A^\mu + A^\mu \partial_\mu) - e^2 A^2$$



$$-ie(p_\lambda + p'_\lambda)$$



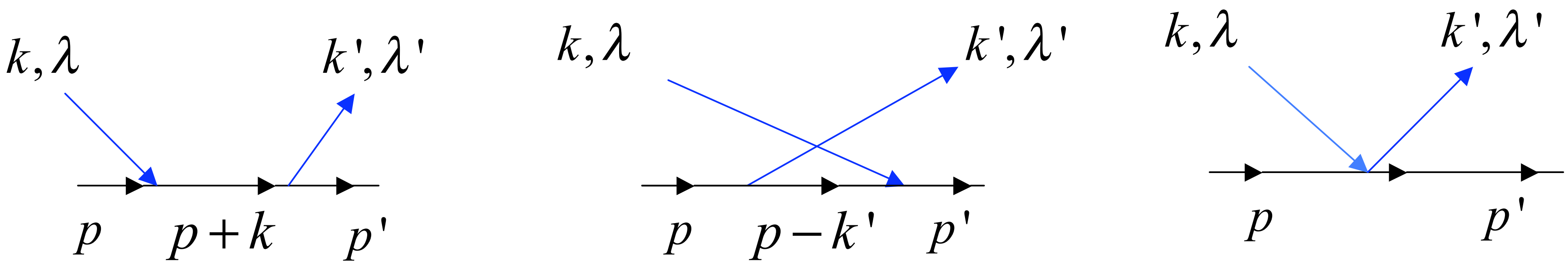
$$ie^2$$



$$\frac{i}{p^2 - m^2}$$

External photon ε^λ

Compton scattering of a π meson

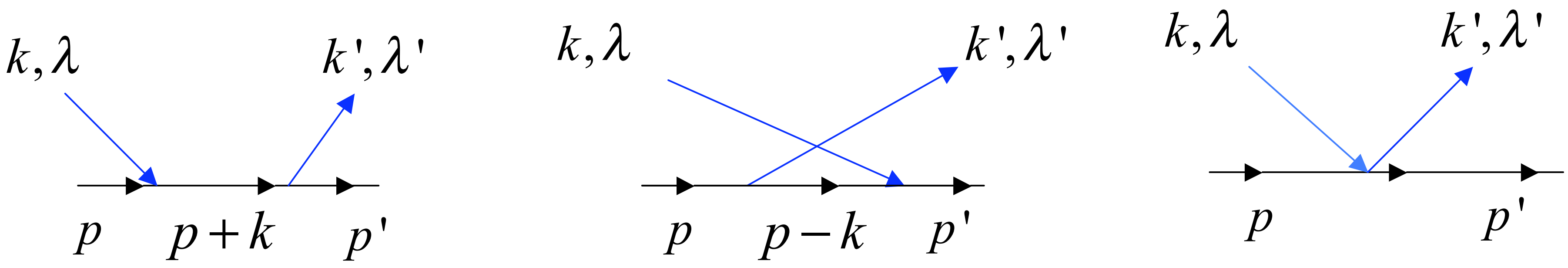


$$i\mathfrak{M}_{fi} = (-ie)^2[\varepsilon.(2p+k)\frac{i}{(p+k)^2-m^2}\varepsilon'.(2p'+k')$$

$$+\varepsilon.(2p'-k)\frac{i}{(p-k')^2-m^2}\varepsilon'.(2p-k')-2i\varepsilon.\varepsilon']$$

$$d\sigma = \frac{V^2}{|\mathbf{v_A}|2E_A 2E_B}\frac{1}{V^4}|\mathfrak{M}|^2\frac{(2\pi)^4}{(2\pi)^6}\delta^4(p_C+p_D-p_A-p_B)\frac{d^3p_C}{2E_C}\frac{d^3p_D}{2E_D}V^2$$

Compton scattering of a π meson



$$\mathfrak{M}_{fi} = \varepsilon.(2p+k)\frac{i}{(p+k)^2-m^2}\varepsilon'.(2p'+k') + \varepsilon.(2p'-k')\frac{i}{(p+k)^2-m^2}\varepsilon'.(2p'-k') - 2i\varepsilon.\varepsilon'$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{\alpha^2}{m^2}\frac{(\varepsilon.\varepsilon')^2}{\left[1+\frac{k}{m}(1-\cos\theta)\right]^2}$$

$(\varepsilon.p = \varepsilon'.p = 0 \text{ gauge})$

$$\sigma_{total}\,|_{k=0} = \int\frac{d\sigma}{d\Omega}.d\Omega = \frac{8\pi\alpha^2}{3m_\pi^2} \simeq 8.10^{-2}\,GeV^{-2} = 3.10^{-2}\,mb$$

$$\sigma_{total}\,|_{k/m>>1} \simeq \frac{2\pi\alpha^2}{mk}$$

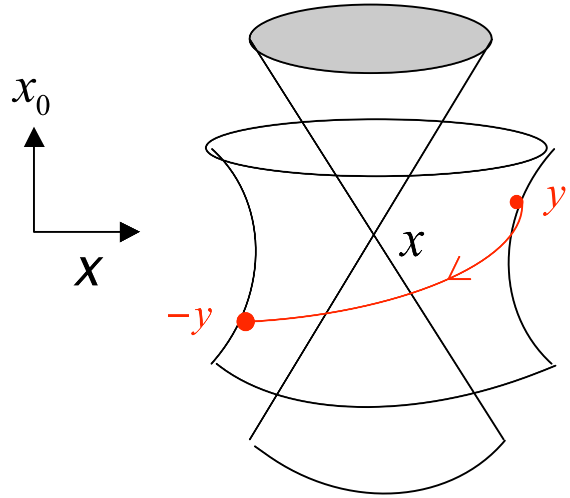
Causality?

QM : $U(x'-x) \propto e^{-m\sqrt{(\mathbf{x}'-\mathbf{x})^2-(t'-t)^2}}$ ✗

See Peskin & Schroeder "Quantum Field Theory" p28

Field theory :

$$\Delta_F(x'-x) = -i\int \frac{d^3p}{(2\pi)^3 2\omega_p} e^{-i\omega_p|t'-t|-i\mathbf{p}.(\mathbf{x}'-\mathbf{x})}$$
$$= D(x-y) - D(y-x)$$



When $(x-y)^2<0$, we can perform a Lorentz transformation taking $(x-y)\rightarrow -(x-y)$
...causality preserved $(e^{-m|r|} - e^{-m|r|})$

No (continuous) transformation possible for $(x\text{-}y)^2 > 0$
...and amplitude nonvanishing $(e^{-imt} - e^{imt})$