# Nuclear polarizability effects in muonic deuterium

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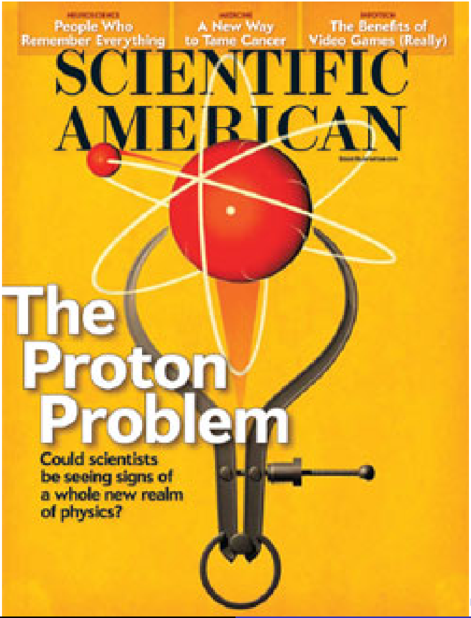
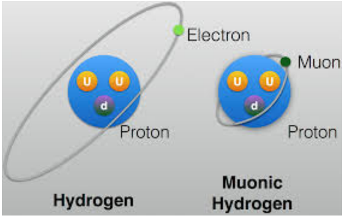
R. Pohl *et al.*, Nature (London) **466**, 213 (2010)

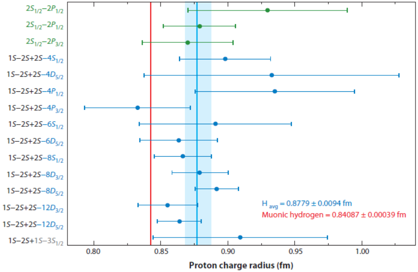
# Proton radius puzzle



Experimental methods

- elecron-proton scattering
- atomic spectroscopy
  - electronic
  - muonic



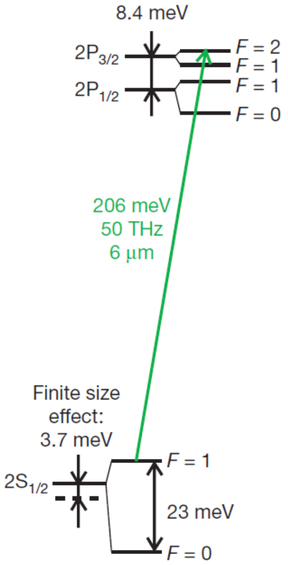




from R. Pohl *et al.*, Annu. Rev. Nucl. Part. Sci. 63, 175 (2013)

# Spectroscopic measurements

Muonic atomic spectroscopy demands

- knowledge of atomic structure
- accurate description of nucleus



# Earlier results

- nonrelativistic nuclear electric dipole polarizability and Coulomb corrections for $\mu$He

  J.L. Friar, Phys. Rev. C 16, 1540 (1977)

$$\delta E = \alpha^2 \left\langle \phi\,\phi_N \left| \frac{\vec{d}\cdot\vec{\rho}}{\rho^3} \frac{1}{E_N + E_0 - H_N - H_0} \frac{\vec{d}\cdot\vec{\rho}}{\rho^3} \right| \phi\,\phi_N \right\rangle,$$

- inelastic contribution for $\mu$D

  W. Leidemann and R. Rosenfelder, Phys. Rev. C 51, 427 (1995)

- absence of the Zemach correction

  K. Pachucki, Phys. Rev. Lett. 106, 193007 (2011)

- complete two-photon exchange contribution

  C. E. Carlson, M. Gorchtein, and M. Vanderhaeghen, Phys. Rev. A 89, 022504 (2014)

# Formalism

- Muon Hamiltonian

$$H_0 = \frac{\vec{p}^{\,2}}{2\,m_r} - \frac{Z\,\alpha}{\rho}$$

where $m_r$ is muon reduced mass

- Nuclear Hamiltonian

$$H_N = \sum_a \frac{\vec{q}_a^{\,2}}{2\,m_a} + V_{\rm nucl},$$

where $m_a$ is a nucleon mass

# Aims and assumptions

- calculate higher order terms
- perturbative formalism
- limitations:
  - simplified model of nuclear interaction with EM field
  - neglect of correction to electric dipole
  - neutron polarizability
  - neglect of isospin

# Calculations

## $2^{nd}$ order Coulomb correction

$$\delta E = \left\langle \phi\,\phi_N \middle| \delta V \frac{1}{E_N + E_0 - H_N - H_0}\, \delta V \middle| \phi\,\phi_N \right\rangle,$$

$\phi$, $\phi_N$ – muon and nucleus wave functions

Perturbation

$$\delta V = \sum_{a=1}^{Z} \frac{\alpha}{|\vec{\rho} - \vec{\rho}_a|} - \frac{Z\,\alpha}{\rho}.$$

## Electric dipole excitation - the dominating contribution

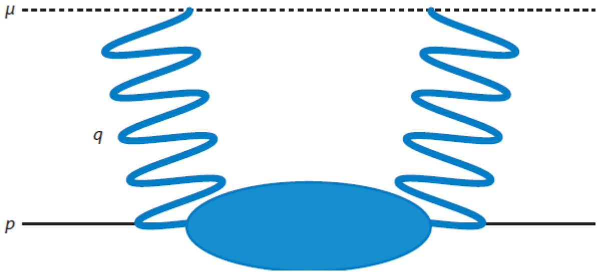
$$\delta E = \alpha^2 \left\langle \phi\,\phi_N \middle| \frac{\vec{d}\cdot\vec{\rho}}{\rho^3}\, \frac{1}{E_N + E_0 - H_N - H_0}\, \frac{\vec{d}\cdot\vec{\rho}}{\rho^3} \middle| \phi\,\phi_N \right\rangle,$$

# Contributions from 2nd order Coulomb correction

Fundamental physical constants are from CODATA and $r_p^2 = 0.8409^2$ fm$^2$, $r_n^2 = -0.1161$ fm$^2$

| Contribution | Value in meV |
|---|---|
| Dipole | 1.910 |
| Coulomb distortion, leading | -0.255 |
| Coulomb distortion, next order | -0.006 |
| Electric monopole excitation | -0.042 |
| Electric dipole excitation | 0.139 |
| Electric quadrupole excitation | -0.061 |
| Finite nucleon size | 0.020 |
| Neutron – proton charge correlation | -0.018 |
| Rel. corr. to dip. contr., long. part | -0.026 |

# 2-photon exchange




$$\delta E = -e^4\,\phi^2(0)\,\frac{1}{3}\int_{E_T} dE\langle\phi_D|\vec{d}|E\rangle^2 \int\frac{d\,\omega}{2\,\pi\,i}$$

$$\times\int^{\epsilon}\frac{d^3k}{(2\,\pi)^3}\,\frac{1}{E+\omega+k^2/(2\,M)}\left(1+\frac{2\,\omega^4}{(\omega^2-k^2)^2}\right)$$

$$\frac{4}{(\omega^2+2\,m\,\omega-k^2)(\omega^2-2\,m\,\omega-k^2)}$$

# Contributions from 2-photon exchange and other corrections

| Contribution | Value in meV |
|---|---|
| Rel. corr. to dip. contr., higher order | 0.004 |
| Magnetic | -0.008 |
| Proton elastic 3rd Zemach moment | 0.0289(15) |
| Proton inelastic polarizability | 0.0140(20) |
| TOGETHER | 1.7041 |

# Other contributions

- finite nucleon size
- In case of the deuteron its magnetic moment is relatively large and thus included

# Numerical results

- AV18 potential
- discrete variable representation

# Results

Nuclear structure corrections in muonic deuterium for 2P-2S transition. Fundamental physical constants are from Ref. [4].

| Contribution | Value in meV |
|---|---|
| Dipole | 1.910 |
| Coulomb distortion, leading | -0.255 |
| Coulomb distortion, next order | -0.006 |
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