

**International Conference on Precision Physics and Fundamental Constants
(FFK-2015)**

12-16 October 2015, Hungarian Academy of Sciences

**Recoil correction
to the proton finite-size contribution
to the Lamb shift
in muonic hydrogen**

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Theory of the Lamb shift in muonic hydrogen

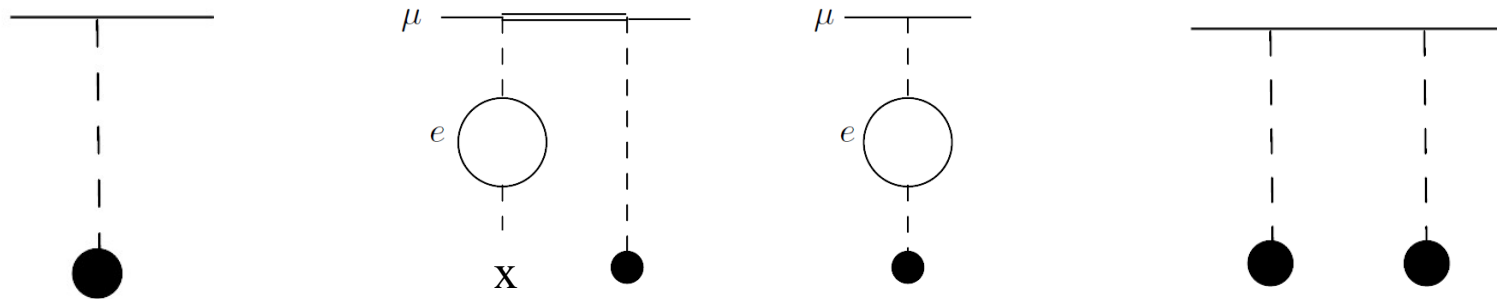
- Levels structure
- Specific QED corrections for μH
- Re-scaled hydrogenic theory
- Proton-line QED

is presented on poster:

“Theory of the Lamb shift in muonic hydrogen”

S. Karshenboim, V. Ivanov, E. Korzinin, and V. Shelyuto

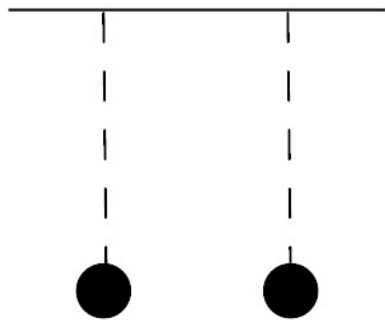
External-field proton-size contribution:



#	Designation	Order	Ref.	ΔE [meV]	
				Value	Estimation
10	PS (NR)	$(Z\alpha)^4 m$		$-5.1974 r_p^2$	-3.7
11	PS (Rel)	$(Z\alpha)^6 m$	[65–67]	$-0.0016 r_p^2 - 0.00004(r_p^2)^2$	-0.0011
12	PS (eVP)	$\alpha(Z\alpha)^4 m$	Eq. (15)	$-0.0282 r_p^2$	-0.020
13	PS (SE)	$\alpha(Z\alpha)^4 m$	[6]	$0.0006 r_p^2$	0.0005
14.1	PS (Fri) term	$(Z\alpha)^5 m$	[69]	$-0.0251(35) + 0.06244 r_p^2$	0.019

External-field proton-size contribution: problem

○ Friar term



$$E(2s_{1/2}) = -\frac{2(Z\alpha)^5 m_r^4}{\pi} I_{\text{Fr}} ,$$

$$\begin{aligned} I_{\text{Fr}} &= \frac{\pi}{48} \int d^3r d^3r' \rho_E(\mathbf{r}) \rho_E(\mathbf{r}') |\mathbf{r} - \mathbf{r}'|^3 \\ &= \frac{\pi}{48} \langle r^3 \rangle_2 , \end{aligned}$$

$$I_{\text{Fr}} = \int_0^\infty \frac{dq}{q^4} \left[(G_E(q^2))^2 - 1 - 2G'_E(0) q^2 \right]$$

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PHYSICS LETTERS

FINITE-SIZE CORRECTIONS TO THE ENERGY LEVELS OF LIGHT MUONIC ATOMS[☆]

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PHYSICAL REVIEW A

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Theory of the Lamb shift in muonic hydrogen

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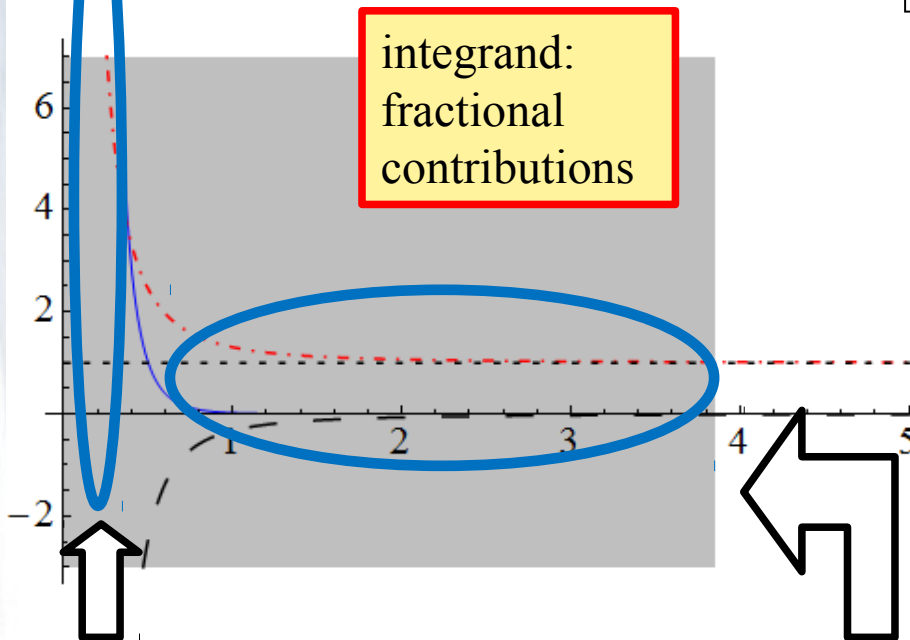
(Received 28 August 1995)

$$G'_E(0) = -\frac{1}{6} R_p^2$$

External-field proton-size contribution: Consistency problem

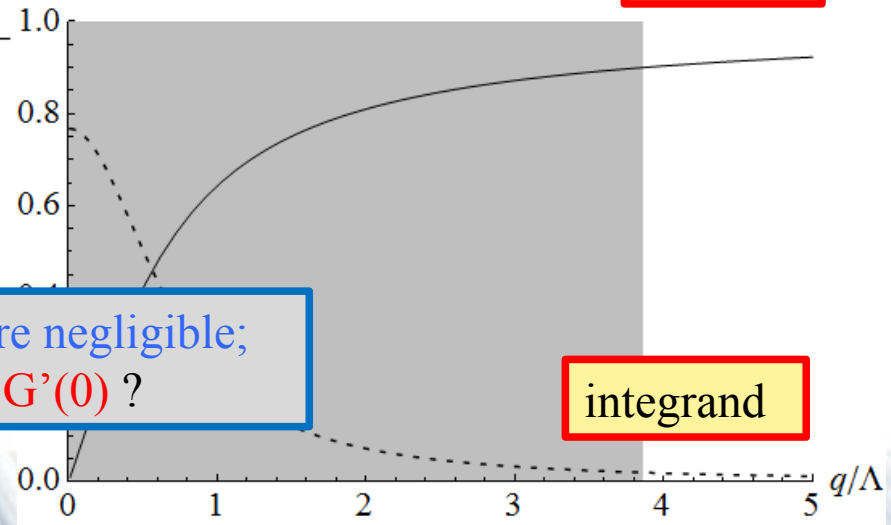
- The integrand includes
Three terms:

$$I_3^E \equiv \int_0^\infty \frac{dq}{q^4} \left[(G_E(q^2))^2 - 1 - 2G'_E(0) q^2 \right]$$



90% of the integral

- $G_E(q^2)$
- -1
- $-2G'(0)q^2$



data are inaccurate;
fit for G ?

data are negligible;
fit for $G'(0)$?

External-field proton-size contribution: method

$$I = \int_0^{\infty} dq... \equiv I_{<} + I_{>} \equiv \int_0^{q_0} dq... + \int_{q_0}^{\infty} dq...$$



ELSEVIER

27 January 1997

PHYSICS LET

Physics Letters A 225 (1997) 97–106

**Nuclear structure-dependent radiative corrections
to the hydrogen hyperfine splitting**

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PHYSICAL REVIEW D **90**, 053012 (2014)

**Self-consistent value of the electric radius of the proton from the Lamb shift
in muonic hydrogen**

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**The recoil correction to the proton-finite-size contribution to the Lamb shift in
muonic hydrogen**

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External-field proton-size contribution: Strategy of the evaluation

- Split the integral

$$I = \int_0^\infty dq \dots \equiv I_{<} + I_{>} \equiv \int_0^{q_0} dq \dots + \int_{q_0}^\infty dq \dots$$

- Low momentum

$$\left(G_E(q^2)\right)^2 \simeq 1 - \frac{R_E^2}{3} q^2 + C^{\text{dip}}(1 \pm 1) q^4$$

- High momentum

$$I_{3>}^E = \int_{q_0}^\infty \frac{dq}{q^4} \left(G_E(q^2)\right)^2 - \frac{1}{3q_0^3} + \frac{1}{3} \frac{R_E^2}{q_0}$$

PHYSICAL REVIEW C 83, 015203 (2011)

Realistic transverse images of the proton charge and magnetization densities

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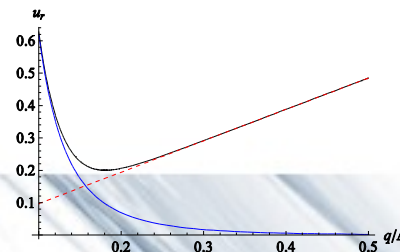
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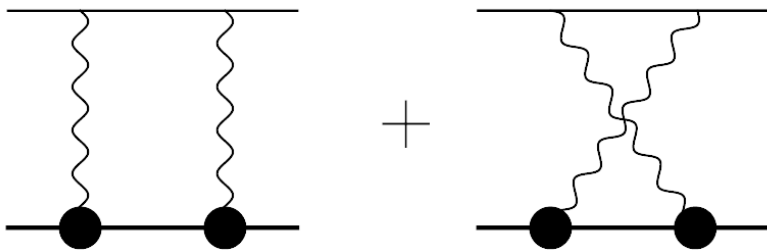
$$\delta I_{3>}^E = \frac{1}{3q_0^3} \frac{2\delta G_E(q_0^2)}{G_E(q_0^2)} \left(G_{\text{dip}}(q_0^2)\right)^2$$

- Minimization of the uncertainty

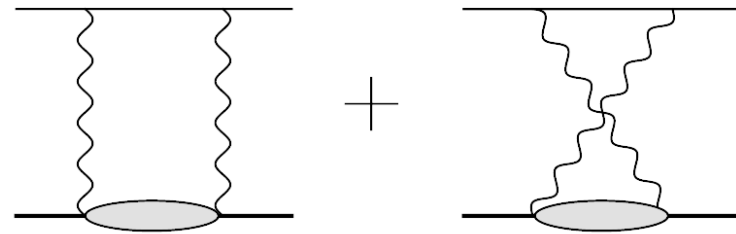


Two photon exchange proton-size contribution:

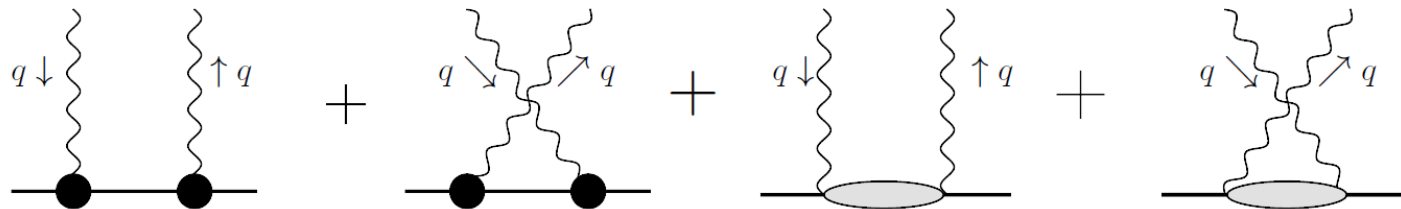
○ Elastic



○ Inelastic

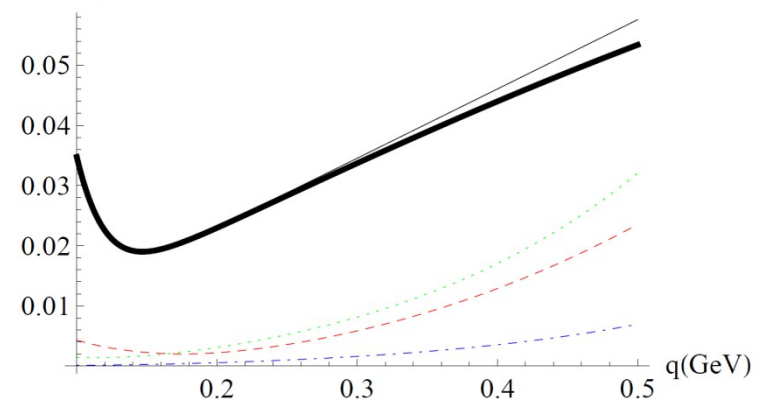
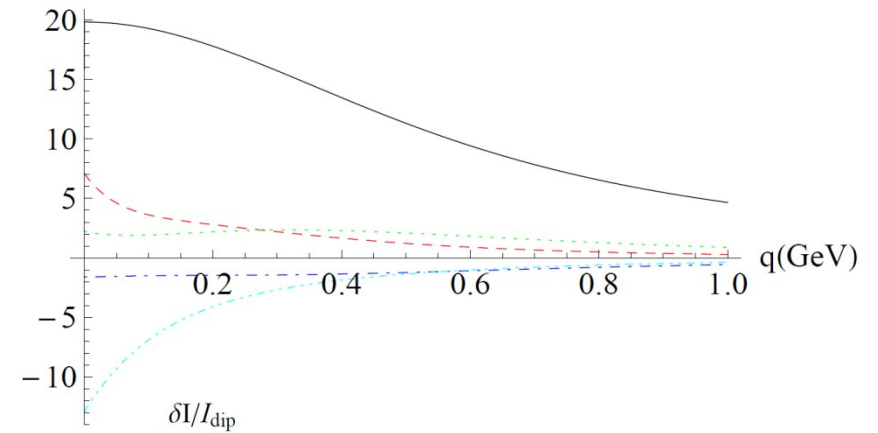


○ Virtual Compton amplitude



Two photon exchange proton-size contribution: Elastic recoil and finite size

$$\begin{aligned}
 I_{\text{rec}} &= I_{\kappa} + I_{\text{EF}} + I_{\text{M1}} + I_{\text{M2}} , \\
 I_{\kappa} &= \kappa \int_0^{\infty} \frac{dq}{q^4} \{ (2 + \kappa) f_{\text{M1}} + f_{\text{M2}} \} , \\
 I_{\text{EF}} &= \int_0^{\infty} \frac{dq}{q^4} f_{\text{EF}}(m, M; q^2) \left[(G_E(q^2))^2 - 1 \right] , \\
 I_{\text{M1}} &= \int_0^{\infty} \frac{dq}{q^4} f_{\text{M1}} \left[(G_M(q^2))^2 - (1 + \kappa)^2 \right] , \\
 I_{\text{M2}} &= \int_0^{\infty} \frac{dq}{q^4} f_{\text{M2}} \left[G_M(q^2) G_E(q^2) - (1 + \kappa) \right] , \quad (21)
 \end{aligned}$$



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PHYSICAL REVIEW A

VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

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 (Received 28 August 1995)

Proton Polarizability and Lamb Shift in the Muonic Hydrogen Atom

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PHYSICAL REVIEW A **84**, 020102(R) (2011)

Higher-order proton structure corrections to the Lamb shift in muonic hydrogen

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 DOI 10.

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Proton polarisability contribution to the Lamb shift in muonic hydrogen at fourth order in chiral perturbation theory

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Two photon exchange proton-size contribution: Elastic recoil and finite size

#	Designation	ΔE [meV]	
		Value	Estimation
14.1	eTPE:Fri	$0.062 r_p^2 - 0.025(4)$	<i>0.019</i>
14.2	eTPE: κ^*	-0.00305	<i>-0.003</i>
14.3	eTPE:EF*	$0.00107 r_p^2 + 0.00136(4)$	<i>0.002</i>
14.4	eTPE:M1*	$0.00188(3)$	<i>0.002</i>
14.5	eTPE:M2*	$-0.000016 r_p^2 - 0.00090$	<i>-0.0009</i>
14	eTPE	$0.064 r_p^2 - 0.026(4)$	<i>0.019</i>

Two photon exchange proton-size contribution: Elastic recoil and finite size

More information one can find in poster:

“Recoil correction to the proton finite-size contribution to the Lamb shift in muonic hydrogen”

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Thank You for attention!