QED nonlinearity affecting charges and dipoles

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Review of results obtained
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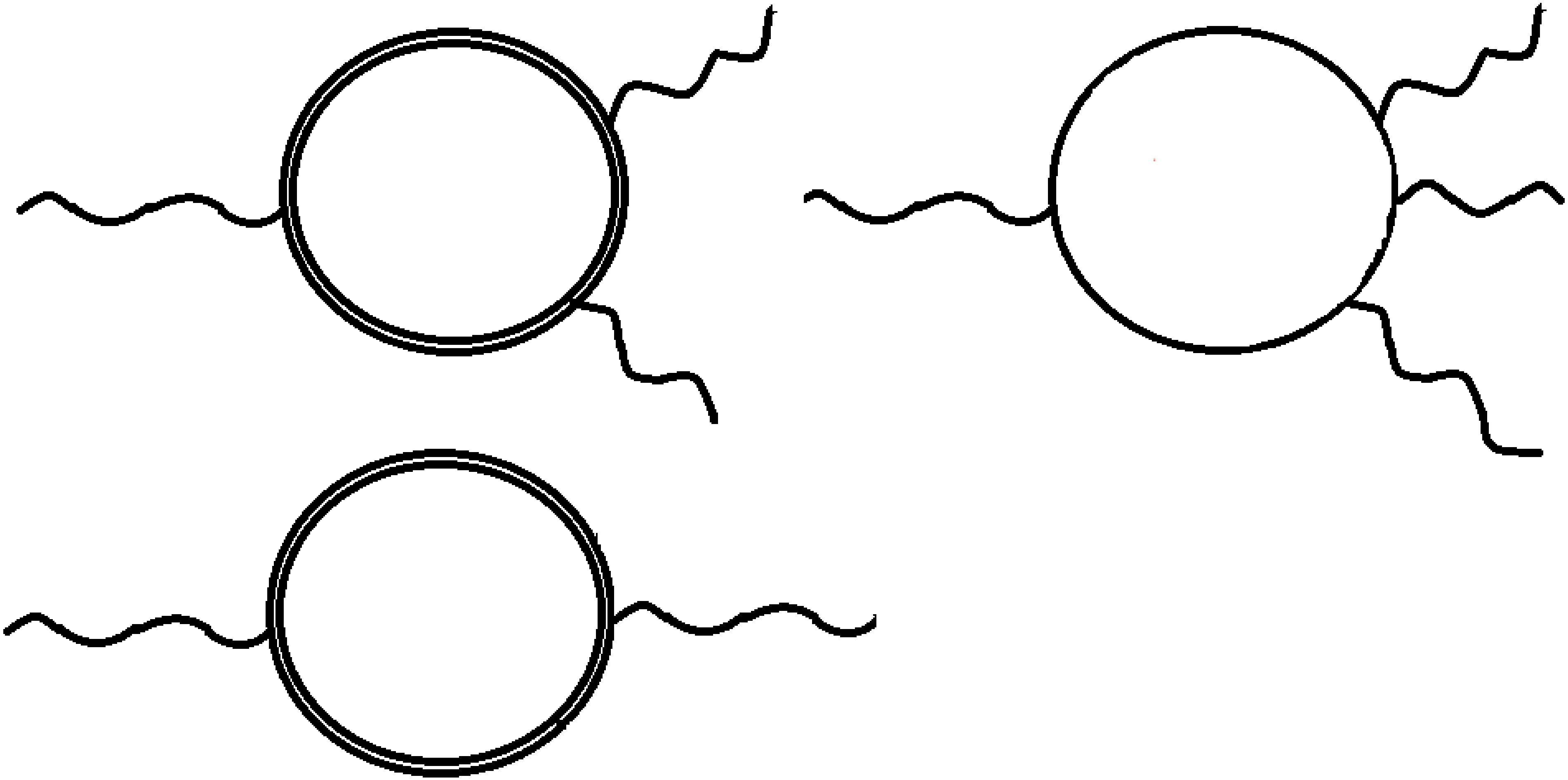


How strong should the fields be?

Electric field E_0
performs over electron charge e
along its Compton length 1/m
the work, equal to its rest energy m

$$E_0 = m^2/e = 1.3 \cdot 10^{16} \text{ V/cm}$$

$$B_0 = m^2/e = 4.4 \cdot 10^{13} \text{ G}$$

Field at the edge of the neutron
due to its magnetic moment
is $3 \cdot 10^{16}$ G

Optically as dense
as:

Air ----- 10^{13}G
Water ----- 10^{16} G
Diamond ------ 27 $\cdot 10^{16}$ G

Old list of nonlinear effects in QED

*Photon splitting in strong magnetic field –
S. Adler, 1971

*Photon capture by magnetic field with formation of positronium, A. Shabad, V. Usov,1972 - 1986

*Screening of the Coulomb field by strong magnetic field. Finiteness of the ground state of Hydrogen when $B \rightarrow \infty$, A. Shabad, V. Usov, 2008

*Positronium collapse in strong magnetic field, A. Shabad, V. Usov, 2006

With background field

*Static charge in electric and magnetic fields becomes also a magnetic monopole, 2015

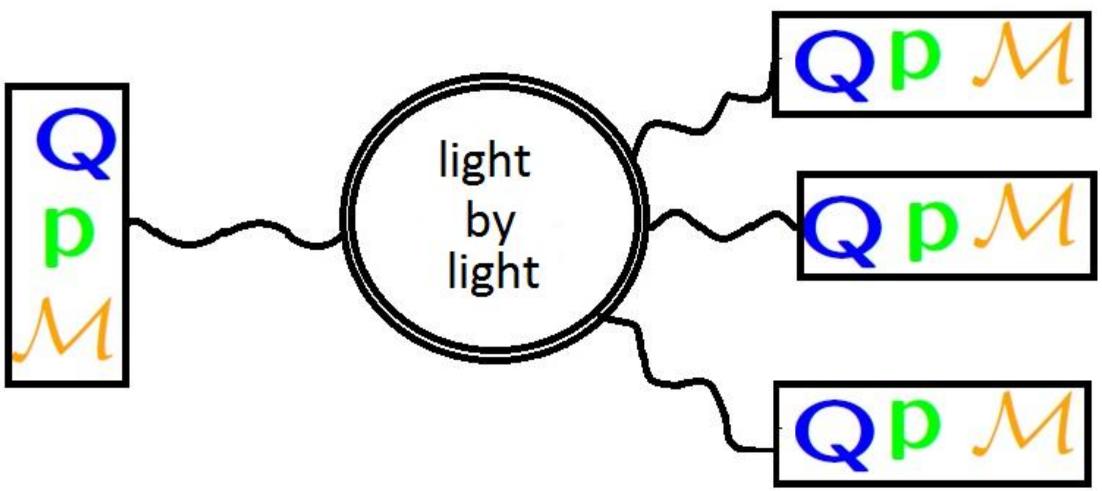
*Static charge in a magnetic field becomes also a magnetic dipole, T. Adorno, D. Gitman, A. Shabad, 2012, 2014

Without any background

*Solitonic solution for the field of a charge, C. Costa, D. Gitman, A. Shabad, A. Shishmarev, 2013-2015

*Nonlinear renormalization of electric and magnetic dipole moments, C. Costa, D. Gitman, A. Shabad, 2013

Cubic corrections to the fields of charge $\mathbf{Q}$, electric $\mathbf{p}$ and magnetic $\mathcal{M}$ dipole



Local action

$$S[A] = \int L(x)\mathbf{d}^4 x$$

$$L = -\mathfrak{F}(x) + \mathcal{L}(\mathfrak{F}(x), \mathfrak{G}(x))$$

Field invariants $\mathfrak{F} = \frac{B^2 - E^2}{2}, \mathfrak{G} = (\mathbf{E} \cdot \mathbf{B})$

Correspondence principle

$$\left.\frac{\partial \mathfrak{L}}{\partial \mathfrak{F}(x)}\right|_{\mathfrak{F}=\mathfrak{G}=0} = \left.\frac{\partial \mathfrak{L}}{\partial \mathfrak{G}(x)}\right|_{\mathfrak{F}=\mathfrak{G}=0} = 0$$

Parity conservation

$$\left.\frac{\partial^2 \mathfrak{L}}{\partial \mathfrak{F}(x) \mathfrak{G}(x)}\right|_{\mathfrak{F}=\mathfrak{G}=0} = 0$$

MAXWELL EQUATIONS (static)

First pair $(\nabla\cdot\mathbf{B}) = 0, \quad [\nabla\times\mathbf{E}] = 0$

Second pair $$\frac{\delta S\left[A\right]}{\delta A^{\mu}\left(x\right)} = j_{\mu}^{\text{lin}}\left(x\right)$$

$$(\boldsymbol{\nabla}\cdot\mathbf{E}) = j_0^{\text{lin}} + j_0^{\text{nl}}, \quad [\boldsymbol{\nabla}\times\mathbf{B}] = \mathbf{j}^{\text{lin}} + \mathbf{j}^{\text{nl}}.$$

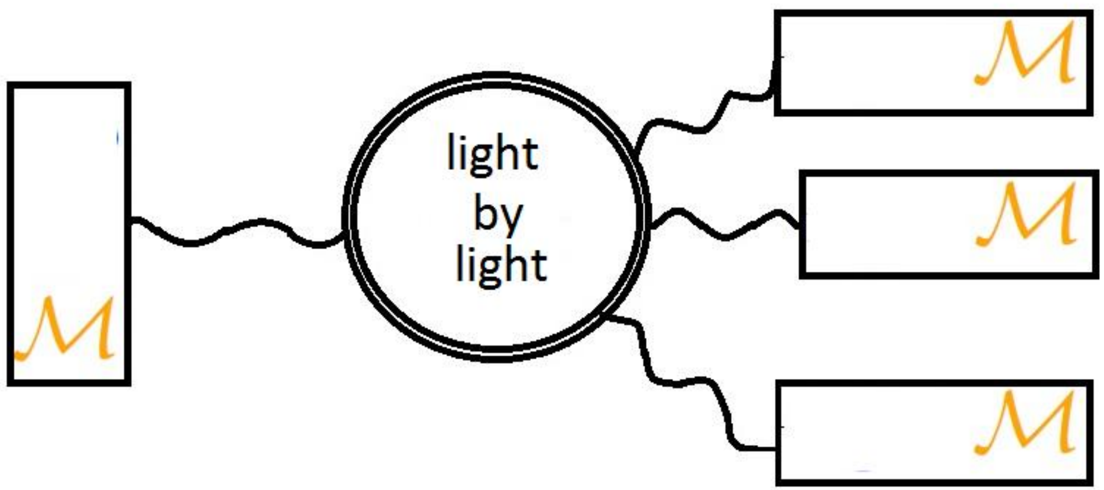
$$j_0^{\text{nl}} = \mathcal{L}_{\mathfrak{F}\mathfrak{F}}\left(\boldsymbol{\nabla}\cdot\mathfrak{F}\mathbf{E}\right) - \mathcal{L}_{\mathfrak{G}\mathfrak{G}}\left(\boldsymbol{\nabla}\cdot\mathbf{B}\right)\mathfrak{G}\,,$$

$$\mathbf{j}^{\text{nl}} = \mathcal{L}_{\mathfrak{F}\mathfrak{F}}\left[\boldsymbol{\nabla}\times\mathbf{B}\right]\mathfrak{F} + \mathcal{L}_{\mathfrak{G}\mathfrak{G}}\left[\boldsymbol{\nabla}\times\mathbf{E}\right]\mathfrak{G}\,.$$

$$\mathcal{L}_{\mathfrak{F}\mathfrak{F}} = \left.\frac{\partial^2\mathfrak{L}(\mathfrak{F}(z),\mathfrak{G}(z))}{\partial\mathfrak{F}^2(z)}\right|_{\mathfrak{F}=\mathfrak{G}=0}, \quad \mathcal{L}_{\mathfrak{G}\mathfrak{G}} = \left.\frac{\partial^2\mathfrak{L}(\mathfrak{F}(z),\mathfrak{G}(z))}{\partial(\mathfrak{G}(z))^2}\right|_{\mathfrak{F}=\mathfrak{G}=0}$$

In QED: $$\mathcal{L}_{\mathfrak{F}\mathfrak{F}} = \frac{4\alpha}{45\pi^2}\left(\frac{e}{m^2}\right)^2, \quad \mathcal{L}_{\mathfrak{G}\mathfrak{G}} = \frac{7\alpha}{45\pi^2}\left(\frac{e}{m^2}\right)^2$$

Cubic corrections to the field of magnetic $\mathcal{M}$ dipole



MAXWELL EQUATIONS (static)

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Second pair
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$$(\boldsymbol{\nabla} \cdot \mathbf{E}) = \cancel{j_0^{\text{lin}}} + \cancel{j_0^{\text{nl}}}, \quad [\boldsymbol{\nabla} \times \mathbf{B}] = \mathbf{j}^{\text{lin}} + \mathbf{j}^{\text{nl}}.$$

$$j_0^{\text{nl}} = \mathcal{L}_{\mathfrak{F}\mathfrak{F}}\left(\boldsymbol{\nabla} \cdot \mathfrak{F}\mathbf{E}\right) - \mathcal{L}_{\mathfrak{G}\mathfrak{G}}\left(\boldsymbol{\nabla} \cdot \mathbf{B}\right)\mathfrak{G}, = 0$$

$$\mathbf{j}^{\text{nl}} = \mathcal{L}_{\mathfrak{F}\mathfrak{F}}\left[\boldsymbol{\nabla} \times \mathbf{B}\right]\mathfrak{F} + \mathcal{L}_{\mathfrak{G}\mathfrak{G}}\left[\boldsymbol{\nabla} \times \mathbf{E}\right]\mathfrak{G}.$$

$$\mathcal{L}_{\mathfrak{F}\mathfrak{F}} = \left.\frac{\partial^2 \mathfrak{L}(\mathfrak{F}(z), \mathfrak{G}(z))}{\partial \mathfrak{F}^2(z)}\right|_{\mathfrak{F}=\mathfrak{G}=0}, \quad \mathcal{L}_{\mathfrak{G}\mathfrak{G}} = \left.\frac{\partial^2 \mathfrak{L}(\mathfrak{F}(z), \mathfrak{G}(z))}{\partial (\mathfrak{G}(z))^2}\right|_{\mathfrak{F}=\mathfrak{G}=0}$$

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Cubic correction to magnetic dipole

E=0 *nonlinear current* $j_0^{\mathsf{nl}}(x) = 0,$

$$\mathbf{j}^{\mathsf{nl}}(x) = \boldsymbol{\nabla} \times \mathfrak{h}(\mathbf{r})$$

$$\mathfrak{h}(\mathbf{r}) = -\frac{1}{2}\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}\mathbf{B}(\mathbf{r})\,B^2(\mathbf{r})$$

The pair of Maxwell equations

$$[\boldsymbol{\nabla} \times B] = [\boldsymbol{\nabla} \times \mathfrak{h}], \quad (\boldsymbol{\nabla} \cdot \mathrm{B}) = 0$$

Solution

$$B_i^{\mathsf{nl}}(\mathbf{r}) = \left(\delta_{ij} - \frac{\nabla_i \nabla_j}{\nabla^2}\right)\mathfrak{h}_j(\mathbf{r}) =$$

$$= \mathfrak{h}_i(\mathbf{r}) + \frac{\nabla_i \nabla_j}{4\pi}\int \frac{\mathfrak{h}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}\mathbf{r}'$$

Seeding bare magnetic dipole

$$\mathbf{B}^{\text{lin}} = -\frac{\boldsymbol{\mathcal{M}}}{r^3} + 3\frac{\mathbf{r}\cdot\boldsymbol{\mathcal{M}}}{r^5}\mathbf{r}, \quad r > R$$

$$\mathbf{B}^{\text{lin}} = \frac{2\boldsymbol{\mathcal{M}}}{R^3}, \qquad r < R$$

with the linear source

$$\mathbf{j}^{\text{lin}}(\mathbf{r}) = \frac{\boldsymbol{\mathcal{M}}\times\mathbf{r}}{r^4}\delta(r-R)$$

into nonlinear current $\mathbf{j}^{\text{nl}}(x) = \boldsymbol{\nabla}\times\mathfrak{h}(\mathbf{r})$

the nonlinearly corrected field in the far-off region is

$$\mathrm{B}(\mathbf{r})|_{r>>R} = B^{\text{lin}}_{r>R}(\mathbf{r})\left(1-\frac{7}{5}\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}\frac{\mathcal{M}^2}{R^6}\right)$$

the corrected magnetic moment is

$$\mathcal{M}^{\text{corrected}} = \mathcal{M}\left(1 - \frac{7}{5}\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}\frac{\mathcal{M}^2}{R^6}\right)$$

In QED

$$\mathbf{B}(\mathbf{r})\big|_{r>>R} = \mathbf{B}^{\text{lin}}_{r>R}(\mathbf{r})\left(1 - \frac{7}{5}\,\frac{\alpha}{45\pi}\left(\frac{B^{\text{lin}}_{\text{surf}}}{B_0}\right)^2\right), \qquad B_0 = \frac{m^2}{e}$$

Estimates: nonlinear correction is beyond the accuracy of the method for baryons, whose magnetic moment is of the order of 2 nuclear magnetons and electromagnetic radius of 1fm, because the magnetic field at the «edge» is almost three orders of magnitude larger than m^2/e.

Better theoretical methods are needed, but the problem of nonlinear renormalization is indicated.

For baryons with lower magn. dipole moments

Ξ^{*0}, Σ^{*0}, and Δ^0. the correction may lie within theor. and exp. errors

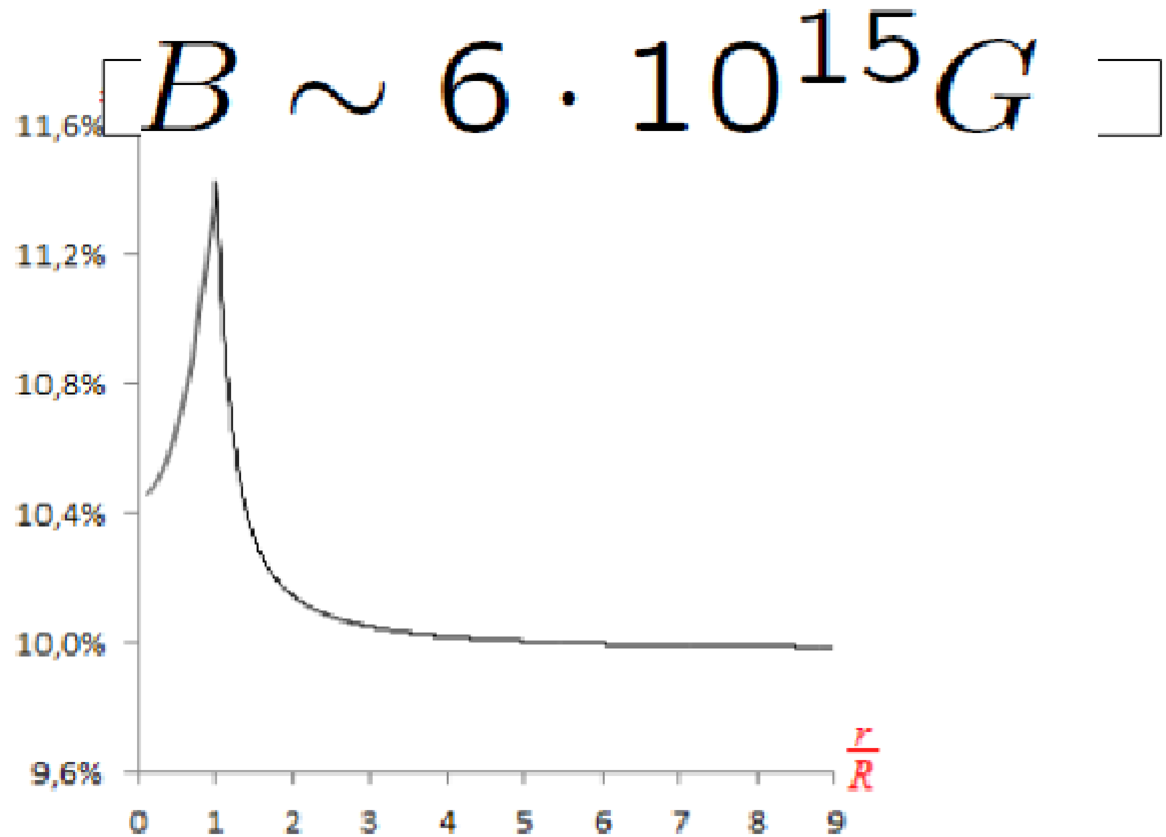
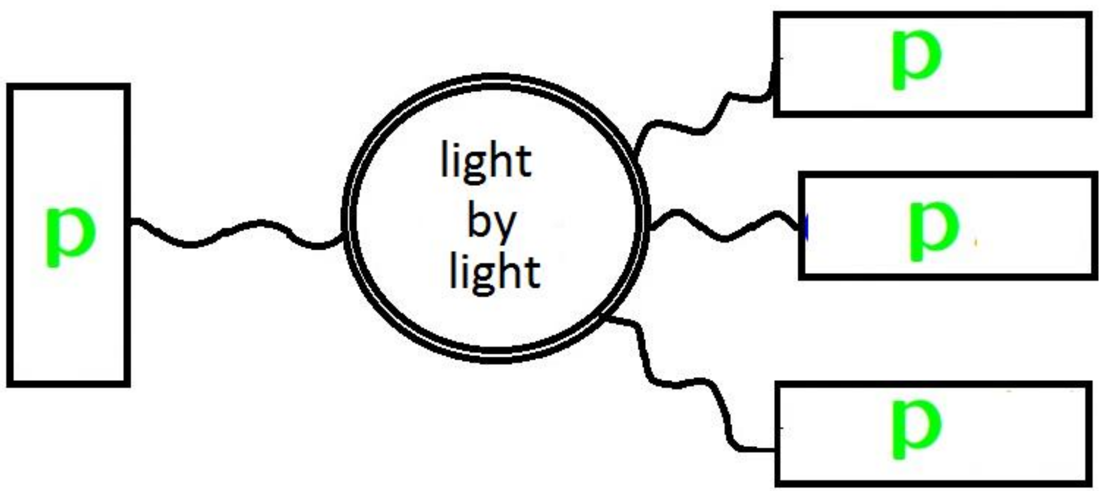


FIG 1. Nonlinear magnetic corrections to the field for Magnetars

Cubic corrections to the fields of charge $\mathbf{Q}$, electric $\mathbf{p}$ and magnetic $\mathcal{M}$ dipole



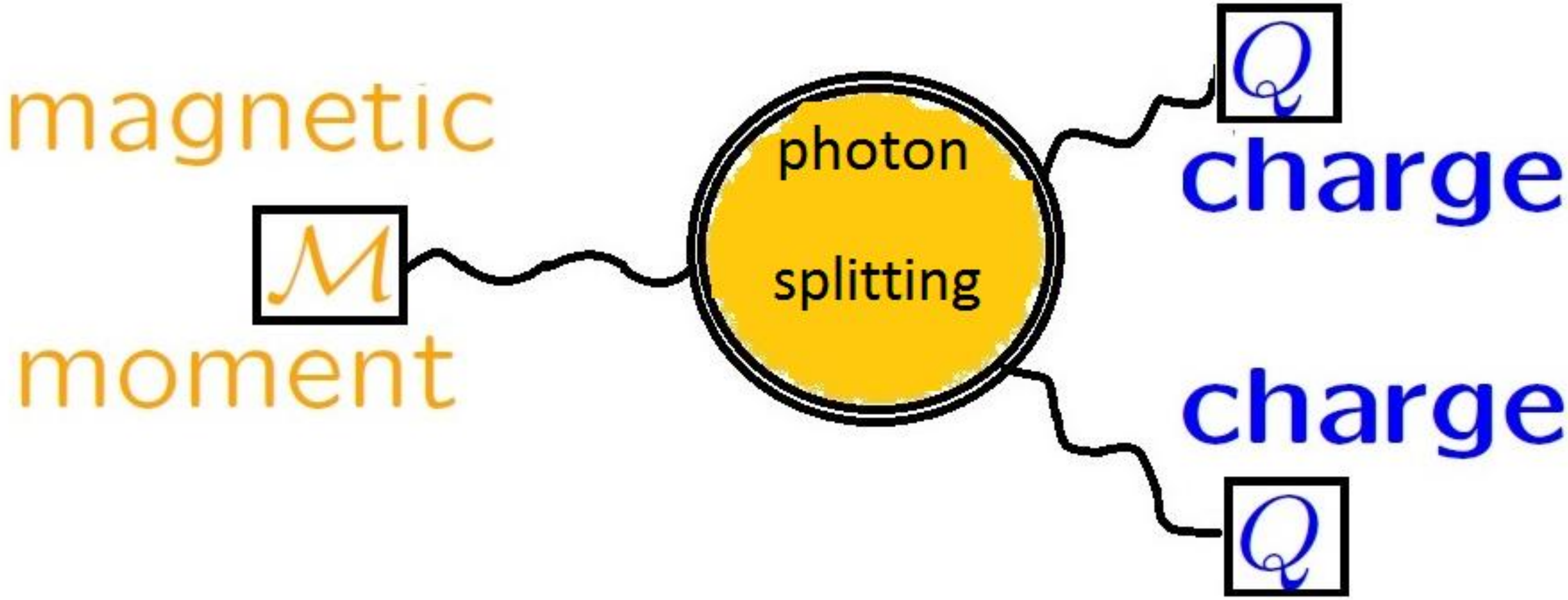
Cubic corrections to electric dipole field

current, producing dipole field $j_0(\mathbf{r}) = 3\frac{\mathbf{r}\cdot\mathbf{p}^{\text{lin}}}{r^4}\delta(r-a)$

Dipole field reproduces itself in the remote region

$$\mathbf{p} = \mathbf{p}^{\text{lin}} - \frac{\mathbf{p}^{\text{lin}}}{10}\mathcal{L}_{\mathfrak{F}\mathfrak{F}}\left(\frac{p^{\,\text{lin}}}{a^3}\right)^2$$

Magnetic (dipole) field produced by static charge



Set: $\mathbf{j}^{\mathrm{lin}} = 0$, $\dot{j}_0^{\mathrm{lin}}$ = spheric charge Q

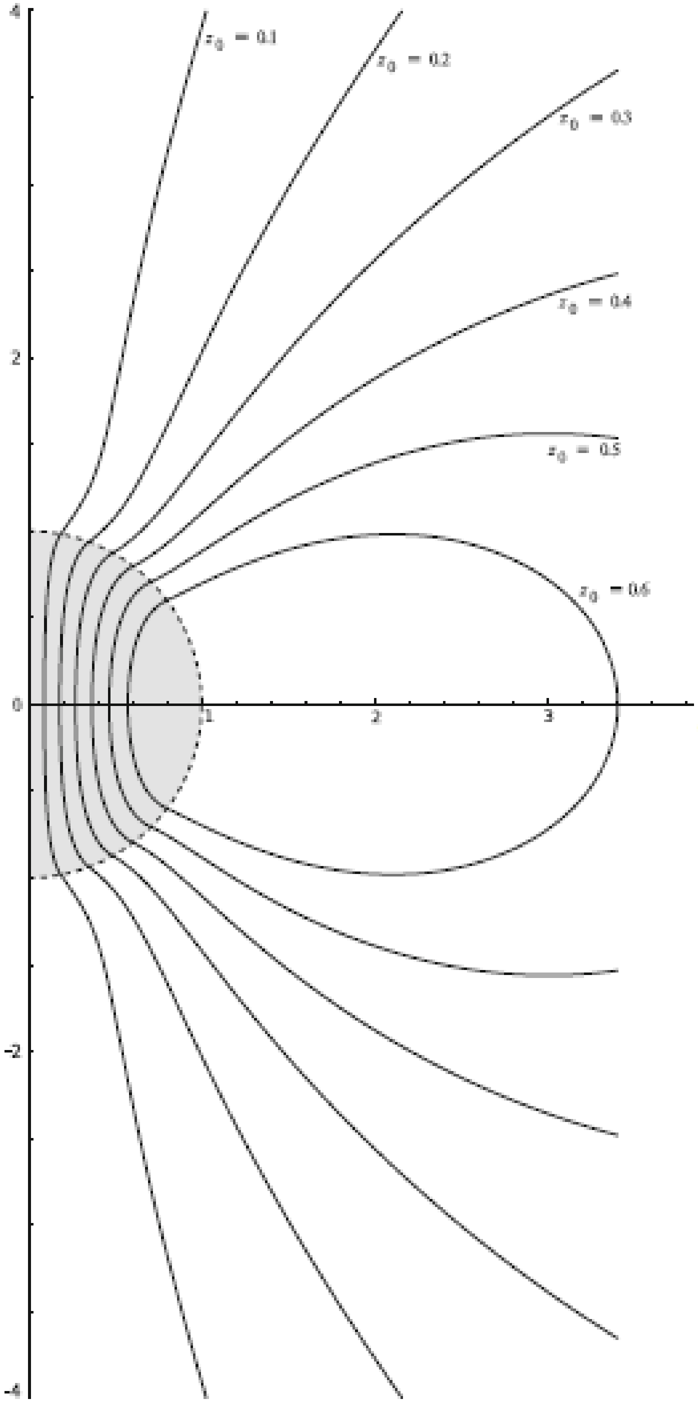
$$\mathbf{E} = \frac{Q}{4\pi r^2}\frac{\mathbf{r}}{r}$$

Static charge is a magnetic dipole

(far from the charge)

$$\mathrm{B} = -\frac{\mathcal{M}}{r^3} + 3\frac{\mathbf{r} \cdot \mathcal{M}}{r^5}\mathbf{r} + \overline{\mathrm{B}}$$

Magnetic moment



$$\boldsymbol{\mathcal{M}}_i = \frac{\left(\frac{Q}{4\pi}\right)^2}{5R}\left(3\mathcal{L}_{\mathfrak{F}\mathfrak{F}} - 2\mathcal{L}_{\mathfrak{G}\mathfrak{G}}\right)\overline{B}_i$$

of charge Q
distributed over sphere R
in presence of constant
magnetic field $\overline{\mathrm{B}}$

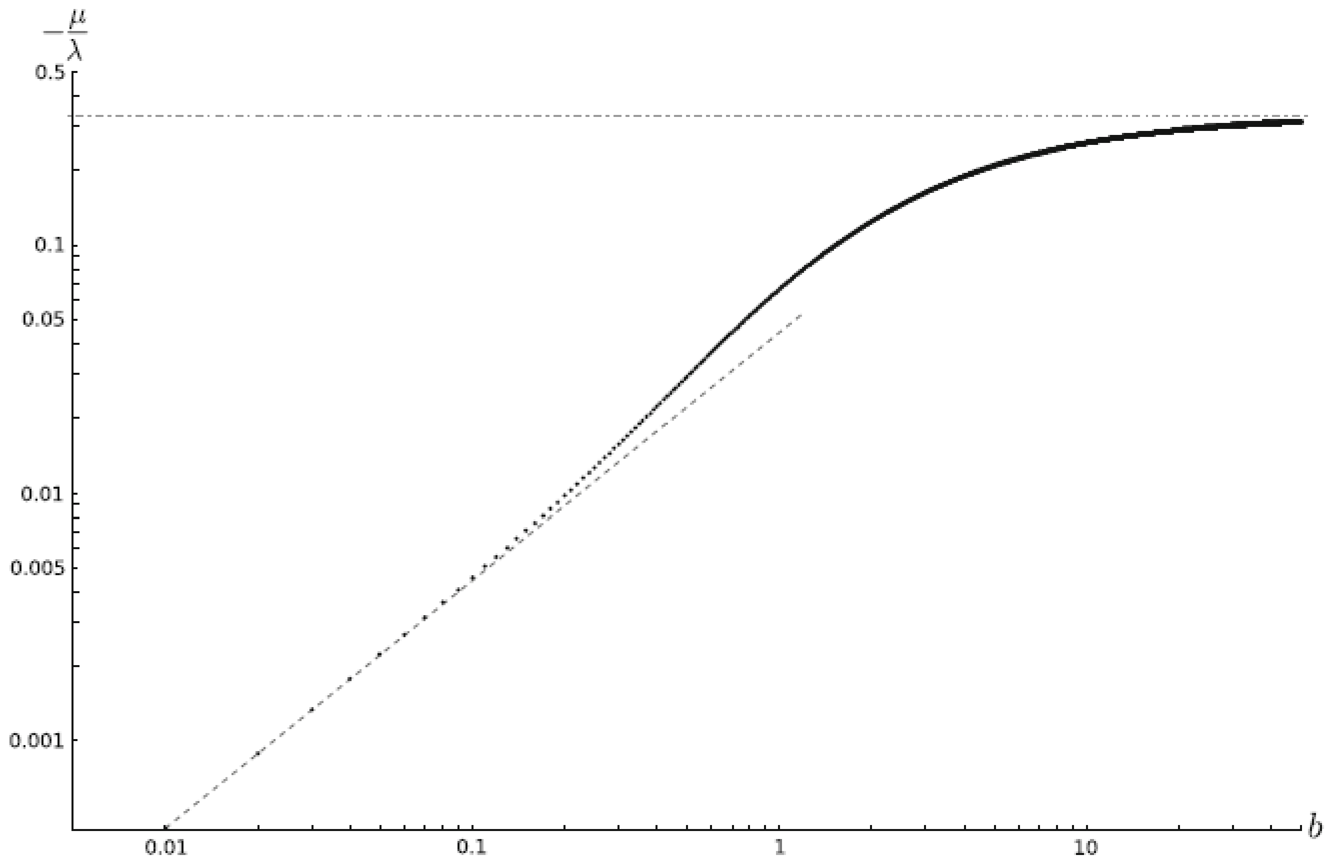


Fig. 1 The magnetic moment (69) of a charge plotted in logarithmic scale against the magnetic field $b = B/B_{\text{Sch}}$ in the range $10^{-2} < b < 50$. The scaling parameter is $\lambda = (Ze/4\pi)^2(\alpha/5\pi R B_{\text{Sch}})$. The *dot-dashed line* corresponds to the large-field constant asymptotic value, while the *dashed line* is the small-field linear asymptotic behavior (73). The step in b is 10^{-2}. The *leftmost value* for the magnetic moment is approximately $-4.4 \times 10^{-3}\lambda$

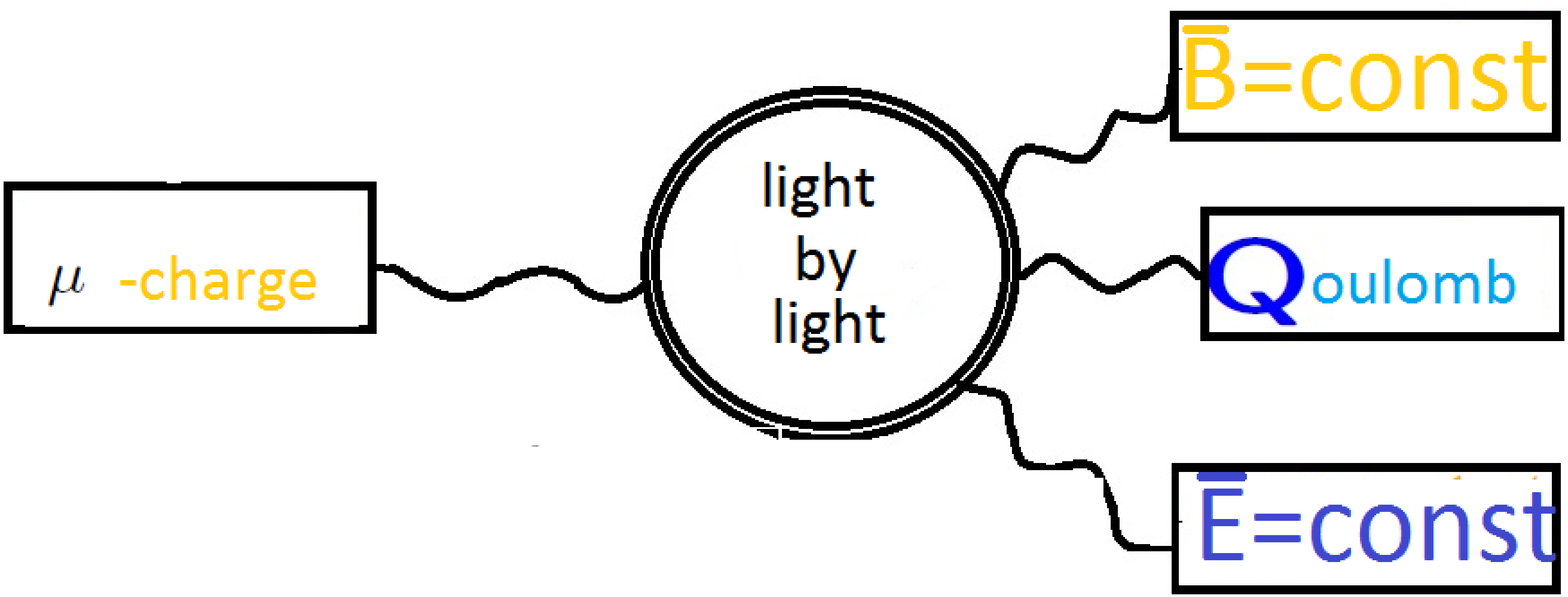
with

This is the so-called second-type Abel first-order differ-

Made-up magnetic charge

standard magnetic monopole $\mathbf{B} = \frac{\mu}{4\pi r^3}\frac{\mathbf{r}}{r}$

μ is a pseudoscalar to be formed as $\mathfrak{G} = (\mathbf{B}\cdot\mathbf{E})$



MAXWELL EQUATIONS (static)

First pair $(\nabla \cdot \mathbf{B}) = 0, \quad [\nabla \times \mathbf{E}] = 0$

Second pair
$$\frac{\delta S[A]}{\delta A^{\mu}(x)} = j_{\mu}^{\rm lin}(x)$$

$$(\boldsymbol{\nabla} \cdot \mathbf{E}) = j_0^{\rm lin} + \cancel{j_0^{\rm nl}}, \quad [\boldsymbol{\nabla} \times \mathbf{B}] = \cancel{\mathbf{j}^{\rm lin}} + \mathbf{j}^{\rm nl}.$$

$$j_0^{\rm nl} = \mathcal{L}_{\mathfrak{F}\mathfrak{F}}(\boldsymbol{\nabla} \cdot \mathfrak{F}\mathbf{E}) - \mathcal{L}_{\mathfrak{G}\mathfrak{G}}(\boldsymbol{\nabla} \cdot \mathbf{B})\,\mathfrak{G}\,,$$
$$\mathbf{j}^{\rm nl} = \mathcal{L}_{\mathfrak{F}\mathfrak{F}}[\boldsymbol{\nabla} \times \mathbf{B}]\,\mathfrak{F} + \mathcal{L}_{\mathfrak{G}\mathfrak{G}}[\boldsymbol{\nabla} \times \mathbf{E}]\,\mathfrak{G}\,.$$

$$\mathcal{L}_{\mathfrak{F}\mathfrak{F}} = \left.\frac{\partial^2 \mathfrak{L}(\mathfrak{F}(z), \mathfrak{G}(z))}{\partial \mathfrak{F}^2(z)}\right|_{\mathfrak{F}=\mathfrak{G}=0}, \quad \mathcal{L}_{\mathfrak{G}\mathfrak{G}} = \left.\frac{\partial^2 \mathfrak{L}(\mathfrak{F}(z), \mathfrak{G}(z))}{\partial (\mathfrak{G}(z))^2}\right|_{\mathfrak{F}=\mathfrak{G}=0}$$

In QED: $\mathcal{L}_{\mathfrak{F}\mathfrak{F}} = \dfrac{4\alpha}{45\pi^2}\left(\dfrac{e}{m^2}\right)^2, \quad \mathcal{L}_{\mathfrak{G}\mathfrak{G}} = \dfrac{7\alpha}{45\pi^2}\left(\dfrac{e}{m^2}\right)^2$

$$B = \overline{B} + \delta B, \quad \overline{B} = const., \quad \delta B \ll \overline{B} \boxed{\ll \frac{m^2}{e}}$$

$$E = \overline{E} + E_Q, \quad \overline{E} = const., \quad E_Q \ll \overline{E} \boxed{\ll \frac{m^2}{e}}$$

$$\boxed{\overline{B} \parallel \overline{E}}$$

Linearized equation for magnetic correction

$$\begin{aligned}
[\nabla \times \delta \mathrm{B}] &= \mathcal{L}_{\mathfrak{F}\mathfrak{F}} \left[\overline{\mathrm{B}} \times \nabla\right] \left(\overline{\mathrm{E}} \cdot \delta \mathrm{E} - \overline{\mathrm{B}} \cdot \delta \mathrm{B}\right) \\
&- \mathcal{L}_{\mathfrak{G}\mathfrak{G}} \left[\overline{\mathrm{E}} \times \nabla\right] \left(\overline{\mathrm{B}} \cdot \delta \mathrm{E} + \overline{\mathrm{E}} \cdot \delta \mathrm{B}\right)
\end{aligned}$$

Ansatz, compatible with Bianchi identity,

$$\delta \mathrm{B} = \mathrm{x} \frac{1}{r^3} f\left(\frac{z}{r}\right)$$

Boundary condition excluding introduction of external magnetic charge

$$f\,(0) \;=\; 0$$

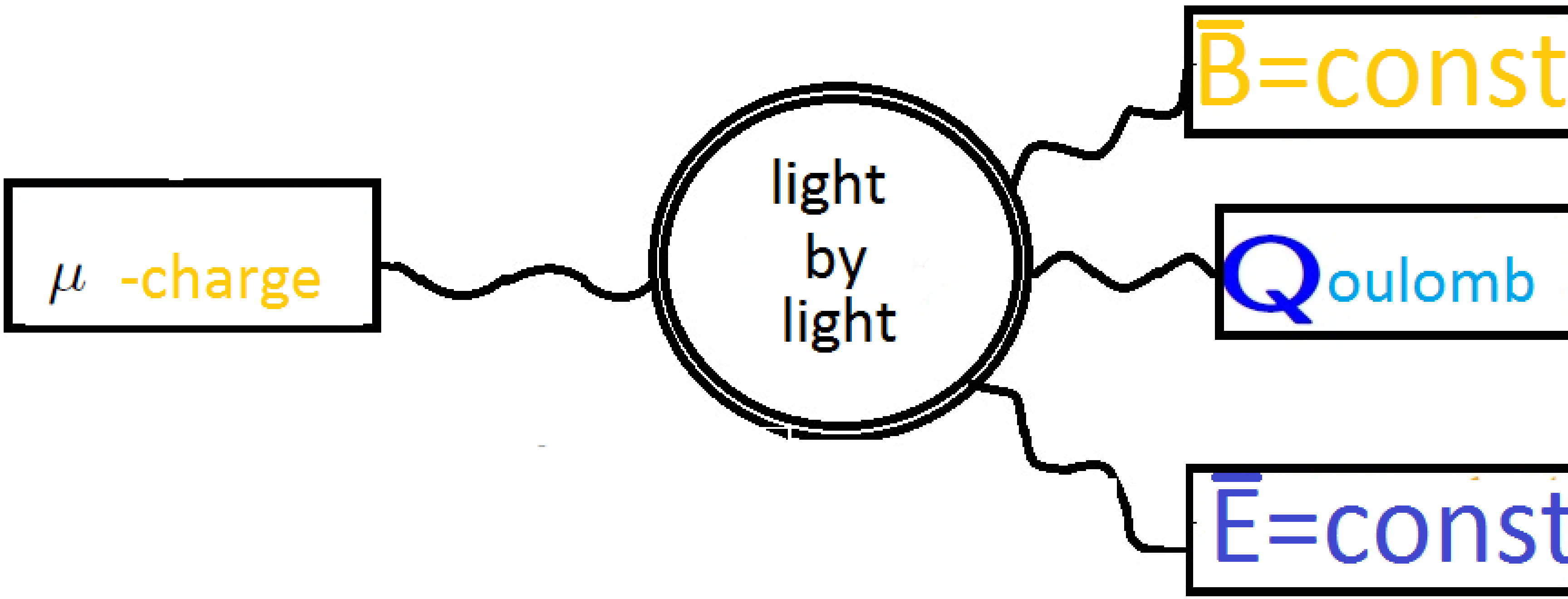
$$\frac{z}{r} = \cos\theta \text{ is a pseudoscalar}$$

$$f\left(\frac{\pm 1}{\sqrt{3}}\right) \;=\; 0 \quad \text{magnetically neutral boundary}$$

condition

Latitude of Moscow ?!

$$\delta \mathrm{B} = \mu \frac{\mathbf{r}}{r} \frac{3}{4\pi} \frac{\cos^2\theta}{r^2}$$



$$\frac{Q}{4\pi r^2} \frac{\mathbf{r}}{r}$$

$$\mu = \int\!\!\int (\mathbf{B}\cdot \mathrm{d}\sigma) = -\frac{Q}{2}(\mathcal{L}_{\mathfrak{F}\mathfrak{F}} + \mathcal{L}_{\mathfrak{G}\mathfrak{G}})\overline{\mathfrak{G}}$$

$$\overline{\mathfrak{G}} = \overline{B}\,\overline{E}$$

Yet to be approved:

When parity is not conserved

$$\left.\frac{\partial^2 \mathfrak{L}}{\partial \mathfrak{F}(x) \mathfrak{G}(x)}\right|_{\mathfrak{F}=\mathfrak{G}=0} \neq 0$$

Magnetic monopole solution may exist without electric field background: $\overline{\mathbf{E}} = 0$

Instead of: $$\mu = -\frac{Q}{2}(\mathcal{L}_{\mathfrak{F}\mathfrak{F}} + \mathcal{L}_{\mathfrak{G}\mathfrak{G}})\overline{\mathfrak{G}}$$ *pseudoscalar*

Now: $$\mu = -\frac{Q}{2}\overline{B}^2 \, \mathfrak{L}_{\mathfrak{F}\mathfrak{G}}\Big|_{\mathfrak{G}=\mathfrak{F}=0}$$ *scalar*

The end


Thank you for attention

Essentially nonlinear
purely electric, B=0,
solution

$$j_0^{\mathrm{nl}} = \mathcal{L}_{\mathfrak{F}\mathfrak{F}} \left(\boldsymbol{\nabla} \cdot \mathfrak{F} \mathbf{E} \right) - \mathcal{L}_{\mathfrak{G}\mathfrak{G}} \left(\boldsymbol{\nabla} \cdot \mathrm{B} \right) \mathfrak{G} \,,$$
$$\mathbf{j}^{\mathrm{nl}} = \mathcal{L}_{\mathfrak{F}\mathfrak{F}} \left[\boldsymbol{\nabla} \times \mathrm{B} \right] \mathfrak{F} + \mathcal{L}_{\mathfrak{G}\mathfrak{G}} \left[\boldsymbol{\nabla} \times \mathbf{E} \right] \mathfrak{G} \,.$$

Set: $\mathbf{j}^{\mathrm{lin}}$ =0 B = 0

Then:

$$j_0^{\mathrm{nl}} = \mathcal{L}_{\mathfrak{F}\mathfrak{F}} \left(\boldsymbol{\nabla} \cdot \mathfrak{F} \mathbf{E} \right)$$

Quartic model

$$L(x) = -\mathfrak{F}(x) + \frac{\gamma}{2}(\mathfrak{F}(x))^2$$

$$\gamma \equiv \mathfrak{L}_{\mathfrak{F}\mathfrak{F}}$$

Nonlinear Maxwell equations

$$\partial_\mu \left[\left(1 - \gamma \mathfrak{F}\left(x \right) \right) F^{\mu\nu} \right] = 0$$

Electrostatic case

$$\nabla \left[\left(1 + \frac{\gamma}{2} E^2 \right) \mathbf{E} \right] = 0$$

Spherical-symmetric solution for point charge Q

$$\left(1 + \frac{\gamma}{2} E^2(r) \right) \mathbf{E}\left(\mathbf{x} \right) = \frac{Q}{4\pi r^2} \frac{\mathbf{x}}{r}$$

$$\boxed{\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\text{lin}}(\mathbf{r}) - \frac{1}{2}\mathcal{L}_{\mathfrak{F}\mathfrak{F}}\mathbf{E}(\mathbf{r})\, E^2(r)\,.}$$

Iteration: $\mathbf{E}(\mathbf{r}) \simeq \mathbf{E}^{\text{lin}}(\mathbf{r})$ in r-h side

With the Euler-Heisenberg Lagrangian $\mathcal{L}_{\mathfrak{F}\mathfrak{F}} = \dfrac{e^4}{45\pi^2 m^4}$

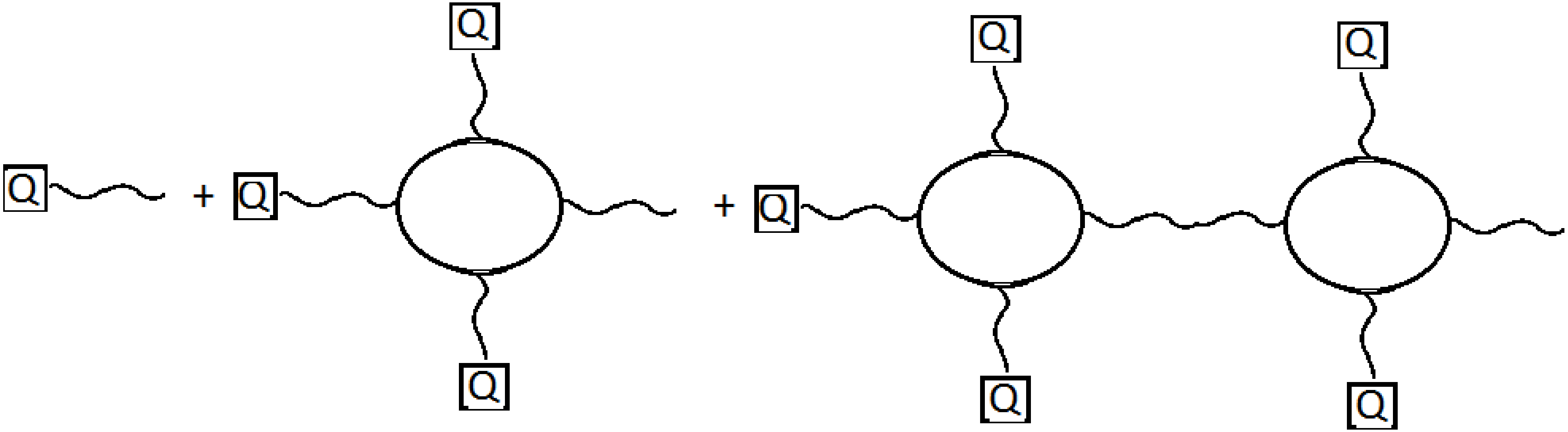
$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\text{lin}}(\mathbf{r}) \left(1 - \frac{2\alpha}{45\pi}\left(\frac{eE^{\text{lin}}(r)}{m^2}\right)^2\right)$$

Point charge $E^{\text{lin}}(r) = \frac{Q}{4\pi r^2}$

$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi r^2}\left(1 - \frac{2\alpha}{45\pi}\left(\frac{e\,Q}{4\pi r^2\, m^2}\right)^2\right)$$

Wichmann and Kroll 1954

One-photon reducible chain



Electrostatic energy of electron charge $Q = e$

$$\Theta^{00} = \frac{E^2}{2} + \frac{3\mathcal{L}_{\mathfrak{F}\mathfrak{F}}E^4}{8}$$

$$M_{Field} = \int \Theta^{00} d^3x = 2.09m$$

Soliton field configuration

$$E = \sqrt[3]{\sqrt{\left(\frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^2 + \left(\frac{2}{3\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^3} + \frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}}$$

$$- \sqrt[3]{\sqrt{\left(\frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^2 + \left(\frac{2}{3\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}\right)^3} - \frac{E_{\text{lin}}}{\mathfrak{L}_{\mathfrak{F}\mathfrak{F}}}}$$

$$E \sim r^{-2/3}$$

$$E_{\text{lin}}(r) = \frac{e}{4\pi r^2}$$

Charge *e* moving with constant speed

$$F_{\mu\nu} = (u_\mu x_\nu - u_\nu x_\mu) g\left(W^2\right)$$

u_μ is the 4-velocity

$$W^2 = (xu)^2 - x^2$$

In the rest frame *W*=*r*

g is subject to equation

$$g^3 + \frac{2g}{\gamma W^2} - \frac{2e}{\gamma W^5} = 0.$$

Energy-momentum

$$T^{\mu\nu} = -\left(1 - \gamma\mathfrak{F}(x)\right) F^{\mu\lambda} F^{\nu}_{\lambda} - g^{\mu\nu} L(x)$$

$$P^{\mu} = \int T^{\mu\nu} u_{\nu} \delta\left(ux\right) \mathrm{d}^4 x = u_{\mu} M_{Field}$$

General local regular monotonous Lagrangian

The field energy of a point charge *e*

$$4\pi \int_0^\infty \Theta^{00} r^2 dr =$$

$$\frac{e^{\frac{3}{2}}}{4\pi^{\frac{1}{2}}} \int_0^\infty \left(-\mathfrak{F}+2\mathfrak{F}\mathfrak{L}_{\mathfrak{F}} - \mathfrak{L}\right) \frac{\left[\mathfrak{L}_{\mathfrak{F}\mathfrak{F}} E^2 + 1 - \mathfrak{L}_{\mathfrak{F}}\right] \mathrm{d}E}{\left(\left(1 - \mathfrak{L}_{\mathfrak{F}}\right) E\right)^{\frac{5}{2}}}$$

converges if

$$\mathfrak{L}(\mathfrak{F}) \sim (-\mathfrak{F})^w , \quad w > \frac{3}{2}$$

or if

$$\mathfrak{L}(\mathfrak{F}) \sim (-\mathfrak{F})^{\frac{3}{2}} \ln^u (-\mathfrak{F}) , \quad u > 2$$

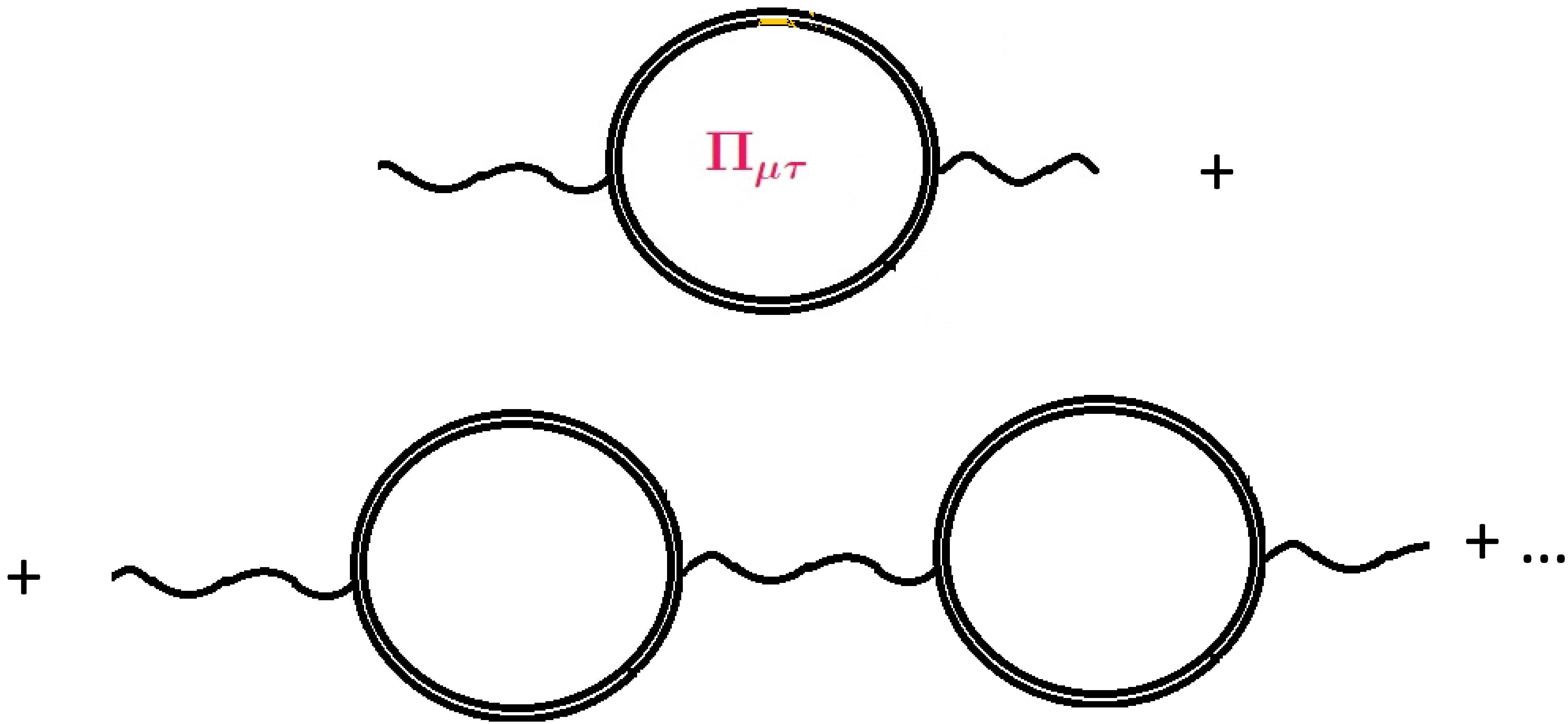
Is obeyed

by truncated QED: polynomial Lagrangian

$\mathfrak{L} \sim \mathfrak{F}^n, \; n \geq 2$

Beyond approximation of local action

One-loop approximation of QED



Characte

$$B_0 = \frac{m^2}{e}$$

Photon propagator

$$\mathbf{D}_{\mu\nu}(\mathbf{k}) = \sum_{\mathbf{a}=1}^{4} \mathcal{D}_{\mathbf{a}}(\mathbf{k}) \frac{\flat_{\mu}^{(a)} \; \flat_{\nu}^{(a)}}{(\flat^{(a)})^2}$$

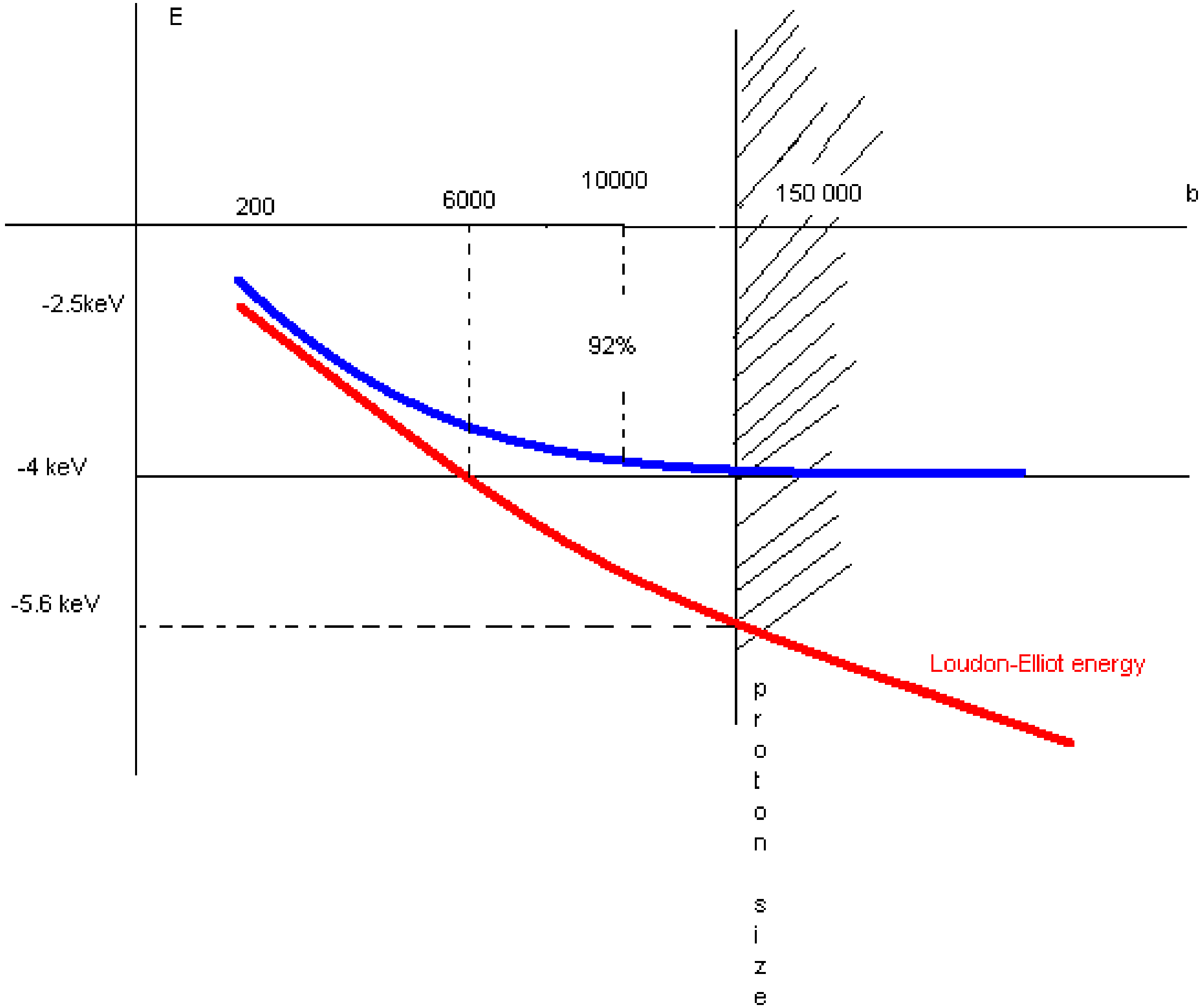
$$\mathcal{D}_{\mathbf{a}}(\mathbf{k}) = -\frac{1}{(k^2 + \kappa_a(k))}, \qquad \mathbf{a} = \mathbf{1}, \mathbf{2}, \mathbf{3}$$

Linear growth with magnetic field

$$\kappa_2 = -\frac{2\alpha b m^2}{\pi} \exp\left(-\frac{k_\perp^2}{2m^2 b}\right) T\left(\frac{k_3^2 - k_0^2}{4m^2}\right)$$

Potential of a point charge

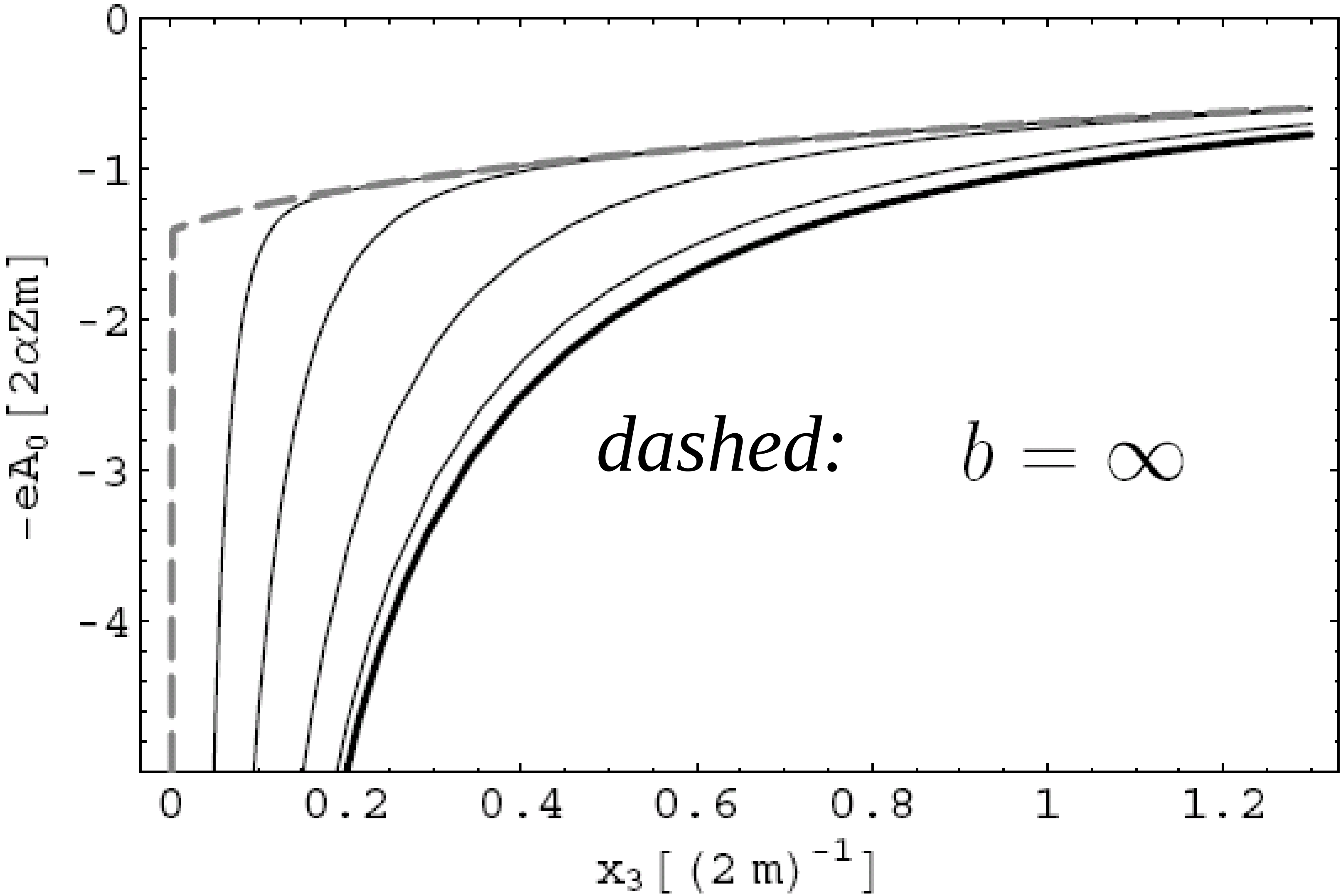
$$A_0(\mathbf{x}) = \frac{q}{(2\pi)^3} \int \frac{\mathrm{e}^{-\mathrm{i}\mathbf{k}\mathbf{x}} \mathrm{d}^3 k}{\mathbf{k}^2 - \kappa_2(0, k_3^2, k_\perp^2)}, \quad A_{1,2,3}(\mathbf{x}) = 0.$$



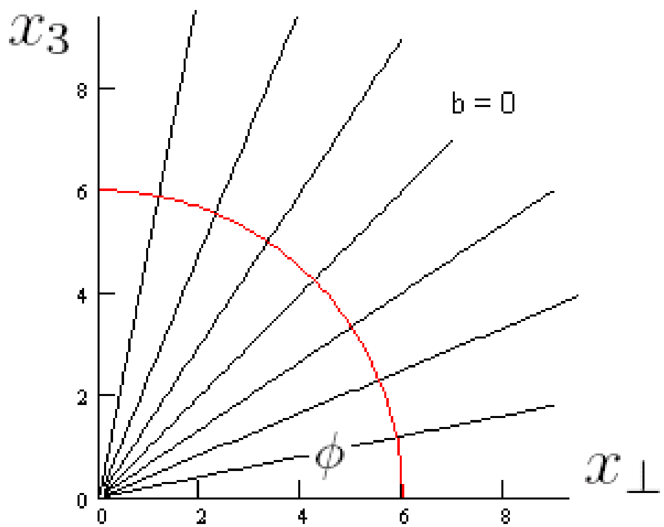
E
b
200
6000
10000
150 000
-2.5keV
-4 keV
-5.6 keV
92%
Loudon-Elliot energy
p r o t o n s i z e

Electron potential energy plotted against longitudinal distance at $X_\perp = 0$

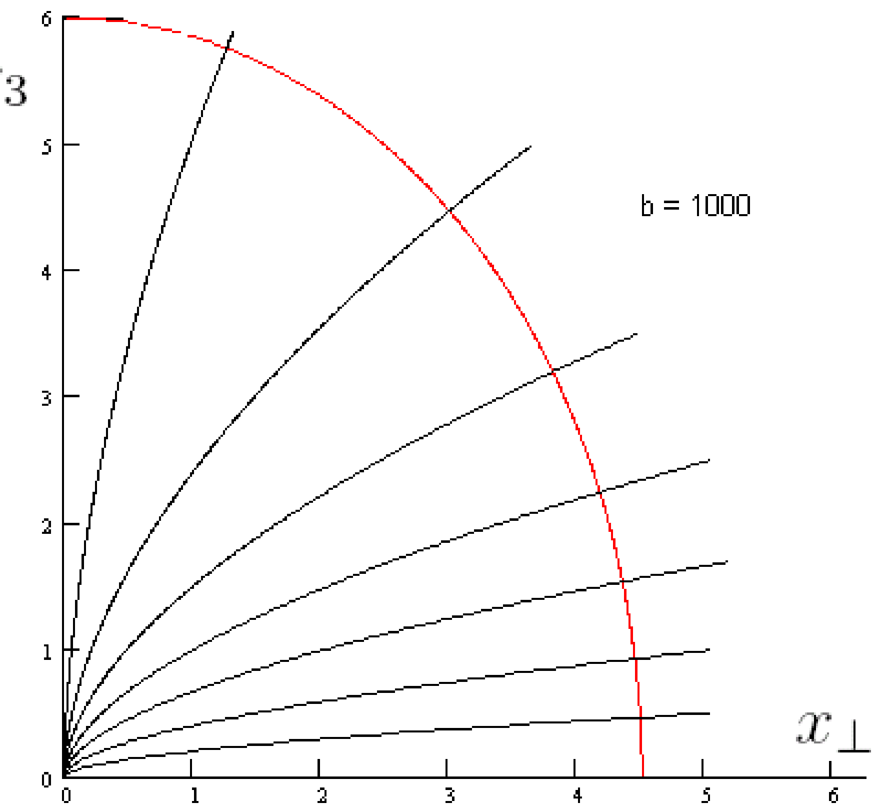
$q = eZ$

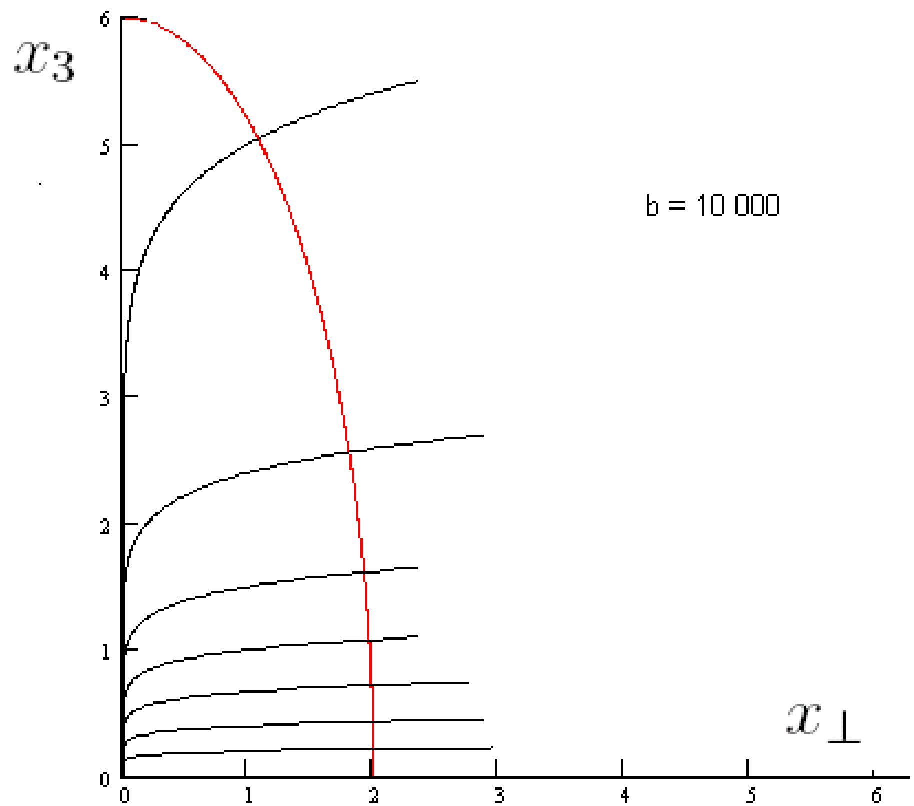


dashed: $b = \infty$



$$x_3 = \tan\phi \; x_\perp^{\left(\frac{1}{1+\frac{\alpha b}{3\pi}}\right)}$$





Potential on the string
(infinite magnetic field)

Long-range part on the axis $X_{\perp} = 0$

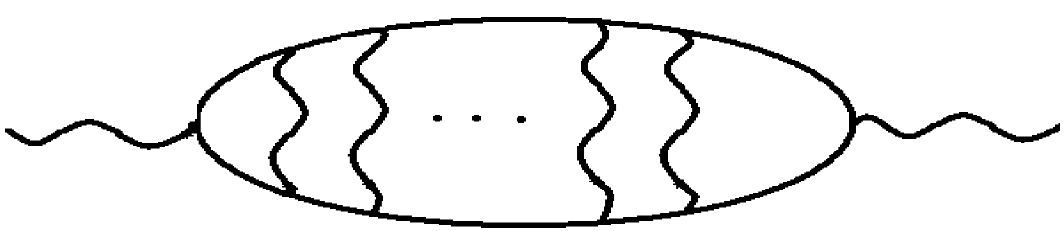
near the charge $2x_3 m \ll 1$

$$A_{\mathrm{l.r.}}(x_3, 0)|_{b=\infty} = \frac{qm}{2\pi}[1.4152 + 0.495 \cdot 2mx_3(\ln(2mx_3) - 0.77)]$$

Short-range part on the axis $X_{\perp}$

$$A_{s.r.}(x_3, 0)|_{b=\infty} = 2.180\frac{q}{2\pi}\delta(x_3)$$

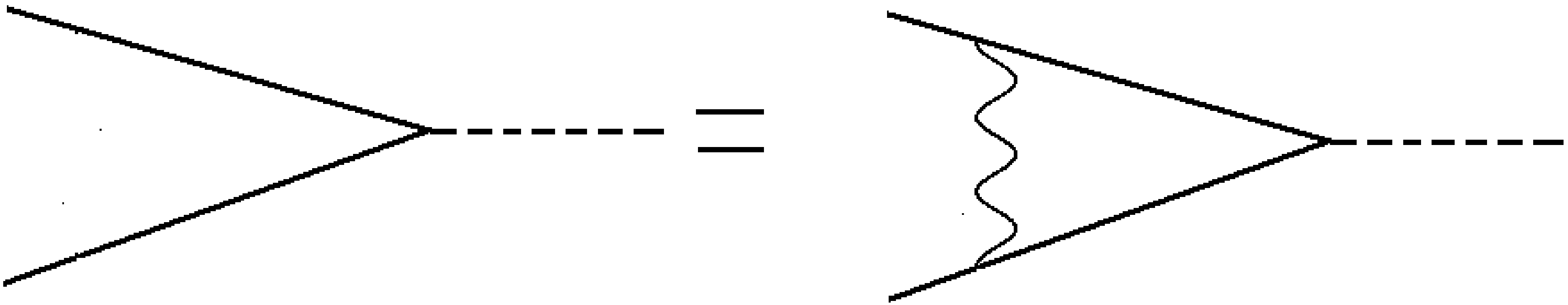
Including positronium state into second-rank polarization tensor



$$(\hbar\omega)^2 - \hbar^2$$

Positronium collapse and maximum magnetic field

Bethe-Salpeter equation



$$D(x,y) = \frac{1}{(x-y)^2}$$

$$B_{\max} = \frac{m^2}{4e}\exp\left\{\frac{\pi^{3/2}}{\sqrt{\alpha}} + 2C_{\mathrm{E}}\right\} \simeq 1.6\times 10^{28} B_0,$$

Summary: List of nonlinear effects in QED

1. *Linearization above a strong background field*

*Photon capture by magnetic field with formation of positronium
*Screening of the Coulomb field by a strong magnetic field
*Finiteness of the ground state of Hydrogen when $B \longrightarrow \infty$
*Magnetic monopole-like perturbation of parallel electric and magnetic fields

Summary: List of nonlinear effects in QED

2. Quadratic effects above a strong background field:

*Static charge in a magnetic field is a magnetic dipole

3. Cubic effects without any background field

*Solitonic solution for the field of a charge

*Nonlinear renormalization of electric and magnetic dipole moments