BEYOND THE PROTON RADIUS

WHAT ELSE CAN MUONIC HYDROGEN TELL US ABOUT THE PROTON?

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 Budapest, Hungary, Oct 12–16, 2015

Outline



Proton Radius Puzzle

Expansion in Radii

Oelta(1232)-resonance and proton deformation Compton scattering and proton polarizabilities

Polarizability contribution to the Lamb shift in Chiral Perturbation Theory

Muonic hydrogen theory and experiment

CREMA Collaboration, Nature (2010); Science (2013)



lutsa — Nucleon at Very Low Q — NStar 2015 — Osaka, May 25-2, 2015





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$$\begin{split} &\operatorname{Im} T_{1}(\nu,Q^{2}) Q^{2} = -(k'-k) \\ &\operatorname{Im} T_{1}(\nu,Q^{2}) Q^{2} = \frac{Q_{\pi^{2}\alpha}^{2} - (k'-k)}{\overline{x} + Q^{2}} = \nu \sigma_{T}(\nu,Q^{2}), \\ &\operatorname{Im} T_{2}(\nu,Q^{2}) x = \frac{4\pi^{2}\alpha}{\nu} Q^{2} / (2M_{N}\nu) \\ &\operatorname{Im} S_{1}(\nu,Q^{2}) x = \frac{4\pi^{2}\alpha}{\nu} (Q^{2}(M_{\mu}Q^{2})) = \frac{Q^{2}\nu}{\nu^{2} + Q^{2}} [\sigma_{T} + \sigma_{L}](\nu,Q^{2}), \\ &\operatorname{Im} S_{1}(\nu,Q^{2}) = \frac{4\pi^{2}\alpha}{\nu} (Q^{2}(M_{\mu}Q^{2})) = \frac{1}{\nu^{2} + Q^{2}} [\sigma_{T} + \sigma_{L}](\nu,Q^{2}), \\ &\operatorname{Im} S_{2}(\nu,Q^{2}) = \frac{1}{2} (\frac{4\pi^{2}\alpha}{\nu^{2}} Q^{2}), \\ & g_{1}(\nu,Q^{2}) = \frac{1}{\nu^{2}} (\rho_{L}^{2} + Q^{2}) = \frac{1}{\nu^{2} + Q^{2}} [\rho_{L}^{2} + Q^{2}] [\nu,Q^{2}), \\ & g_{1}(\nu,Q^{2}) = \frac{1}{\nu^{2}} (\rho_{L}^{2} + Q^{2}) = \frac{1}{\nu^{2} + Q^{2}} [\rho_{L}^{2} + Q^{2}] [\nu,Q^{2}), \\ & g_{1}(\nu,Q^{2}) = \frac{1}{\nu^{2}} (\rho_{L}^{2} + Q^{2}) = \frac{1}{\nu^{2} + Q^{2}} [\rho_{L}^{2} + Q^{2}] [\nu,Q^{2}), \\ & g_{2}(\nu,Q^{2}) = \frac{1}{\nu^{2}} (\rho_{L}^{2} + Q^{2}) = \frac{1}{\nu^{2} + Q^{2}} [\rho_{L}^{2} + Q^{2}] [\nu,Q^{2}), \\ & g_{2}(\nu,Q^{2}) = \frac{1}{\nu^{2}} (\rho_{L}^{2} + Q^{2}) = \frac{1}{\nu^{2} + Q^{2}} [\rho_{L}^{2} + Q^{2}] [\nu,Q^{2}), \\ & g_{2}(\nu,Q^{2}) = \frac{1}{\nu^{2}} (\rho_{L}^{2} + Q^{2}) = \frac{1}{\nu^{2} + Q^{2}} [\rho_{L}^{2} + Q^{2}] [\rho_{L}^{2} + Q^{2}] [\nu,Q^{2}), \\ & g_{2}(\nu,Q^{2}) = \frac{1}{\nu^{2}} (\rho_{L}^{2} + Q^{2}) = \frac{1}{\nu$$

These unitarity relations hold in the physical region, where the Bjorken variable is unit interval: $x \in [0, 1]$.

(i) Elastic part given averson averson

The structure functions describing the purely elastic scattering are given in term $f_1^{\rm el}(\nu, Q^2) = \frac{1}{2}G_M^2(Q^2)\,\delta(1-x),$

$$\begin{split} f_{2}^{\mathrm{el}}(\nu,Q^{2}) & f_{1}^{\mathrm{el}}(\underline{\nu},Q^{2}) \stackrel{1}{=} \frac{1}{2} G_{1E}^{2}(Q^{2}) \stackrel{\delta(1-x)}{=} \tau G_{M}^{2}(Q^{2}) \left[\delta(1-x), \right. \\ & \left. f_{2}^{\mathrm{el}}(\nu,Q^{2}) \stackrel{1}{=} \tau \frac{1}{2} F_{1}(Q^{2}) \stackrel{\delta(1-x)}{=} \left. f_{2}^{\mathrm{el}}(Q^{2}) \stackrel{\delta(1-x)}{=} \tau \frac{1}{2} F_{1}(Q^{2}) \stackrel{\delta(1-x)}{=} \left. f_{2}^{\mathrm{el}}(Q^{2}) \stackrel{\delta(1-x)}{=} \left. f_{2}^{\mathrm{el}}(Q^{2}) \stackrel{\delta(1-x)}{=} \right] \right. \\ & \left. g_{1}^{\mathrm{el}}(\nu,Q^{2}) \stackrel{e}{=} \tau \frac{1}{2} F_{1}(Q^{2}) \stackrel{\delta(1-x)}{=} \left. f_{2}^{\mathrm{el}}(Q^{2}) \stackrel{\delta(1-x)}{=} \right] \right. \\ & \left. g_{2}^{\mathrm{el}}(\nu,Q^{2}) \stackrel{g}{=} \tau \frac{1}{2} F_{2}(Q^{2}) \stackrel{\delta(1-x)}{=} \left. f_{2}^{\mathrm{el}}(Q^{2}) \stackrel{\delta(1-x)}{=} \right] \right. \\ & \left. g_{2}^{\mathrm{el}}(\nu,Q^{2}) \stackrel{g}{=} \tau \frac{1}{2} \frac{$$

where $Q^{\neq}/4MQ^{2}and M_{E}(Q^{2}and G_{M}Q^{2}Q^{2}and G_{M}Q^{2}Q^{2}and G_{M}Q^{2}Q^{2}and G_{M}Q^{2}and G_{$

$$G_F = F_1 + \tau F_2, \quad G_M = F_1 + F_2.$$

Proton Form Factors and RMS Radii

FF interpretation: Fourier transforms of charge and magnetization distributions

$$\rho(r) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} G(\boldsymbol{q}^2) e^{-i\boldsymbol{q}\boldsymbol{r}}$$

$$G_E(Q^2) = 1 - \frac{1}{6} R_E^2 Q^2 + \cdots$$

root-mean-square (rms) charge radius:

$$R_E = \sqrt{\langle r^2 \rangle_E}$$

$$\langle r^2 \rangle_E \equiv \int d\mathbf{r} \, r^2 \, \rho_E(\mathbf{r}) = -6 \frac{d}{dQ^2} G_E(Q^2) \Big|_{Q^2=0}$$

 $R_E = 0.879(5)_{\text{stat}}(4)_{\text{syst}}(2)_{\text{model}}(4)_{\text{group}} \text{ fm},$ $R_M = 0.777(13)_{\text{stat}}(9)_{\text{syst}}(5)_{\text{model}}(2)_{\text{group}} \text{ fm}.$ J. C. Bernauer *et al.*, Phys. Rev. C**90**,015206 (2014).



Proton Radius — Historical Perspective History of proton rms charge radius



Current status of RE and RM of the proton Present Status



[5] J. C. Bernauer *et al.*, Phys. Rev. C**90**,015206 (2014).

Muonic hydrogen more sensitive to proton structure





$$\Delta E_{\rm HFS}^{\rm exp} = 22.8089(51) \,\mathrm{meV}$$

$$\Delta E_{\rm HFS}^{\rm th} = 22.9763(15) - 0.1621(10) \left(R_Z/\mathrm{fm}\right) + \Delta E_{\rm HFS}^{\rm (pol)}$$

Zemach radius: $R_Z = -\frac{4}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left[\frac{G_E(Q^2)G_M(Q^2)}{1+\varkappa} - 1 \right]$

from 2S HFS: 1.082(37) [fm]

Muonic Hydrogen Lamb shift

 $\Delta E_{\rm LS}^{\rm th} = 206.0668(25) - 5.2275(10) \, (R_E/{\rm fm})^2$

theory uncertainty: $2.5 \,\mu eV$

numerical values reviewed in: A. Antognini *et al.*, Annals Phys. **331**, 127-145 (2013).



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Lame (Servisit in terms of VVCS amplitudes



 $\Delta E_{nS}^{(\text{pol})}$

$$= -4\alpha_{em}\phi_n^2 \int_0^\infty \frac{dQ}{Q^2} w \left(Q^2/4m_\ell^2\right) \left[T_2^{(\text{NB})}(0,Q^2) - T_1^{(\text{NB})}(0,Q^2)\right]$$

empirically known

where unpolarized, **forward** Doubly-Virtual Compton scattering (VVCS) amplitude:

$$T^{\mu\nu}(p,q) = \frac{i}{8\pi M} \int d^4x \, e^{iqx} \langle p|Tj^{\mu}(x)j^{\nu}(0)|p\rangle$$

= $\left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right) T_1(\nu,Q^2)$
+ $\frac{1}{M^2} \left(p^{\mu} - \frac{p \cdot q}{q^2}q^{\mu}\right) \left(p^{\nu} - \frac{p \cdot q}{q^2}q^{\nu}\right) T_2(\nu,Q^2)$

NB stands for non-Born, i.e. w/o elastic FFs $T_1^{(\rm NB)}(0,Q^2) \simeq Q^2 \beta_{M1}$ $T_2^{(\text{NB})}(0,Q^2) \simeq Q^2(\alpha_{E1} + \beta_{M1}), \text{ for low } Q$



 $\phi_n^2(0) = m_r^3 \alpha^3 / (\pi n^3)$

 Q^2)

Status of proton polarizabilities





Polarizability contribution in ChPT

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Regular Article - Theoretical Physics

Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution

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Proton polarizability effect in mu-H

	Heavy-Baryon (HB)ChPT					[Alarcon, Lensky & VP, EPJC (2014)]	
(μeV)	Pachucki [9]	Martynenko [10]	Nevado and Pineda [11]	Carlson and Vanderhaeghen [12]	Birse and McGovern [13]	Gorchtein et al. [14]	LO-BχPT [this work]
$\Delta E_{2S}^{(\mathrm{subt})}$	1.8	2.3	_	5.3 (1.9)	4.2 (1.0)	$-2.3 (4.6)^{a}$	-3.0
$\Delta E_{2S}^{(\text{inel})}$	-13.9	-13.8	_	-12.7 (5)	-12.7 (5) ^b	-13.0 (6)	-5.2
$\Delta E_{2S}^{(\text{pol})}$	-12 (2)	-11.5	-18.5	-7.4 (2.4)	-8.5 (1.1)	-15.3 (5.6)	$-8.2(^{+1.2}_{-2.5})$

^a Adjusted value; the original value of Ref. [14], +3.3, is based on a different decomposition into the 'elastic' and 'polarizability' contributions ^b Taken from Ref. [12]

- [9] K. Pachucki, Phys. Rev. A 60, 3593 (1999).
- [10] A. P. Martynenko, Phys. Atom. Nucl. 69, 1309 (2006).
- [11] D. Nevado and A. Pineda, Phys. Rev. C 77, 035202 (2008).
- [12] C. E. Carlson and M. Vanderhaeghen, Phys. Rev. A 84, 020102 (2011).
- [13] M. C. Birse and J. A. McGovern, Eur. Phys. J. A 48, 120 (2012).
- [14] M. Gorchtein, F. J. Llanes-Estrada and A. P. Szczepaniak, Phys. Rev. A 87, 052501 (2013).

$$\Delta E_{2S}^{(\text{pol})}(\text{LO-HB}\chi\text{PT}) \approx \frac{\alpha_{\text{em}}^5 m_r^3 g_A^2}{4(4\pi f_\pi)^2} \frac{m_\mu}{m_\pi} (1 - 10G + 6\log 2) = -16.1 \ \mu\text{eV}, \quad G \simeq 0.9160 \text{ is the Catalan constant.}$$

ChPT of Compton scattering off protons





Unpolarized cross sections for RCS



Vladimir Pascalutsa — Beyond the proton radius — FKK-15 — Budapest, Oct 13, 2015

Proton polarizabilities



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Predictions of BChPT for VVCS

Alarcon, Lensky & VP, PRC (2014)



BChPT for polarised VVCS (deltaLT puzzle)

Alarcon, Lensky & VP, PRC (2014)



HFS calculation in ChPT

Hagelstein & V.P., in progress



Delta(1232) and proton deformation



Physica 96A (1979) 27-30 © North-Holland Publishing Co.

THE UNMELLISONANT QUARK

SHELDON L. GLASHOW*

Quadrupole N-> Delta transitions signatures of nucleon deformation

Proton radius puzzle: possible explanations



Beyond Standard Model

Summary

Polarizability contribution to mu-H Lamb shift



Polarizability contribution to mu-H 2S HFS



mu-H HFS yields access to magnetisation distr. and deformation of the proton

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Collaborators

Franziska Hagelstein (Mainz)

Jose Alarcon (Bonn) Vadim Lensky (Mainz)

Judith McGovern (Manchester) Marc Vanderhaeghen (Mainz)

Backup slides

UV dependence in HB- vs B-ChPT



$$M_N \sim m_\pi^3$$
$$\kappa \sim m_\pi$$
$$\beta_M \sim \frac{1}{m_\pi}$$

Heavy-Baryon expansion fails for quantities where the leading chiral-loop effects scales with a negative power of pion mass

E.g.: the effective range parameters of the NN force are such quantities -- hope for "perturbative pions" (KSW) in BChPT

From beam asymmetry

PRL 110, 262001 (2013) PHYSICAL REVIEW LETTERS

week ending 28 JUNE 2013

Separation of Proton Polarizabilities with the Beam Asymmetry of Compton Scattering

Nadiia Krupina and Vladimir Pascalutsa

PRISMA Cluster of Excellence Institut für Kernphysik, Johannes Gutenberg–Universität Mainz, 55128 Mainz, Germany (Received 3 April 2013; published 25 June 2013)



Vladimir Pascalutsa — A few moments in ChPT — Workshop on Tagged Structure Functions — JLab, Jan 16-18, 2014 28

New Mainz data for Compton beam asymmetry

Data taken: 28.05. – 17.06.2013, 327 h



Predictions of HBChPT vs BChPT



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