Exclusive decays $J/\psi \to D_{(s)}^{(*)\,-}\ell\nu_\ell$ in the Covariant Confined Quark Model

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Content

Recent Experimental Results

- **Weak decays of J/ψ are rare processes**
- **CMS and LHCb:**
  $\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$ **and**
  $\mathcal{B}(B^0 \to \mu^+\mu^-) = (3.9^{+1.6}_{-1.4}) \times 10^{-10}$

V. Khachatryan *et al.*, Nature (2015)

- **BESIII Collaboration (10^{10} J/ψ events/year): at 90% C.L.**
  $\mathcal{B}(J/\psi \to D_s^{*-} e^+ \nu_e + c.c.) < 1.8 \times 10^{-6}$ **(first time) and**
  $\mathcal{B}(J/\psi \to D_s^{-} e^+ \nu_e + c.c.) < 1.3 \times 10^{-6}$ **(30 times more stringent)**

M. Ablikim *et al.*, Phys. Rev. D 90, 112014 (2014)

- **Standard Model prediction:** $\approx 10^{-8} - 10^{-10}$

SanchisLozano:1993, Wang:2007, Shen:2008

Theoretical studies

Theoretical predictions in SM:

- **(Approximate) spin symmetry of heavy mesons**
  Inclusive BF $\approx 10^{-8}$; $R \equiv \mathcal{B}(J/\psi \to D_s^* \ell\nu)/\mathcal{B}(J/\psi \to D_s \ell\nu) \approx 1.5$

  Sanchis-Lonzano:1993

- **QCD sum rules (QCD SR)**
  Exclusive BFs $\approx 10^{-10}$; $R \approx 3.1$

  Wang:2007

- **Covariant light-front quark model (LFQM)**
  BFs are $2 - 8$ times larger than Wang's

  Shen:2008

Our aim:

- **Crosscheck!**
- **Covariant Confined Quark Model**

M. A. Ivanov and C. T. Tran:2015

Covariant Confined Quark Model of hadrons

G. V. Efimov, M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner, P. Santorelli,...

- **Main assumption: hadrons interact via quark exchange only**
- **Interaction Lagrangian**

$$\mathcal{L}_{int} = g_H \cdot H(x) \cdot J_H(x)$$

- **Quark current**

$$J_H(x) = \int dx_1 \int dx_2 \, F_H(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \, \Gamma_H \, q^a_{f_2}(x_2)$$

- **Vertex Function**

$$F_H(x; x_1, x_2) = \delta(x - w_1 x_1 - w_2 x_2) \, \Phi_H((x_1 - x_2)^2)$$

where $w_i = m_{q_i}/(m_{q_1} + m_{q_2})$
Translational invariant: $F_H(x + c; x_1 + c, x_2 + c) = F_H(x; x_1, x_2)$

- **Nonlocal Gaussian-type vertex functions with fall-off behavior in Euclidean space to temper high energy divergence of quark loops**

$$\widetilde{\Phi}_H(-k^2) = \int dx \, e^{ikx} \Phi_H(x^2) = e^{k^2/\Lambda_H^2}$$

where Λ_H **characterizes the meson size.**

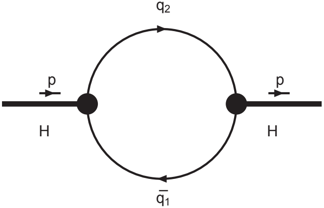
- **Compositeness condition** $Z_H = 0$

Salam 1962; Weinberg 1963

Z_H – wave function renormalization constant of the meson H.

$$Z_H^{1/2} = < H_{\rm bare} | H_{\rm dressed} > = 0$$

- $Z_H = 1 - \tilde{\Pi}'(m_H^2) = 0$ **where** $\tilde{\Pi}(p^2)$ **is the meson mass operator.**



$$\Pi_P(p) = 3g_P^2 \int \frac{dk}{(2\pi)^4 i} \widetilde{\Phi}_P^2(-k^2) {\rm tr}[S_1(k+w_1p)\gamma^5 S_2(k-w_2p)\gamma^5]$$

$$\Pi_V(p) = g_V^2[g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}] \int \frac{dk}{(2\pi)^4 i} \widetilde{\Phi}_V^2(-k^2) {\rm tr}\,[S_1(k+w_1p)\gamma_\mu S_2(k-w_2p)\gamma_\nu]$$

The matrix elements

- Matrix elements are described by a set of Feynman diagrams which are convolutions of quark propagators and vertex functions.
- Let Π be the matrix element corresponding to the Feynman diagram:

  j external momenta;
  n quark propagators;
  ℓ loop integrations;
  m vertices.
  In the momentum space it will be represented as

$$\Pi(p_1,\ldots,p_j) = \int [d^4k]^\ell \prod_{i_1=1}^{m} \Phi_{i_1+n}\left(-K^2_{i_1+n}\right) \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3} + \tilde{p}_{i_3})$$

$$K^2_{i_1+n} = \sum_{i_2} (\tilde{k}^{(i_2)}_{i_1+n} + \tilde{p}^{(i_2)}_{i_1+n})^2$$

$\tilde{k}_i$ are linear combinations of the loop momenta k_i

$\tilde{p}_i$ are linear combinations of the external momenta p_i

- **Use the Schwinger representation of the propagator:**

$$\frac{m+\not{k}}{m^2-k^2} = (m+\not{k})\int\limits_0^\infty d\alpha\,\exp[-\alpha(m^2-k^2)]$$

- **Choose a simple Gaussian form for the vertex function**

$$\Phi(-K^2) = \exp\left(K^2/\Lambda^2\right)$$

where the parameter Λ characterizes the hadron size.

- **We imply that the loop integration k proceed over Euclidean space:**

$$k^0 \to e^{i\frac{\pi}{2}}k_4 = ik_4\,, \qquad k^2 = (k^0)^2 - \vec{k}^2 \to -k_E^2 \le 0\,.$$

- **We also put all external momenta p to Euclidean space:**

$$p^0 \to e^{i\frac{\pi}{2}}p_4 = ip_4\,, \qquad p^2 = (p^0)^2 - \vec{p}^2 \to -p_E^2 \le 0$$

so that the quadratic momentum form in the exponent becomes negative-definite and the loop integrals are absolutely convergent.

Infrared confinement

- **Convert the loop momenta in the numerator into derivatives over external momenta:**

$$k_i^{\mu} e^{2kr} = \frac{1}{2}\frac{\partial}{\partial r_{i\,\mu}} e^{2kr},$$

- **Move the derivatives outside of the loop integrals.**
- **Calculate the scalar loop integral:**

$$\prod_{i=1}^{n} \int \frac{d^4 k_i}{i\pi^2} e^{kAk+2kr} = \prod_{i=1}^{n} \int \frac{d^4 k_i^E}{\pi^2} e^{-k_E A k_E - 2k_E r_E} = \frac{1}{|A|^2} e^{-rA^{-1}r}$$

where a symmetric $n \times n$ real matrix A is positive-definite.

- **Use the identity**

$$P\left(\frac{1}{2}\frac{\partial}{\partial r}\right) e^{-rA^{-1}r} = e^{-rA^{-1}r} P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1}r\right)$$

to move the exponent to the left.

Infrared confinement

- **Employ the commutator**

$$[\frac{\partial}{\partial r_{i\,\mu}}, r_{j\,\nu}] = \delta_{ij}\, g_{\mu\nu}$$

to make differentiation in

$$P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1} r\right)$$

for any polynomial P. The necessary commutations of the differential operators are done by a FORM program.

- **One obtains**

$$\Pi = \int\limits_0^\infty d^n\alpha\, F(\alpha_1, \ldots, \alpha_n)\,,$$

where F stands for the whole structure of a given diagram.

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$$\Pi = \int_0^\infty d^n\alpha \, F(\alpha_1, \ldots, \alpha_n) \, ,$$

where F stands for the whole structure of a given diagram.
The set of Schwinger parameters α_i can be turned into a simplex by introducing an additional t–integration via the identity

$$1 = \int_0^\infty dt \, \delta(t - \sum_{i=1}^n \alpha_i)$$

$$\Pi = \int_0^\infty dt t^{n-1} \int_0^1 d^n\alpha \, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) \, F(t\alpha_1, \ldots, t\alpha_n) \, .$$

Cut off the upper integration at $1/\lambda^2$

$$\Pi^c = \int_0^{1/\lambda^2} dt t^{n-1} \int_0^1 d^n\alpha \, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) \, F(t\alpha_1, \ldots, t\alpha_n)$$

Formalism

- **Matrix element**

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cq} \left\langle D^- \left| \bar{q} O_\mu c \right| J/\psi \right\rangle \left[\bar{\nu_\ell} O^\mu \ell \right], \text{where } q = s, d,\ O^\mu = \gamma^\mu (1 - \gamma_5).$$

- **Decay width**

$$\Gamma \left(J/\psi \to D_{(s)}^{(*)\,-} \ell^+ \nu_\ell \right) = \frac{G_F^2}{(2\pi)^3} \frac{|V_{cq}|^2}{64\, m_1^3} \int_{m_\ell^D}^{(m_{J/\psi} - m_2)^2} dq^2 \int_{s_1^-}^{s_1^+} ds_1\ \frac{1}{3} H_{\mu\nu} L^{\mu\nu},$$

where $s_1 = (p_D + p_\ell)^2$.

- **Lepton tensor**

$$L^{\mu\nu} = \mathrm{tr} \left[(\not{p}_\ell + m_\ell) O^\mu \not{p}_{\nu_\ell} O^\nu \right]$$

- **Hadron tensor**

$$H_{\mu\nu} = \begin{cases} T^{VP}_{\mu\alpha} \left(-g^{\alpha\alpha'} + \frac{p_1^\alpha p_1^{\alpha'}}{m_1^2} \right) T^{VP\,\dagger}_{\nu\alpha'} & \text{for } V \to P \\ T^{VV}_{\mu\alpha\beta} \left(-g^{\alpha\alpha'} + \frac{p_1^\alpha p_1^{\alpha'}}{m_1^2} \right) \left(-g^{\beta\beta'} + \frac{p_2^\beta p_2^{\beta'}}{m_2^2} \right) T^{VV\,\dagger}_{\nu\alpha'\beta'}, & \text{for } V \to V \end{cases}$$

Formalism

$V \to P$ transition

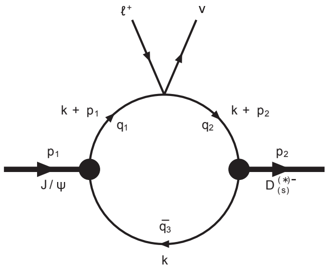
$$\left\langle D^-_{(s)}(p_2)\left|\bar{q}O_\mu c\right|J/\psi(\epsilon_1, p_1)\right\rangle = \frac{\epsilon_1^\alpha}{m_1 + m_2} \times$$
$$\times \left[-g_{\mu\nu} pq A_0(q^2) + p_\mu p_\nu A_+(q^2) + q_\mu p_\nu A_-(q^2) + i\varepsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma V(q^2)\right],$$

where $q = p_1 - p_2$, $p = p_1 + p_2$, $m_1 \equiv m_{J/\psi}$, $m_2 \equiv m_{D_{(s)}}$.

Formalism

$V \to V$ transition

$$\left\langle D_{(s)}^{*-}(\epsilon_2, p_2)\,|\bar{q}O_\mu c|\,J/\psi(\epsilon_1, p_1)\right\rangle$$
$$= \varepsilon_{\mu\nu\alpha\beta}\epsilon_1^{\alpha}\epsilon_2^{*\beta}\left[\left(p^\nu - \frac{m_1^2 - m_2^2}{q^2}q^\nu\right)A_1(q^2) + \frac{m_1^2 - m_2^2}{q^2}q^\nu A_2(q^2)\right]$$
$$+ \frac{i}{m_1^2 - m_2^2}\varepsilon_{\mu\nu\alpha\beta}p_1^{\alpha}p_2^{\beta}\left[A_3(q^2)\epsilon_1^{\nu}\epsilon_2^{*}\cdot q - A_4(q^2)\epsilon_2^{*\nu}\epsilon_1\cdot q\right]$$
$$+ (\epsilon_1\cdot\epsilon_2^*)\left[-p_\mu V_1(q^2) + q_\mu V_2(q^2)\right]$$
$$+ \frac{(\epsilon_1\cdot q)(\epsilon_2^*\cdot q)}{m_1^2 - m_2^2}\left[\left(p_\mu - \frac{m_1^2 - m_2^2}{q^2}q_\mu\right)V_3(q^2) + \frac{m_1^2 - m_2^2}{q^2}q_\mu V_4(q^2)\right]$$
$$- (\epsilon_1\cdot q)\epsilon_{2\,\mu}^{*}V_5(q^2) + (\epsilon_2^*\cdot q)\epsilon_{1\,\mu}V_6(q^2)\,.$$



$$\left\langle D^-_{(s)}(p_2)\,|\bar{q}O_\mu c|\,J/\psi(\epsilon_1,p_1)\right\rangle = \epsilon_1^\alpha T^{VP}_{\mu\alpha}$$

$$T^{VP}_{\mu\alpha} = 3g_{J/\psi}g_P\int\frac{d^4k}{(2\pi)^4 i}\,\widetilde{\Phi}_{J/\psi}[-(k+w_{13}\,p_1)^2]\,\widetilde{\Phi}_P[-(k+w_{23}p_2)^2]$$
$$\times\ \mathrm{tr}\,[S_2(k+p_2)O_\mu S_1(k+p_1)\gamma_\alpha S_3(k)\gamma_5]\,,$$

$$\left\langle D^{*-}_{(s)}(\epsilon_2,p_2)\,|\bar{q}O_\mu c|\,J/\psi(\epsilon_1,p_1)\right\rangle = \epsilon_1^\alpha\epsilon_2^{*\,\beta}T^{VV}_{\mu\alpha\beta}$$

$$T^{VV}_{\mu\alpha\beta} = 3g_{J/\psi}g_V\int\frac{d^4k}{(2\pi)^4 i}\,\widetilde{\Phi}_{J/\psi}[-(k+w_{13}\,p_1)^2]\,\widetilde{\Phi}_V[-(k+w_{23}\,p_2)^2]$$
$$\times\ \mathrm{tr}\,[S_2(k+p_2)O_\mu S_1(k+p_1)\gamma_\alpha S_3(k)\gamma_\beta]\,.$$

$\epsilon_1\cdot p_1=\epsilon_2^*\cdot p_2=p_i^2-m_i^2=0$ and $w_{ij}=m_{q_j}/(m_{q_i}+m_{q_j}),\ w_{ij}+w_{ji}=1.$

Numerical Results

- **Model parameters (all in GeV)**

$m_{u/d}$	m_s	m_c	λ	$\Lambda_{J/\psi}$	Λ_{D^*}	$\Lambda_{D_s^*}$	Λ_D	Λ_{D_s}
0.241	0.428	1.67	0.181	1.74	1.53	1.56	1.60	1.75

- **Leptonic decay constants f_H (in MeV)**

	This work	Other	Ref.
$f_{J/\psi}$	415.0	418 ± 9	LAT & QCD SR Becirevic:2013
f_D	206.1	204.6 ± 5.0	PDG:2014
f_{D^*}	244.3	$245(20)^{+3}_{-2}$	LAT Becirevic:1998
		$278\pm13\pm10$	LAT Becirevic:2012
		$252.2\pm22.3\pm4$	QCD SR Lucha:2014
f_{D_s}	257.5	257.5 ± 4.6	PDG:2014
$f_{D_s^*}$	272.0	$272(16)^{+3}_{-20}$	LAT Becirevic:1998
		311 ± 9	LAT Becirevic:2012
		$305.5\pm26.8\pm5$	QCD SR Lucha:2014
f_{D_s}/f_D	1.249	1.258 ± 0.038	PDG:2014
$f_{D_s^*}/f_{D^*}$	1.113	$1.16\pm0.02\pm0.06$	LAT Becirevic:2012

Form factors

- **Double–pole parametrization**

$$\mathbf{F(q^2) = \frac{F(0)}{1 - as + bs^2}}, \quad \mathbf{s = \frac{q^2}{m^2_{J/\psi}}}, \qquad (1)$$

- **Relative error relative to the exact results is less than 1% over the entire q^2 range.**

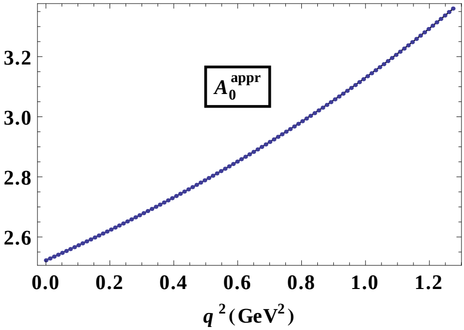


Figure : Comparison of $\mathbf{A_0(q^2)}$ form factor for $\mathbf{J/\psi \to D_s}$ transition calculated by FORTRAN code (dotted) with the parametrization (solid).

Form factors

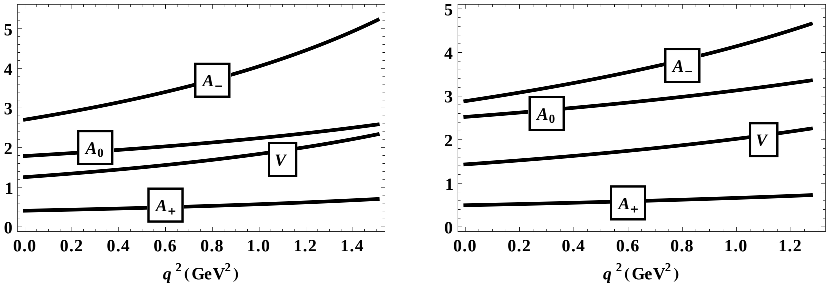


Figure : Our results for the form factors of the $\mathbf{J/\psi \to D}$ (left) and $\mathbf{J/\psi \to D_s}$ (right) transitions.

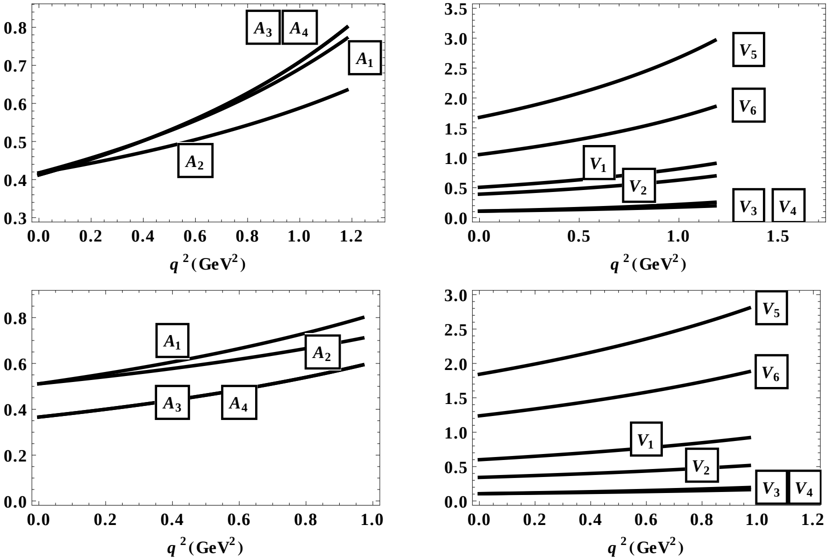


Figure : Our results for the form factors of the $\mathbf{J/\psi \to D^*}$ (up) and $\mathbf{J/\psi \to D_s^*}$ (down) transitions. One has to note that in the left panel $\mathbf{A_1(0) = A_2(0)}$ and $\mathbf{A_3(q^2) \equiv A_4(q^2)}$.

Form factors

	$J/\psi \to D: \quad q^2 = 0$				$J/\psi \to D_s: \quad q^2 = 0$			
	A_0	A_+	A_-	V	A_0	A_+	A_-	V
QCD SR	**1.09**	**0.34**	—	**0.81**	**1.71**	**0.35**	—	**1.07**
LFQM	**2.75**	**0.18**	—	**1.6**	**3.05**	**0.13**	—	**1.8**
our	**1.79**	**0.41**	**2.71**	**1.26**	**2.52**	**0.50**	**2.88**	**1.43**

Table : Comparison of $J/\psi \to D_{(s)}$ form factors at maximum recoil with those obtained in QCD SR [Wang:2007] and LFQM [Shen:2008].

	A_1	A_2	A_3	A_4	V_1	V_2	V_3	V_4	V_5	V_6
	$J/\psi \to D^*: \quad q^2 = 0$									
QCDSR	**0.40**	**0.44**	**0.86**	**0.91**	**0.41**	**0.63**	**0.22**	**0.26**	**1.37**	**0.87**
our	**0.42**	**0.42**	**0.41**	**0.41**	**0.51**	**0.39**	**0.11**	**0.11**	**1.68**	**1.05**
	$J/\psi \to D_s^*: \quad q^2 = 0$									
QCDSR	**0.53**	**0.53**	**0.91**	**0.91**	**0.54**	**0.69**	**0.24**	**0.26**	**1.69**	**1.14**
our	**0.51**	**0.51**	**0.37**	**0.37**	**0.60**	**0.34**	**0.11**	**0.11**	**1.84**	**1.24**

Table : Comparison of $J/\psi \to D^*_{(s)}$ form factors at maximum recoil with those obtained in QCD SR [Wang:2007ys].

Branching Fractions

Mode	Unit	This work	QCD SR	LFQM
$J/\psi \to D^- e^+ \nu_e$	10^{-12}	17.1	$7.3^{+4.3}_{-2.2}$	$51 \sim 57$
$J/\psi \to D^- \mu^+ \nu_\mu$	10^{-12}	16.6	$7.1^{+4.2}_{-2.2}$	$47 \sim 55$
$J/\psi \to D_s^- e^+ \nu_e$	10^{-10}	3.3	$1.8^{+0.7}_{-0.5}$	$5.3 \sim 5.8$
$J/\psi \to D_s^- \mu^+ \nu_\mu$	10^{-10}	3.2	$1.7^{+0.7}_{-0.5}$	$5.5 \sim 5.7$
$J/\psi \to D^{*-} e^+ \nu_e$	10^{-11}	3.0	$3.7^{+1.6}_{-1.1}$	—
$J/\psi \to D^{*-} \mu^+ \nu_\mu$	10^{-11}	2.9	$3.6^{+1.6}_{-1.1}$	—
$J/\psi \to D_s^{*-} e^+ \nu_e$	10^{-10}	5.0	$5.6^{+1.6}_{-1.6}$	—
$J/\psi \to D_s^{*-} \mu^+ \nu_\mu$	10^{-10}	4.8	$5.4^{+1.6}_{-1.5}$	—

$$R \equiv \frac{\mathcal{B}(J/\psi \to D_s^* \ell \nu)}{\mathcal{B}(J/\psi \to D_s \ell \nu)} = \begin{cases} 1.5 & \text{M.A. Sanchis-Lonzano} \\ 3.1 & \text{Y.M. Wang} \\ 1.5 & \text{This work} \end{cases}$$

$$R_1 \equiv \frac{\mathcal{B}(J/\psi \to D_s \ell \nu)}{\mathcal{B}(J/\psi \to D \ell \nu)} \quad \text{and} \quad R_2 \equiv \frac{\mathcal{B}(J/\psi \to D_s^* \ell \nu)}{\mathcal{B}(J/\psi \to D^* \ell \nu)}$$

$\simeq |V_{cs}|^2/|V_{cd}|^2 \simeq 18.4$ under SU(3) flavor symmetry limit. Wang's: $R_1 \simeq 24.7$ and $R_2 \simeq 15.1$. This work: $R_1 \simeq 19.3$ and $R_2 \simeq 16.6$.

Summary and Conclusions

- **New hope for experimental observation of rare weak decays of J/ψ.**
- **Introduce the Covariant Confined Quark Model.**
- **$J/\psi \to D_{(s)}^{(*)}$ transition form factors in full range of momentum transfer.**
- **Predictions for branching fractions and their ratios.**