http://arxiv.org/pdf/0705.4264

Particle Physics
The Standard Model

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4. QUANTUM CHROMODYNAMICS

- QCD Lagrangian
- Running Coupling
- Asymptotic Freedom
- α_s Measurements

QUANTUM CHROMODYNAMICS

FREE QUARKS:

$$N_C = 3$$

$$\mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

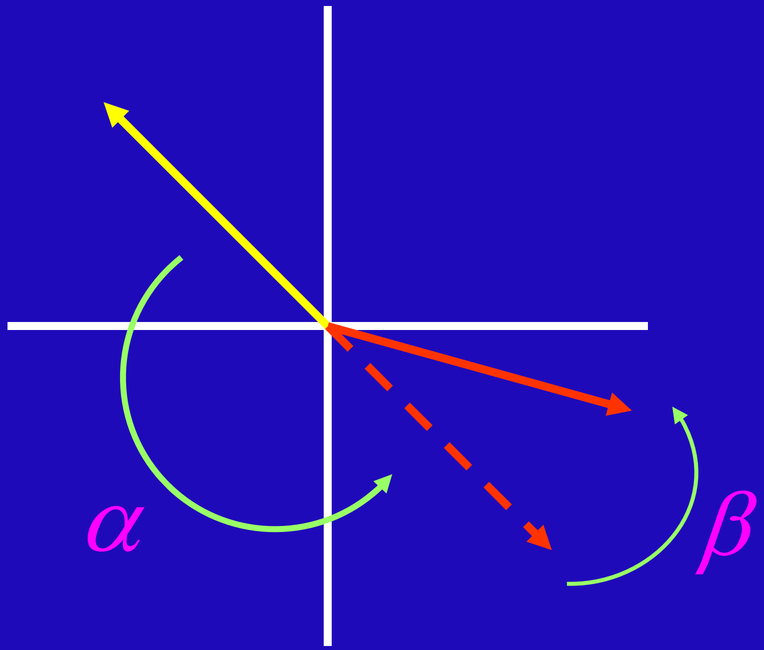
$$\mathcal{L} = \bar{\mathbf{q}} \, [\, i \gamma^\mu \partial_\mu - m \,] \, \mathbf{q}$$

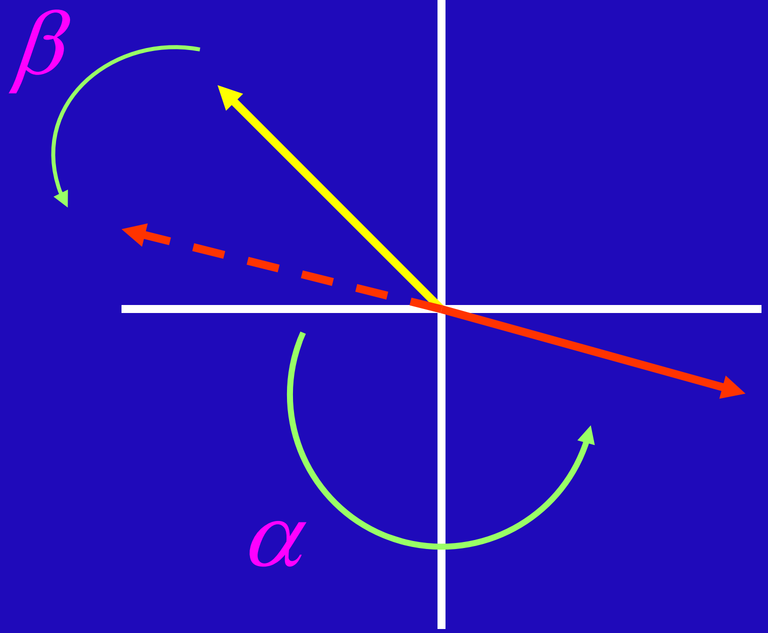
SU(3) Colour Symmetry:

$$\mathbf{q} \to \mathbf{U} \, \mathbf{q} \qquad ; \qquad \bar{\mathbf{q}} \to \bar{\mathbf{q}} \, \mathbf{U}^\dagger$$

$$\mathbf{U} \, \mathbf{U}^\dagger = \mathbf{U}^\dagger \mathbf{U} = \mathbf{1} \qquad ; \qquad \det \mathbf{U} = 1$$

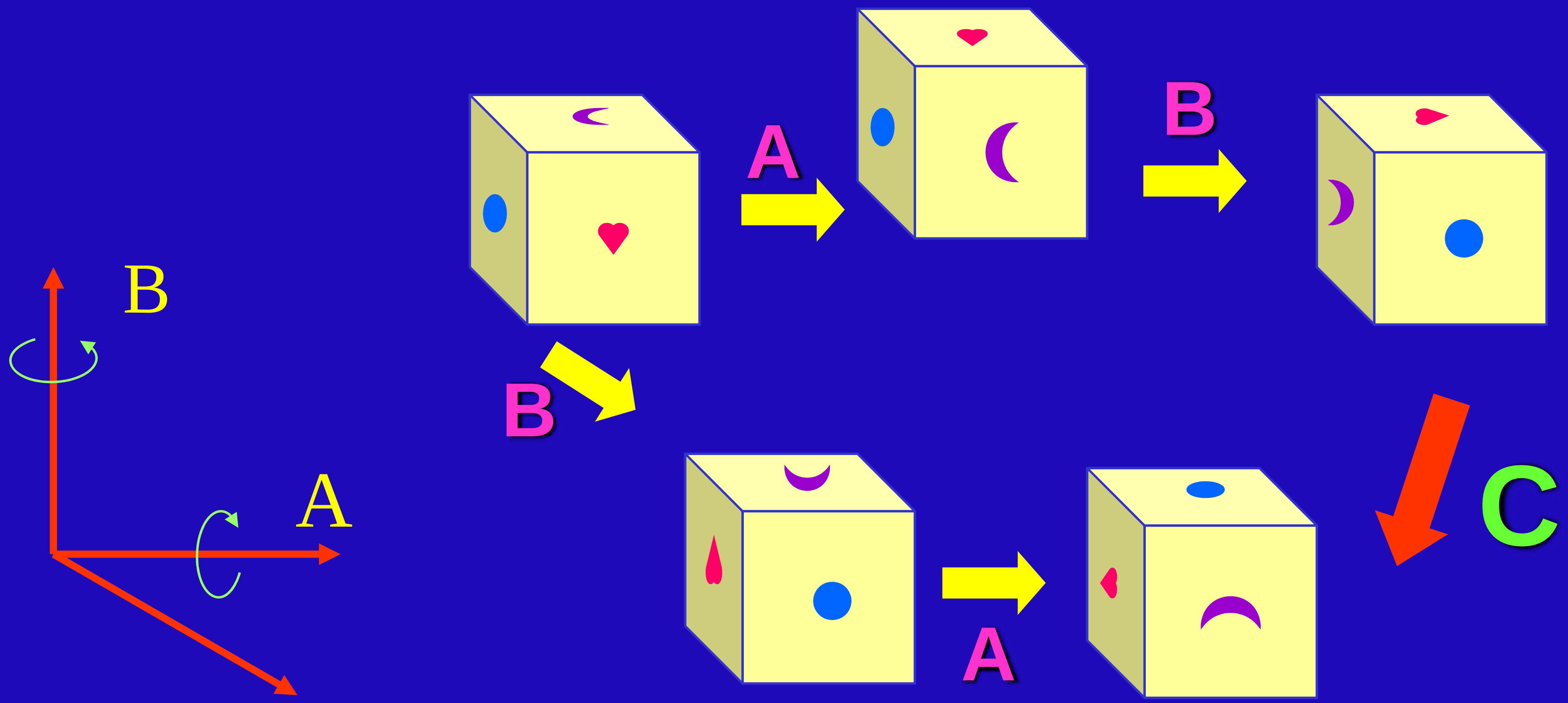
ABELIAN ROTATIONS





NON ABELIAN ROTATIONS

AB - BA = C



SU(N) ALGEBRA

$N \times N$ matrices: $\quad \mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\,\mathbf{U} = \mathbf{1} \quad ; \quad \det \mathbf{U} = 1$

$\mathbf{U} = \exp\left\{ i\, \mathbf{T}^a \theta_a \right\} \quad ; \quad \mathbf{T}^a = \mathbf{T}^{a\dagger} \quad ; \quad \mathrm{Tr}\,(\mathbf{T}^a) = 0 \quad ; \quad a = 1, \cdots, N^2 - 1$

Commutation Relation: $\quad \left[\mathbf{T}^a, \mathbf{T}^b \right] = i\, f^{abc}\, \mathbf{T}^c$

Structure Constants $\quad f^{abc} \quad$ real , antisymmetric

Fundamental Representation: $\quad \mathbf{T}^a_F = \frac{1}{2} \lambda^a \qquad N \times N$

Adjoint Representation: $\quad (\mathbf{T}^a_A)_{bc} = -i\, f^{abc} \qquad (N^2 - 1) \times (N^2 - 1)$

$$\mathrm{Tr}\,(\mathbf{T}^a_F\, \mathbf{T}^b_F) = T_F\, \delta_{ab} \quad ; \quad \mathrm{Tr}\left(\mathbf{T}^a_A\, \mathbf{T}^b_A\right) = C_A\, \delta_{ab} \quad ; \quad \left(\mathbf{T}^a_F\, \mathbf{T}^a_F\right)_{\alpha\beta} = C_F\, \delta_{\alpha\beta}$$

$$T_F = \frac{1}{2} \quad ; \quad C_A = N \quad ; \quad C_F = \frac{N^2 - 1}{2N}$$

SU(2)

2×2 matrices: $\quad \mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\,\mathbf{U} = \mathbf{1} \quad ; \quad \det \mathbf{U} = 1$

$$\mathbf{U} = \exp\left\{ i\, \mathbf{T}^a \theta_a \right\} \quad ; \quad \mathbf{T}^a = \mathbf{T}^{a\dagger} \quad ; \quad \mathrm{Tr}\,(\mathbf{T}^a) = 0 \quad ; \quad \mathrm{a} = 1, \cdots, 3$$

Commutation Relation: $\left[\mathbf{T}^a, \mathbf{T}^b\right] = i\,\varepsilon^{abc}\,\mathbf{T}^c$

Fundamental Representation: $\boxed{\mathbf{T}^a_F = \frac{1}{2}\sigma^a}$ **Pauli**

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad ; \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

SU(3)

$$[\mathbf{T}^a, \mathbf{T}^b] = i\, f^{abc}\, \mathbf{T}^c$$

Fundamental Representation:

$$\mathbf{T}_F^a = \frac{1}{2}\lambda^a$$

Gell-Mann

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \; ; \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \; ; \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \; ; \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\frac{1}{2} f^{123} = f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{\sqrt{3}} f^{458} = \frac{1}{\sqrt{3}} f^{678} = \frac{1}{2}$$

QUANTUM CHROMODYNAMICS

FREE QUARKS: $N_C = 3$

$$\mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

$$\mathcal{L} = \bar{\mathbf{q}}\,[\,i\gamma^\mu\partial_\mu - m\,]\,\mathbf{q}$$

SU(3) Colour Symmetry:

$$\mathbf{q} \to \mathrm{U}\,\mathbf{q} = \exp\left\{i\,\frac{\lambda^a}{2}\,\theta_a\right\}\,\mathbf{q}$$

Gauge Principle: Local Symmetry $\theta_a = \theta_a(x)$

$$\mathbf{D^\mu q} \equiv (\,\mathbf{I}_3\,\partial^\mu + i\,g_s\,\mathbf{G}^\mu\,)\,\mathbf{q} \;\to\; \mathbf{U}\;\mathbf{D^\mu q}$$

$$\mathbf{D}^\mu \to \mathbf{U}\;\mathbf{D}^\mu\;\mathbf{U}^\dagger \quad ; \quad \mathbf{G}^\mu \to \mathbf{U}\;\mathbf{G}^\mu\;\mathbf{U}^\dagger + \frac{i}{g_s}(\partial^\mu\mathbf{U})\;\mathbf{U}^\dagger$$

$$[\mathbf{G}^\mu]_{\alpha\beta} \equiv \frac{1}{2}\,(\lambda^a)_{\alpha\beta}\;G_a^\mu(x)$$

8 Gluon Fields

Infinitesimal SU(3) Transformation:

$$\delta \mathrm{q}^{\alpha} = i\ \delta\theta_a \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} \mathrm{q}^{\beta} \qquad ; \qquad \delta G_a^{\mu} = -\frac{1}{g_s}\,\partial^{\mu}(\delta\theta_a) - f^{abc}\,\delta\theta_b\, G_c^{\mu}$$

Non Abelian Group: $f^{abc} \neq 0$

- δG_a^{μ} depends on G_a^{μ}
- Universal g_s
- No Colour Charges

Kinetic Term:

$$\mathbf{G}^{\mu\nu} \equiv -\frac{i}{g_s}\,[\mathbf{D}^\mu, \mathbf{D}^\nu] = \partial^\mu \mathbf{G}^\nu - \partial^\nu \mathbf{G}^\mu + i\, g_s\, [\mathbf{G}^\mu, \mathbf{G}^\nu] \quad \to \quad \mathbf{U}\; \mathbf{G}^{\mu\nu}\; \mathbf{U}^\dagger$$

$$\mathbf{G}^{\mu\nu} \equiv \frac{\lambda^a}{2}\, G_a^{\mu\nu} \quad ; \quad G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s\, f^{abc}\, G_b^\mu\, G_c^\nu$$

$$\mathcal{L}_K = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\mathbf{G}_{\mu\nu}) = -\frac{1}{4}\, G_a^{\mu\nu}\, G^a_{\mu\nu}$$

Mass Term:

$$\mathcal{L}_M = \frac{1}{2}\, m_G^2\, G_a^\mu\, G^a_\mu$$

Not Gauge Invariant ➡ $m_G = 0$

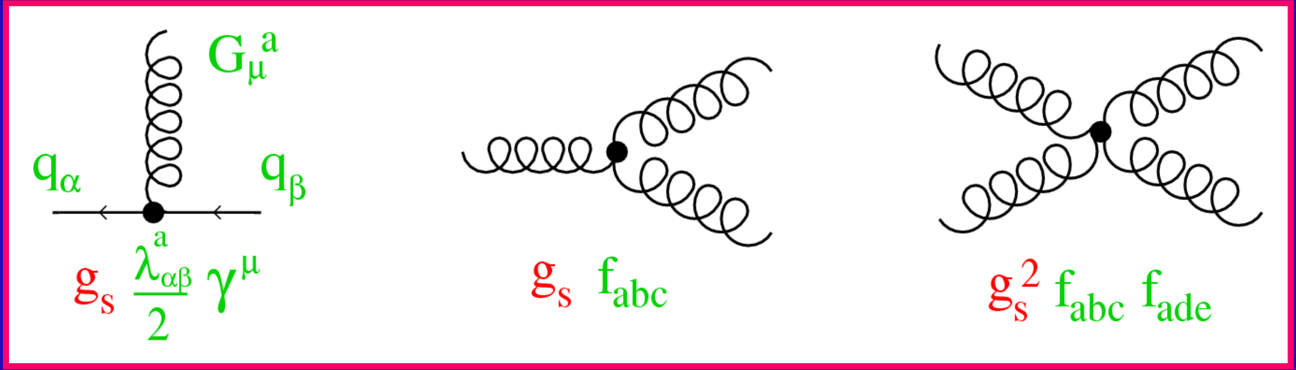
Massless Gluons

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}\,{\rm Tr}\,(\mathbf{G}^{\mu\nu}\,\mathbf{G}_{\mu\nu}) + \bar{\mathbf{q}}\,[\,i\,\gamma^\mu\,\mathbf{D}_\mu - m_q\,]\,\mathbf{q}$$

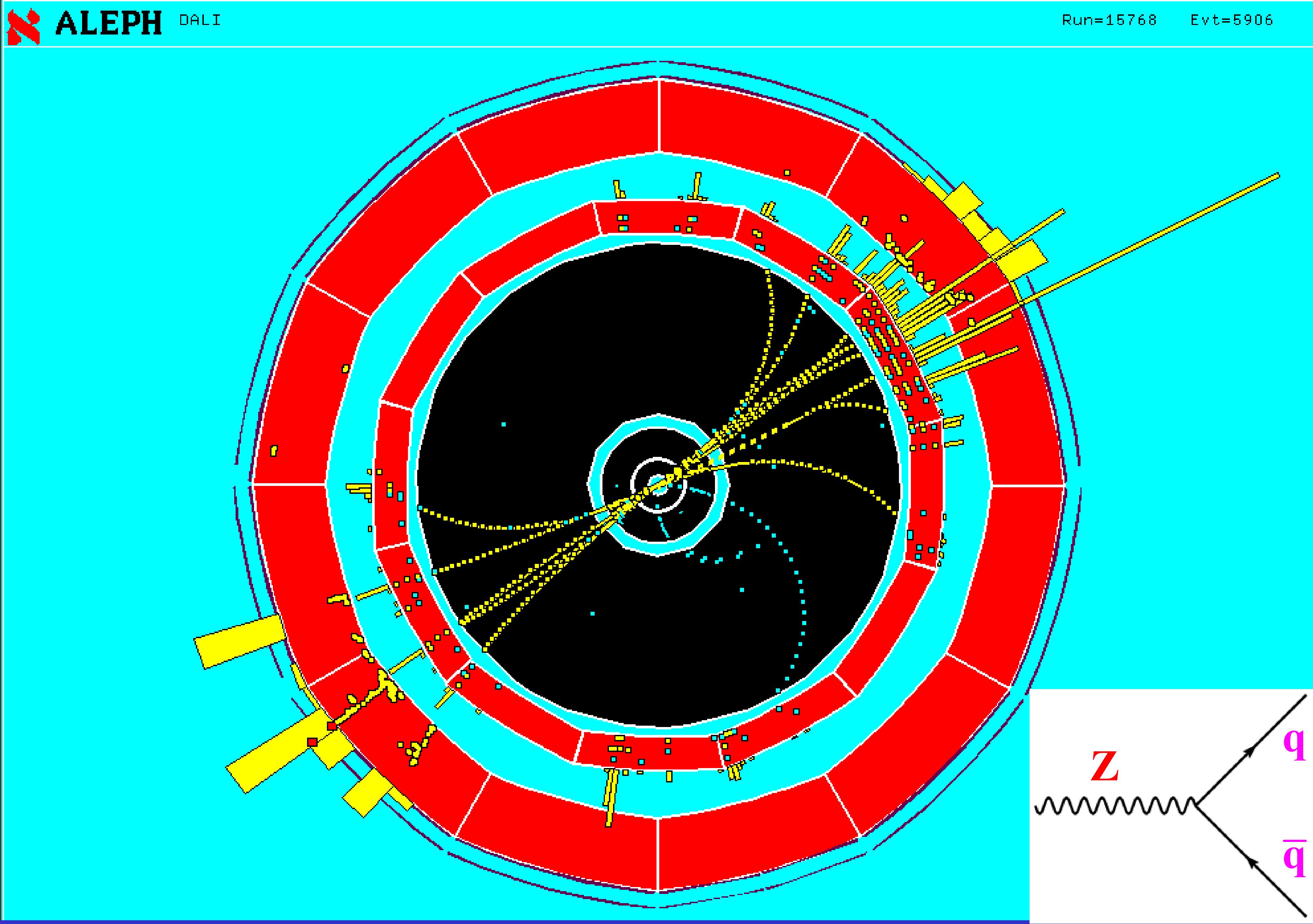
$$= -\frac{1}{4}\left(\partial^\mu G^\nu_a - \partial^\nu G^\mu_a\right)\left(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu\right) + \sum_q \bar{q}_\alpha\,[\,i\,\gamma^\mu\partial_\mu - m_q\,]\,q_\alpha$$

$$-\frac{1}{2}\,g_s\,G^a_\mu\,\sum_q\,[\,\bar{q}_\alpha\,(\lambda^a)_{\alpha\beta}\,\gamma^\mu q_\beta\,]$$

$$+\frac{1}{2}\,g_s\,f_{abc}\,(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu)\,G^\mu_b\,G^\nu_c - \frac{1}{4}\,g_s^2\,f_{abc}\,f_{ade}\,G^\mu_b\,G^\nu_c\,G^d_\mu\,G^e_\nu$$



- **Gluon Self – interactions** $\quad G^3\ ,\ G^4$
- **Universal Coupling** $\quad g_s$ $\quad$ **(No Colour Charges)**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DALI
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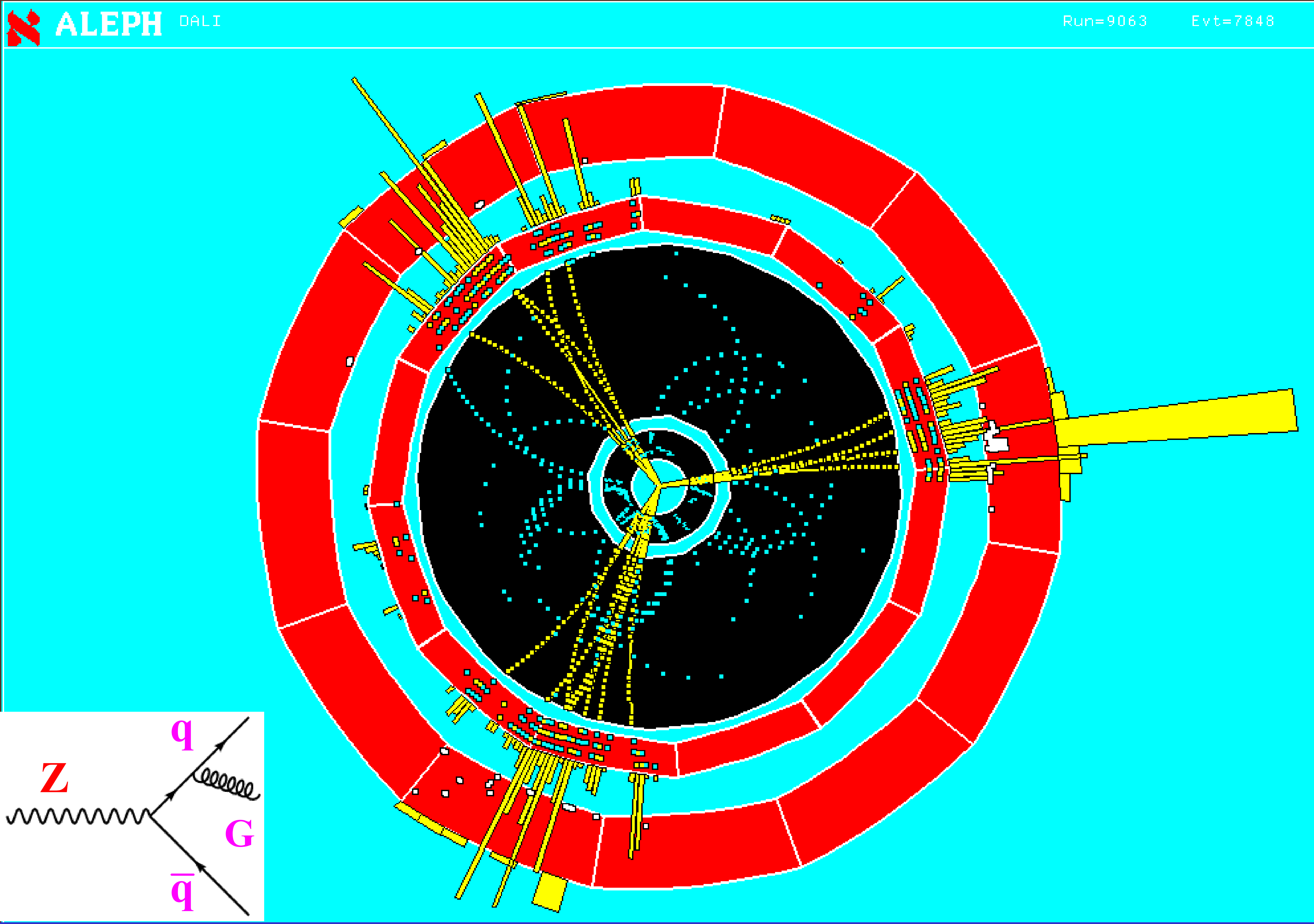
Evt=5906
Z
q
$\bar{q}$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q
Z
G
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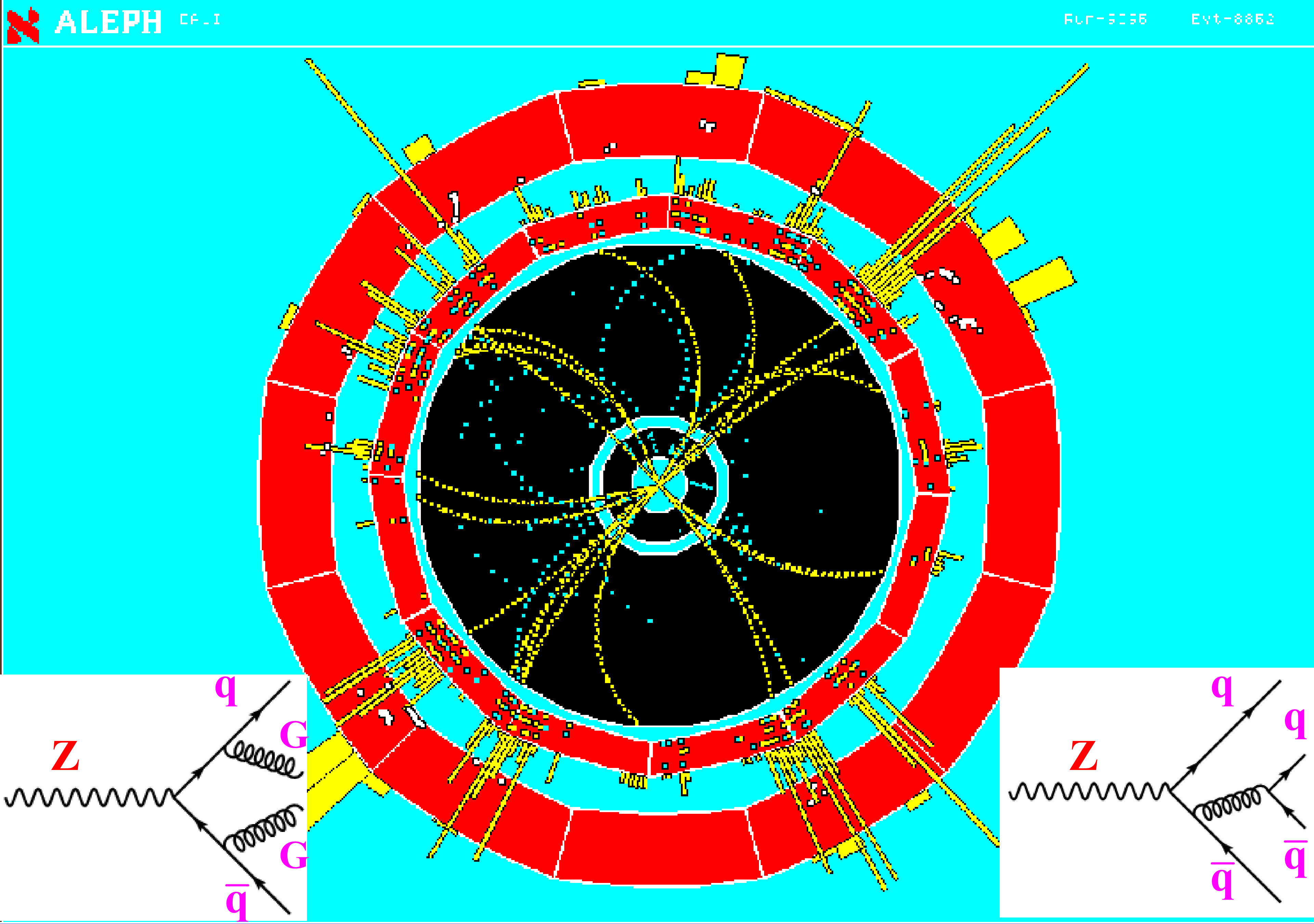
ALEPH
Z
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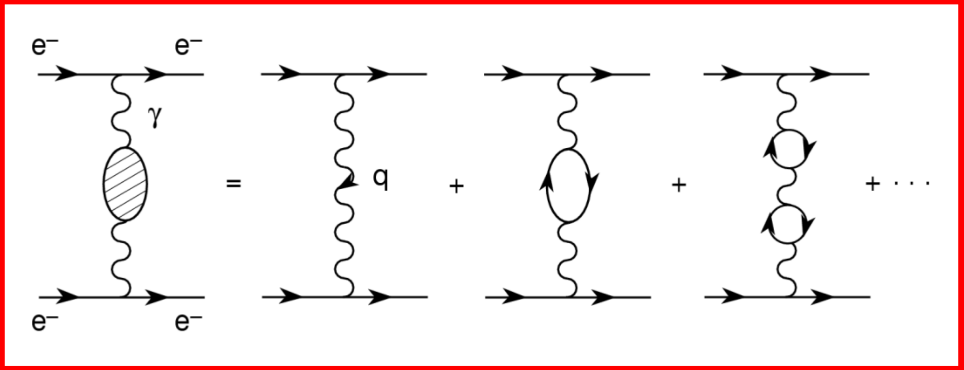
QUANTUM CORRECTIONS



$$T(Q^2) \sim \frac{\alpha}{Q^2} \left\{ 1 + \Pi(Q^2) + \Pi(Q^2)^2 + \cdots \right\} \sim \frac{\alpha(Q^2)}{Q^2}$$

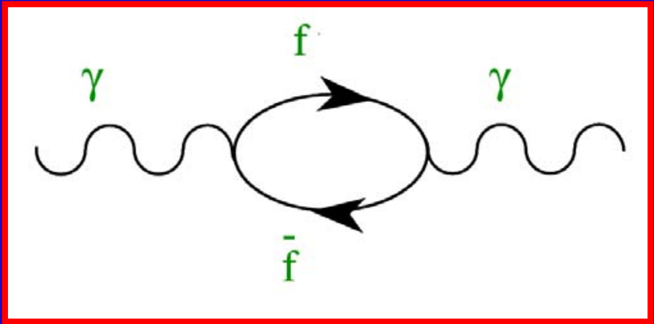
Effective ***(Running)*** **Coupling:**

$$\alpha(Q^2) = \frac{\alpha}{1 - \Pi(Q^2)} \approx \frac{\alpha}{1 - \frac{\alpha}{3\pi} \log\left(\frac{Q^2}{m^2}\right)}$$

SCREENING

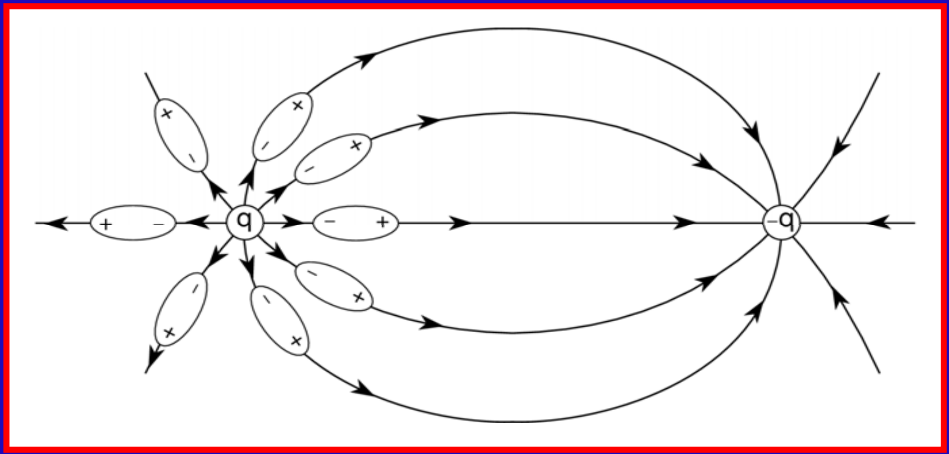
$\alpha(Q^2)$ Increases with $Q^2 \equiv -q^2$ ➡ **Decreases at Large Distances**

VACUUM POLARIZATION



The Photon Couples to *Virtual* $f\,\bar{f}$ *Pairs*

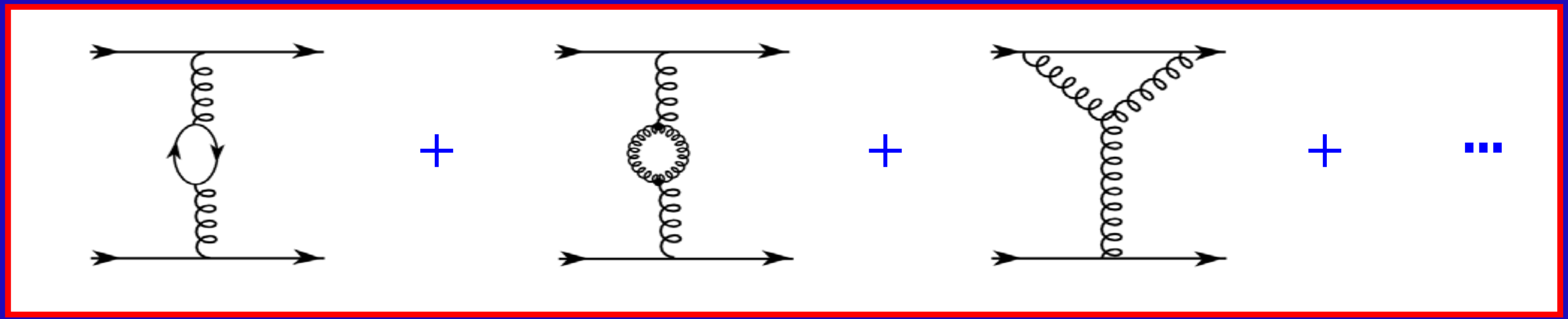
Vacuum ⟷ Polarized Dielectric Medium



$$\alpha^{-1} = \alpha(m_e^2)^{-1} = 137.035999710\ (96) \quad ; \quad \alpha(M_Z^2)^{-1} = 128.93 \pm 0.05$$

($l^- l^+$ and $q\bar{q}$ contributions included)

QCD RUNNING COUPLING



$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha_s(Q_0^2)}{2\pi} \log\left(\frac{Q^2}{Q_0^2}\right)}$$

$$\beta_1 = \frac{1}{3} N_F - \frac{11}{6} N_C$$

quarks gluons

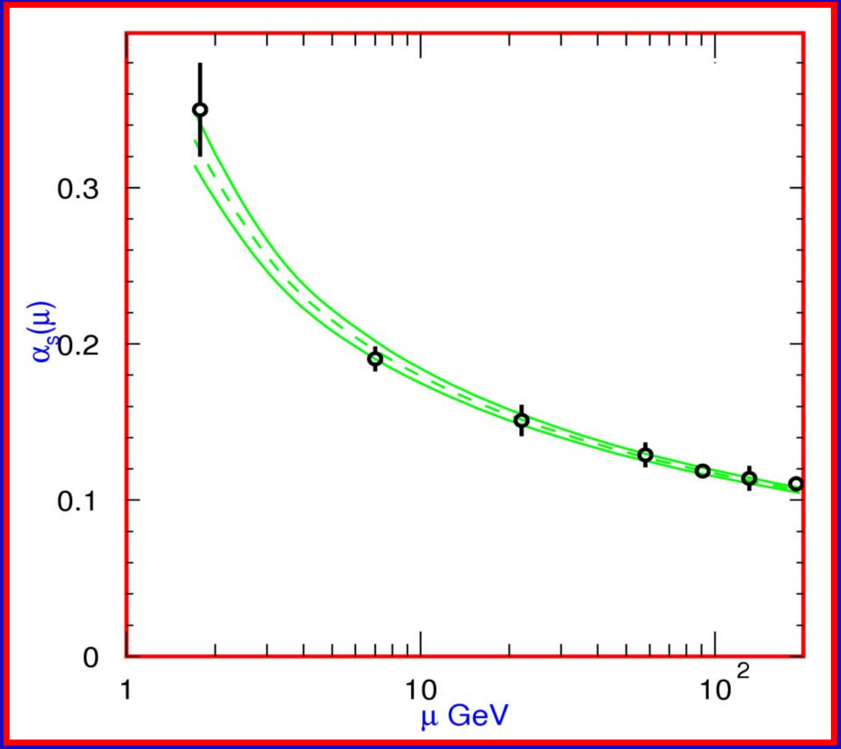
$N_C = 3$, $N_F = 6$ ➡ $\beta_1 < 0$; $Q^2 > Q_0^2$ ➡ $\alpha_s(Q^2) < \alpha_s(Q_0^2)$

$\alpha_s(Q^2)$ Decreases at Short Distances **ANTI-SCREENING**

ASYMPTOTIC FREEDOM

$$\alpha_s(M_Z^2) = 0.119 \pm 0.002$$

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha_s(Q_0^2)}{2\pi} \log\left(\frac{Q^2}{Q_0^2}\right)}$$



$$\beta_1 < 0 \quad \longrightarrow \quad \lim_{Q^2\to\infty} \alpha_s(Q^2) = 0$$

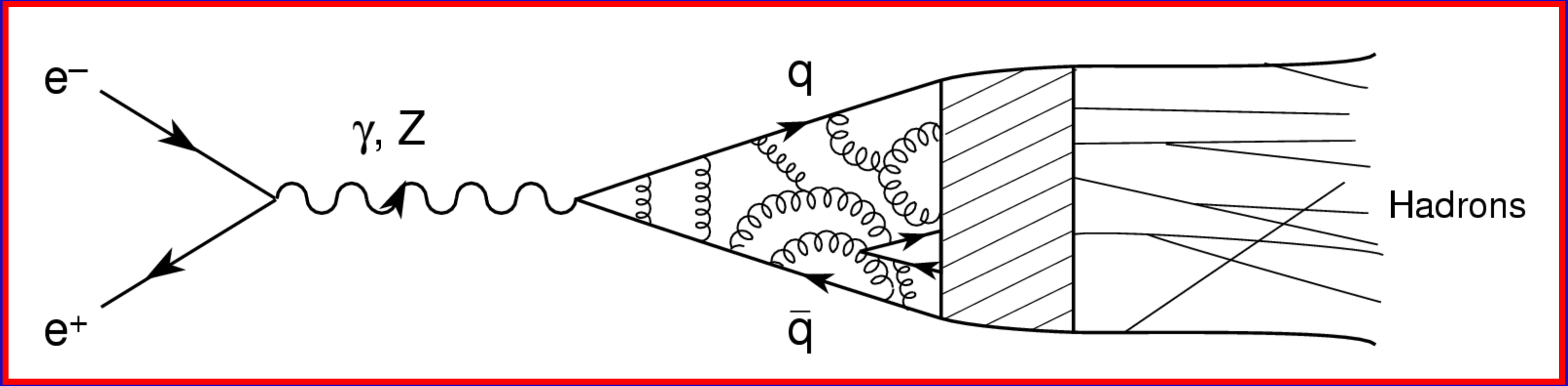
$\alpha_s(Q^2)$ increases at low energies

$\alpha_s \simeq O(1)$ at 1 GeV

Non–Perturbative Region

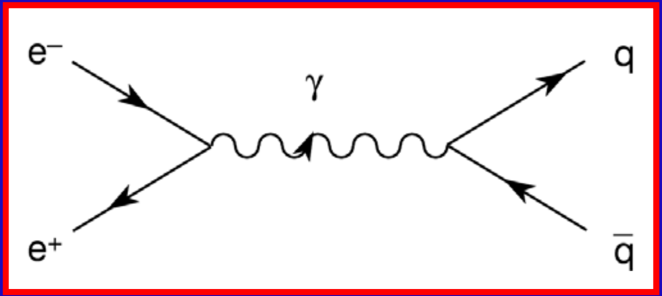
→ CONFINEMENT ?

Confinement ⟷ Probability Hadronization = 1



$$\sigma(e^+e^- \to \text{hadrons}) = \sigma(e^+e^- \to q\bar{q} + q\bar{q}G + q\bar{q}GG + q\bar{q}q\bar{q} + \cdots)$$

$$\mathbf{T}\left(e^+e^- \to q\,\bar{q}\right) \quad =$$



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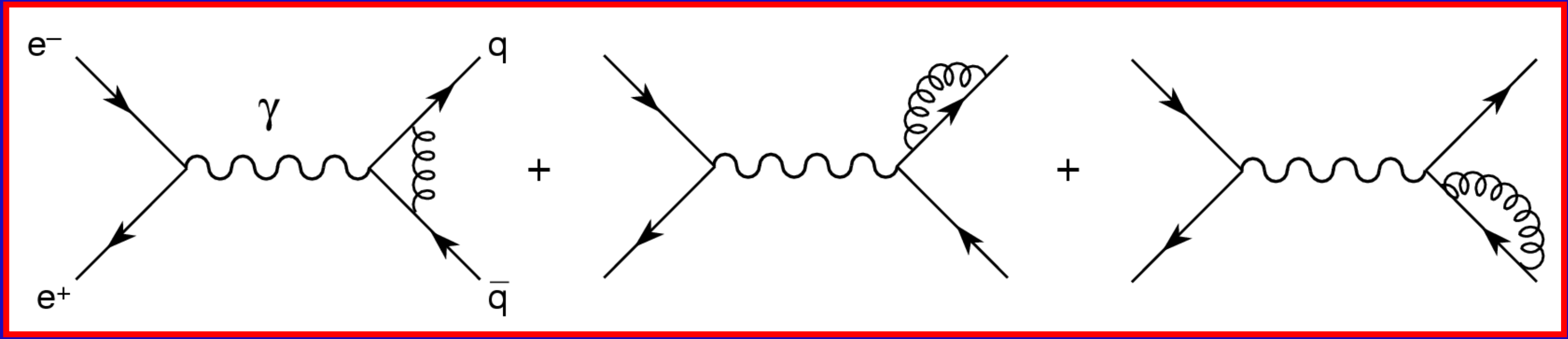
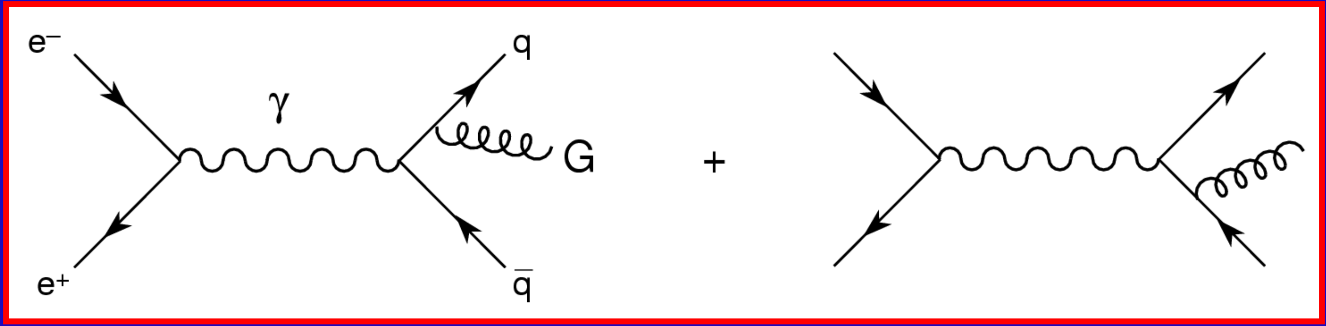
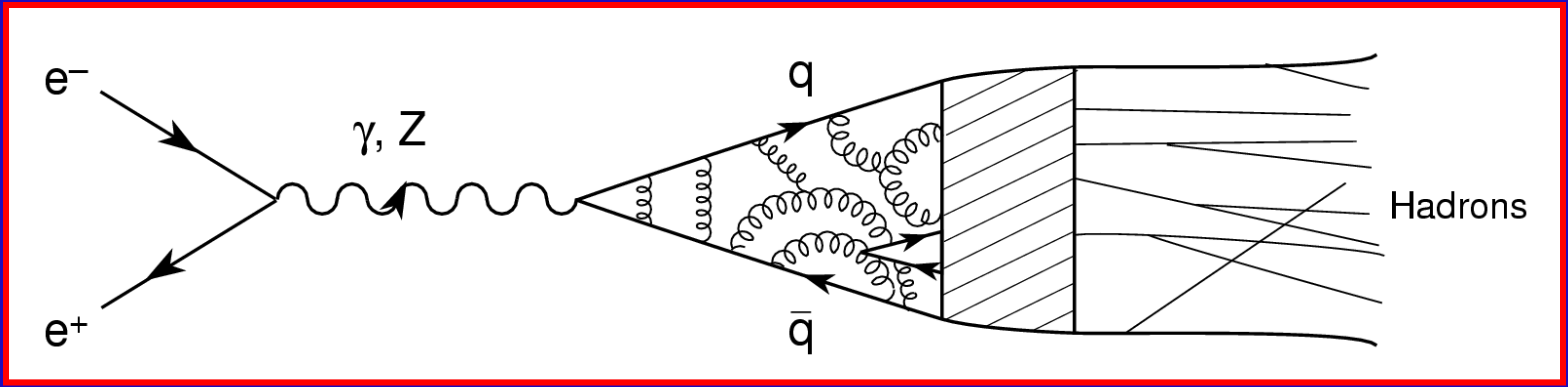


$$\mathbf{T}\left(e^+e^- \to q\,\bar{q}\,G\right) \quad =$$



Confinement ⟷ Probability Hadronization = 1

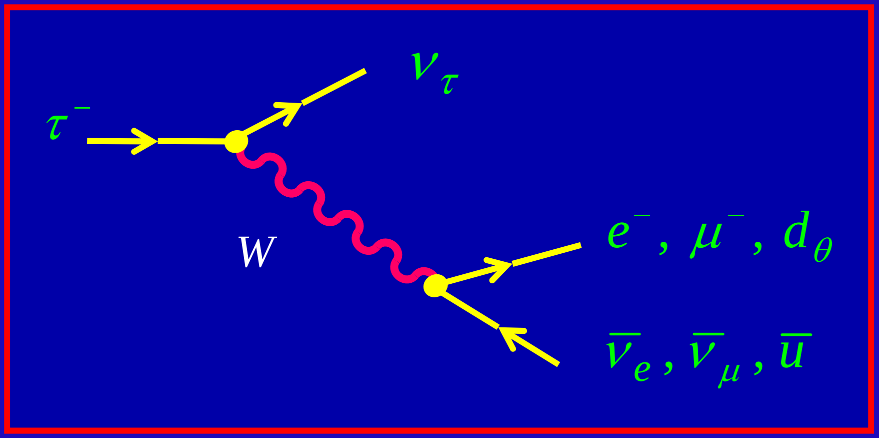


$$\sigma(e^+e^- \to \text{hadrons}) = \sigma(e^+e^- \to q\bar{q} + q\bar{q}G + q\bar{q}GG + q\bar{q}q\bar{q} + \cdots)$$

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q Q_q^2 \; N_C \; \left\{ 1 + \frac{\alpha_s(s)}{\pi} + \cdots \right\}$$

$$R_Z \equiv \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to e^+e^-)} = R_Z^{EW} \; N_C \; \left\{ 1 + \frac{\alpha_s(M_Z^2)}{\pi} + \cdots \right\}$$

$\tau^- \to \nu_\tau + \text{Hadrons}$



$$d_\theta = \cos\theta_C\, d + \sin\theta_C\, s$$

$$B_l \equiv \text{Br}(\tau^- \to \nu_\tau\, l^- \bar{\nu}_l) \approx \frac{1}{2+N_C} = \frac{1}{5} = 20\%$$

$$B_e = (17.84 \pm 0.05)\,\%$$

$$B_\mu = (17.36 \pm 0.05)\,\%$$

QCD:

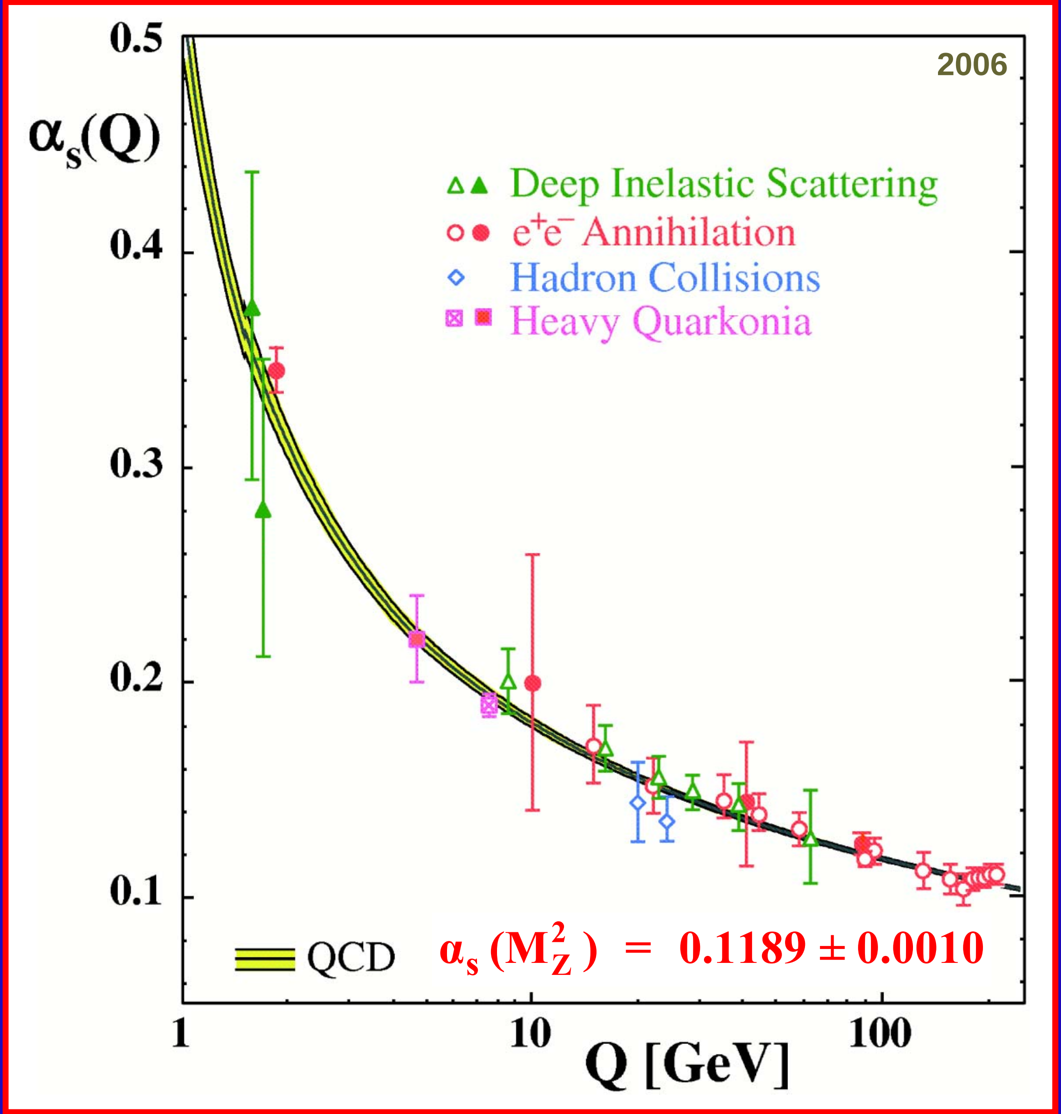
$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau\, e^- \bar{\nu}_e\right)} = N_C \left\{ 1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \cdots \right\}$$

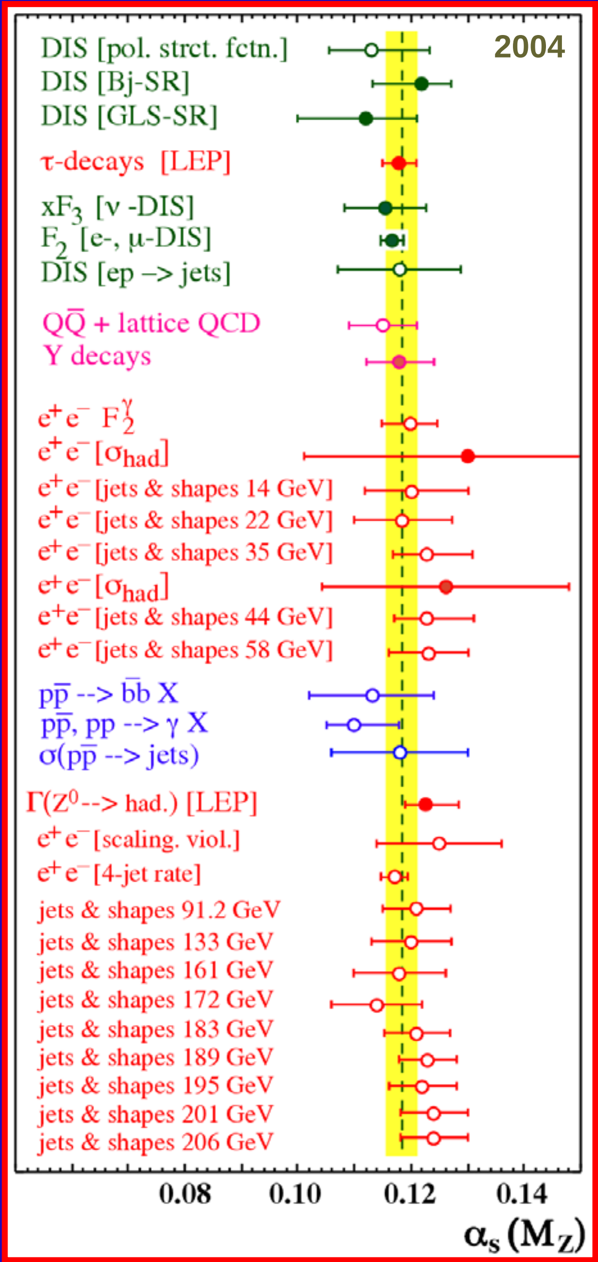
$R_\tau = 3.632 \pm 0.013$ ➡ $\alpha_s(m_\tau^2) = 0.344 \pm 0.010 > \alpha_s(M_Z^2) = 0.119 \pm 0.002$

MEASUREMENTS OF α_s

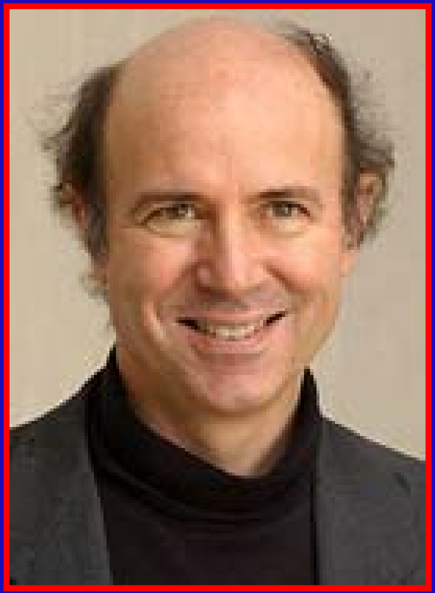
$\alpha_s(M_Z^2) = 0.1182 \pm 0.0027$

S. Bethke



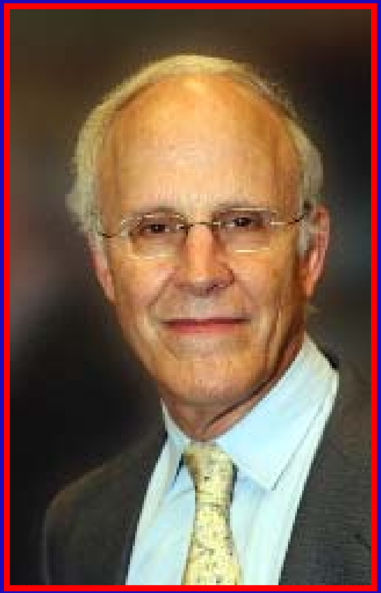


NOBEL PRIZE IN PHYSICS 2004



Frank Wilczek

Massachusetts
Institute of Technology



David Gross

Kavli Institute
University of California

David Politzer

California
Institute of Technology