

# Particle Physics

# The Standard Model

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# 5. Electroweak Unification

- **Experimental Facts**
- $SU(2)_L \otimes U(1)_Y$  **Gauge Theory**
- **Charged Current Interaction**
- **Neutral Current Interaction**
- **Gauge Self-Interactions**

# Quarks



up



down



charm



strange



top



beauty

# Leptons



electron



neutrino  $e$



muon



neutrino  $\mu$



tau



neutrino  $\tau$



photon



gluon



$Z^0$   $W^\pm$



Higgs

# CHIRALITY

Chirality Projectors:  $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix} ; (\gamma_5)^2 = I_4$

$$P_R \equiv \frac{1+\gamma_5}{2} ; P_L \equiv \frac{1-\gamma_5}{2} \quad P_R^2 = P_R ; P_L^2 = P_L ; P_R P_L = P_L P_R = 0$$

$$\psi(x) = (P_L + P_R)\psi(x) \equiv \psi_L(x) + \psi_R(x)$$

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi$$

$$= \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R - m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

# EXPERIMENTAL FACTS

Three Families

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix}, \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix}, \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

Family Structure

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \left\{ \left( \begin{matrix} \nu_l \\ l^- \end{matrix} \right)_L, (\nu_l)_R, l^-_R \right\}; \left\{ \left( \begin{matrix} q_u \\ q_d \end{matrix} \right)_L, (q_u)_R, (q_d)_R \right\}$$

Charged Currents

$$W^\pm \begin{cases} \text{Left-handed Fermions only} \\ \text{Flavour Changing: } \nu_l \Leftrightarrow l, q_u \Leftrightarrow q_d \end{cases}$$

Neutral currents

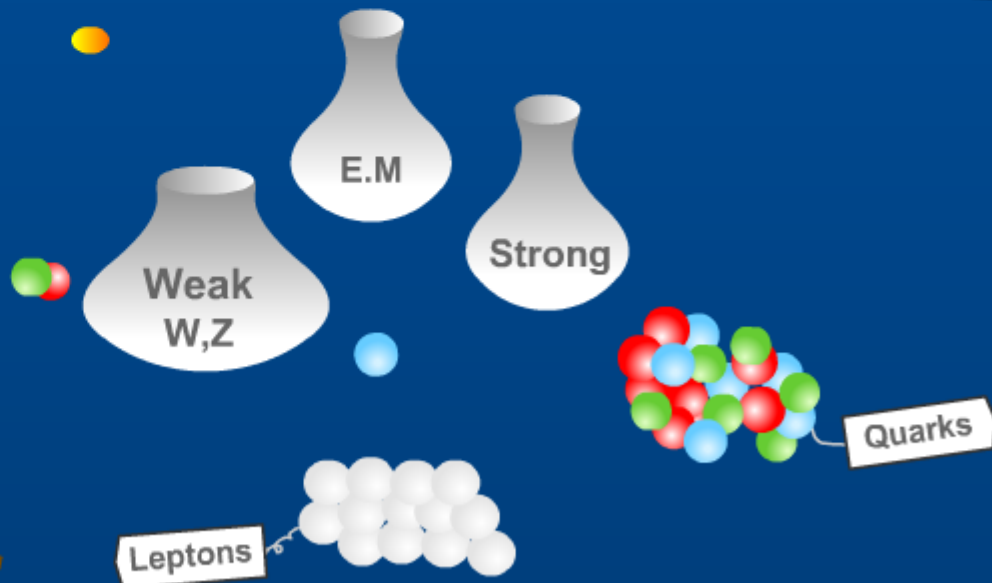
$$\gamma, Z \quad \text{Flavour Conserving}$$

Universality

(Family – Independent Couplings)

$$(\nu_l)_R \quad ?$$

# Standard Model



mer

$$SU(2)_L \otimes U(1)_Y$$

# GAUGE THEORY

Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

Free Lagrangian for Massless Fermions:

$$\mathcal{L}_0 = \sum_j i \bar{\psi}_j \gamma^\mu \partial_\mu \psi_j$$

$$SU(2)_L \otimes U(1)_Y$$

Flavour Symmetry:

$$U_L \equiv \exp \left\{ i \frac{\vec{\sigma}}{2} \vec{\alpha} \right\}$$

$$\psi_1 \rightarrow e^{i y_1 \beta} U_L \psi_1 \quad ; \quad \psi_2 \rightarrow e^{i y_2 \beta} \psi_2 \quad ; \quad \psi_3 \rightarrow e^{i y_3 \beta} \psi_3$$

$$\bar{\psi}_1 \rightarrow \bar{\psi}_1 U_L^\dagger e^{-i y_1 \beta} \quad ; \quad \bar{\psi}_2 \rightarrow \bar{\psi}_2 e^{-i y_2 \beta} \quad ; \quad \bar{\psi}_3 \rightarrow \bar{\psi}_3 e^{-i y_3 \beta}$$

# Gauge Principle:

$$\vec{\alpha} = \vec{\alpha}(x) \quad , \quad \beta = \beta(x)$$

$$\mathbf{D}_\mu \psi_1 \equiv \left[ \partial_\mu + i g \mathbf{W}_\mu(x) + i g' y_1 B_\mu(x) \right] \psi_1 \rightarrow e^{i y_1 \beta(x)} \mathbf{U}_L(x) \mathbf{D}_\mu \psi_1$$

$$\mathbf{D}_\mu \psi_k \equiv \left[ \partial_\mu + i g' y_k B_\mu(x) \right] \psi_k \rightarrow e^{i y_k \beta(x)} \mathbf{D}_\mu \psi_k \quad (k = 2, 3)$$

$$B_\mu(x) \rightarrow B_\mu(x) - \frac{1}{g'} \partial_\mu \beta(x)$$

$$\mathbf{W}_\mu(x) \rightarrow \mathbf{U}_L(x) \mathbf{W}_\mu(x) \mathbf{U}_L^\dagger(x) + \frac{i}{g} \partial_\mu \mathbf{U}_L(x) \mathbf{U}_L^\dagger(x)$$

$$\mathbf{U}_L(x) \equiv \exp \left\{ i \frac{\vec{\sigma}}{2} \vec{\alpha}(x) \right\} \quad ; \quad \mathbf{W}_\mu(x) \equiv \frac{\vec{\sigma}}{2} \vec{W}_\mu(x) \quad ; \quad \delta W_\mu^i = -\frac{1}{g} \partial_\mu (\delta \alpha^i) - \varepsilon^{ijk} \delta \alpha^j W_\mu^k$$

## 4 Massless Gauge Bosons

$$W_\mu^\pm, W_\mu^3, B_\mu^0$$



# CHARGED CURRENTS

$$\sum_j i \bar{\psi}_j \gamma^\mu D_\mu \psi_j \quad \rightarrow \quad -g \bar{\psi}_1 \gamma^\mu \mathbf{W}_\mu \psi_1 - g' B_\mu \sum_j y_j \bar{\psi}_j \gamma^\mu \psi_j$$

$$\mathbf{W}_\mu \equiv \frac{\vec{\sigma}}{2} \cdot \vec{W}_\mu = \frac{1}{2} \begin{pmatrix} W_\mu^3 & \sqrt{2} W_\mu^+ \\ \sqrt{2} W_\mu^- & -W_\mu^3 \end{pmatrix} ; \quad W_\mu \equiv (W_\mu^1 + i W_\mu^2) / \sqrt{2}$$

$$\mathcal{L}_{\text{CC}} = -\frac{g}{2\sqrt{2}} W_\mu^+ \left[ \bar{q}_u \gamma^\mu (1-\gamma_5) q_d + \bar{\nu}_l \gamma^\mu (1-\gamma_5) l \right] + \text{h.c.}$$

**Quark / Lepton Universality**

**Left – Handed Interaction**

# NEUTRAL CURRENTS

$$\mathcal{L}_{\text{NC}} = -g W_\mu^3 \bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - g' B_\mu \sum_j y_j \bar{\psi}_j \gamma^\mu \psi_j$$

Massless Fields  $\rightarrow$  Arbitrary Combination

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

$A_\mu$  has the QED Interaction IF  $g \sin \theta_W = g' \cos \theta_W = e$

$$y_1 = Q_u - \frac{1}{2} = Q_d + \frac{1}{2} \quad ; \quad y_2 = Q_u \quad ; \quad y_3 = Q_d$$

Electroweak  
Unification

$$\mathcal{L}_{\text{NC}} = -e A_\mu \sum_j \bar{\psi}_j \gamma^\mu Q_j \psi_j + \mathcal{L}_{\text{NC}}^Z$$

$$Q_1 = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix} \quad ; \quad Q_2 = Q_u \quad ; \quad Q_3 = Q_d$$

$$\mathcal{L}_{\text{NC}}^Z = - \frac{e}{\sin \theta_W \cos \theta_W} Z_\mu \left\{ \bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - \sin^2 \theta_W \sum_j \bar{\psi}_j \gamma^\mu Q_j \psi_j \right\}$$

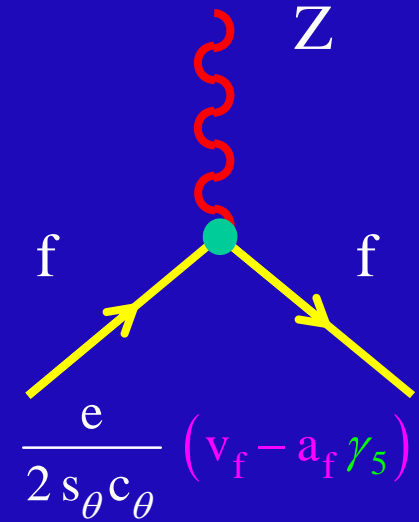
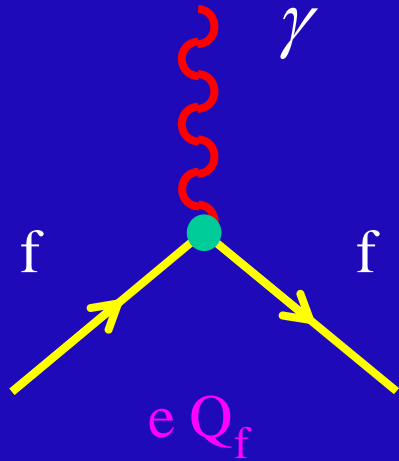
$$= - \frac{e}{2 \sin \theta_W \cos \theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu [v_f - a_f \gamma_5] f$$

	$q_u$	$q_d$	$\nu_l$	$l^-$
$2 v_f$	$1 - \frac{8}{3} \sin^2 \theta_W$	$-1 + \frac{4}{3} \sin^2 \theta_W$	1	$-1 + 4 \sin^2 \theta_W$
$2 a_f$	1	-1	1	-1

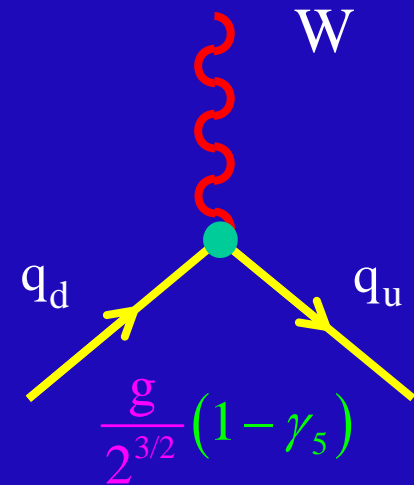
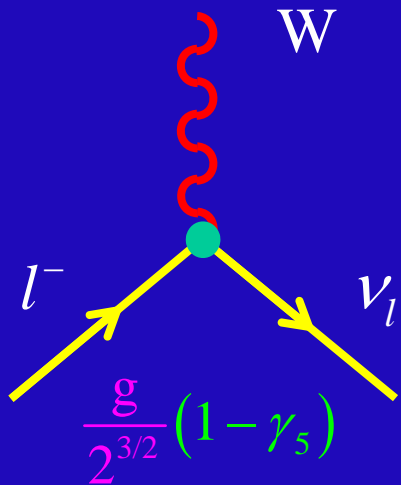
IF  $\nu_R$  do exist:  $y(\nu_R) = Q_\nu = 0 \quad \Rightarrow \quad$  No  $\nu_R$  Interactions

**Sterile Neutrinos**

# NEUTRAL CURRENTS



# CHARGED CURRENTS



$$\mathbf{W}_{\mu\nu} \equiv -\frac{i}{g} [\mathbf{D}_\mu, \mathbf{D}_\nu] \equiv \frac{\vec{\sigma}}{2} \cdot \vec{W}_{\mu\nu} \rightarrow \mathbf{U}_L \mathbf{W}_{\mu\nu} \mathbf{U}_L^\dagger \quad ; \quad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \rightarrow B_{\mu\nu}$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g \varepsilon^{ijk} W_\mu^j W_\nu^k$$

$$\mathcal{L}_K = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr}(\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}) = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu} \vec{W}_{\mu\nu} = \mathcal{L}_{\text{kin}} + \mathcal{L}_3 + \mathcal{L}_4$$

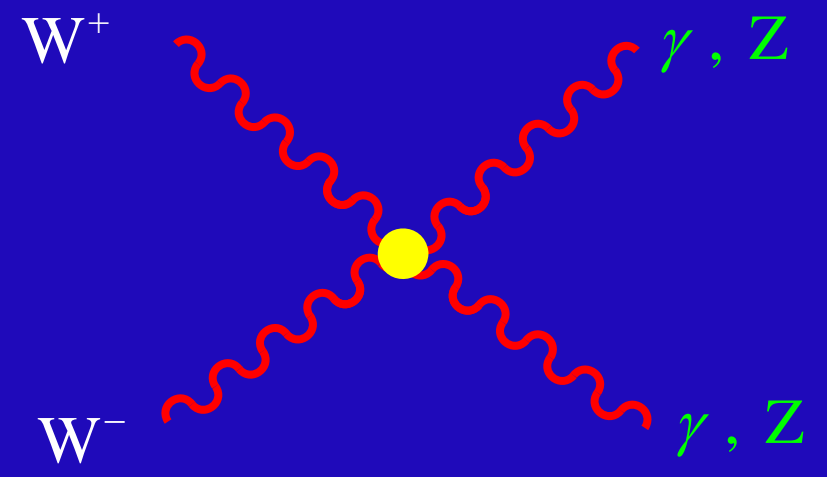
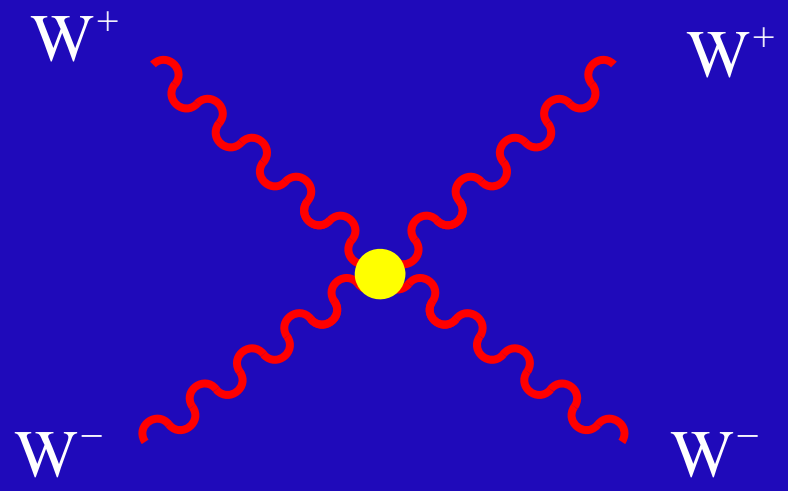
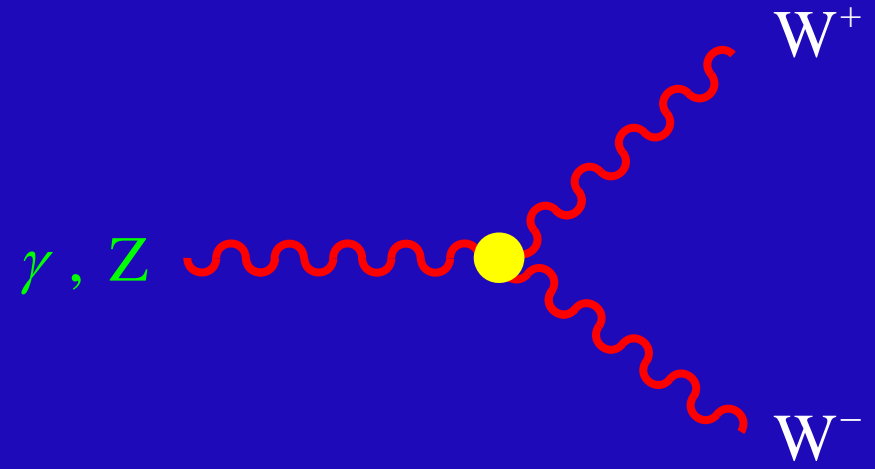
$$\mathcal{L}_3 = i e \cot \theta_W \left\{ (\partial^\mu W^\nu - \partial^\nu W^\mu) W_\mu^\dagger Z_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}) W_\mu Z_\nu + W_\mu W_\nu^\dagger (\partial^\mu Z^\nu - \partial^\nu Z^\mu) \right\}$$

$$+ i e \left\{ (\partial^\mu W^\nu - \partial^\nu W^\mu) W_\mu^\dagger A_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}) W_\mu A_\nu + W_\mu W_\nu^\dagger (\partial^\mu A^\nu - \partial^\nu A^\mu) \right\}$$

$$\mathcal{L}_4 = -\frac{e^2}{2 \sin^2 \theta_W} \left\{ (W_\mu^\dagger W^\mu)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu \right\} - e^2 \cot^2 \theta_W \left\{ W_\mu^\dagger W^\mu Z_\nu Z^\nu - W_\mu^\dagger Z^\mu W_\nu Z^\nu \right\}$$

$$- e^2 \cot \theta_W \left\{ 2 W_\mu^\dagger W^\mu Z_\nu A^\nu - W_\mu^\dagger Z^\mu W_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu Z^\nu \right\} - e^2 \left\{ W_\mu^\dagger W^\mu A_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu A^\nu \right\}$$

# GAUGE SELF-INTERACTIONS



# PROBLEM WITH MASS SCALES

Gauge Symmetry



$$m_\gamma = 0$$

Good

$$M_W = M_Z = 0$$

Bad!



$$M_W = 80.40 \text{ GeV}$$

$$M_Z = 91.19 \text{ GeV}$$

Moreover

$$\mathcal{L}_{m_f} \equiv -m_f \bar{f} f = -m_f (\bar{f}_L f_R + \bar{f}_R f_L)$$

Also Forbidden by Gauge Symmetry



$$m_f = 0$$

$\forall f$

## All Particles Massless

# Quarks



up



down



charm



strange



top



beauty

# Leptons



electron



neutrino  $e$



muon



neutrino  $\mu$



tau



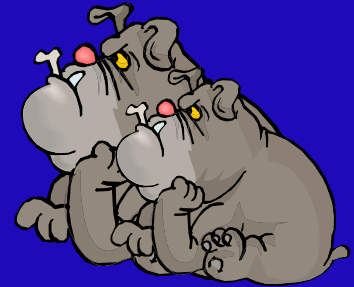
neutrino  $\tau$



photon



gluon



$Z^0$   $W^\pm$



Higgs