http://arxiv.org/pdf/0705.4264

Particle Physics
The Standard Model

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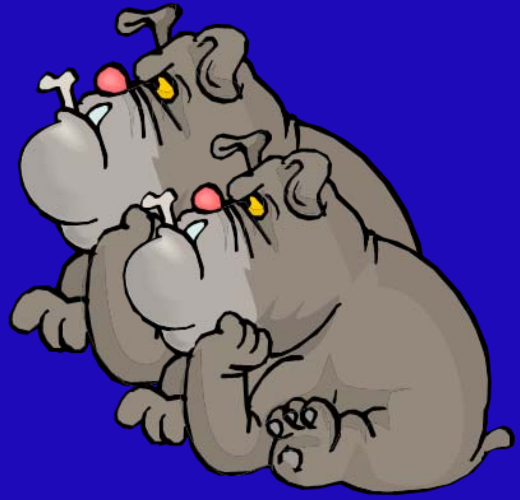
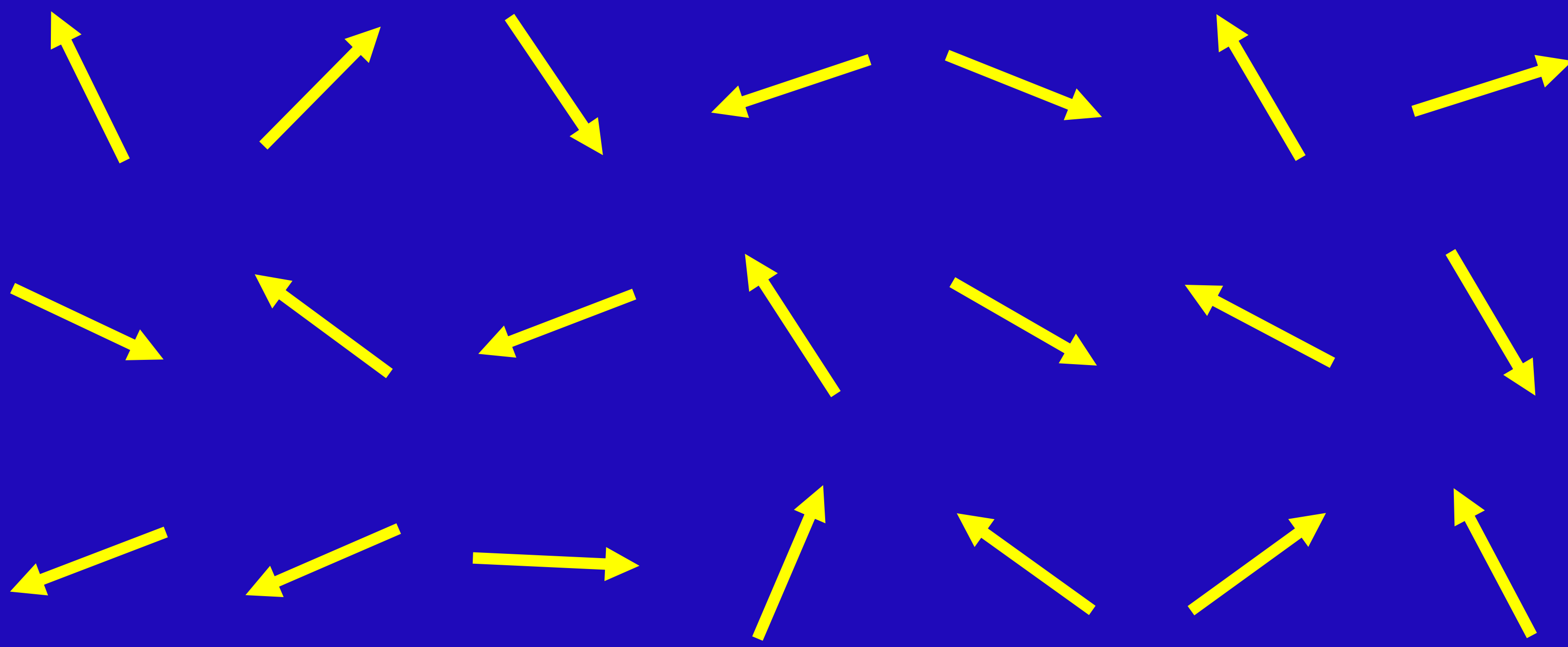
6. Symmetry Breaking

- **Spontaneous Symmetry Breaking**
- **Scalar Potential**
- **Ground State Symmetry**
- **Higgs Mechanism**
- **The Higgs Boson**
- **Fermion Masses**



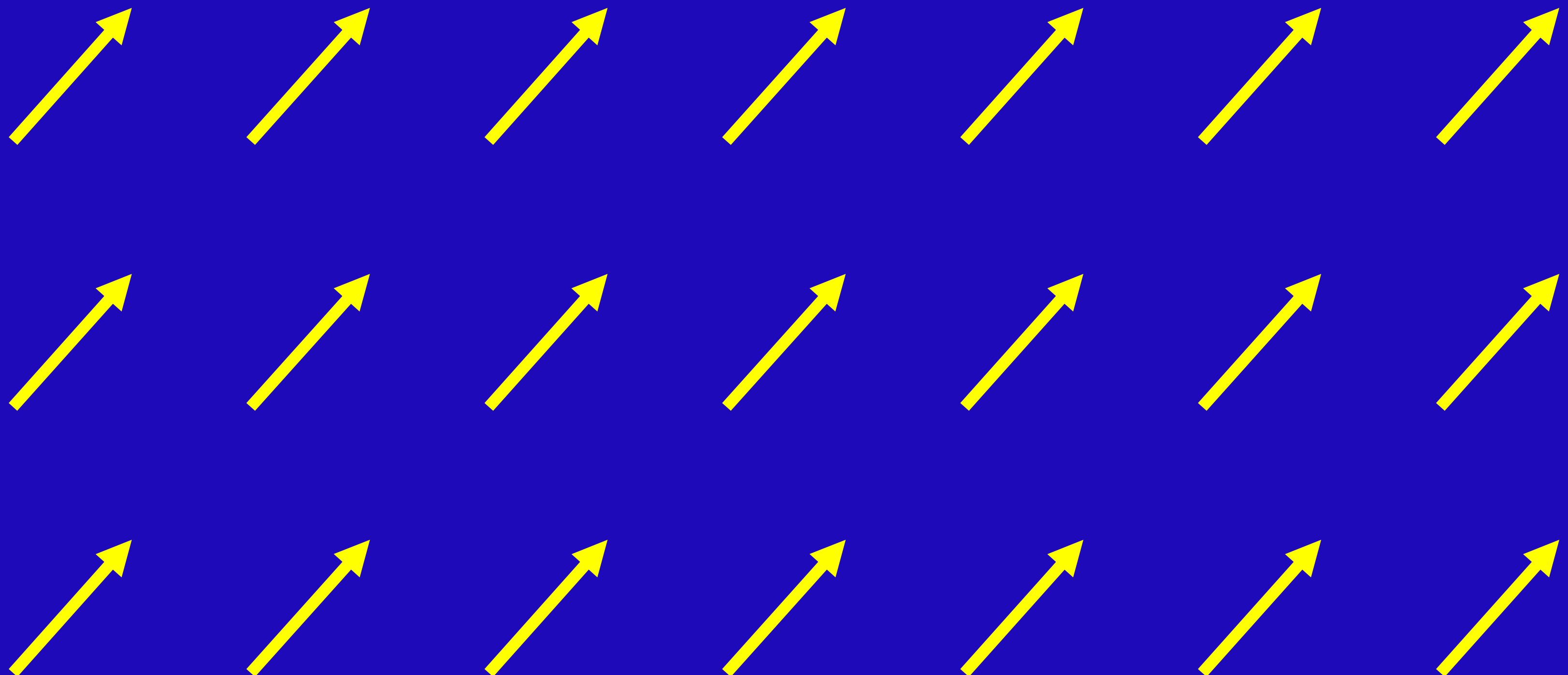
FERROMAGNET

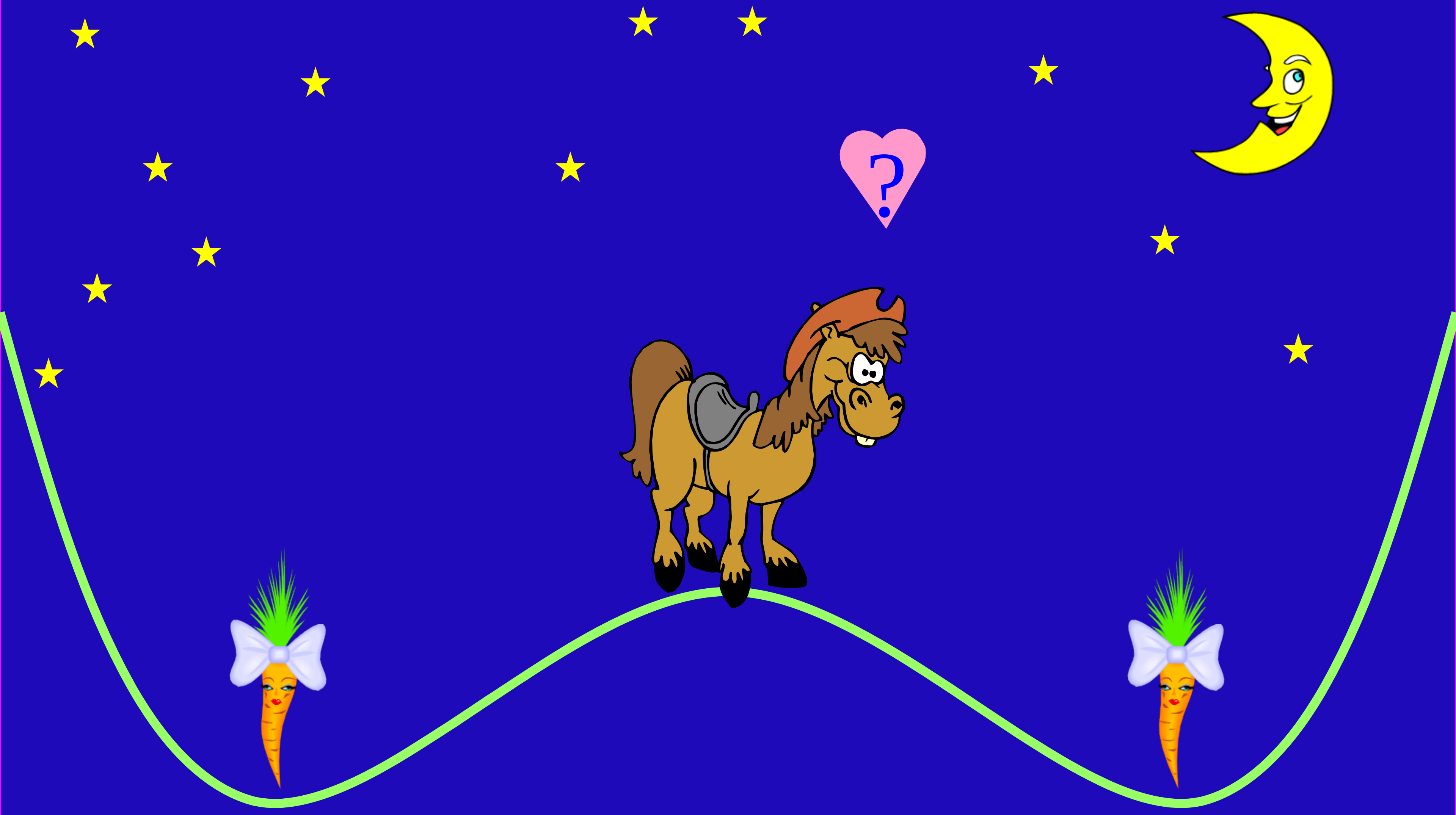
$T > T_C$

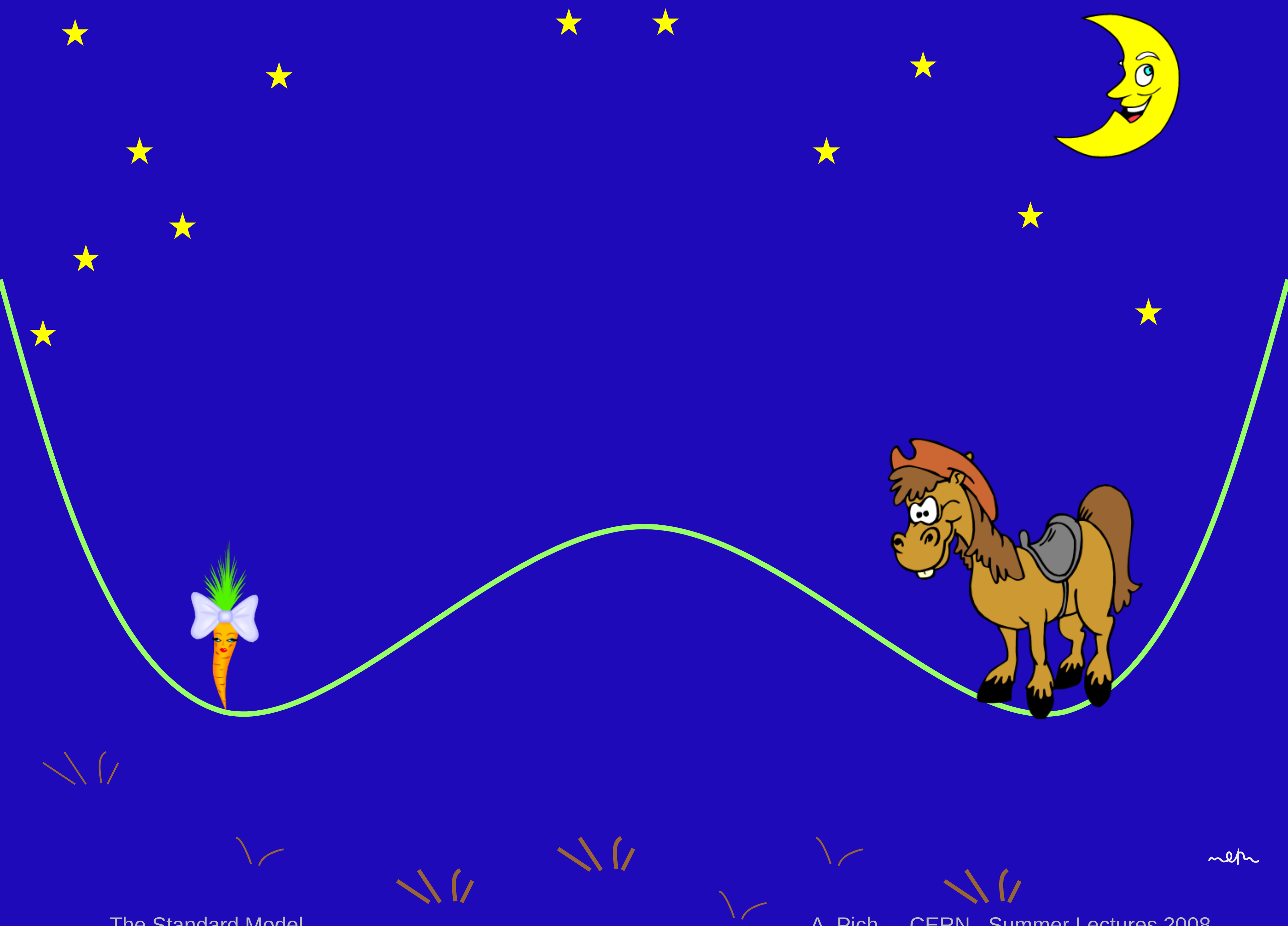


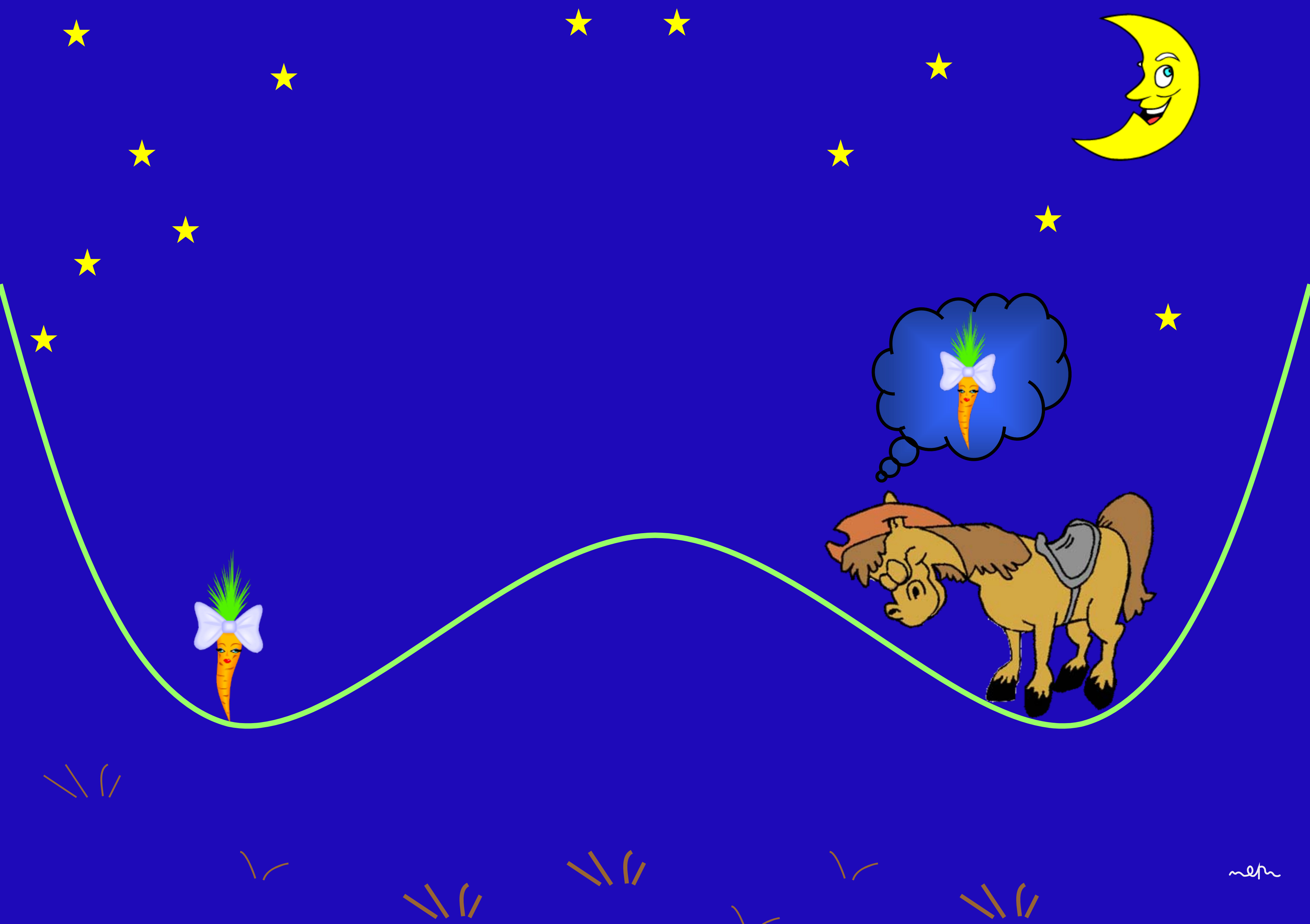
FERROMAGNET

$T < T_c$









SCALAR POTENCIAL

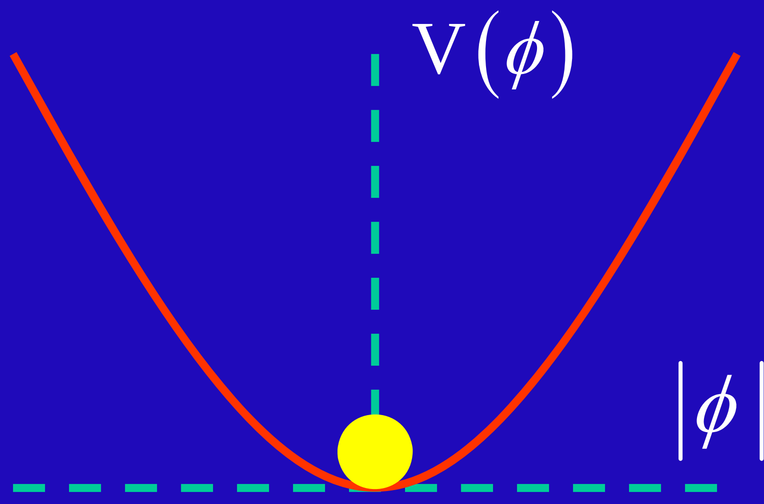
$$\mathcal{L}(\phi) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)$$

$$\mathrm{V}(\phi) = \mu^2 \, \phi^\dagger \phi + h \, (\phi^\dagger \phi)^2$$

Phase Symmetry: $\phi(x) \to \mathrm{e}^{i\theta} \, \phi(x)$

$h > 0 \quad ; \quad \mu^2 > 0$

$M_\phi = \mu$



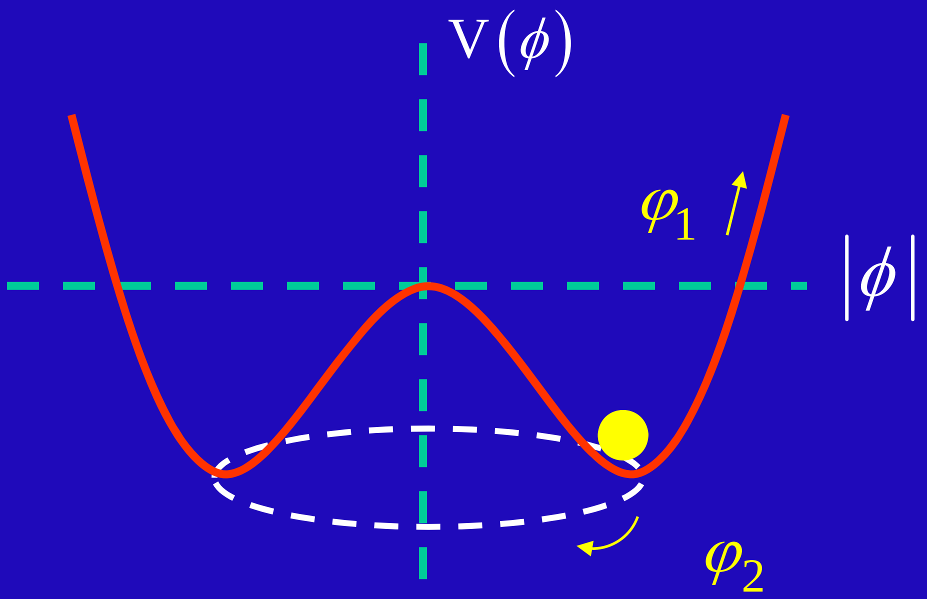
Trivial Minimum (Ground State / Vacuum): $\phi = \phi_0 = 0$

$$V(\phi) = \mu^2 \, \phi^\dagger \phi + h \, (\phi^\dagger \phi)^2$$

Phase Symmetry:

$$\phi(x) \to e^{i\theta} \, \phi(x)$$

$$\mu^2 < 0$$



Degenerate Minima

(Ground State / Vacuum)

$$|\phi_0| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}} > 0 \quad ; \quad V(\phi_0) = -\frac{1}{4} h v^4$$

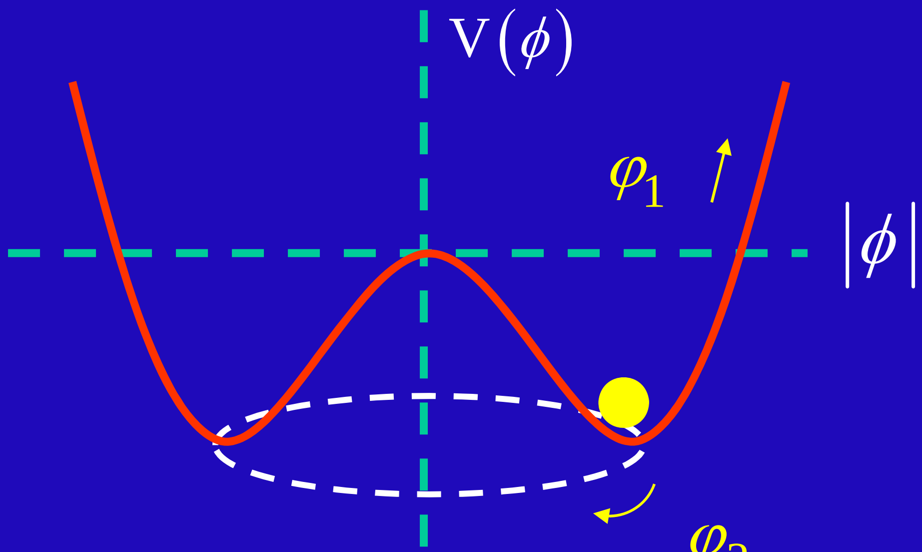
Spontaneous Symmetry Breaking:

$$\phi \equiv \frac{1}{\sqrt{2}} \left[v + \varphi_1(x) \right] e^{i \varphi_2(x)/v}$$

Vacuum Choice

Spontaneous Symmetry Breaking

$\mu^2 < 0$



$$\phi \equiv \frac{1}{\sqrt{2}}\left[\mathrm{v} + \varphi_1(x)\right]\, \mathrm{e}^{i\varphi_2(x)/\mathrm{v}}$$

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\varphi_1\,\partial^\mu\varphi_1 + \frac{1}{2}\left(1+\frac{\varphi_1}{\mathrm{v}}\right)^2 \partial_\mu\varphi_2\,\partial^\mu\varphi_2 - V(\phi)$$

$$V(\phi) = V(\phi_0) + \frac{1}{2}M^2_{\varphi_1}\,\varphi_1^2 + h\,\mathrm{v}\,\varphi_1^3 + \frac{1}{4}h\,\varphi_1^4$$

$$M^2_{\varphi_1} = -2\mu^2 > 0 \quad ; \quad M^2_{\varphi_2} = 0$$

1 Massless Goldstone Boson

ELECTROWEAK SSB

New Scalar Doublet $\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix}$; $y_\phi = Q_\phi - T_3 = \frac{1}{2}$

$$\mathcal{L}(\phi) = (\mathbf{D}_\mu \phi)^\dagger \, \mathbf{D}^\mu \phi - \mu^2 \, \phi^\dagger \phi - h \, (\phi^\dagger \phi)^2$$

$$\mathbf{D}^\mu \phi = \left[\partial^\mu + i\, g\, \mathbf{W}^\mu + i\, g'\, y_\phi \, B^\mu \right] \phi \qquad ; \qquad \mathbf{W}^\mu = \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu$$

$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ Symmetry

Degenerate Vacuum States: $(\mu^2 < 0 \; , \; h > 0)$

$$\left| \langle 0 | \, \phi^{(0)} \, | 0 \rangle \right| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{\mathrm{v}}{\sqrt{2}}$$

Spontaneous Symmetry Breaking: $\phi(x) = \exp\left\{ i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathrm{v} + H(x) \end{pmatrix}$

HIGGS MECHANISM

$$\phi(x) = \exp\left\{ i\,\frac{\vec{\tau}}{2}\cdot\vec{\theta}\,(x)\right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathrm{v} + H(x) \end{pmatrix}$$

$\mathrm{SU(2)_L}$ Invariance ➡ $\vec{\theta}(x)$ can be rotated away

Unitary Gauge: $\vec{\theta}(x) = 0$

$$(\mathrm{D}_\mu\phi)^\dagger\,\mathrm{D}^\mu\phi \;\to\; \frac{1}{2}\,\partial_\mu H\,\partial^\mu H + \frac{g^2}{4}(\mathrm{v}+H)^2 \left\{ W_\mu^\dagger W^\mu + \frac{1}{2\cos^2\theta_W}\,Z_\mu Z^\mu \right\}$$

➡
$$M_Z \cos\theta_W = M_W = \frac{1}{2}\,\mathrm{v}\,g$$

Massive Gauge Bosons

Bosonic Degress of Freedom

Massless $W^{\pm}, Z$

3 x 2 polarizations **= 6**

+

3 Goldstones $\vec{\theta}$

SSB ↓

Massive $W^{\pm}, Z$

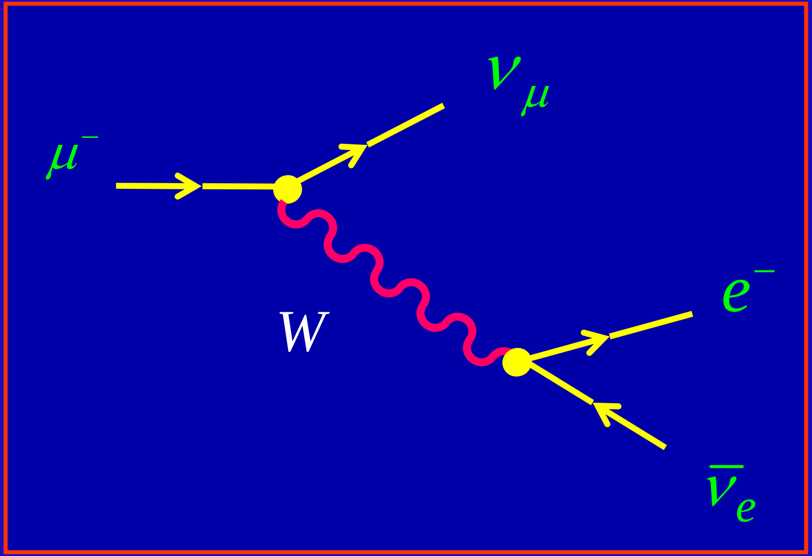
3 x 3 polarizations **= 9**

SAME PHYSICS

$$M_Z \cos\theta_W = M_W = \frac{1}{2} \, \mathrm{v} \, g$$

$$M_Z = 91.1875 \text{ GeV} > M_W = 80.398 \text{ GeV} \quad \Rightarrow \quad \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223$$

$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} \equiv 4\sqrt{2}\, G_F$$



$$\frac{1}{\tau_\mu} \equiv \Gamma = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}$$

$$G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$$

$$g = \frac{e}{\sin\theta_W} \quad , \quad M_W$$

$$\Rightarrow \quad \sin^2\theta_W = 0.215$$

$$\mathrm{v} = \left(\sqrt{2}\, G_F\right)^{-1/2} = 246 \text{ GeV}$$

THE HIGGS BOSON

$$\mathcal{L}_S = \frac{h\,\mathrm{v}^4}{4} + \mathcal{L}_H + \mathcal{L}_{HG^2}$$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H\,\partial^\mu H - \frac{1}{2}M_H^2\,H^2 - \frac{M_H^2}{2\mathrm{v}}\,H^3 - \frac{M_H^2}{8\mathrm{v}^2}\,H^4$$

$$\mathcal{L}_{HG^2} = \left[M_W^2\,W_\mu^\dagger W^\mu + \frac{1}{2}M_Z^2\,Z_\mu Z^\mu\right]\left\{1 + \frac{2}{\mathrm{v}}H + \frac{H^2}{\mathrm{v}^2}\right\}$$

1 Scalar Particle H^0 to be Discovered

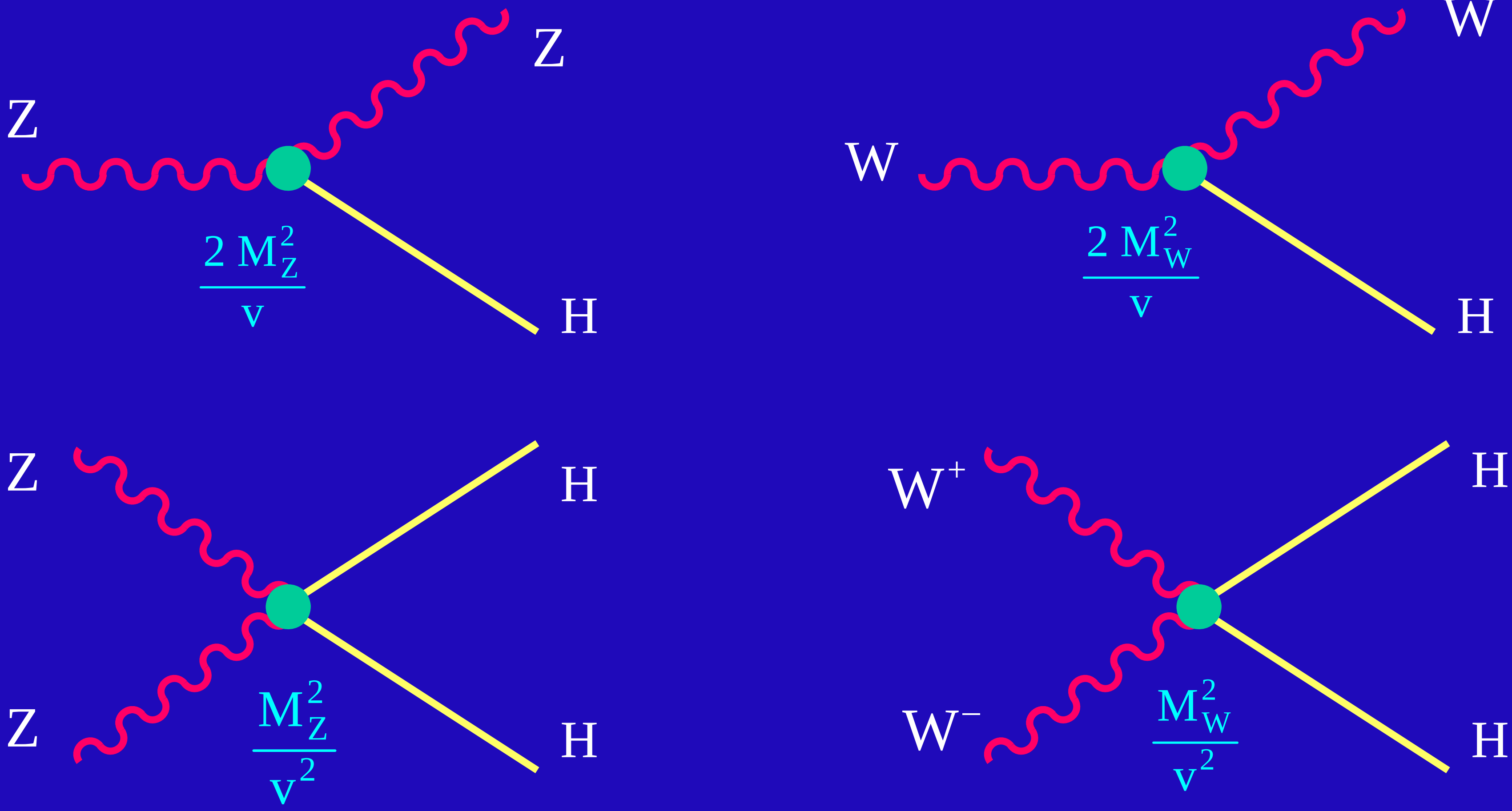
$$M_H = \sqrt{-2\mu^2} = \sqrt{2h}\,\mathrm{v}$$

Free Parameter

LEP: $114\ \mathrm{GeV} < M_H < 160\ \mathrm{GeV}$ (95% CL)

(Direct) (Indirect)

Higgs Couplings $\propto$ Masses



$$v = \left(\sqrt{2}\, G_F\right)^{-1/2} = 246 \;\; \text{GeV}$$

FERMION MASSESS

Scalar – Fermion Couplings allowed by Gauge Symmetry

$$\mathcal{L}_Y = -\,(\bar{q}_u\,,\bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] - (\bar{\nu}_l,\bar{l})_L \; c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

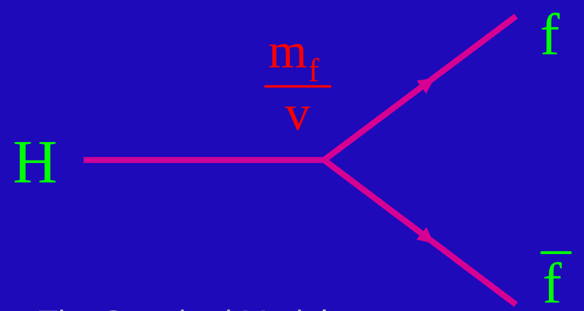
SSB ⬇

$$\mathcal{L}_Y = -\left(1+\frac{H}{\mathrm{v}}\right) \left\{ m_{q_d}\; \bar{q}_d\, q_d + m_{q_u}\; \bar{q}_u\, q_u + m_l\; \bar{l}\, l \right\}$$

Fermion Masses are

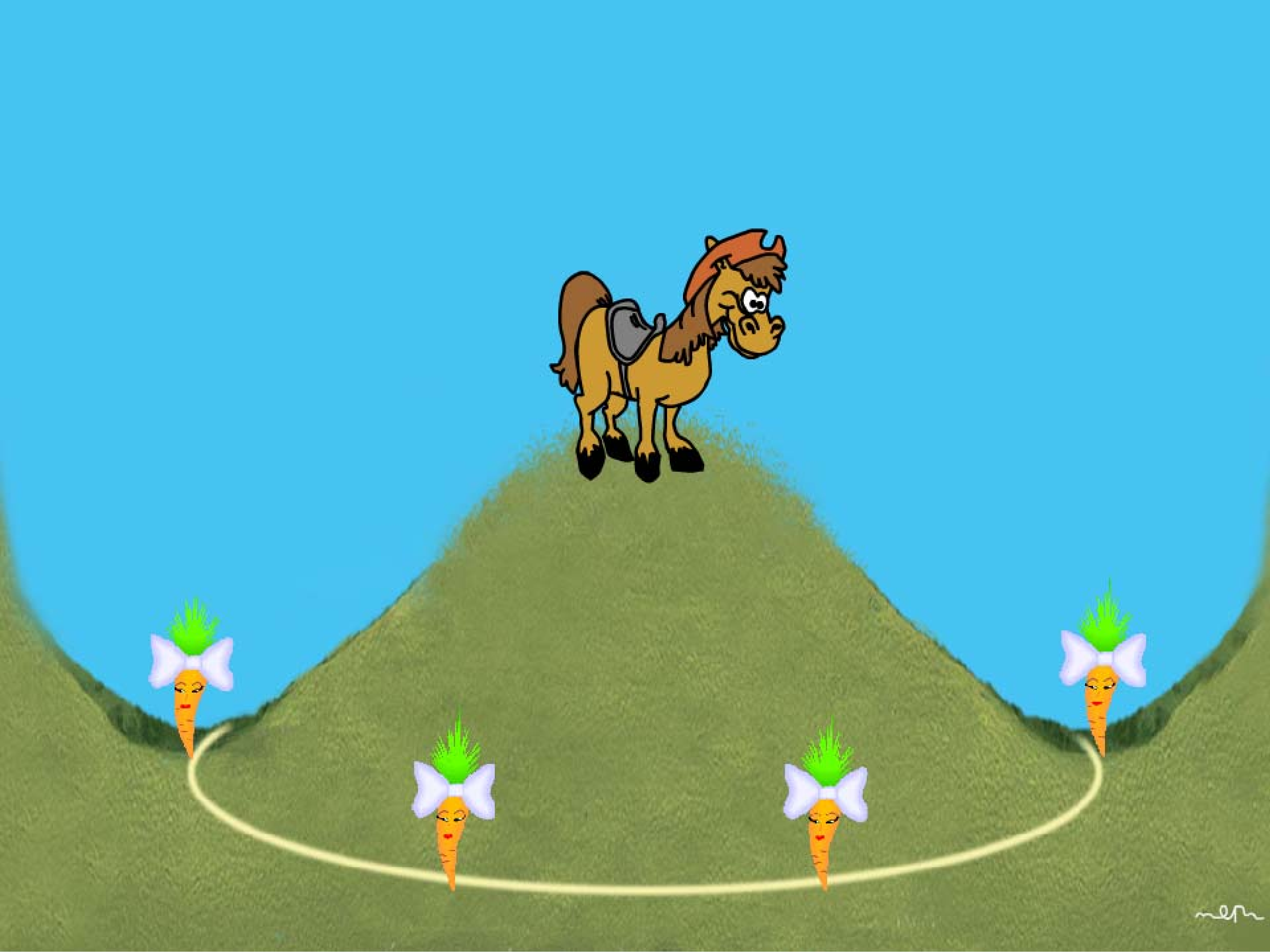
New Free Parameters

$$\left[m_{q_d}\,,\; m_{q_u}\,,\; m_l \right] = \left[c^{(d)}\,,\; c^{(u)}\,,\; c^{(l)} \right] \frac{\mathrm{v}}{\sqrt{2}}$$



Couplings Fixed: $\quad g_{Hf\bar{f}} = \frac{m_f}{v}$


Quarks
Leptons
Bosons
up
down
electron
neutrino e
e
photon
gluon
charm
strange
muon
neutrino μ
μ
top
beauty
tau
neutrino τ
τ
Z0 W±
Higgs



FERMION GENERATIONS

$N_G = 3$ Identical Copies

Masses are the only difference

$$Q=0 \qquad Q=-1 \qquad \begin{pmatrix} \nu'_j & u'_j \\ l'_j & d'_j \end{pmatrix} \qquad Q=+2/3 \qquad Q=-1/3$$

$$(j = 1, \cdots, N_G)$$

WHY ?

$$\mathcal{L}_Y = -\sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j\right)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] - \left(\bar{\nu}'_j, \bar{l}'_j\right)_L c^{(l)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

↓ **SSB**

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ \bar{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \bar{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \bar{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.} \right\}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$\left[\mathbf{M}'_d \, , \, \mathbf{M}'_u \, , \, \mathbf{M}'_l \right]_{jk} = \left[c^{(d)}_{jk} \, , \, c^{(u)}_{jk} \, , \, c^{(l)}_{jk} \right] \frac{v}{\sqrt{2}}$$

DIAGONALIZATION OF MASS MATRICES

$$\mathbf{M}'_d = \mathbf{H}_d \cdot \mathbf{U}_d = \mathbf{S}_d^\dagger \cdot \mathcal{M}_d \cdot \mathbf{S}_d \cdot \mathbf{U}_d$$
$$\mathbf{M}'_u = \mathbf{H}_u \cdot \mathbf{U}_u = \mathbf{S}_u^\dagger \cdot \mathcal{M}_u \cdot \mathrm{S}_u \cdot \mathbf{U}_u$$
$$\mathbf{M}'_l = \mathbf{H}_l \cdot \mathbf{U}_l = \mathbf{S}_l^\dagger \cdot \mathcal{M}_l \cdot \mathbf{S}_l \cdot \mathbf{U}_l$$

$$\mathbf{H}_\mathrm{f} = \mathbf{H}_\mathrm{f}^\dagger$$
$$\mathbf{U}_\mathrm{f} \cdot \mathbf{U}_\mathrm{f}^\dagger = \mathbf{U}_\mathrm{f}^\dagger \cdot \mathbf{U}_\mathrm{f} = 1$$
$$\mathbf{S}_\mathrm{f} \cdot \mathbf{S}_\mathrm{f}^\dagger = \mathbf{S}_\mathrm{f}^\dagger \cdot \mathbf{S}_\mathrm{f} = 1$$

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right)\left\{\bar{\mathrm{d}} \cdot \mathcal{M}_d \cdot \mathrm{d} + \bar{\mathrm{u}} \cdot \mathcal{M}_u \cdot \mathrm{u} + \bar{l} \cdot \mathcal{M}_l \cdot l\right\}$$

$$\mathcal{M}_u = \mathrm{diag}\,(m_u, m_c, m_t) \quad ; \quad \mathcal{M}_d = \mathrm{diag}\,(m_d, m_s, m_b) \quad ; \quad \mathcal{M}_l = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

$$\mathrm{d}_L \equiv \mathbf{S}_d \cdot \mathrm{d}'_L \quad ; \quad \mathrm{u}_L \equiv \mathbf{S}_u \cdot \mathrm{u}'_L \quad ; \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$
$$\mathrm{d}_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot \mathrm{d}'_R \quad ; \quad \mathrm{u}_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot \mathrm{u}'_R \quad ; \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

Mass Eigenstates
$\neq$
Weak Eigenstates

$$\bar{\mathrm{f}}'_L\, \mathrm{f}'_L = \bar{\mathrm{f}}_L\, \mathrm{f}_L \quad ; \quad \bar{\mathrm{f}}'_R\, \mathrm{f}'_R = \bar{\mathrm{f}}_R\, \mathrm{f}_R \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{NC}} = \mathcal{L}_{\mathrm{NC}}$$

$$\bar{\mathrm{u}}'_L\, \mathrm{d}'_L = \bar{\mathrm{u}}_L \cdot \mathbf{V} \cdot \mathrm{d}_L \quad ; \quad \mathbf{V} \equiv \mathbf{S}_u \cdot \mathbf{S}_d^\dagger \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{CC}} \neq \mathcal{L}_{\mathrm{CC}}$$

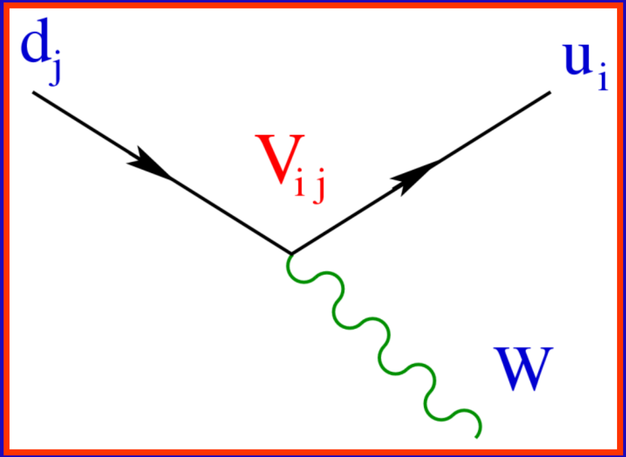
QUARK MIXING

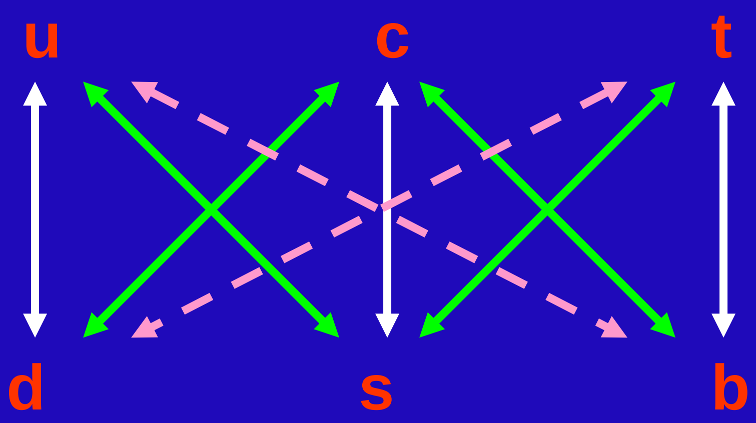
$$\mathcal{L}_{NC}^{Z} = -\frac{e}{2\sin\theta_W \cos\theta_W}\, Z_\mu \sum_f \bar{f}\,\gamma^\mu \left[v_f - a_f\,\gamma_5\right] f$$

Flavour Conserving Neutral Currents

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\, W_\mu^\dagger \left[\sum_{ij} \bar{u}_i\,\gamma^\mu (1-\gamma_5)\, \mathbf{V}_{ij}\, d_j + \sum_l \bar{\nu}_l\,\gamma^\mu (1-\gamma_5)\, l\right] + \text{h.c.}$$

Flavour Changing Charged Currents





LEPTON MIXING

$$\mathcal{L}_{CC}^{(l)} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \sum_{ij} \bar{\nu}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij}^{(l)} l_j + \text{h.c.}$$

- **IF** $m_{\nu_i} = 0$ ➡ $\mathcal{L}_{CC}^{(l)} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \sum_{l} \bar{\nu}_l \gamma^\mu (1-\gamma_5) l + \text{h.c.}$

  $\bar{\nu}_{l_j} \equiv \bar{\nu}_i \mathbf{V}_{ij}^{(l)}$

Separate Lepton Number Conservation (Minimal SM without ν_R)

- **IF** ν_R^i exist and $m_{\nu_i} \neq 0$

  $\not{L}_e , \not{L}_\mu , \not{L}_\tau$ $\quad (L_e + L_\mu + L_\tau$ Conserved$)$

BUT $\text{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$; $\text{Br}(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$

(90 % CL)

QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

- $2 N_G - 1$ **arbitrary phases:**

$$u_i \to e^{i\phi_i} u_i \quad ; \quad d_j \to e^{i\theta_j} d_j \quad \Longrightarrow \quad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)} \mathbf{V}_{ij}$$

$\Downarrow$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G (N_G - 1)$ **Moduli** ; $\frac{1}{2} (N_G - 1) (N_G - 2)$ **phases**