A very short introduction to a $SU(3)_L \otimes U(1)_N$ (331) Gauge Model (Charged Higgs Conference - 15-19,09,2008, Uppsala, Sweden)

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Abstract

The following is a simple overview of a SU(3)$_L\otimes$U(1)$_N$ (3-3-1 for short) electroweak model. The model can be better seen in References within this small introduction.

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I. OVERVIEW OF THE MODEL

The underlying eletroweak symmetry group is $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$, where N is the quantum number of the $U(1)$ group. Therefore, the left-handed lepton matter content is $\begin{pmatrix} \nu'_a & \ell'_a & \mathtt{L}'_a \end{pmatrix}^{\mathtt{T}}_L$ transforming as $(\mathbf{3}, 0)$, where $a = e, \mu, \tau$ is a family index (we are using primes for the interaction eigenstates). $\mathtt{L}'_{aL}$ are lepton fields which can be the charge conjugates ${\ell'_{aR}}^C$ [1, 2] or the antineutrinos ${\nu'_{La}}^C$ or heavy leptons P'^+_{aL} $\left(P'^+_{aL} = E'^+_L, M'^+_L, T'^+_L\right)$ [3].

The model of Ref. [3] has the simplest scalar sector of the class of 3-3-1 Models. In this version the charge operator is given by

$$\frac{Q}{e} = \frac{1}{2}\left(\lambda_3 - \sqrt{3}\lambda_8\right) + N, \tag{1}$$

where λ_3 and λ_8 are the diagonal Gell-Mann matrices and e is the elementary electric charge. The right-handed charged leptons are introduced in singlet representation of $\mathrm{SU}(3)_L$ as $\ell'^-_{aR} \sim (\mathbf{1}, -1)$ and $P'^+_{aR} \sim (\mathbf{1}, 1)$.

The quark sector is given by

$$Q_{1L} = \begin{pmatrix} u'_1 \\ d'_1 \\ J_1 \end{pmatrix}_L \sim \left(\mathbf{3}, \frac{2}{3}\right), \qquad Q_{\alpha L} = \begin{pmatrix} d'_\alpha \\ u'_\alpha \\ J'_\alpha \end{pmatrix}_L \sim \left(\mathbf{3}^*, -\frac{1}{3}\right), \tag{2}$$

where $\alpha = 2, 3$, J_1 and J_α are exotic quarks with electric charge $5/3$ and $-4/3$ respectively. It must be notice that the first quark family transforms differently from the two others under the gauge group, which is essential for the anomaly cancellation mechanism [1, 2].

The physical fermionic eigenstates rise by the transformations

$$\ell'^-_{aL(R)} = A^{L(R)}_{ab} \ell^-_{bL(R)}, \quad P'^+_{aL(R)} = B^{L(R)}_{ab} P^+_{bL(R)}, \tag{3a}$$

$$U'_{L(R)} = \mathcal{U}^{L(R)} U_{L(R)}, \quad D'_{L(R)} = \mathcal{D}^{L(R)} D_{L(R)}, \quad J'_{L(R)} = \mathcal{J}^{L(R)} J_{L(R)}, \tag{3b}$$

where $U_{L(R)} = \begin{pmatrix} u & c & t \end{pmatrix}_{L(R)}$, $D_{L(R)} = \begin{pmatrix} d & s & b \end{pmatrix}_{L(R)}$, $J_{L(R)} = \begin{pmatrix} J_1 & J_2 & J_3 \end{pmatrix}_{L(R)}$ and $A^{L(R)}$, $B^{L(R)}$, $\mathcal{U}^{L(R)}$, $\mathcal{D}^{L(R)}$, $\mathcal{J}^{L(R)}$ are arbitrary mixing matrices.

The minimal scalar sector contains the three scalar triplets

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^-_1 \\ \eta^+_2 \end{pmatrix} \sim (\mathbf{3}, 0), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (\mathbf{3}, 1), \quad \chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (\mathbf{3}, -1). \tag{4}$$

The most general, gauge invariant and renormalizable Higgs potential, which conserves the leptobaryon number [5], is

$$
\begin{aligned}
V(\eta,\rho,\chi) \;=\; & \mu_1^2\eta^\dagger\eta+\mu_2^2\rho^\dagger\rho+\mu_3^2\chi^\dagger\chi+\lambda_1\left(\eta^\dagger\eta\right)^2+\lambda_2\left(\rho^\dagger\rho\right)^2+\lambda_3\left(\chi^\dagger\chi\right)^2+\\
&+\left(\eta^\dagger\eta\right)\left[\lambda_4\left(\rho^\dagger\rho\right)+\lambda_5\left(\chi^\dagger\chi\right)\right]+\lambda_6\left(\rho^\dagger\rho\right)\left(\chi^\dagger\chi\right)+\lambda_7\left(\rho^\dagger\eta\right)\left(\eta^\dagger\rho\right)+\\
&+\lambda_8\left(\chi^\dagger\eta\right)\left(\eta^\dagger\chi\right)+\lambda_9\left(\rho^\dagger\chi\right)\left(\chi^\dagger\rho\right)+\frac{1}{2}\left(f\epsilon^{ijk}\eta_i\rho_j\chi_k+\text{H. c.}\right) \qquad (5)
\end{aligned}
$$

.

The neutral components of the scalars triplets (4) develop non zero vacuum expectation values (VEV's) $\langle\eta^0\rangle=v_\eta$, $\langle\rho^0\rangle=v_\rho$ and $\langle\chi^0\rangle=v_\chi$, with $v_\eta^2+v_\rho^2=v_W^2=(246\text{ GeV})^2$. The pattern of symmetry breaking is $\text{SU}(3)_L\otimes\text{U}(1)_N \xrightarrow{\langle\chi\rangle} \text{SU}(2)_L\otimes\text{U}(1)_Y \xrightarrow{\langle\eta,\rho\rangle} \text{U}(1)_{\text{em}}$. Therefore, we can expect $v_\chi \gg v_\eta, v_\rho$. In the potential (5), f and μ_j $(j=1,2,3)$ are constants with dimension of mass and the λ_i $(i=1,\ldots,9)$ are adimensional constants. The masses of the Higgs bosons were calculated by shifting the neutral fields of the potential (5) around its minimum as $\varphi=v_\varphi+\xi_\varphi+i\zeta_\varphi$, with $\varphi=\eta^0,\ \rho^0,\ \chi^0$ and diagonalizing the bilinear terms. These procedures are showed in Ref. [6] under the conditions $v_\chi\approx -f$, leading a following results for the masses of the neutral physical scalars

$$
m_{H_1^0}^2\approx 4\frac{\lambda_2 v_\rho^4-2\lambda_1 v_\eta^4}{v_\eta^2-v_\rho^2},\quad m_{H_2^0}^2\approx\frac{v_W^2 v_\chi^2}{2v_\eta v_\rho},\quad m_{H_3^0}^2\approx-\lambda_3 v_\chi^2,\quad m_h^2=-\frac{f v_\chi}{v_\eta v_\rho}\left[v_W^2+\left(\frac{v_\eta v_\rho}{v_\chi}\right)^2\right], \qquad (6a)
$$

for the singly charged ones

$$
m_{\pm1}^2=\frac{v_W^2}{2v_\eta v_\rho}\left(f v_\chi-2\lambda_7 v_\eta v_\rho\right),\qquad m_{\pm2}^2=\frac{v_\eta^2+v_\chi^2}{2v_\eta v_\chi}\left(f v_\rho-2\lambda_8 v_\eta v_\chi\right)\ , \qquad (6b)
$$

and for the doubly charged Higgs bosons

$$
m_{\pm\pm}^2=\frac{v_\rho^2+v_\chi^2}{2v_\rho v_\chi}\left(f v_\eta-2\lambda_9 v_\rho v_\chi\right)\ . \qquad (6c)
$$

After the process of diagonalization of the Higgs potential (5) we obtain the physical neutral scalar eigenstates H_i^0 $(i=1,2,3)$ and h which are related to the shifted fields as

$$
\begin{pmatrix}\xi_\eta\\ \xi_\rho\end{pmatrix}\approx\frac{1}{v_W}\begin{pmatrix}c_\omega & s_\omega\\ s_\omega & c_\omega\end{pmatrix}\begin{pmatrix}H_1^0\\ H_2^0\end{pmatrix},\qquad \xi_\chi\approx H_3^0,\qquad \zeta_\chi\approx ih, \qquad (7a)
$$

where $c_\omega=\cos\omega=v_\eta/\sqrt{v_\eta^2+v_\rho^2}$ and $s_\omega=\sin\omega$. For the charged physical eigenstates $H_1^\pm$, $H_2^\pm$ and $H^{\pm\pm}$ we have

$$\eta_1^+ = s_\omega H_1^+, \qquad \eta_2^+ = s_\varphi H_2^+, \qquad \rho^+ = c_\omega H_1^+, \qquad \chi^+ = c_\varphi H_2^+, \tag{7b}$$

$$\rho^{++} = s_\phi H^{++}, \qquad \chi^{++} = c_\phi H^{++}, \tag{7c}$$

with $c_\varphi=\cos\varphi=v_\eta/\sqrt{v_\eta^2+v_\chi^2}$, $s_\varphi=\sin\varphi$, $c_\phi=\cos\phi=v_\rho/\sqrt{v_\rho^2+v_\chi^2}$ and $s_\phi=\sin\phi$.

The Yukawa interactions for leptons and quarks are, respectively,

$$-\mathcal{L}_\ell = G_{ab}\overline{\psi}_{aL}\ell'_{bR}\rho + G'_{ab}\overline{\psi}_{aL}P'_{bR}\chi + \text{H. c.}, \tag{8a}$$

$$\begin{aligned}-\mathcal{L}_Q = {} & \overline{Q}_{1L}\sum_i \left[G_{1i}^u U'_{iR}\eta + G_{1i}^d D'_{iR}\rho + \sum_\alpha \overline{Q}_{\alpha L}\left(F_{\alpha i}^u U'_{iR}\rho^* + F_{\alpha i}^d D'_{iR}\eta^*\right)\right] + \\ & + G^j\overline{Q}_{1L}J_{1R}\chi + \sum_{\alpha\beta} G_{\alpha\beta}^j \overline{Q}_{\alpha L}J'_{\beta R}\chi^* + \text{H. c.}\end{aligned} \tag{8b}$$

In Eqs. (8) $a,b=e,\mu,\tau$ and $\alpha=2,3$. We are assuming the masses of exotic fermions are of order of v_χ.

Beyond the standard particles γ, Z and $W^\pm$ the model predicts, in the gauge sector, one neutral (Z'), two single charged ($V^\pm$) and two double charged ($U^{\pm\pm}$) gauge bosons. The interactions between the gauge and Higgs bosons are given by the covariant derivative

$$\mathcal{D}_\mu\varphi_i = \partial_\mu\varphi_i - ig\left(\vec{W}_\mu.\frac{\vec{\lambda}}{2}\right)_i^j\varphi_j - ig'N_\varphi\varphi_i B_\mu, \tag{9}$$

where N_φ are the U(1) charges for the φ Higgs triplets ($\varphi=\eta,\rho,\chi$). $\vec{W}_\mu$ and B_μ are field tensors of SU(2) and U(1), respectively, $\vec{\lambda}$ are Gell-Mann matrices and g and g' are coupling constants for SU(2) and U(1), respectively. Diagonalization of the covariant derivative (9), after symmetry breaking, furnishes the masses of the exotic gauge bosons, *i. e.*,

$$m_{Z'}^2 \approx \left(\frac{ev_\chi}{s_W}\right)^2\frac{2(1-s_W^2)}{3(1-4s_W^2)}, \qquad m_V^2 = \left(\frac{e}{s_W}\right)^2\frac{v_\eta^2+v_\chi^2}{2}, \quad m_U^2 = \left(\frac{e}{s_W}\right)^2\frac{v_\rho^2+v_\chi^2}{2}, \tag{10}$$

where $s_W=\sin\theta_W$.

Introducing the eigenstates (3) and (7) in the Lagrangeans (8) we obtain the Yukawa interactions as function of the physical eigenstates, *i. e.*,

$$-\mathcal{L}_{\ell p} = \frac{1}{2}\left\{\frac{1}{v_\rho}\left[c_\omega\overline{\nu}\mathcal{U}^{\nu e}H_1^+ + \left(v_\rho + s_\omega H_1^0 - c_\omega H_2^0\right)\overline{e^-} + s_\phi\overline{P^+}\mathcal{U}^{Pe}H^{++}\right]M^e G_R e^- + \right.$$

$$+\frac{1}{v_\chi}\left[c_\omega\overline{\nu}\mathcal{V}^{\nu P}H_2^- + c_\phi\overline{e^-}\mathcal{V}^{eP}H^{--} + \left(v_\chi + H_3^0 + ih\right)\overline{P^+}\right]M^E G_R P^+\Big\} +$$
$$+\text{H. c.}, \tag{11a}$$

$$-\mathcal{L}_{Qp} = \frac{1}{2}\Big\{\overline{U}G_R\left[1+\left[\frac{s_\omega}{v_\rho}+\left(\frac{c_\omega}{v_\eta}+\frac{s_\omega}{v_\rho}\right)\mathcal{V}^u\right]H_1^0+\left[-\frac{c_\omega}{v_\rho}+\left(\frac{s_\omega}{v_\eta}-\frac{c_\omega}{v_\rho}\right)\mathcal{V}^u\right]H_2^0\right]M^u U+$$
$$+\overline{D}G_R\left[1+\left[\frac{c_\omega}{v_\eta}+\left(\frac{s_\omega}{v_\rho}-\frac{c_\omega}{v_\eta}\right)\mathcal{V}^D\right]H_1^0+\left[\frac{s_\omega}{v_\eta}-\left(\frac{c_\omega}{v_\rho}+\frac{s_\omega}{v_\eta}\right)\mathcal{V}^D\right]H_2^0\right]M^d D+$$
$$+\overline{U}G_R\left[\frac{s_\omega}{v_\eta}V_{\text{CKM}}^\dagger H_1^- + \left(\frac{c_\omega}{v_\eta}-\frac{s_\omega}{v_\rho}\right)\mathcal{V}^{ud}H_1^+\right]M^d D+$$
$$+\overline{D}G_R\left[\frac{c_\omega}{v_\rho}V_{\text{CKM}}H_1^+ + \left(\frac{s_\omega}{v_\eta}-\frac{c_\omega}{v_\rho}\right)\mathcal{V}^{ud\dagger}H_1^-\right]M^u U\Big\} + \text{H. c.}, \tag{11b}$$

$$-\mathcal{L}_J = \frac{1}{2}\left[\overline{J}G_R\mathcal{J}^{L\dagger}\left(\mathcal{N}\mathcal{U}^L M^u U + \mathcal{R}\mathcal{D}^L M^d D\right)+\right.$$
$$\left.+\left(\overline{U}\mathcal{U}^{L\dagger}\mathcal{X}_1 + \overline{D}\mathcal{D}^{L\dagger}\mathcal{X}_2 + \overline{J}\mathcal{J}^{L\dagger}\mathcal{X}_0\right)\mathcal{J}^L M^J G_R J\right] + \text{H. c.}, \tag{11c}$$

where $G_R = 1+\gamma_5$, $V_L^U V_L^D = V_{\text{CKM}}$, is the Cabibbo-Kobayashi-Maskawa mixing matrix, $\mathcal{U}^{\nu e}$, $\mathcal{U}^{Pe}$, $\mathcal{V}^{\nu e}$, $\mathcal{V}^{eP}$, $\mathcal{V}^u = V_L^U \Delta V_L^{U\dagger}$, $\mathcal{V}^d = V_L^D \Delta V_L^{D\dagger}$ and $\mathcal{V}^{ud} = V_L^U \Delta V_L^{D\dagger}$ are arbitrary mixing matrices, $M^e = \text{diag}\begin{pmatrix} m_e & m_\mu & m_\tau\end{pmatrix}$, $M^P = \text{diag}\begin{pmatrix} m_E & m_M & m_T\end{pmatrix}$, $M^u = \text{diag}\begin{pmatrix} m_u & m_c & m_t\end{pmatrix}$, $M^d = \text{diag}\begin{pmatrix} m_d & m_s & m_b\end{pmatrix}$ and $M^J = \text{diag}\begin{pmatrix} m_{J_1} & m_{J_2} & m_{J_3}\end{pmatrix}$. In Eq. (11c) we have defined

$$\mathcal{N} = \begin{pmatrix} s_\omega H_2^+/v_\eta & 0 & 0 \\ 0 & s_\phi H^{--}/v_\rho & s_\phi H^{--}/v_\rho \\ 0 & s_\phi H^{--}/v_\rho & s_\phi H^{--}/v_\rho \end{pmatrix}, \quad \mathcal{X}_0 \approx \frac{v_\chi + H_3^0 + ih}{v_\chi}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1\end{pmatrix}, \tag{12a}$$

$$\mathcal{R} = \begin{pmatrix} s_\phi H^{++}/v_\rho & 0 & 0 \\ 0 & s_\omega H_2^-/v_\eta & s_\omega H_2^-/v_\eta \\ 0 & s_\omega H_2^-/v_\eta & s_\omega H_2^-/v_\eta \end{pmatrix}, \quad \mathcal{X}_1 = \frac{1}{v_\chi}\begin{pmatrix} c_\omega H_2^- & 0 & 0 \\ 0 & c_\phi H^{++} & c_\phi H^{++} \\ 0 & c_\phi H^{++} & c_\phi H^{++}\end{pmatrix}, \tag{12b}$$

$$\mathcal{X}_2 = \frac{1}{v_\chi}\begin{pmatrix} c_\phi H^{--} & 0 & 0 \\ 0 & c_\omega H_2^+ & c_\omega H_2^+ \\ 0 & c_\omega H_2^+ & c_\omega H_2^+\end{pmatrix}. \tag{12c}$$

It should be noticed that non standard field interactions violate leptonic number, as can be seen from the Lagrangians (8) and (9). However the total leptonic number is conserved [1, 2].

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