# Tree-level Vaccum stability in THDM

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NExT

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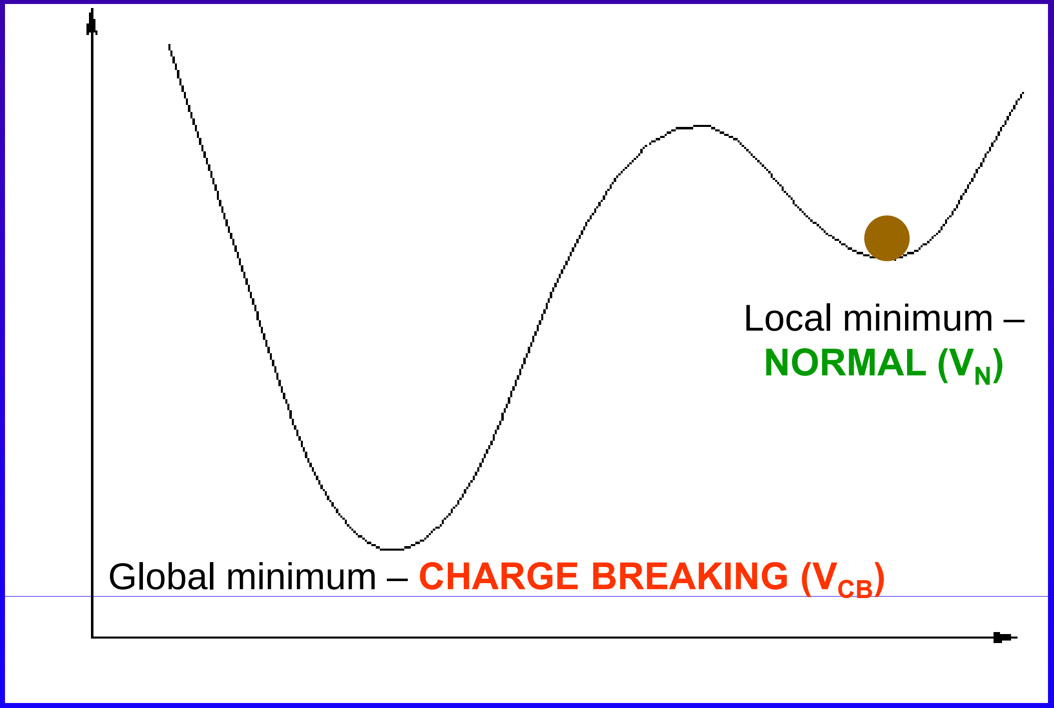
# Can the SM potential have a Normal and a CB minima simultaneously?




~~$m_\gamma = 0$~~

$$m_\gamma \neq 0 \ !$$

# Charge breaking in the SM

$$\Phi_{SM} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

**What you see in books!**

$$Q_{SM}\Phi_{SM} = \frac{1}{2}[\sigma_3 + Y]\Phi_{SM} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} = 0$$

$$\Phi^{new}_{SM} = \begin{pmatrix} v_1 + i\, v_2 \\ v_3 + i\, v_4 \end{pmatrix}$$

$$D_\mu \Phi^{new}_{SM} = \left[\partial_\mu - \frac{i}{2} g\, \sigma_a\, W^a_\mu - \frac{i}{2} g' Y\, B_\mu\right]\Phi^{new}_{SM}$$

$$(D_\mu \Phi^{new}_{SM})^+ (D_\mu \Phi^{new}_{SM})$$

**What you don't see in books!**

$$\begin{bmatrix} \frac{g^2v^2}{4} & 0 & 0 & \frac{g\,g'Y}{2}(v_1v_3 + v_2v_4) \\ 0 & \frac{g^2v^2}{4} & 0 & \frac{g\,g'Y}{2}(v_1v_4 - v_2v_3) \\ 0 & 0 & \frac{g^2v^2}{4} & \frac{g\,g'Y}{4}(v_1^2 + v_2^2 - v_3^2 - v_4^2) \\ \frac{g\,g'Y}{2}(v_1v_3 + v_2v_4) & \frac{g\,g'Y}{2}(v_1v_4 - v_2v_3) & \frac{g\,g'Y}{4}(v_1^2 + v_2^2 - v_3^2 - v_4^2) & \frac{g'^2v^2Y^2}{4} \end{bmatrix}$$

# Charge breaking in the SM

$$m_1^2 = m_2^2 = \frac{g^2 v^2}{4}$$

$$m_3^2 = \frac{v^2}{4}\left(g^2 + g'^2 Y^2\right)$$

$$m_4^2 = 0$$

**Gauge boson masses in the most general case**

Look, it's the PHOTON!

**U(1) always survives in the SM!**

Electric charge is conserved!

$$v = \sqrt{v_1^2 + v_2^2 + v_3^2 + v_4^2}$$

The Weinberg angle is the same!

## Conclusion: No Charge Breaking in the SM!

## What about CB in THDM?

$\Phi_1 \quad \Phi_2$ ← Now we have two doublets

$$\Phi_k = \begin{pmatrix} v_1^k + i\, v_2^k \\ v_3^k + i\, v_4^k \end{pmatrix}$$

8 possible vevs

$$\Phi_1 = \begin{pmatrix} v_a e^{i\theta_a} \\ v_b e^{i\theta_b} \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} v_c e^{i\theta_c} \\ v_d e^{i\theta_d} \end{pmatrix} \xrightarrow{SU(2)\otimes U(1)} \begin{pmatrix} v_2 \\ v_3 \end{pmatrix} \begin{pmatrix} 0 \\ v_1 e^{i\theta} \end{pmatrix}$$

# Charge breaking in the THDM

$\begin{pmatrix} v_2 \\ v_3 \end{pmatrix}$ $\begin{pmatrix} 0 \\ v_1 e^{i\theta} \end{pmatrix}$ ⇐ **most simple configuration**

**mass eigenvalues**

$$m_1^2 = m_2^2 = \frac{g^2 v^2}{4}$$

$$m_3^2 = \frac{1}{8}\left[ v^2\left(g^2 + g'^2 Y^2\right) + \sqrt{v^4\left(g^2 + g'^2 Y^2\right)^2 - 16\, g^2 g'^2\, v_1^2\, v_2^2 Y^2} \right]$$

$$m_4^2 = \frac{1}{8}\left[ v^2\left(g^2 + g'^2 Y^2\right) - \sqrt{v^4\left(g^2 + g'^2 Y^2\right)^2 - 16\, g^2 g'^2\, v_1^2\, v_2^2 Y^2} \right]$$

$$v = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

Look It's the… PHOTON?

**two ways to recover the photon**

$v_1 = 0$ ⇒ **SM**

$v_2 = 0$ ⇒ **ALIGNMENT**

**OR ELSE…**

# CB is possible in THDM!

## Suppose we live in a THDM!

# Are we in DANGER?!...

**Three minimum field configurations**

► NORMAL

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \text{ e } \Phi_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

► CHARGE BREAKING

$$\Phi_1 = \begin{pmatrix} 0 \\ v'_1 \end{pmatrix} \text{ e } \Phi_2 = \begin{pmatrix} \alpha \\ v'_2 \end{pmatrix}$$

► CP BREAKING

$$\Phi_1 = \begin{pmatrix} 0 \\ v'_1 + i\delta \end{pmatrix} \text{ e } \Phi_2 = \begin{pmatrix} 0 \\ v'_2 \end{pmatrix}$$

## For a safer THDM

Some algebra

$$V_{CB} - V_N = \frac{M^2_{H^\pm}}{2v^2}\underbrace{\left[(v'_1 v_2 - v'_2 v_1)^2 + \alpha^2 v_1^2\right]}_{>0}$$

$\frac{M^2_{H^\pm}}{2v^2}$ → Calculated at the Normal stationary point.

$V_N$ is a MINIMUM → $\frac{M^2_{H^\pm}}{2v^2} > 0$ → $V_N < V_{CB}$

Bonus → $V_{CB}$ is a Saddle Point

Are we in DANGER?!... → No! $V_N$ is below $V_{CB}$ and tunnelling will not happen!

# And for CP

$$V_{CP} - V_N = \frac{M_A^2}{2v^2}\underbrace{\left[\left(v''_1 v_2 - v''_2 v_1\right)^2 + \delta^2 v_1^2\right]}_{>0}$$

$\frac{M_A^2}{2v^2}$ → **Calculated at the Normal stationary point.**

**$V_N$ is a MINIMUM** → $\frac{M_A^2}{2v^2} > 0$ → $V_N < V_{CP}$

**Bonus** → **$V_{CP}$ is a Saddle Point**

# Too safe?

## However

If $V_{CP}$ is a MINIMUM

- Any competing N SP is a saddle point above it
- Any competing CB SP is a saddle point above it

**The THDM build on $V_{CP}$ is also safe!**

# No comments!

For charge breaking

$$V_{CB} - V_N = \frac{M_{H^\pm}^2}{2v^2}\left[(v'_1 v_2 - v'_2 v_1)^2 + \alpha^2 v_1^2\right]$$

For CP breaking

$$V_{CP} - V_N = \frac{M_A^2}{2v^2}\left[(v''_1 v_2 - v''_2 v_1)^2 + \delta^2 v_1^2\right]$$

For 2 competing normal minima

$$V_{N_2} - V_{N_1} = \frac{1}{2}\left\{\left(\frac{M_{H^\pm}^2}{v^2}\right)_{N_1} - \left(\frac{M_{H^\pm}^2}{v^2}\right)_{N_2}\right\}\left[(v''_1 v_2 - v''_2 v_1)^2 + \delta^2 v_1^2\right]$$

# Pause! The global picture

1. THDM bounded from below have no maxima.
2. THDM have at most two minima.
3. Minima of different nature never coexist.
4. CB and CP minima are uniquely determined.
5. If a THDM has only one normal minimum than this is the absolute minimum – all other if they exist are saddle points and above it.

**Ivan Ivanov has helped to complete the picture! Proved 2. and the part of 3. we didn't prove - that when the CP SP was above the Normal minimum it was always a saddle point – we only proved the reverse statement which is obvious from the mass expressions**

But how many normal minima?

# Normal minima

Two competing normal minima

$$V_{N_2} - V_{N_1} = \frac{1}{2}\left\{ \left( \frac{M_{H^\pm}^2}{v^2} \right)_{N_1} - \left( \frac{M_{H^\pm}^2}{v^2} \right)_{N_2} \right\} \left[ \left( v''_1 v_2 - v''_2 v_1 \right)^2 + \delta^2 v_1^2 \right]$$

The potential with the largest charged Higgs mass wins!

# Two competing N minima?

So, if

$$\left(\frac{M_{H^\pm}^2}{v^2}\right)_{N_1} = \left(\frac{M_{H^\pm}^2}{v^2}\right)_{N_2}$$

$$V_{N_2} = V_{N_1}$$

$$\left(M_h^2\right)_{N_1} \neq \left(M_h^2\right)_{N_2}$$

**Equal but different?**

Can we have 2 Normal minima with the same depth, with equal charged Higgs masses but with some of the other masses different?

No!

# The potential we love

$$V = m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - m_{12}^2 \left[\phi_1^\dagger \phi_2 + h.c.\right] + \frac{\lambda_1}{2}\left(\phi_1^\dagger \phi_1\right)^2$$

$$+ \frac{\lambda_2}{2}\left(\phi_2^\dagger \phi_2\right)^2 + \lambda_3\left(\phi_1^\dagger \phi_1\right)\left(\phi_2^\dagger \phi_2\right) + \lambda_4\left(\phi_1^\dagger \phi_2\right)\left(\phi_2^\dagger \phi_1\right) + \frac{\lambda_5}{2}\left\{\left(\phi_1^\dagger \phi_2\right)^2 + h.c.\right\}$$

The potential has 8 parameters:

It has a softly broken Z2 symmetry

It can break CP

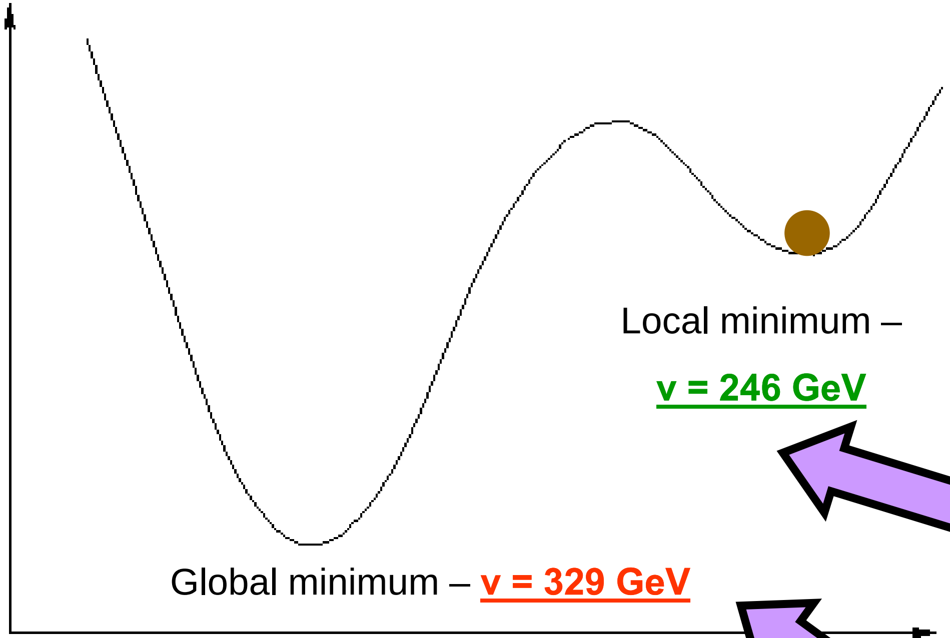
It can break charge

But once we are at a Normal minimum, CB and CP will not occur. What about two normal minima for the same parameter set?

# Bad news?

Is there a Normal minimum below?

~~$m_W = 80.4 \text{ GeV}$~~

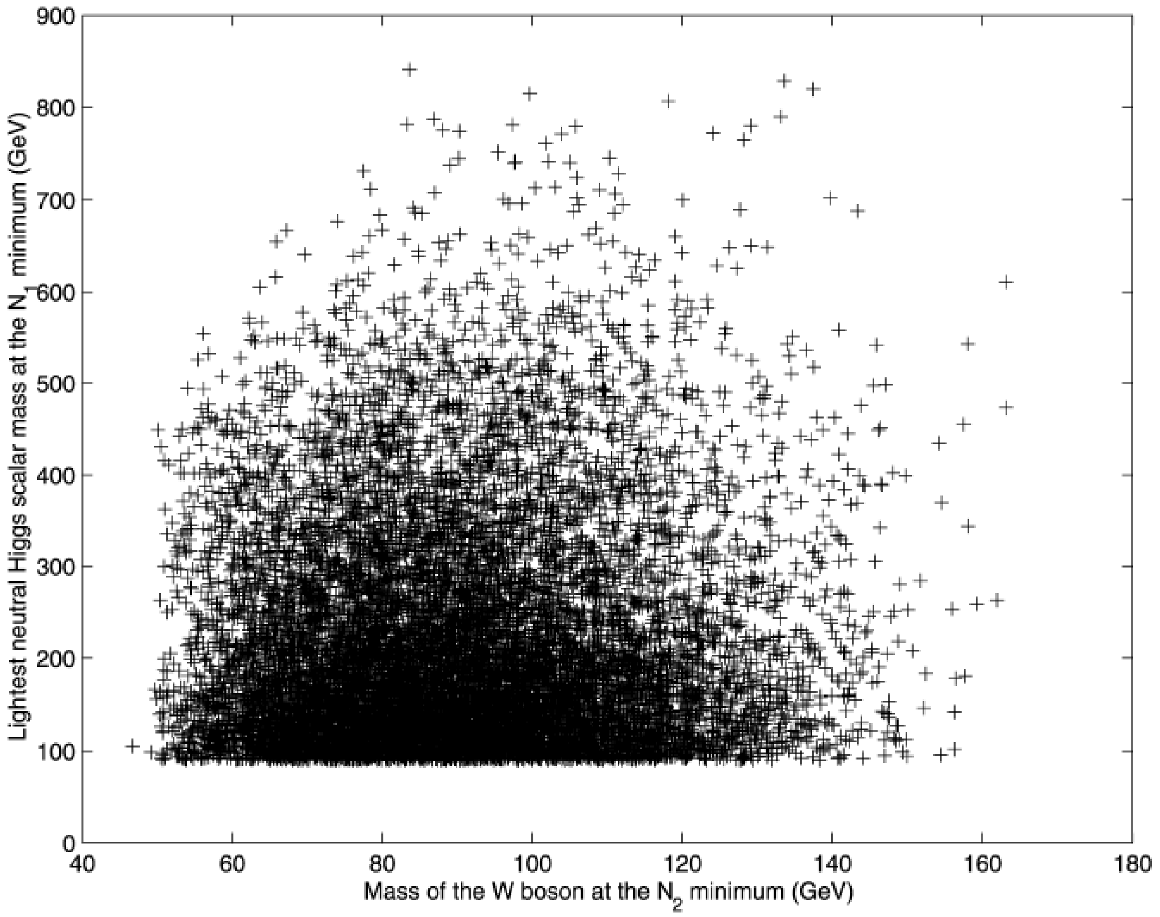



$m_A = m_H = 436 \text{ GeV}$

$m_{H^\pm} = 365 \text{ GeV}$

$m_h = 190 \text{ GeV}$

# Well…




No pattern

No bounds

$N_1$ is "our" minimum

$N_2$ is the other minimum

**Note however that this potential is fine for CP-violating minima- in this case the minimum is uniquely determined!**

# Exact $Z_2$ - the perfect THDM for Normal minima

$$m_{12}^2 = 0 \Rightarrow \begin{cases} v_1^2 = \dfrac{2\lambda_2 m_1^2 - (\lambda_3+\lambda_5)m_2^2}{(\lambda_3+\lambda_5)^2 - 4\lambda_1\lambda_2} \\ v_2^2 = \dfrac{2\lambda_1 m_2^2 - (\lambda_3+\lambda_5)m_1^2}{(\lambda_3+\lambda_5)^2 - 4\lambda_1\lambda_2} \end{cases}$$

$$\begin{cases} v_1^2 = 0 \\ v_2^2 = -\dfrac{m_2^2}{2\lambda_2} \end{cases} \Rightarrow m_h^2 \propto \frac{2\lambda_1 m_2^2 - (\lambda_3+\lambda_5)m_1^2}{2\lambda_1}$$

Just one! Normal minimum

$$m_h^2 < 0$$

Using the condition for the potential to be bounded from below we show that it is always above and it is a saddle point

# $Z_2$, CP and FCNC and U(1)

Force the CP minima not to exist (minimum equations have no solution).

2 easy ("natural") ways of avoiding that the CP minimum equations have a solution:

- $m_{12}=0$ and $\lambda_5 \neq 0$ $(\Phi_1 \to -\Phi_1 ; \Phi_2 \to \Phi_2)$
- $\lambda_5 = 0$ and $m_{12} \neq 0$ $(\Phi_1 \to e^{i\theta} \Phi_1 ; \Phi_2 \to \Phi_2)$ softy broken (otherwise axion)

**Two 7-parameters models** Z2 and global U(1)

Extending the symmetry to the fermions

Implies flavour conservation at tree level.

# Conclusions I

When it exists, the N minimum is always below. The CB SP is a saddle point.

**The very quiet world of the (tree-level) two-Higgs doublet model!**

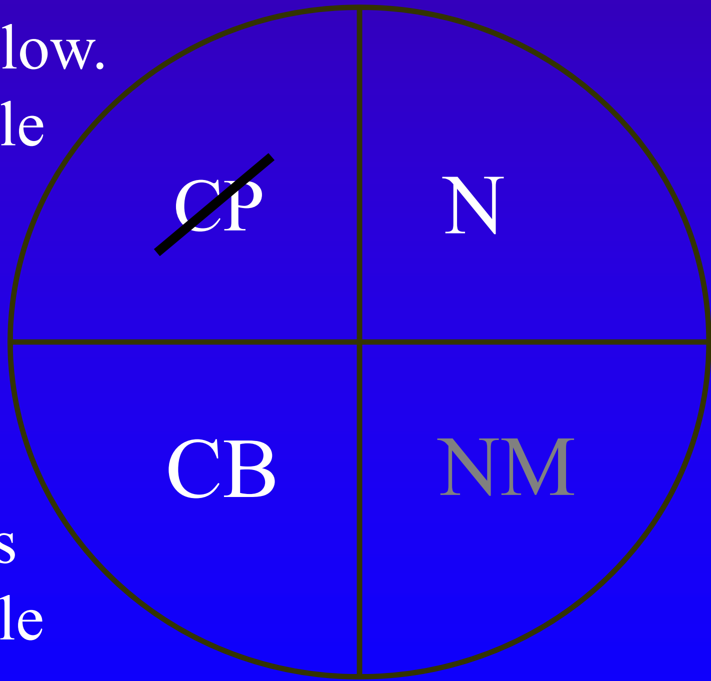
$V_{CB}$ vs $V_N$

When it exists, the N minimum is always below. The CP SP is a saddle point.

$V_{CP}$ vs $V_N$

$V_{CB}$ vs $V_{CP}$

When it exists, the CP minimum is always below. The CB SP is a saddle point (and so is the N SP).



# Conclusions II

- The conclusions are valid for all renormalizable THDM at tree level.
- The soft broken ($Z_2$) potential is fine for competing minima of different nature and <u>for CP violating minima</u> (which are unique) but shows "problems" for two competing Normal minima. Exact $Z_2$ potential is doing great for normal minima but it doesn't allow for CP violation.
- The stability of the Normal minimum is not guaranteed. At one-loop the CB minimum could be below the Normal one.

Thank you

BUP

# Many Higgs

$$V_{CP} - V_N = \frac{1}{2}\left(M_A^2\right)_{ij} s_i^\dagger s_j$$

Yes!

No!

$$V_{CB} - V_N = \frac{1}{2}\left(M_{H^\pm}^2\right)_{ij} c_i^\dagger c_j + (\text{terms that may be negative})$$

# Based on

- PLB322 (1994) Tree Level Vacuum Stability in THDM
- PLB603 (2004) Stability of the tree level vacuum in THDM against charge or CP spontaneous violation
- PLB632 (2006) Charge and CP breaking in THDM
- PRD74 (2006) Stability of the normal vacuum in MHDM
- PLB652 (2007) Charge and CP breaking in THDM