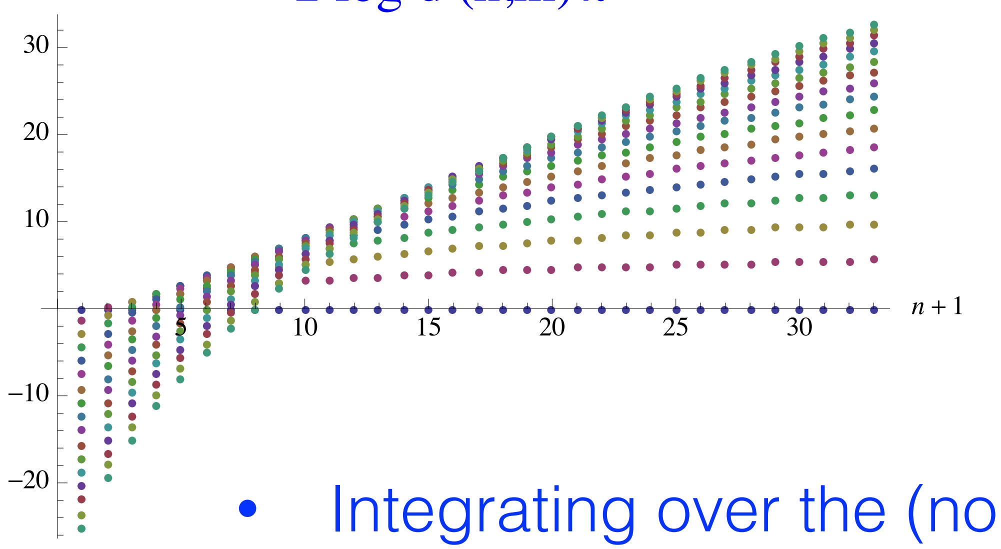
V.V. Khoze — *Rising cross sections for multi-H,W,Z production*

- At high energies it becomes kinematically possible to produce multiple Higgs and massive vector W, Z bosons. These amplitudes grow factorially with the number of particles — overcome the suppression by powers of perturbative couplings.

$$\mathcal{A}_{h^*\to n\times h+m\times Z_L} \;=\; (2v)^{1-n-m}\, n!\, m!\, d(n,m)\, \exp\left[-\frac{7}{6}\, n\,\varepsilon_h - 1.7\, m\,\varepsilon_V\right]$$

2 log d (n,m) κ^m



$$E = n(1+\varepsilon_h)m_h + m(1+\varepsilon_V)m_V$$

$$\kappa := \frac{g}{2\sqrt{2\lambda}} = \frac{m_V}{m_h} \sim 80/125 \sim 0.65$$

- Integrating over the (non-relativistic) phase-space get the cross section

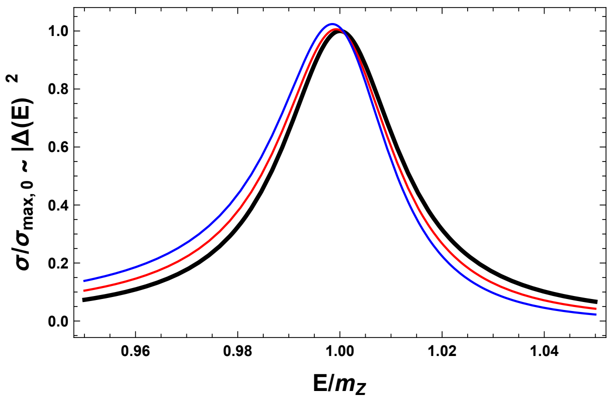
$$\sigma_{n,m} \;=\; \int d\Phi_{n,m}\, \frac{1}{n!\,m!}\, \left|\mathcal{A}_{h^*\to n\times h+m\times Z_L}\right|^2$$

$$\sim\; \exp\Big[2\log(\kappa^m d(n,m)) \,+\, n\log\frac{\lambda n}{4} \,+\, m\log\frac{\lambda m}{4} \,+\, \frac{n}{2}\left(3\log\frac{\varepsilon_h}{3\pi}+1\right)$$

$$+\, \frac{m}{2}\left(3\log\frac{\varepsilon_V}{3\pi}+1\right) \,-\, \frac{25}{12}\, n\,\varepsilon_h \,-\, 3.15\, m\,\varepsilon_V \,+\, \mathcal{O}(n\varepsilon_h^2 + m\varepsilon_V^2)\Big]$$

- 1990's: Voloshin 9409344; Son 9505338; Libanov, Rubakov,Troitsky -1997
- 2014: VVK1404.4876 & 1411.2925; Jaeckel, VVK 1411.5633

- The perturbative cross section grows with energy, ultimately violating:
  - naive perturbative unitarity: $$\sum_{n,\text{inelastic}} \int d\Phi_{n,m} |\mathcal{A}_{n,m}|^2 \le 8\pi (l_{\max}+1)^2 \;\sim^{?}\; 8\pi$$
  - the observed form of the Z-peak via the Kallen-Lehmann spectral representation:



$$\Delta(p) = \frac{Z}{p^2 - m^2} + \sum_{n\ge 2} \int_{(nm)^2}^{\infty} ds\, \frac{\int d\Phi_n |\mathcal{A}(1\to n)|^2(s)}{p^2 - s}$$

$$\int_{(nm)^2}^{\infty} ds\, \frac{\int d\Phi_n |\mathcal{A}(1\to n)|^2(s)}{p^2 - s} = \frac{1}{m^2}\mathcal{C}_1 + \frac{p^2}{m^4}\mathcal{C}_2 + \dots \quad , \quad |\mathcal{C}_1| < 1 \quad , \quad |\mathcal{C}_2| < 1$$

  - the cosmic ray limit: upper bound comes from assuming that the effective cross section for inelastic scattering of cosmic rays is of the size of the universe. In this case high energy cosmic rays will be severely attenuated in conflict w observation.

  - pert. SM cross sections exceed these bounds at energies:

$$\begin{array}{llll} E & \lesssim & 812\ \text{TeV} & \text{naive unitarity limit} \\ E & \lesssim & 830\ \text{TeV} & \text{cosmic limit} \\ E & \lesssim & 299\ \text{TeV} & \text{asymptotic series truncation heuristic} \\ E & \lesssim & 100\ \text{TeV} & \text{adding } \simeq n\varepsilon^2 \text{ term in the exponent in } \sigma \end{array}$$

Perturbative SM exhibits not simply a formal breakdown but will also be in conflict with observation.
Two options:

- Jaeckel, VVK 1411.5633

1. ***At high energies (multiplicities) the SM is fundamentally non-perturbative.***
2. ***New physics beyond the SM has to set in before the cross sections become large.***