Possible Implication of a Single Nonextensive p_T Distribution for Hadron Production in High-Energy pp Collisions

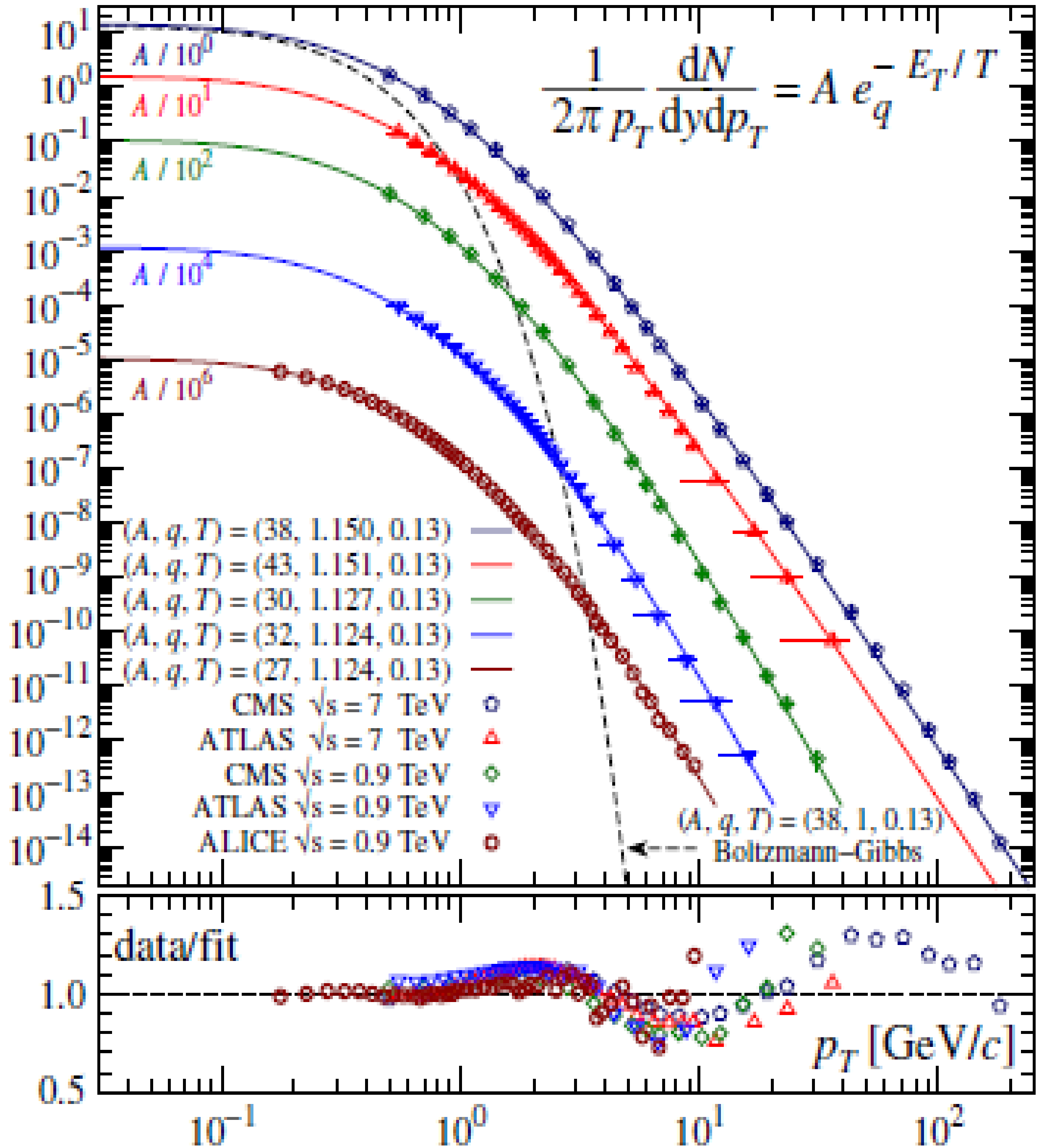
Grzegorz Wilk

National Centre for Nuclear Research, Warsaw;

in collaboration with : Cheuk-Yin Wong (Oak Ridge National Laboratory)

and Leonardo J. L. Cirto and Constantino Tsallis (CBPF, Rio deJaneiro)

11-th Polish Workshop on Relativistic Heavy-Ion Collisions; 17-18 January 2015
organized by Heavy Ion Reactions Group, Faculty of Physics, Warsaw University of Technology



LHC high energy data for pp → pi…
Fits at midrapidity, one simple formula used,

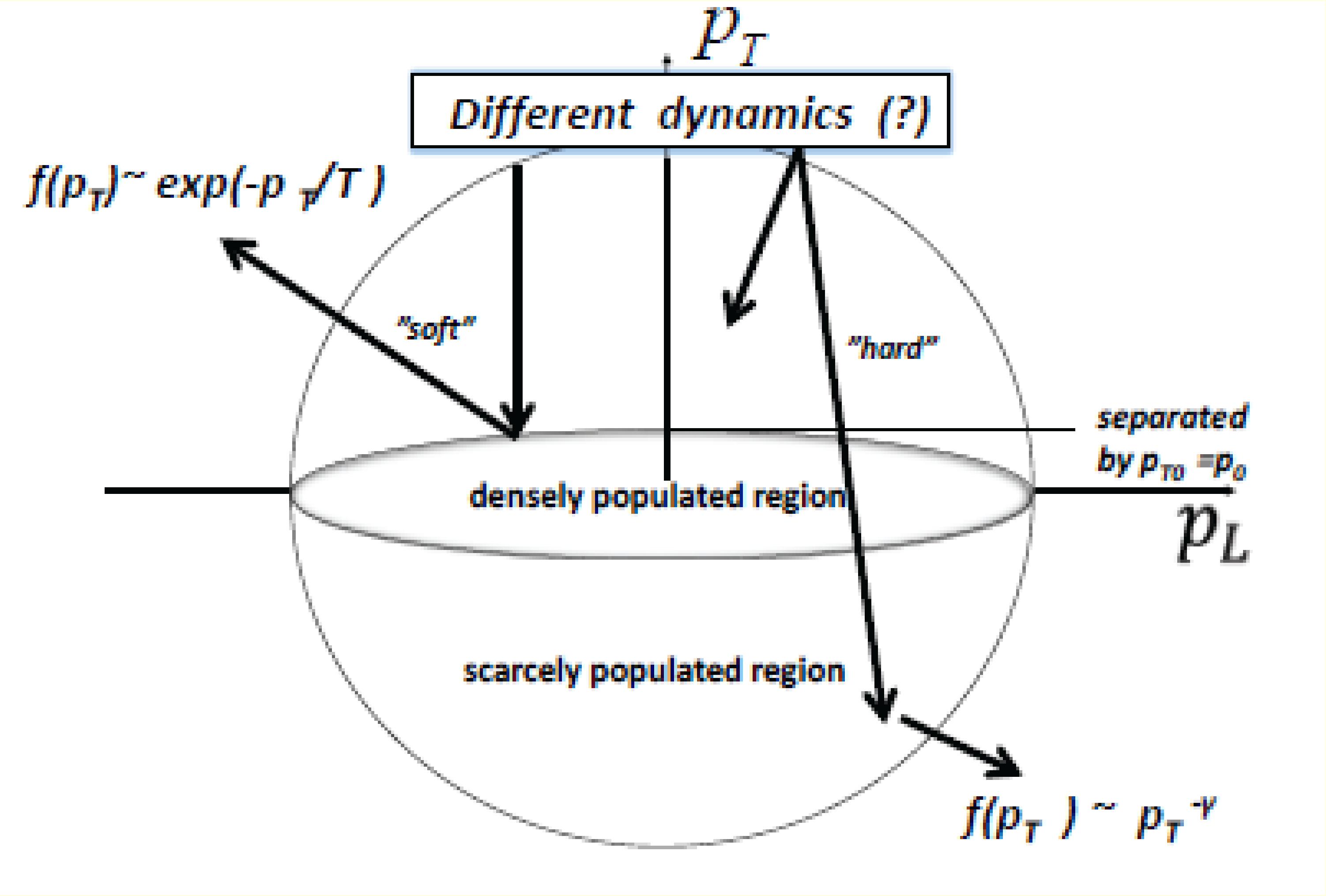
$$e_q\,(E_T) = \left[1-(1-q)\,\frac{E_T}{T}\right]^{\frac{1}{(1-q)}}$$

q~1.1, T=130 MeV

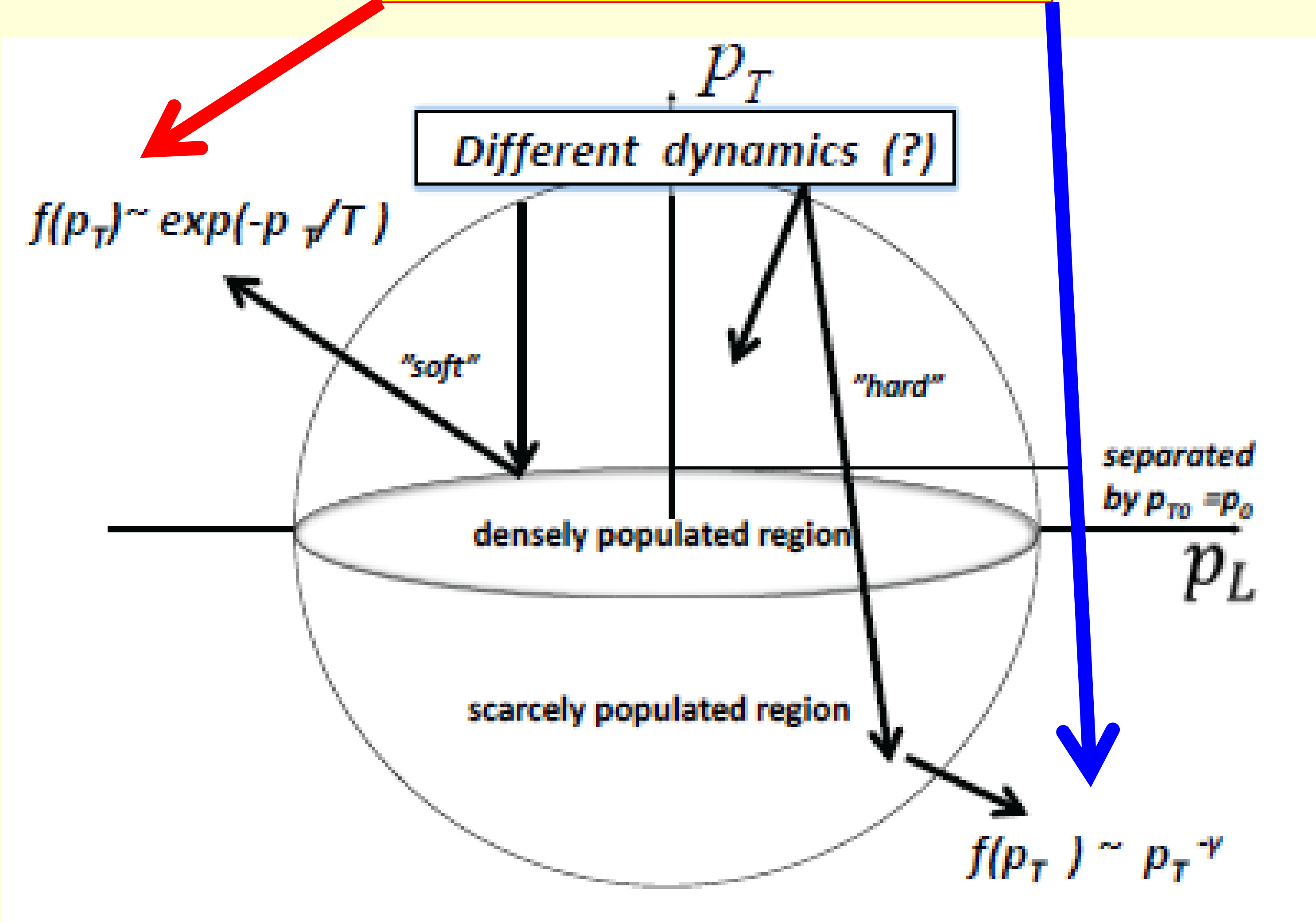
?

APPB43(2012)2047
PRD87(2013)114007
1409.3278
1412.0474

Schematic view of details of the phase space in multiparticle production.



$$f(p_T) = C \cdot \left(1 + \frac{p_T}{np_0}\right)^{-n}$$



C. Michael, L. Vanryckeghem, J. Phys. G 3, L151;,1977); C. Michael, Prog. Part. Nucl. Phys. 2, 1 (1979); R. Hagedorn, Riv. Nuovo Cimento 6, 1 (198).

$$f(p_T) = C \cdot \left(1 + \frac{p_T}{np_0}\right)^{-n} \rightarrow \begin{cases} exp\left(-\frac{p_T}{p_0}\right) & for\ \ p_T \rightarrow 0 \\ (p_T)^{-n} & for\ p_T \rightarrow \infty \end{cases}$$

$p_0 = nT$, $n = 1/(q-1)$

$$f(p_T) = C \cdot \left[1 - (1-q)\frac{p_T}{T}\right]^{\frac{1}{(1-q)}} \xrightarrow[q \rightarrow 1]{} C \cdot exp\left(-\frac{p_T}{T}\right)$$

C. Tsallis, J. Stat. Phys. 52, 479 (1988) and Eur. Phys. J. A40, 257 (2009)

Example
-small and medium p_T

$$E\frac{d\sigma}{d^3p} = \frac{A}{\left(1+\frac{m_T - m}{nT}\right)^n};$$

$$n = \frac{1}{q-1}$$

Phenix Coll., PRD 83, 052004 (2011)

Fig. 12
Invariant differential cross sections of different particles measured in p p collisions at √s = 200 GeV in various decay modes.

q=1.1 n=10

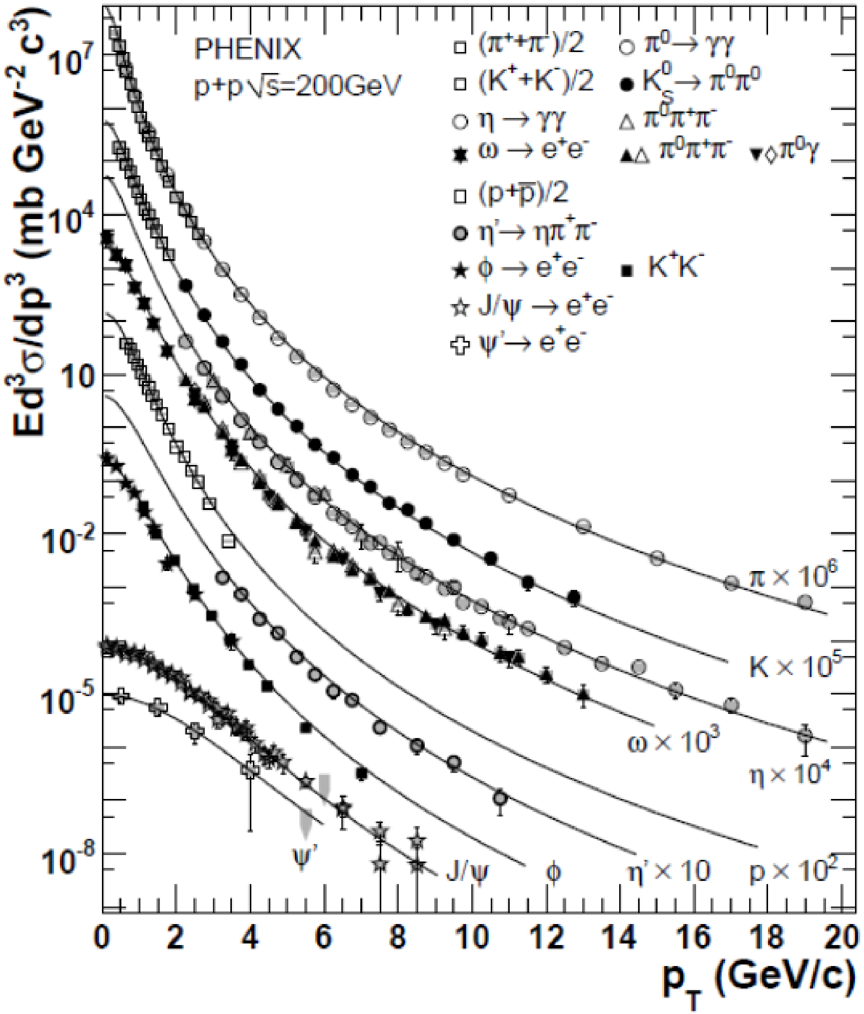
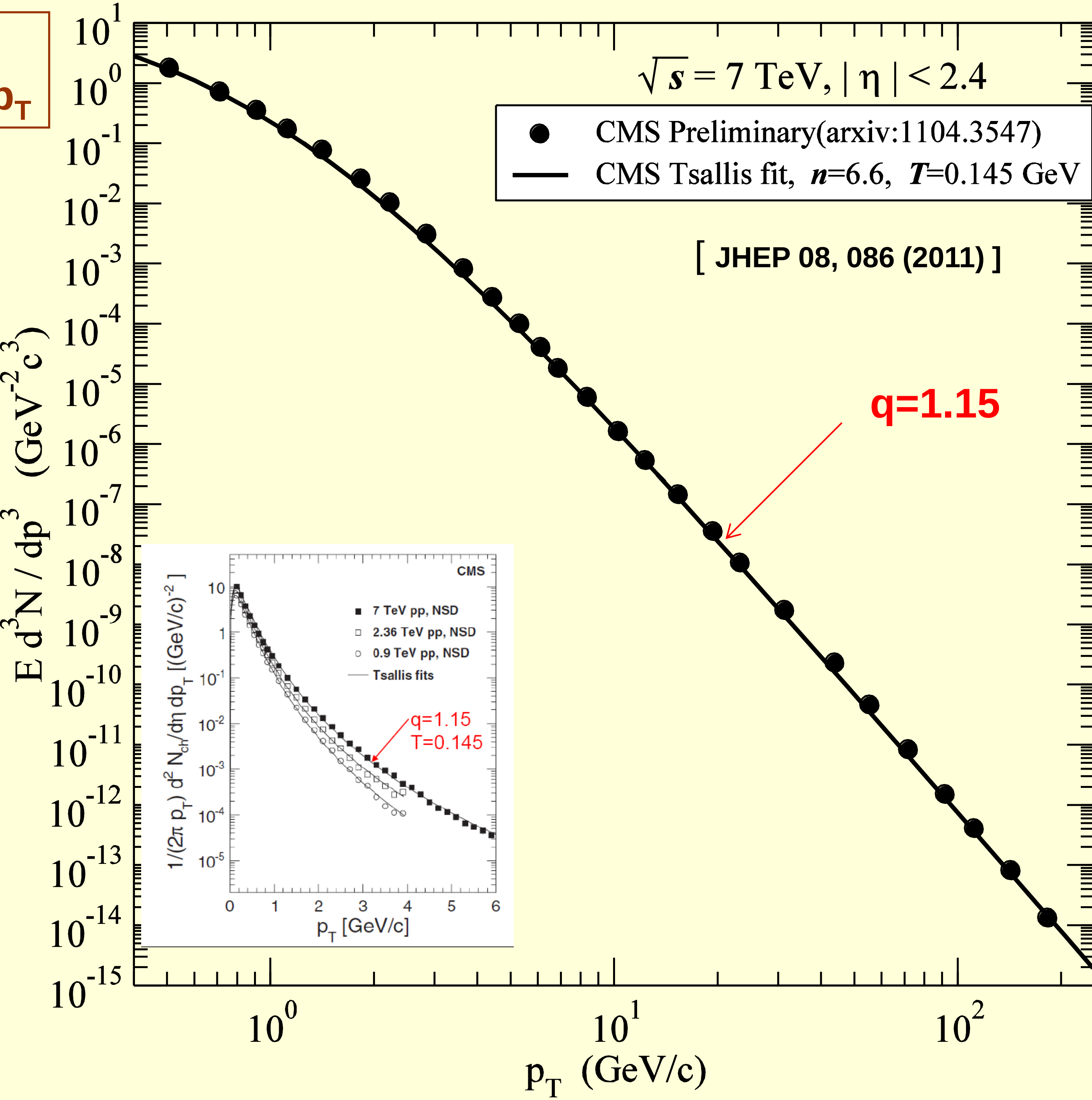


FIG. 12: Invariant differential cross sections of different particles measured in $p+p$ collisions at $\sqrt{s}$ =200 GeV in various decay modes. The spectra published in this paper are shown with closed symbols, previously published results are shown with open symbols. The curves are the fit results discussed in the text.

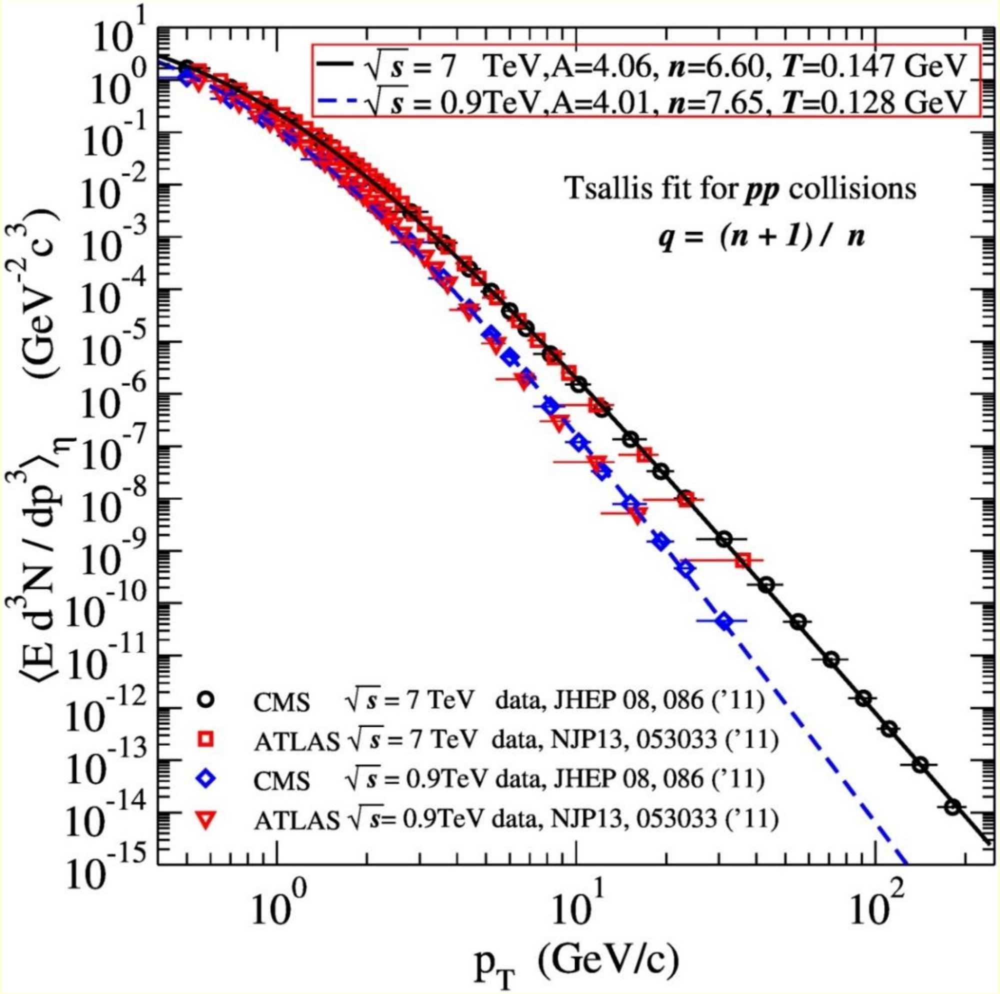
$$q \simeq 1.10$$

Example
- very large p_T



CMS fits recovered in APPB43(2012)2047:

$$E\frac{d\sigma}{d^3p}=\frac{A}{\left(1+\frac{m_T-m}{nT}\right)^n};\quad n=\frac{1}{q-1}$$



Good Tsallis fits have been obtained

$$\begin{cases} \text{for } \sqrt{s}=\mathbf{7}\text{ TeV}, & n=6.60 \\ & q=1.15 \\ \text{for } \sqrt{s}=\mathbf{0.9}\text{ TeV}, & n=7.65 \\ & q=1.13 \end{cases}$$

Questions:

- What is the physical meaning of n ?
- If n is the power index of $1/p_T^n$, then why is $n \sim 7$, whereas pQCD predicts $n \sim 4$?
- Why are there only few degrees of freedom over such a large p_T domain ?
- Do multiple parton collisions play any role in modifying the power index n?
- Does the hard scattering process contribute significantly to the production of low-p_T hadrons?
- What is the origin of low-p_T part of Tsallis fits ?

CYW,GW: *- Phys. Rev. D87 (2013) 114007;*
CYW,GW, LC, CT – arXiv: 1412.0474

Parton Multiple Scatterings:

(*) The contribution from the single collision dominates, but high multiple collisions comes in at lower $\mathbf{p_T}$ and for more central collisions

However,

(*) One expects then that for more and more central collisions, contributions with a greater number of multiple parton collisions gains in importance. As a consequence, the power index **n** is expected to become greater when we select more central collisions (subject to experimental verification).

PMS do not play any substantial role in modyfying power index n

Simplest way to model parton collisions basing on QCD ideas: use the

Relativistic Hard Scattering Model:

$$E_p \frac{d\sigma(AB \to cX)}{d^3p} = \sum_{ab} \int dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b)\, E_c \frac{d\sigma(ab \to cd)}{d^3p}$$

The basic differential cross section is

$$E_c \frac{d\sigma(ab \to cd)}{d^3p} = \frac{\hat{s}}{\pi} \frac{d\sigma(ab \to cd)}{dt} \delta(\hat{s} + \hat{t} + \hat{u}).$$

We assume: $x_a G_{a/A}(x_a) = A_a (1 - x_a)^g,$

For central rapidity, $|\eta| \approx 0,$ we obtain

$$E_p \frac{d^3\sigma(AB \to cX)}{d^3c} = \sum_{ab} \frac{A_a A_b}{\sqrt{\pi g}} (1 - x_{a0})^g (1 - x_{b0})^g \times$$

$$\times \frac{1}{\sqrt{\tau_c}} \left\{ \frac{1 - x_c}{1 - \tau_c^2 / x_c} \right\}^{1/4} \sqrt{\frac{(1 - x_{b0})}{1 - (x_{b0} + \tau_c^2 / x_c)/2}} \frac{d\sigma(ab \to cd)}{dt}$$

$$x_c = \frac{c_0 + c_z}{\sqrt{s}}, \quad \tau_c = \frac{c_T}{\sqrt{s}}, \quad x_{a0} = x_c + \tau_c \sqrt{\frac{1 - \tau_c^2 / x_c}{1 - x_c}}, \quad x_{b0} = \frac{\tau_c^2}{x_c} + \tau_c \sqrt{\frac{1 - \tau_c^2 / x_c}{1 - x_c}}$$

The Power Index in Jet Production

For $gg \to gg, qq' \to qq'$, and $qg \to qg$, $\quad \frac{d\sigma(ab \to cd)}{dt} \propto \frac{\alpha_s^2(c_T)}{c_T^4}$

The analytical formula is

$$E_c \frac{d\sigma(AB \to cX)}{d^3c} \propto \frac{1}{(c_T/\sqrt{s})^{1/2}} \frac{\alpha_s^2(c_T)}{c_T^4} (1 - x_{a0}(c_T))^g (1 - x_{b0}(c_T))^g$$

For $\eta \sim 0$, $x_{a0}(c_T) = x_{b0}(c_T) = 2x_c(c_T) = 2c_T/\sqrt{s}$, the analytical formula is

$$E_c \frac{d\sigma(AB \to cX)}{d^3c} \propto \frac{\alpha_s^2(c_T)\ (1 - 2x_c(c_T))^g (1 - 2x_c(c_T))^g}{c_T^{4+1/2} / (\sqrt{s})^{1/2}}$$

We change notations $\quad c \to p \quad$ and introduce power index n

$$E_p \frac{d\sigma(AB \to cX)}{d^3p} \propto \frac{\alpha_s^2(p_T)\ (1 - 2x_c(p_T))^g (1 - 2x_c(p_T))^g}{p_T^n / (\sqrt{s})^{1/2}}$$

where $\quad n = 4 + 1/2 \quad$ for LO pQCD. $\quad g_{a,b} = g = 6 - 10 \quad$ (we take $g = 6$ [18])

[18] D. W. Duke and J. F. Owens, Phy. Rev D30, 49 (1984).

How to get n from data ?

At a fixed $\sqrt{s}$, consider running coupling constant

$$\alpha_s(Q(c_T)) = \frac{12\pi}{27\ln(C + Q^2/\Lambda_{QCD}^2)},$$

$$\Lambda_{QCD} = 0.25\,\text{GeV} \Rightarrow \alpha_S\left(M_Z^2\right) = 0.1184;$$

$$C = 10 \Rightarrow \alpha_S\left(Q \propto \Lambda_{QCD}\right) \approx 0.6 \text{ in hadron spectroscopy studies [17]}$$

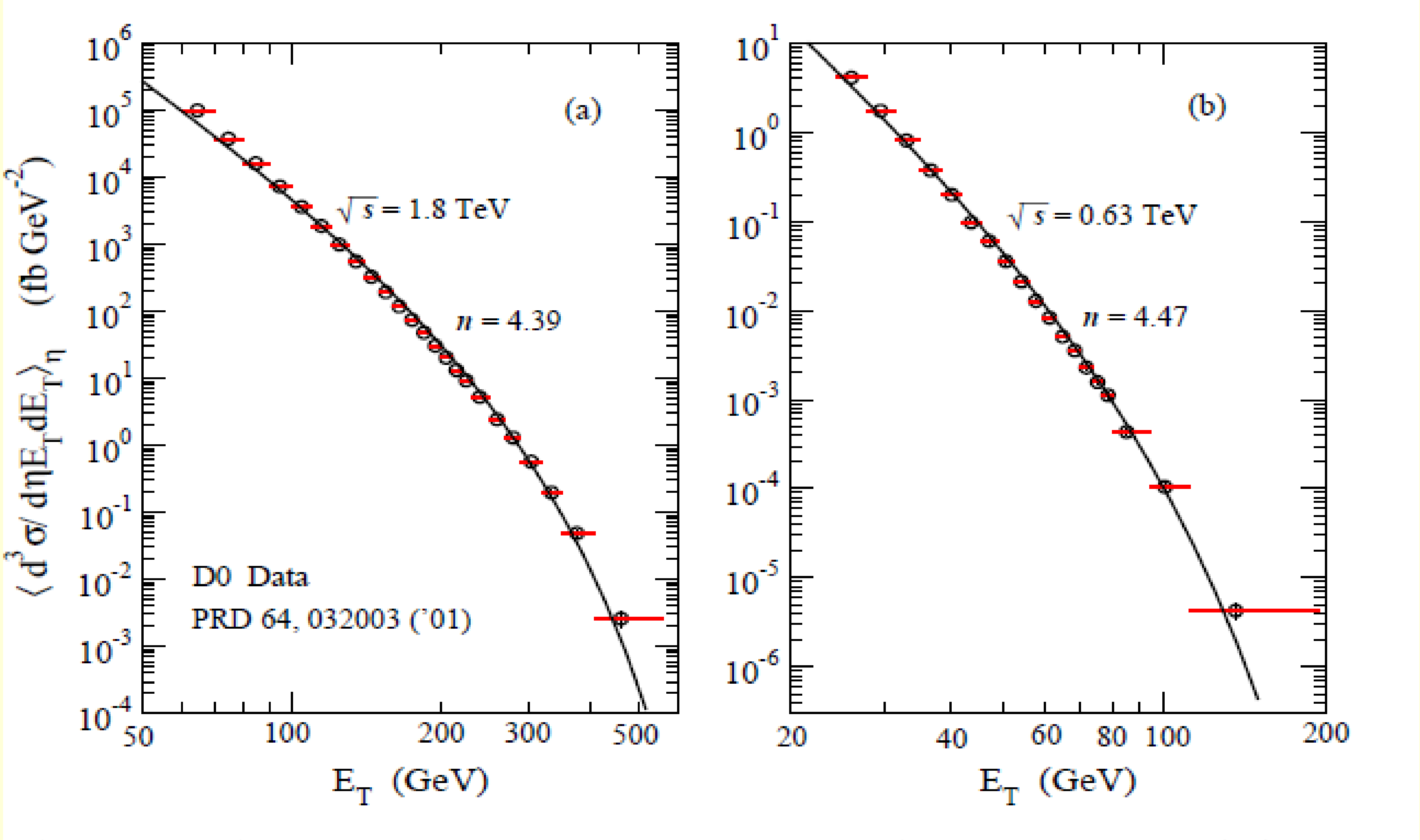
For $Q = c_T$ search for n by fitting experimental data at $\eta \approx 0$

$$E_c \frac{d\sigma^3(AB \to cX)}{dc^3} = \frac{A\alpha_s^2(Q^2(c_T))\ (1 - x_{a0})^{g_a + \frac{1}{2}}(1 - x_{b0})^{g_b + \frac{1}{2}}}{c_T^n \sqrt{1 - x_c}}$$

Notice: parameter C regularizes the coupling constant for small values of $Q(c_T)$.

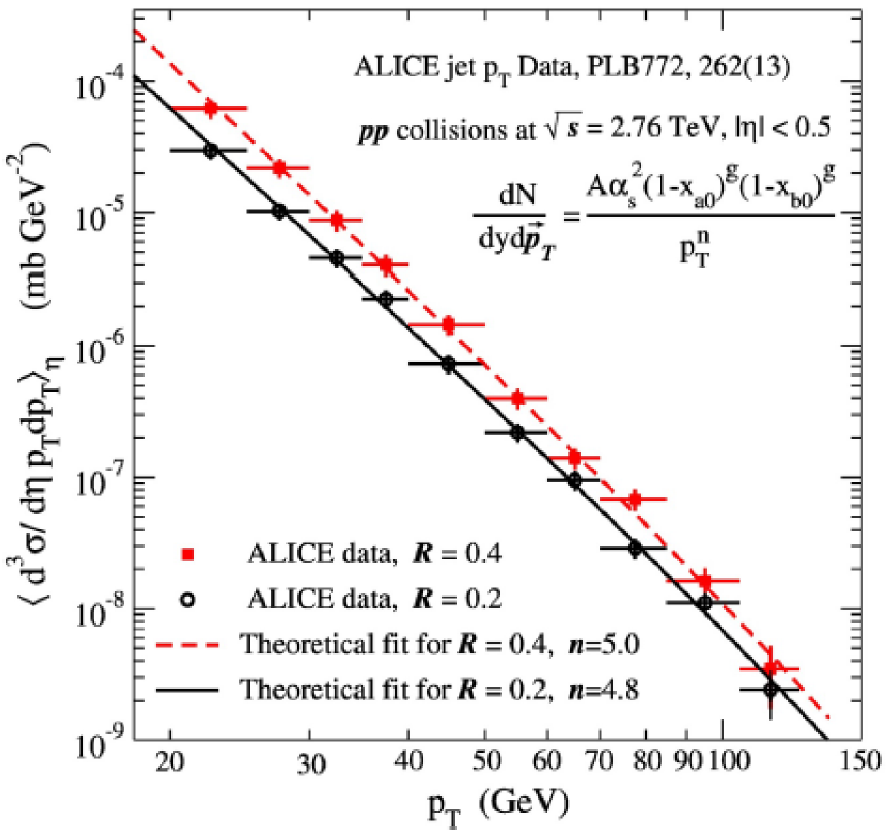
[17] C. Y. Wong, E. S. Swanson, and T. Barnes, Phy. Rev. C, 65, 014903 (2001).

Fits to D0 jet data

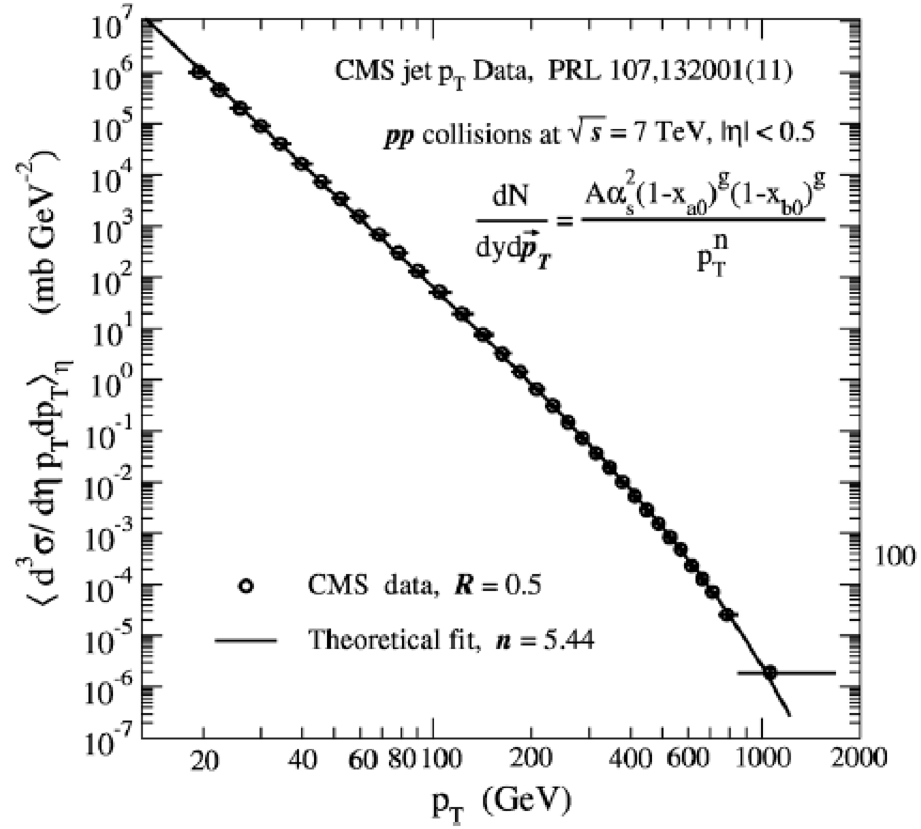


Comparison of the relativistic hard-scattering model results for jet production, Eq. (22) (solid curves), with experimental $d\sigma/d\eta\, E_T\, dE_T$ data from the D0 Collaboration, for hadron jet production within $|\eta|<0.5$, in $\bar{p}p$ collision at (a) $\sqrt{s}$=1.80 TeV, and (b) $\sqrt{s}$=0.63 TeV.

Fits to ALICE and CMS jet data



n predicted from pQCD is 4.5



n predicted from pQCD is 4.5

(without regularization of $\alpha(p_T)$; R is clustering parameter used in jet finding algorithms)

Except for the CMS data at 7 TeV that may need further re-examination, the power indices extracted for hadron jet production are in approximate agreement with the value of **n=4.5** in Eq. (19) and with previous analysis in [10], indicating the approximate validity of the hard scattering model for jet production in hadron-hadron collisions, with the predominant α_s^2 / c_T^4 parton-parton differential cross section as predicted by pQCD.

Collaboration	$\sqrt{s}$	R	η	n
D0	$\bar{p}p$ at 1.80 TeV	0.7	$\lvert\eta\rvert < 0.7$	4.39
D0	$\bar{p}p$ at 0.63 TeV	0.7	$\lvert\eta\rvert < 0.7$	4.47
ALICE	pp at 2.76 TeV	0.2	$\lvert\eta\rvert < 0.5$	4.78
ALICE	pp at 2.76 TeV	0.4	$\lvert\eta\rvert < 0.5$	4.98
CMS	pp at 7 TeV	0.5	$\lvert\eta\rvert < 0.5$	5.39

[10] F. Arleo, S. Brodsky, D. S. Hwang, and A. M. Sickles, Phys. Rev. Lett. 105, 062002 (2010).

Change of the Power Index *n* from Jet Production to Hadron Production

– one has to account for:

(i) showering (and/or fragmentation),

(ii) hadronization

Jet production

.

$$E_c \frac{d\sigma^3(AB \rightarrow cX)}{dc^3} = \frac{A\alpha_s^2(Q^2(c_T))\ (1-x_{a0})^{g_a+\frac{1}{2}}(1-x_{b0})^{g_b+\frac{1}{2}}}{c_T^n\sqrt{1-x_c}}$$

hadrons

Simple kinematics:

(*) A jet c evolves by fragmentation, showering, and hadronization to turn the jet into a large numbers of hadrons in a cone along the jet axis. The showering of the partons will go through many generations of branching:

$$c_T \rightarrow p_T^{(1)} \rightarrow p_T^{(2)} \rightarrow p_T^{(3)} \rightarrow \ldots \rightarrow p_T^{(\lambda)}$$

(*) Each branching will kinematically degrade the momentum of the showering parton by a momentum fraction, $\zeta = p_T^{(j+1)} / p_T^{(j)}$

(*) At the end of the terminating λ-th generation of the showering, and hadronization, the p_T of a produced hadron is related to the c_T of the parent parton jet by $p_T / c_T = \zeta^{\lambda}$

If the generation number λ and the fragmentation fraction ζ are independent of the jet c_T, then the power law and the power index for the p_T distribution are unchanged.

Dynamical ingredients:

(*) In addition to the kinematic decrease of p_T *, the showering generation number* **λ** *is also governed by the virtuality, which measures the degree of the off-the mass-shell property of the parton.*

(*) From the different parton showering schemes (PYTHIA , HERWIG, ARIADNE) a general picture emerges that the initial parton with a large initial virtuality **Q** *decreases its virtuality by showering until a limit of* **Q_0** *is reached.*

*(*) As a result of the virtuality ordering and virtuality cut-off (**dynamical ingredients**):*

the hadron fragment transverse momentum p_T is related to the parton momentum c_T nonlinearly by an exponent $1-\mu$, where μ is a parameter that can be searched to fit the data:

$$\frac{dc_T}{dp_T} = \frac{1}{1-\mu}\left(\frac{p_T}{c_{T0}}\right)^{\frac{2\mu}{1-\mu}}$$

After the fragmentation and showering of the parton c to hadron p, the hard-scattering cross section for the scattering in terms of hadron momentum p_T becomes

$$\frac{d^3\sigma(AB \to pX)}{dy d\mathbf{p}_T} = \frac{d^3\sigma(AB \to cX)}{dy d\bar{\mathbf{c}}_\mathbf{T}} \frac{d\bar{\mathbf{c}}_\mathbf{T}}{d\mathbf{p}_T}$$

$$\propto \frac{\alpha_S^2(\bar{\mathbf{c}}_\mathbf{T})(1 - x_{a0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}(1 - x_{b0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}}{c_T^{4+1/2}} \frac{d\bar{\mathbf{c}}_\mathbf{T}}{d\mathbf{p}_T}$$

where $$\frac{d\bar{\mathbf{c}}_\mathbf{T}}{d\mathbf{p}_T} = \frac{1}{1-\mu}\left(\frac{p_T}{c_{T0}}\right)^{\frac{2\mu}{1-\mu}}$$

$$\frac{\mathbf{d^3\sigma(AB \to pX)}}{\mathbf{dydp}_\mathbf{T}} \propto \frac{\boldsymbol{\alpha}_\mathbf{S}^\mathbf{2}(\bar{\mathbf{c}}_\mathbf{T})(\mathbf{1 - x_{a0}}(\bar{\mathbf{c}}_\mathbf{T}))^{\mathbf{g_a}}(\mathbf{1 - x_{b0}}(\bar{\mathbf{c}}_\mathbf{T}))^{\mathbf{g_a}}}{\mathbf{p}_\mathbf{T}^{\mathbf{n'}}}$$

$$\mathbf{n'} = \left(\frac{\mathbf{n - 2\boldsymbol{\mu}}}{\mathbf{1 - \boldsymbol{\mu}}}\right) \quad \textbf{with} \quad \mathbf{n = 4 + 1/2}$$

After all this one gets power-law behavior

but this is not Tsallis formula

because low p_T behaviour s not correct

$$\frac{d^3\sigma(AB \to pX)}{dy\,dp_T} \propto \frac{\alpha_S^2(\bar{c}_T)\left(1 - x_{a0}(\bar{c}_T)\right)^{g_a}\left(1 - x_{b0}(\bar{c}_T)\right)^{g_a}}{p_T^{n'}}$$

$$n' = \left(\frac{n - 2\mu}{1 - \mu}\right) \quad \text{with} \quad n = 4 + 1/2$$

Regularization of the Hard-Scattering Integral:

The obtained power-law applies for high p_T. To apply it to the whole range of E_T, we need to regularize it by the replacement,

$$\frac{1}{p_T} \rightarrow \frac{1}{p_{T0} + p_T} \rightarrow \frac{1}{p_{T0}} \cdot \frac{1}{1 + p_T/p_{T0}}$$

The quantity p_{T0} measures the average transverse mass of the detected hadron in the hard-scattering process.

$$\frac{d^3\sigma(AB \rightarrow pX)}{dy\,dp_T} \propto \frac{\alpha_S^2(\bar{c}_T)\left(1 - x_{a0}(\bar{c}_T)\right)^{g_a}\left(1 - x_{b0}(\bar{c}_T)\right)^{g_a}}{\left(1 + \frac{m_T}{m_{T0}}\right)^n}; \;\; m_T = \sqrt{m^2 + p}$$

Notice:

that we have just assumed

the form of Tsallis/Hagedorn

formula showed at the beginning:

$$E\frac{d\sigma}{d^3p}=\frac{A}{\left(1+\frac{m_T-m}{nT}\right)^n}$$

$$\frac{d^3\sigma(AB\to pX)}{dy\,dp_T}\propto\frac{\alpha_S^2(\bar{c}_T)\left(1-x_{a0}(\bar{c}_T)\right)^{g_a}\left(1-x_{b0}(\bar{c}_T)\right)^{g_a}}{\left(1+\frac{m_T}{m_{T0}}\right)^n};\quad m_T=\sqrt{m^2+p_T^2}$$

Notice: *in addition to regularization we were using (following CYW et al. PRC65(2001)14903*

$$\boldsymbol{\alpha_s(p_T) = \frac{12\pi}{27\ln(C + p_T^2/\Lambda_{QCD}^2)}},$$

$$\Lambda_{\mathrm{QCD}} = 0.25\,\mathrm{GeV} \Rightarrow \alpha_S\left(M_Z^2\right) = 0.1184;$$

$$C = 10 \Rightarrow \alpha_S\left(Q \propto \Lambda_{QCD}\right) \approx 0.6 \text{ in hadron spectroscopy studies}$$

Final result for QCD based derivation:

$$\frac{d^3N(AB \rightarrow pX)}{dyd\mathbf{p}_T} = \sqrt{1-\frac{m^2}{m_T^2\cosh^2 y}}\,A\,\frac{\alpha_S^2(\bar{\mathbf{c}}_\mathbf{T})(1-x_{a0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}(1-x_{b0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}}{\left(1+\frac{m_T}{m_{T0}}\right)^n} =$$

$$= \sqrt{1-\frac{m^2}{m_T^2\cosh^2 y}}\,A\alpha_S^2(\bar{\mathbf{c}}_\mathbf{T})(1-x_{a0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}(1-x_{b0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}\,\cdot$$

$$\bullet\left[1-(1-q)\frac{m_T}{T}\right]^{\frac{1}{1-q}}$$

where $\mathrm{q} = 1+\frac{1}{\mathrm{n}}$ and $T = \frac{m_{T0}}{q-1}$

Looks like Tsallis

Final result for QCD based derivation:

$$\frac{d^3N(AB\rightarrow pX)}{dy\,d\mathbf{p}_T}=\sqrt{1-\frac{m^2}{m_T^2\cosh^2 y}}\,A\,\frac{\alpha_S^2(\bar{\mathbf{c}}_\mathbf{T})(1-x_{a0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}(1-x_{b0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}}{\left(1+\frac{m_T}{m_{T0}}\right)^n}=$$

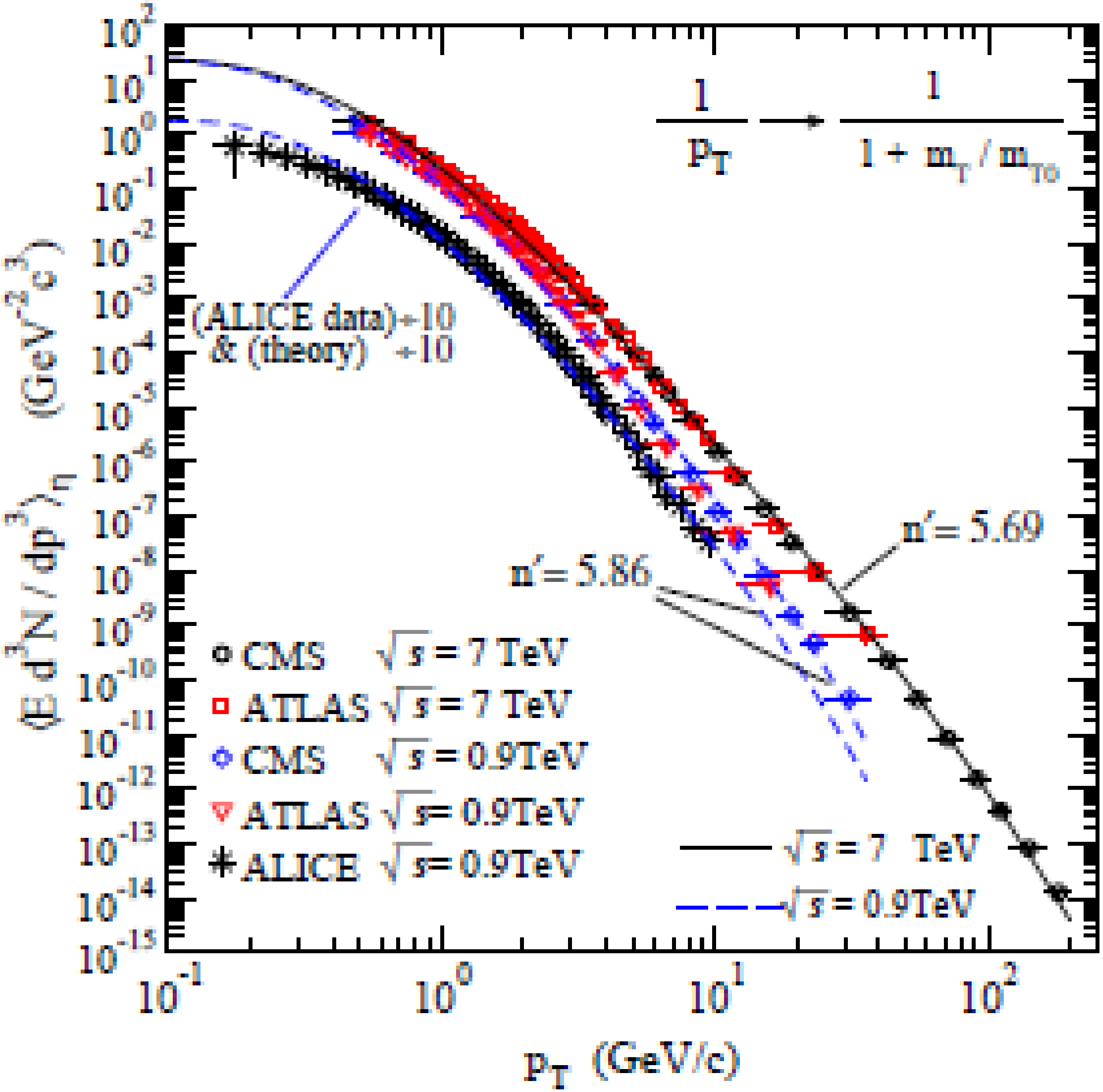
$$=\sqrt{1-\frac{m^2}{m_T^2\cosh^2 y}}\,A\alpha_S^2(\bar{\mathbf{c}}_\mathbf{T})(1-x_{a0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}(1-x_{b0}(\bar{\mathbf{c}}_\mathbf{T}))^{g_a}\,\cdot$$

$$\bullet\left[1-(1-q)\frac{m_T}{T}\right]^{\frac{1}{1-q}}$$

where $\mathrm{q}=1+\frac{1}{\mathrm{n}}$ and $T=\frac{m_{T0}}{q-1}$

… but with quite complicated prefactor

Looks like Tsallis



The experimental data gives *n′*=5.69 , m_{T0} =0.804 GeV for √*s*=7 TeV and *n′*=5.86, m_{T0} =0.634 GeV for √*s*=0.9 TeV.

There is indeed a systematic change of the power index *n* from jet production to a larger value *n′* in hadron production.

(The fits to the low p_T region for the ALICE data can be improved with a larger power index *n′*)

Further Approximation of the Hard-Scattering Integral

For $\sqrt{s} >> c_T$ and at $y \sim 0$ one has:

$$\frac{[1-x_{a0}(c_T)]^{g_a+1/2}[1-x_{b0}(c_T)]^{g_b+1/2}}{\sqrt{[1-x_c(c_T)]}} \sim [1-x_c(c_T)]^{2g_a+3/4} \sim \frac{1}{\left[1+\frac{m_T}{m_{T0}}\right]^{n_g}}$$

$$n_g = \frac{2(2g_a+3/4)m_{T0}}{\sqrt{s}}\left\langle\frac{1}{z}\right\rangle \sim 0.04\ (for\ 0.9\ TeV)\ and\ 0.007\ (for\ 7\ TeV)$$

$$\alpha_S[Q^2(c_T)] \propto \frac{1}{\left[1+\frac{m_T}{m_{T0}}\right]^{n_\alpha}}$$

(where n_α can be chosen to minimize errors by matching α_S at two points of p_T ; it is ~0.3-0.5)

$$\frac{d^3\sigma(AB\to pX)}{dy d^2p_T} = F(p_T) \sim \frac{1}{\left[1+\frac{m_T}{m_{T0}}\right]^{n}} \quad with\ \ n = n' + n_g + n_\alpha$$

As a consequence of all these simplifying approximations,
we can write the hard-scattering integral

$$\frac{d^3\sigma(AB \to pX)}{dy\,dp_T} \propto \frac{\alpha_S^2(\bar{c}_T)(1 - x_{a0}(\bar{c}_T))^{g_a}(1 - x_{b0}(\bar{c}_T))^{g_a}}{\left(1 + \frac{m_T}{m_{T0}}\right)^n}$$

in the approximate form

$$\frac{d^3\sigma(AB\to pX)}{dy\,d^2p_T} = F(p_T) \sim \frac{1}{\left[1+\frac{m_T}{m_{T0}}\right]^n} \quad \textit{with} \;\; n = n' + n_g + n_\alpha$$

n' - the power index after taking into account the fragmentation process,
n_g - the power index from the structure function,
n_α - the power index from the coupling constant.

Predominant change of the power index from jet production to hadron production arises from the fragmentation process .

Nonextensive Distribution as a Lowest-Order Approximation

of the Hard-scattering Integral:

Identifying in

$$\frac{d^3\sigma(AB\to pX)}{dy\,d^2p_T} = F(p_T) \sim \frac{1}{\left[1+\frac{m_T}{m_{T0}}\right]^n}$$

$$n \to \frac{1}{q-1} \quad and \quad m_{T0} \to \frac{T}{q-1} = nT$$

and considering produced particles as relativistic (with $m_T \sim E_T \sim p_T$ and $E_T \sim E$ at mid-rapidity **y~0)**,

we obtain the nonextensive distribution

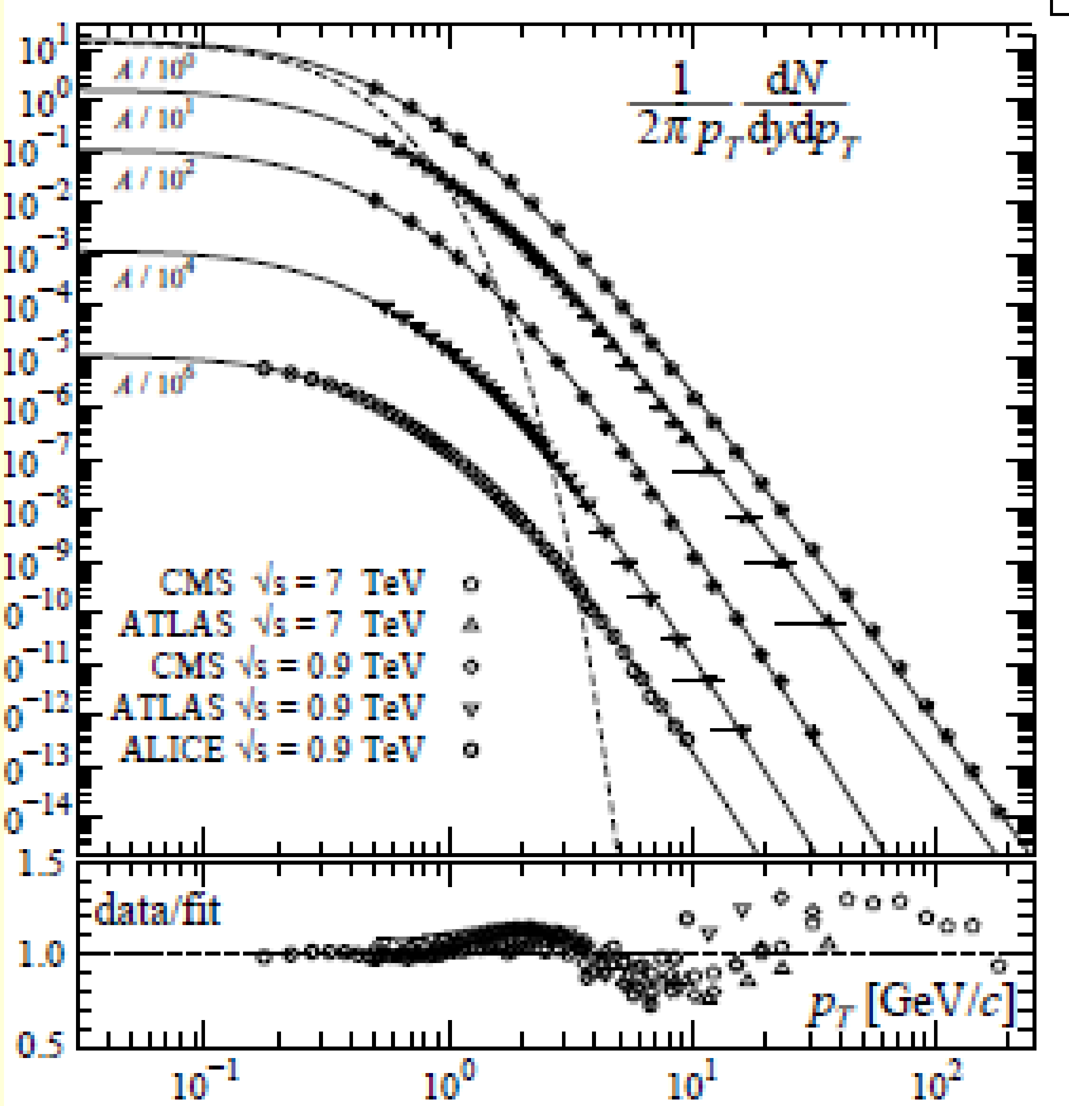
$$F(p_T) = A\left[1-(1-q)\frac{p_T}{T}\right]^{1/(1-q)}$$

as the lowest-order approximation for the QCD-based hard-scattering integral.

$$\frac{dN}{dy\,dp_T}\bigg|_{y\sim 0} = \frac{1}{2\pi p_T}\frac{dN}{dy\,dp_T}\bigg|_{y\sim 0} = A e_q^{-E/T},$$

$$e_q^{-E/T} \equiv [1-(1-q)\,E/T]^{1/(1-q)}, \quad e_1^{-E/T} = e^{-E/T},$$

Collaboration	$\sqrt{s}$ [TeV]	A	q	$n=1/(q-1)$
CMS [17]	7	38	1.150	6.67
ATLAS [18]	7	43	1.151	6.62
CMS [17]	0.9	30	1.127	7.87
ATLAS [18]	0.9	32	1.124	8.06
ALICE [19]	0.9	27	1.124	8.06



Fits to the experimental transverse momentum distribution of hadrons in *pp* collisions at central rapidity y. The temperature *T* = 0.13 GeV, the normalization constant is in units of GeV^{-2}/c^3. The corresponding Boltzmann-Gibbs (purely exponential) fit is presented by the dashed curve. The ratios data/fit show (?) a roughly log-periodic behavior iobserved on top of the *q*-exponential one.

Possible interpretation:

The convergence of hard scattering and nonextensivity equations can be considered from the viewpoint of ***the reduction of a microscopic description to a statistical-mechanical description.***

(*) From the microscopic perspective, the hadron production in a pp collision is a very complicated process, as evidenced by the complexity of the evolution dynamics in the evaluation of the p_T *spectra in Monte Carlo programs.*

(*) Starting from the initial condition of two colliding nucleons, there are many intermediate and complicated processes entering into the dynamics, each of which contain a large set of microscopic and stochastic degrees of freedom.

Possible interpretation (cont):

(*) There are stochastic elements :

- *in the picking of the degree of inelasticity,*
- *in the picking of the colliding parton momenta from the parent nucleons,*
- *in the scattering of the partons,*
- *in the showering evolution of scattered partons,*
- *in the hadronization of the fragmented partons.*

(*) Some of these stochastic elements cannot be definitive and many different models, sometimes with untestable assumptions, have been put forth.

In spite of all these complicated stochastic dynamics, the final result of the single-particle distribution can be approximated to depend only on three degrees of freedom, after all is done, put together, and integrated.

Possible interpretation (cont):

(*) The simplification can be considered as a "no hair" reduction from the microscopic description to nonextensive statistical mechanics in which all the complexities in the microscopic description "disappear" and are subsumed behind the stochastic processes and integrations.

(*) In line with statistical mechanics and in analogy with the Boltzmann-Gibbs distribution, we can cast the hard-scattering integral in the non-extensive form in the lowest-order approximation .

(*) The good agreement with data in the whole range of p_T with a single nonextensive distribution and demonstration that the hard-scattering integral can be written as a nonextensive distribution in the lowest-order approximation means that it is reasonable to infer that the dominant mechanism of hadron production in this case is the hard-scattering process.

Conclusions

(*) Particle production in high-energy pp collisions at central rapidity is a complex process that can be viewed from two different and complementary perspectives:

(1) Microscopic description based on pQCD and nonperturbative hadronization at the parton level.

(2) Statistical mechanics, where the single-particle distribution can be cast into a form that exhibit all the essential features of the process with only three degrees of freedom.

(*) The final result of the hard scattering process can be summarized, in the lowest-order approximation, by :

- *power index **n** which can be represented by a nonextensivity parameter **q=(n + 1)/n**,*
- *taverage transverse momentum m_{T0}, which can be represented by an effective temperature $\boldsymbol{T=m_{T0}/n}$,*
- *constant **A** that is related to the multiplicity .*

(*) One uncovers the dominance of the hard-scattering hadron production and the approximate validity of a "no-hair" statistical-mechanical description for the whole transverse momentum region in pp collision at high-energies

Thank you