FINITE SIZE OF HADRONS
AND
3-BODY BOSE-EINSTEIN CORRELATIONS

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(preliminary version)

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FINITE SIZE OF HADRONS IMPLIES SPACE-TIME CORRELATIONS

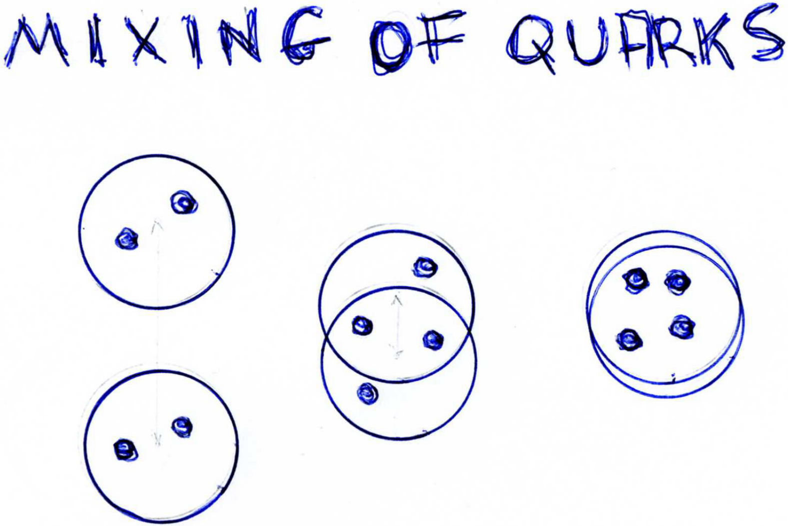
IDEA: WHEN HADRONS ARE TOO CLOSE TO EACH OTHER THEY ARE *NOT* HADRONS ANYMORE!!! (BECAUSE THEIR CONSTITUENTS ARE MIXING AND THEIR WAVE FUNCTIONS ARE NOT WELL-DETERMINED).

SINCE HBT EXPERIMENTS MEASURE QUANTUM INTERFERENCE BETWEEN THE WAVE FUNCTIONS OF PIONS, THEY CANNOT SEE PIONS WHICH ARE TOO CLOSE TO EACH OTHER.

THEREFORE THE SOURCE FUNCTION MUST VANISH AT SMALL DISTANCE BETWEEN PIONS, IMPLYING THAT THEIR POSITIONS ARE *CORRELATED*.

[AB & K.Zalewski, Phys. Lett. B727 (2013) 182]

PICTURE



SOURCE FUNCTIONS

SINCE THE SOURCE FUNCTIONS MUST VANISH AT SMALL DISTANCE BETWEEN PIONS, IT IS CONVENIENT TO WRITE THEM AS

(i) TWO-BODY:

$$S(p_1, p_2; x_1, x_2) = S(p_1, x_1)S(p_2, x_2)[1 - D(x_1, x_2)] \tag{1}$$

(ii) THREE-BODY:

$$\begin{aligned} S(p_1, p_2, p_3; x_1, x_2, x_3) &= S(p_1, x_1)S(p_2, x_2)S(p_3, x_3) \\ &[1 - D(x_1, x_2)]\,[1 - D(x_2, x_3)][1 - D(x_3, x_1)] \end{aligned} \tag{2}$$

WHERE THE CUT-OFF FUNCTION $D(x, y)$, RESPONSIBLE FOR THE CORRELATIONS, APPROACHES 1 AT SMALL DISTANCE (BELOW, SAY, 1-2 fm) AND VANISHES AT LARGER DISTANCES. IF THERE ARE NO CORRELATIONS, $D(x, y) \equiv 0$.

DISTRIBUTION OF TWO PIONS

$$N_2(p_1,p_2) = \int dx_1 dx_2 S(p_1,p_2;x_1,x_2) + \\ + \int dx_1 dx_2 S(p_{12},p_{12};x_1,x_2) e^{i(x_1-x_2)q_{12}}$$

WHERE $p_{12} = (p_1+p_2)/2$**;** $q_{12} = p_1 - p_2$**.**

$$C_2(p_{12},q_{12}) \equiv \frac{N_2(p_1,p_2)}{N(p_1)N(p_2)} = C_2^{uncorrelated}(p_{12},q_{12}) - \eta_1 - \eta_1';$$

$$C_2^{uncorrelated} = 1 + \frac{\int dx_1 dx_2 S(p_{12},x_1) S(p_{12};x_2) e^{i(x_1-x_2)q_{12}}}{N(p_1)N(p_2)};$$

$$\eta_1 = \frac{\int dx_1 dx_2 S(p_1,x_1) S(p_2;x_2) D(x_1,x_2)}{N(p_1)N(p_2)};$$

$$\eta_1' = \frac{\int dx_1 dx_2 S(p_{12},x_1) S(p_{12};x_2) D(x_1,x_2) e^{i(x_1-x_2)q_{12}}}{N(p_1)N(p_2)}. \quad (3)$$

NOTE: $C_2^{uncorrelated} \geq 1$**;** $q_{12} = 0$ **IMPLIES** $C_2 = 2(1-\eta_1)$.

DATA L3

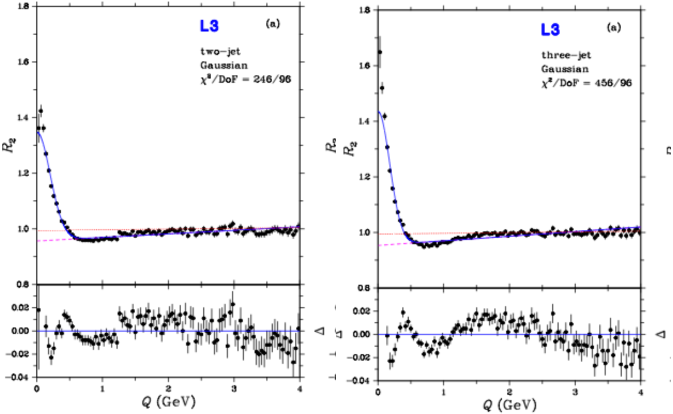


Figure: L3 data for two-jet and three-jet events.

DATA CMS 1

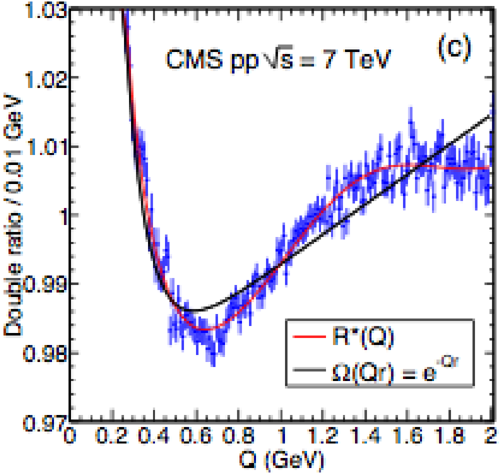


Figure: Two-pion correlation function from CMS (pp at 7 TeV)

DISTRIBUTION OF THREE PIONS

$N_3(p_1,p_2,p_3) = \int dx_1 dx_2 dx_3 Z(p_1,p_2,p_3;x_1,x_2,x_3)$, **WITH**

$$Z(p_1,p_2,p_3;x_1,x_2,x_3) = S(p_1,p_2,p_3;x_1,x_2,x_3)+$$
$$+S(p_{12},p_{12},p_3;x_1.x_2,x_3)e^{i(x_1-x_2)q_{12}}+$$
$$+S(p_1,p_{23},p_{23};x_1.x_2,x_3)e^{i(x_2-x_3)q_{23}}+$$
$$+S(p_{13},p_2,p_{13};x_1.x_2,x_3)e^{i(x_3-x_1)q_{31}}+$$
$$+2S(p,p,p;x_1.x_2,x_3)\Re e^{ix_1q_{23}+ix_2q_{31}+ix_3q_{12}},$$

WHERE $p_{ij}=(p_i+p_j)/2$, $q_{ij}=p_i-p_j$, $p=(p_1+p_2+p_3)/3$.
THIS EXPRESSION CONSISTS OF $6\times 8 = 48$ **TERMS!**

CONSIDER $p_1=p_2=p_3=p$. **WE HAVE**

$$C_3 = \frac{N_3(p,p,p)}{N(p)^3} = 6[1-3\eta_1+3\eta_2-\eta_3] \qquad (4)$$

$$\eta_2 = \frac{\int dx_1 dx_2 dx_3 S(p;x_1)S(p;x_2)S(p,x_3)D(x_1,x_2)D(x_2,x_3)}{N(p)^3};$$

$$\eta_3 = \frac{\int dx_1 dx_2 dx_3 S(p;x_1)S(p;x_2)S(p,x_3)D(x_1,x_2)D(x_2,x_3)D(x_3,x_1)}{N(p)^3} \qquad (5)$$

TWO-PION VS THREE-PION HBT CORRELATIONS

SUGGESTION BY HEINZ AND ZHANG: CONSTRUCT THE RATIO

$$r_3 \equiv \frac{2 + C_3(p_1,p_2,p_3) - C_2(p_1,p_2) - C_2(p_2,p_3) - C_2(p_3,p_1)}{\sqrt{(C_2(p_1,p_2)-1)(C_2(p_2,p_3)-1)(C_2(p_3,p_1)-1)}} \quad (6)$$

THE INTEREST IN THIS RATIO IS THAT AT THE POINT $p_1 = p_2 = p_3 = p$, IF THERE ARE NO SPACE-TIME CORRELATIONS (IGNORED BY HEINZ AND ZHANG),

$$r_3 = 2\ .$$

THUS $r_3 < 2$ SUGGESTS *COHERENT COMPONENT*.

WHEN THE SPACE-TIME CORRELATIONS ARE INCLUDED THE RESULT IS DIFFERENT. WE HAVE

$$r_3 = \frac{2 + 6[1 - 3\eta_1 + 3\eta_2 - \eta_3] - 3\times 2[1-\eta_1]}{(1-2\eta_1)^{3/2}} \approx 2 - 6\eta_1 \quad (7)$$

ESTIMATE OF η_1 FOR *pp* COLLISIONS

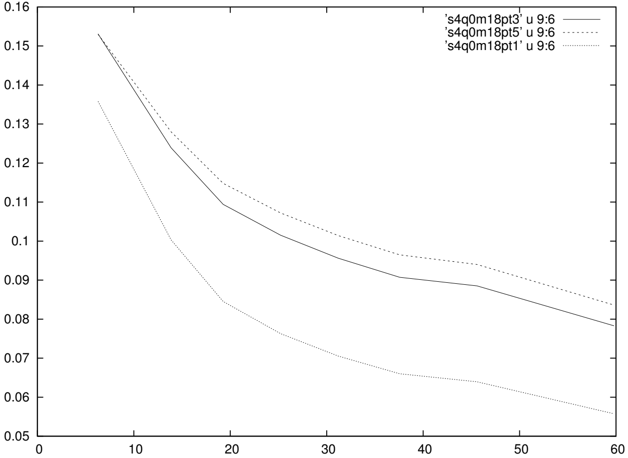


Figure: (preliminary) η_1 for *pp* collisions vs N_{ch}. $k_\perp = 163, 349$ and 548 MeV; $D(x,y) = e^{-(x-y)^2/\Delta^2}$; $\Delta = 1.5$ fm. η_1 increases with incrreasing $k_\perp$.

SUMMARY

(i)The consequences of the observation that hadrons are NOT POINT-LIKE, and thus their distribution in space-time MUST BE CORRELATED, were studied for two-body and three-body HBT distributions.
(ii) For the two-body distributions the most striking implication is that the observed HBT correlation function may take values which are strictly forbidden when the space-time correlations are ignored.
(iii) For the three-body distributions, we have shown that the presence of space-time correlations modifies the Heinz-Zhang relation between the two-particle and three-particle HBT correlation functions.
(iv) The scale of the effect depends strongly on the ratio of the length of cut region (characterized by the function $D(x,y)$) to the total size of the system. Consequently, it is expected to be small in case of AA collisions but seems to be considerable in pp and perhaps even in pA collisions.

EFFECT OF THE FOURIER TRANSFORM