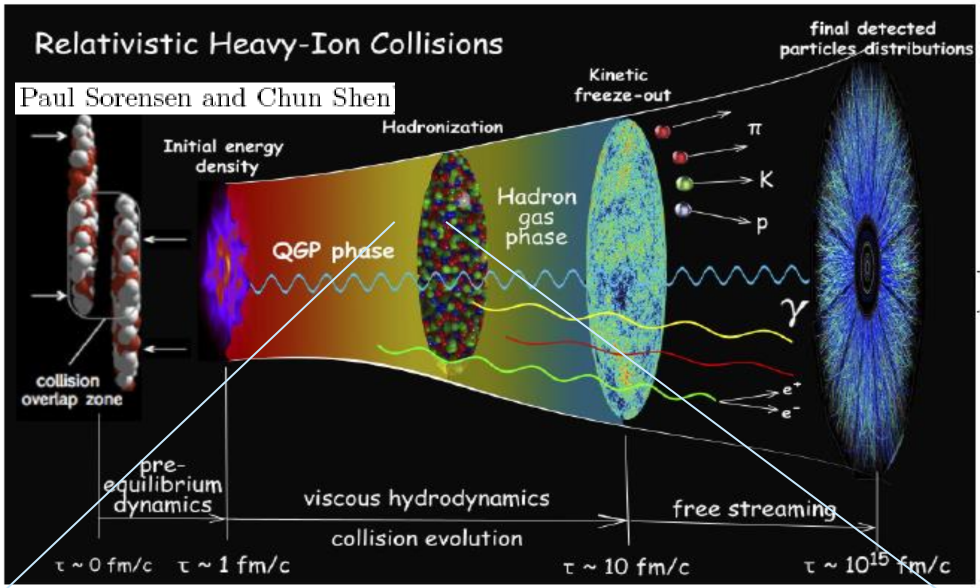
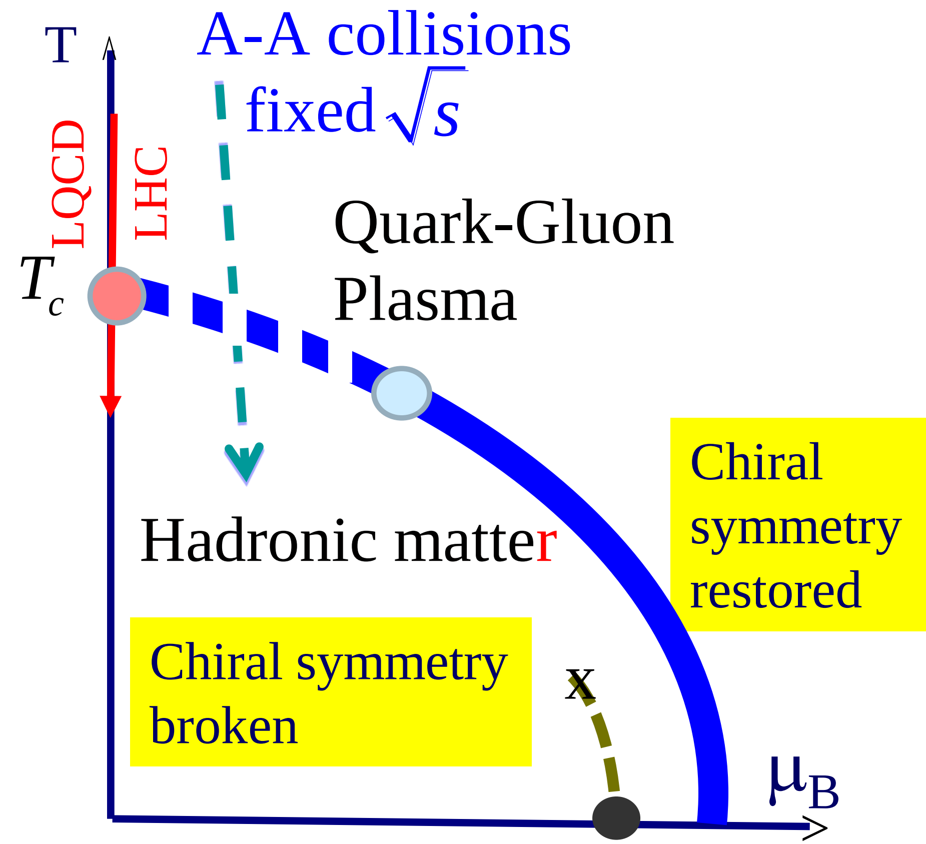
Fluctuations at the LHC and QCD phase diagram

Krzysztof Redlich, Uni Wroclaw





- Fluctuations of conserved charges at the LHC and LQCD results

P. Braun-Munzinger, A. Kalweit and J. Stachel

1st principle calculations:

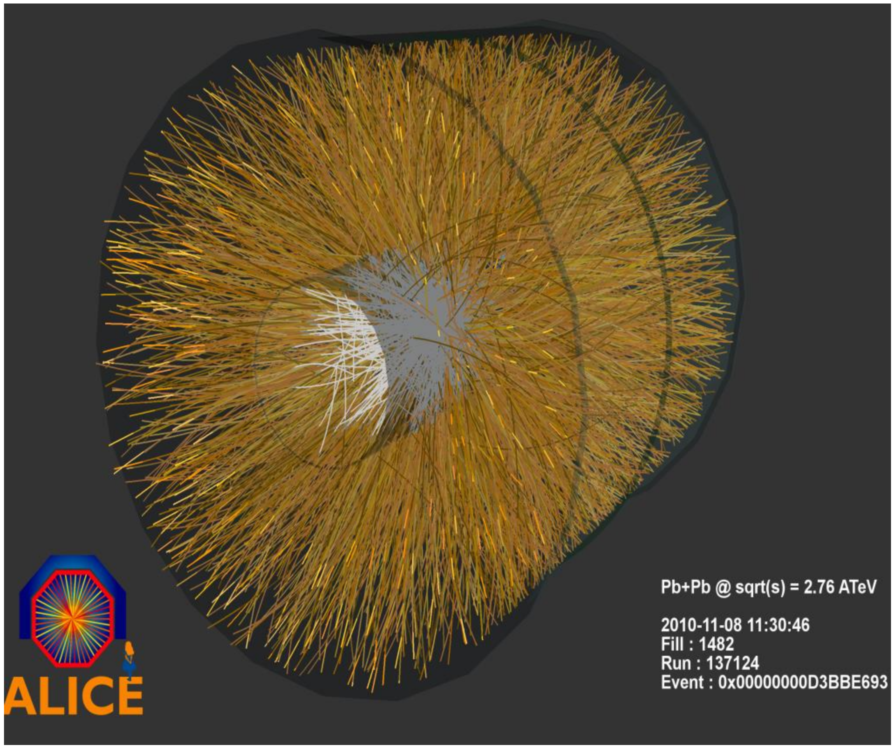
$\mu, T << \Lambda_{QCD}$: χ – perturbation theory

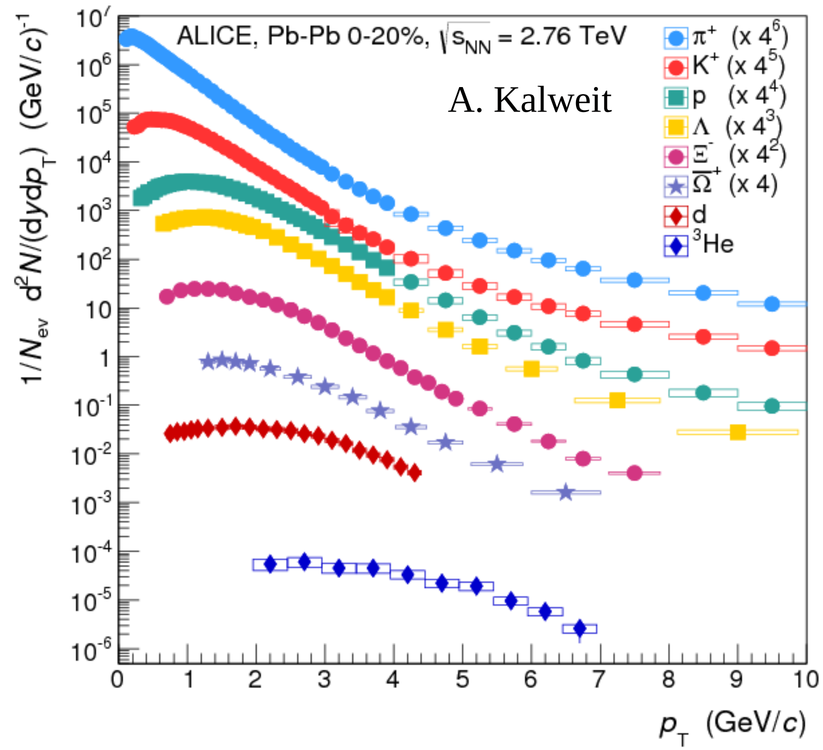
$\mu, T >> \Lambda_{QCD}$: pQCD >

$\mu_q < T$: LGT

Excellent data of ALICE Collaboration for particle yields

ALICE Collaboration





ALICE Time Projection Chamber (TPC), Time of Flight Detector (TOF), High Momentum Particle Identification Detector (HMPID) together with the Transition Radiation Detector (TRD) and the Inner Tracking System (ITS) provide information on the flavour composition of the collision fireball, vector meson resonances, as well as charm and beauty production through the measurement of leptonic observables.

Consider fluctuations and correlations of conserved charges

- They are quantified by susceptibilities:

  If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2}=\frac{\partial^2(P)}{\partial(\mu_N)^2} \qquad \frac{\chi_{NM}}{T^2}=\frac{\partial^2(P)}{\partial\mu_N\partial\mu_M}$$

$$N=N_q-N_{-q},\;\; N,M=(B,S,Q), \quad \mu=\mu/T, \quad P=P/T^4$$

- Susceptibility is connected with variance

$$\frac{\chi_N}{T^2}=\frac{1}{VT^3}(<N^2>-<N>^2)$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$<N^n>=\sum_N N^n \mathrm{P}(N)$$

Consider special case:

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

$< N_q > \equiv \overline{N}_q$ =>

- Charge and anti-charge uncorrelated and Poisson distributed, then
- $P(N)$ the Skellam distribution

$$P(N) = \left(\frac{\overline{N}_q}{\overline{N}_{-q}} \right)^{N/2} I_N \left(2\sqrt{\overline{N}_{-q}\overline{N}_q}\right) \exp[-(\overline{N}_{-q} + \overline{N}_q)]$$

- Then the susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

Consider special case: particles carrying $q = \pm 1, \pm 2, \pm 3$

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

- The probability distribution

$$< S_{-q} > \equiv \bar{S}_{-q}$$

$$q = \pm 1, \pm 2, \pm 3$$

$$\begin{aligned} P(S) \; = \; & (\frac{\bar{S}_1}{\bar{S}_{\bar{1}}})^{\frac{S}{2}} \exp[\sum_{n=1}^{3}(\bar{S}_n + \bar{S}_{\bar{n}})] \\ & \sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} (\frac{\bar{S}_3}{\bar{S}_{\bar{3}}})^{k/2} I_k(2\sqrt{\bar{S}_3 \bar{S}_{\bar{3}}}) \\ & (\frac{\bar{S}_2}{\bar{S}_{\bar{2}}})^{i/2} I_i(2\sqrt{\bar{S}_2 \bar{S}_{\bar{2}}}) \\ & (\frac{\bar{S}_1}{\bar{S}_{\bar{1}}})^{-i-3k/2} I_{2i+3k-S}(2\sqrt{\bar{S}_1 \bar{S}_{\bar{1}}}) \end{aligned}$$

Fluctuations

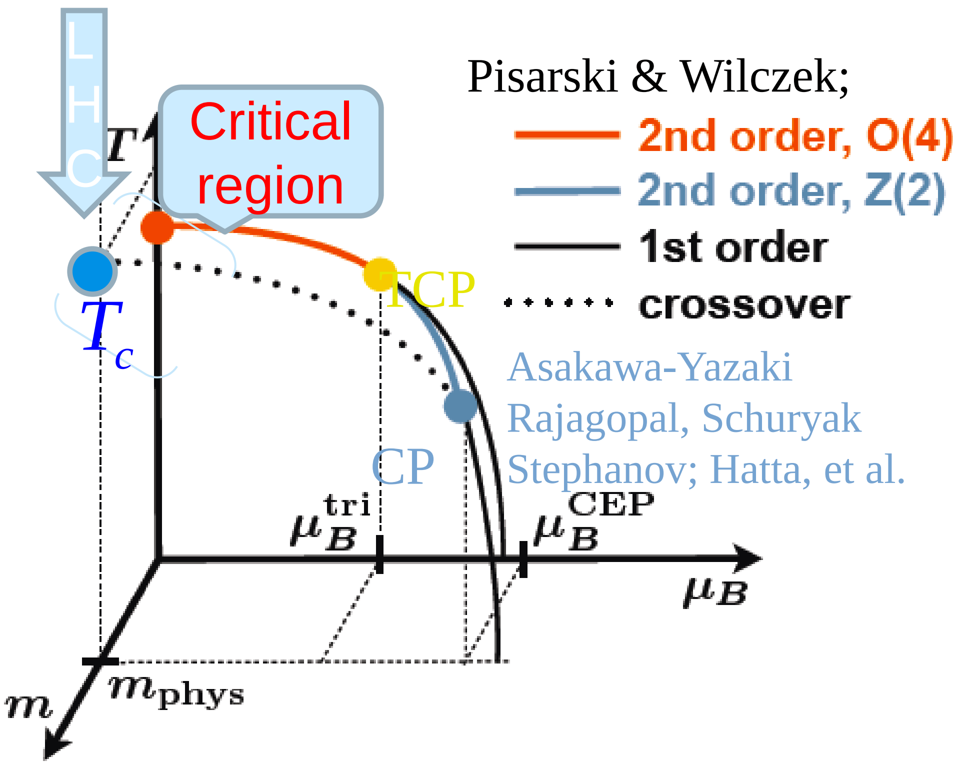
$$\frac{\chi_S}{T^2} = \frac{1}{VT^3} \sum_{n=1}^{|q|} n^2 (\langle S_n \rangle + \langle S_{-n} \rangle)$$

Correlations

$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} \sum_{n=-q_N}^{q_N} \sum_{m=-q_M}^{q_M} nm \langle N_{n,m} \rangle$$

$\langle N_{n,m} \rangle$, is the mean number of particles
; carrying charge $N = n$ and $M = m$.

Deconfinement and chiral symmetry restoration in QCD



- The QCD chiral transition is crossover Y.Aoki, et al Nature (2006)
  and appears in the O(4) critical region

  O. Kaczmarek et.al. Phys.Rev. D83, 014504 (2011)

- Chiral transition temperature

  $$T_c = 155(1)(8) \text{ MeV}$$

  T. Bhattacharya et.al.
  Phys. Rev. Lett. 113, 082001 (2014)

- Deconfinement of quarks sets in at the chiral crossover

  A.Bazavov, Phys.Rev. D85 (2012) 054503

- The shift of T_c with chemical potential

$$T_c(\mu_B) = T_c(0)[1 - 0.0066 \cdot (\mu_B / T_c)^2]$$

Ch. Schmidt Phys.Rev. D83 (2011) 014504

See also:
Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, *et al.*
JHEP, 0906 (2009)

Probing O(4) chiral criticality with charge fluctuations

- Due to expected O(4) scaling in QCD the free energy:

$$P = P_R(T, \mu_q, \mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta\delta/\nu} h)$$

- Generalized susceptibilities of net baryon number

$$c_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = c_R^{(n)} + c_S^{(n)} \quad \text{with} \quad \begin{aligned} c_s^{(n)}\big|_{\mu=0} &= d\ h^{(2-\alpha-n/2)/\beta\delta}\ f_\pm^{(n)}(z) \\ c_s^{(n)}\big|_{\mu\neq 0} &= d\ h^{(2-\alpha-n)/\beta\delta}\ f_\pm^{(n)}(z) \end{aligned}$$

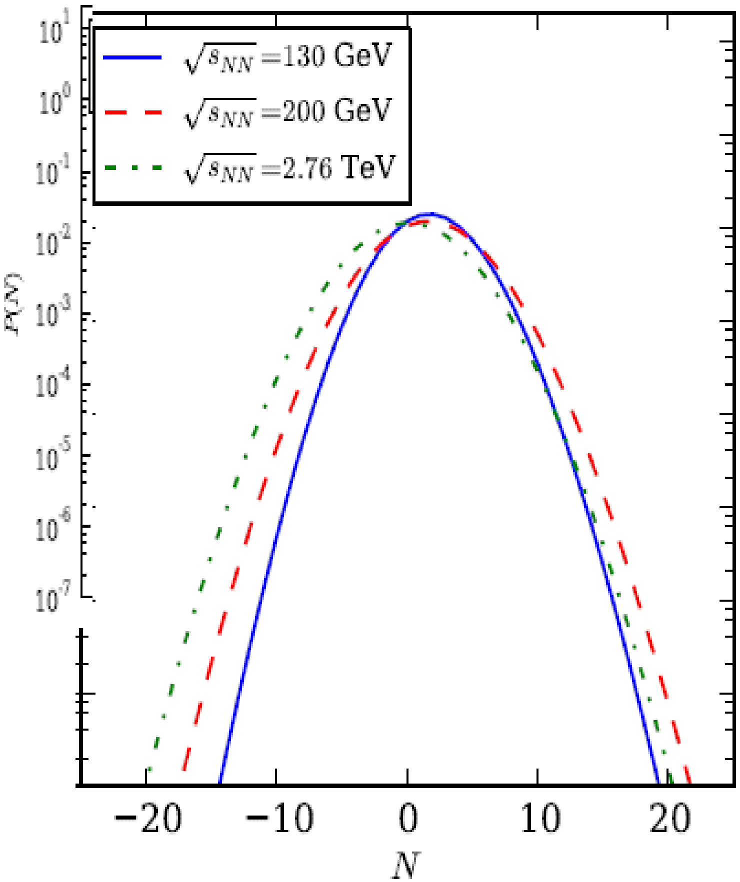
- At $\mu = 0$ only $c_B^{(n)}$ with $n \geq 6$ receive contribution from $c_S^{(n)}$
- At $\mu \neq 0$ only $c_B^{(n)}$ with $n \geq 3$ receive contribution from $c_S^{(n)}$

- $c_B^{n=2} = \chi_B / T^2$ Generalized susceptibilities of the net baryon number never critical with respect to ch. sym.

Variance at 200 GeV AA central coll. at RHIC

STAR Collaboration data in central coll. 200 GeV

P. Braun-Munzinger, et al.
Nucl. Phys. A880 (2012) 48)



- Consistent with Skellam distribution

$$\frac{<p>+<\bar{p}>}{\sigma^2} = 1.022 \pm 0.016 \qquad \frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$

- Consider ratio of cumulants in in the whole momentum range:

$$\frac{\sigma^2}{p-\bar{p}} = 6.18 \pm 0.14 \;\; in \;\; 0.4 < p_t < 0.8 GeV$$

$$\frac{p+\bar{p}}{p-\bar{p}} = 7.67 \pm 1.86 \;\; in \;\; 0.0 < p_t < \infty \; GeV$$

Constructing net charge fluctuations and correlation from ALICE data

- Net baryon number susceptibility

$$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3}(\langle p\rangle + \langle N\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + \langle \Xi^-\rangle + \langle \Xi^0\rangle + \langle \Omega^-\rangle + \overline{par}\ )$$

- Net strangeness

$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + \langle K_S^0\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + 4\langle \Xi^-\rangle + 4\langle \Xi^0\rangle + 9\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-} + \Gamma_{\varphi\to K_S^0} + \Gamma_{\varphi\to K_L^0})\langle \varphi\rangle\ )$$

- Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + 2\langle \Xi^-\rangle + 3\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-})\langle \varphi\rangle - (\Gamma_{K_0^*\to K^+} + \Gamma_{K_0^*\to K^-})\langle K_0^*\rangle\ )$$

χ_B , χ_S , χ_{QS} constructed from ALICE particle yields

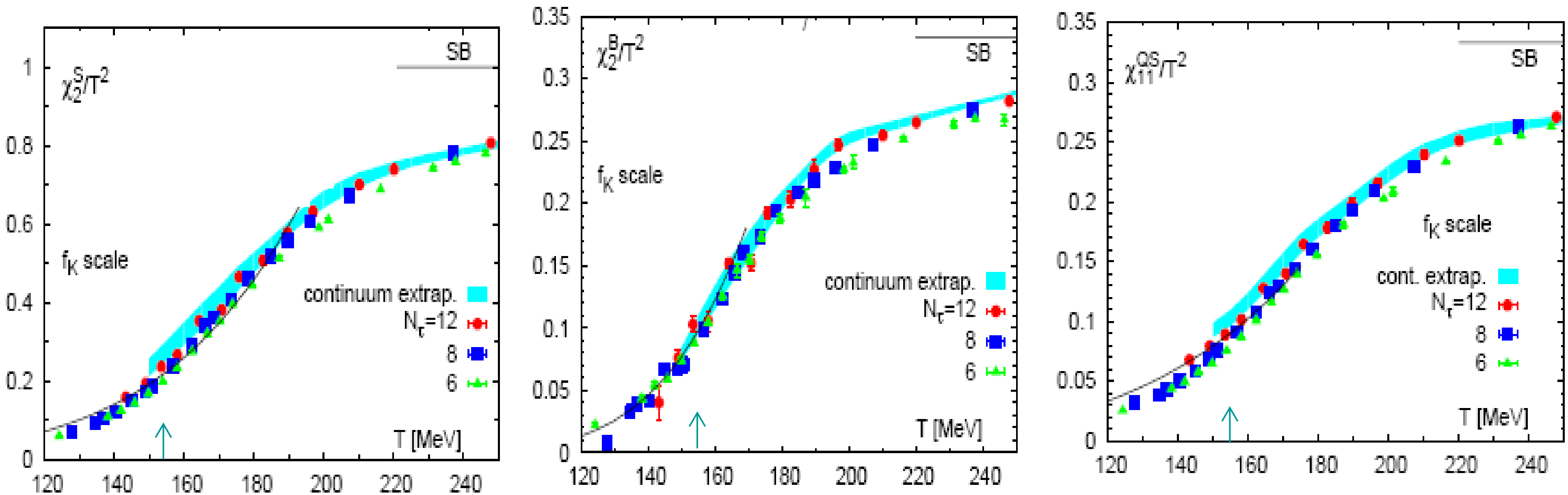
- use also $\Sigma^0 / \Lambda = 0.278$ from pBe at $\sqrt{s} = 25\ GeV$
- Net baryon fluctuations $$\frac{\chi_B}{T^2} = \frac{1}{VT^3}(203.7 \pm 11.4)$$
- Net strangeness fluctuations $$\frac{\chi_S}{T^2} = \frac{1}{VT^3}(504.2 \pm 16.8)$$
- Charge-Strangeness corr. $$\frac{\chi_{QS}}{T^2} = \frac{1}{VT^3}(191 \pm 12)$$
- Ratios is volume independent

$$\frac{\chi_B}{\chi_S} = 0.404 \pm 0.026$$ and $$\frac{\chi_B}{\chi_{QS}} = 1.066 \pm 0.09$$

Compare the ratio with LQCD data:

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee

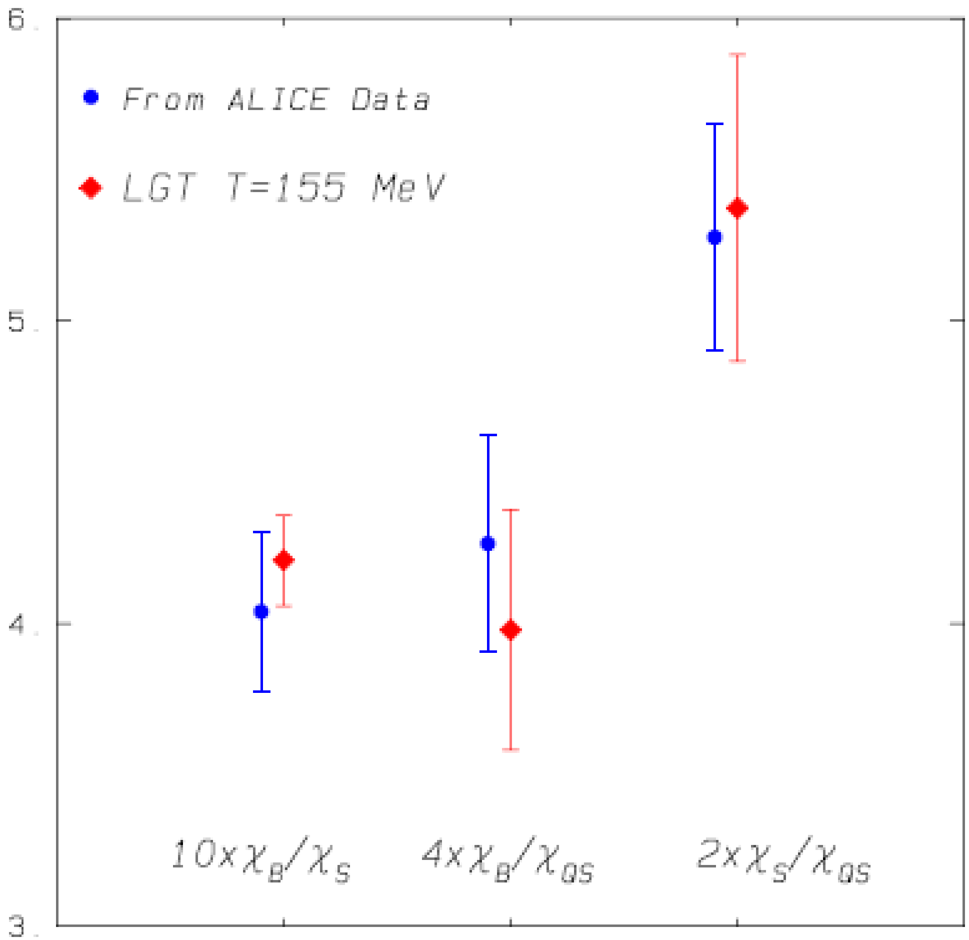
Phys.Rev.Lett. 113 (2014) and HotQCD Coll. A. Bazavov et al. Phys.Rev. D86 (2012) 034509



- Is there a temperature where calculated ratios from ALICE data agree with LQCD?

Baryon number strangeness and Q-S correlations

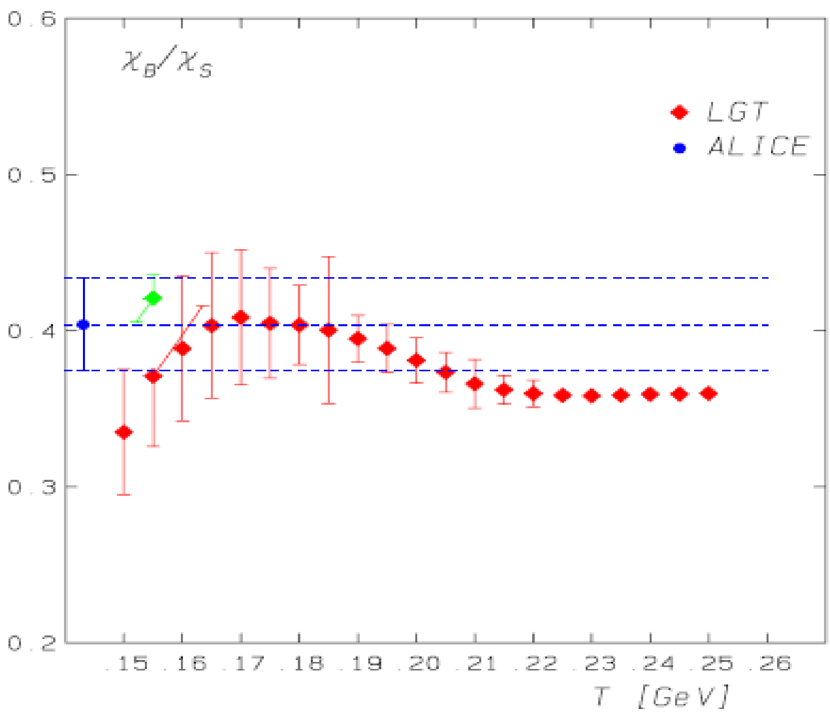
Compare at chiral crossover

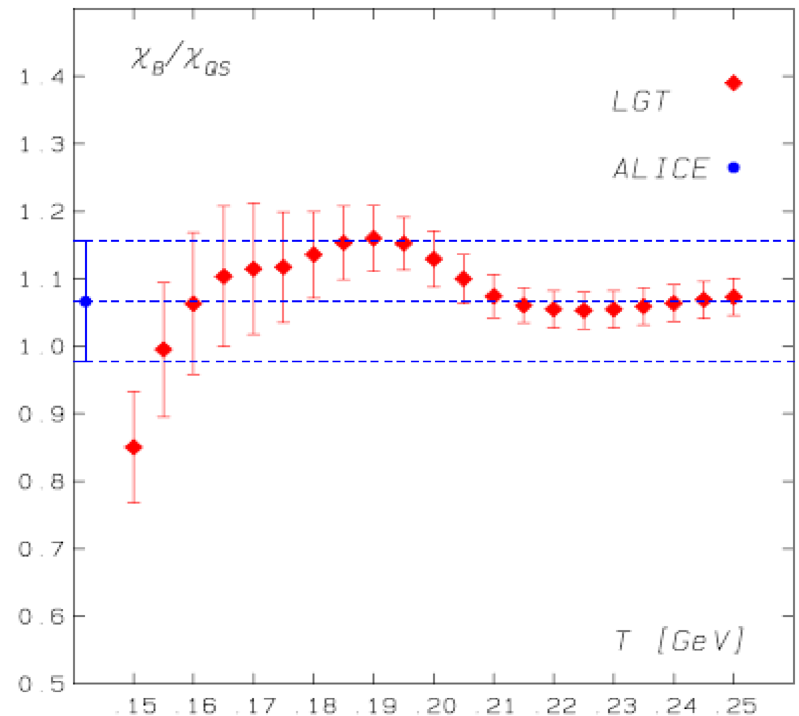


- There is a very good agreement, within systematic uncertainties, between extracted susceptibilities from ALICE data and LQCD at the chiral crossover

- How unique is the determination of the temperature at which such agreement holds?

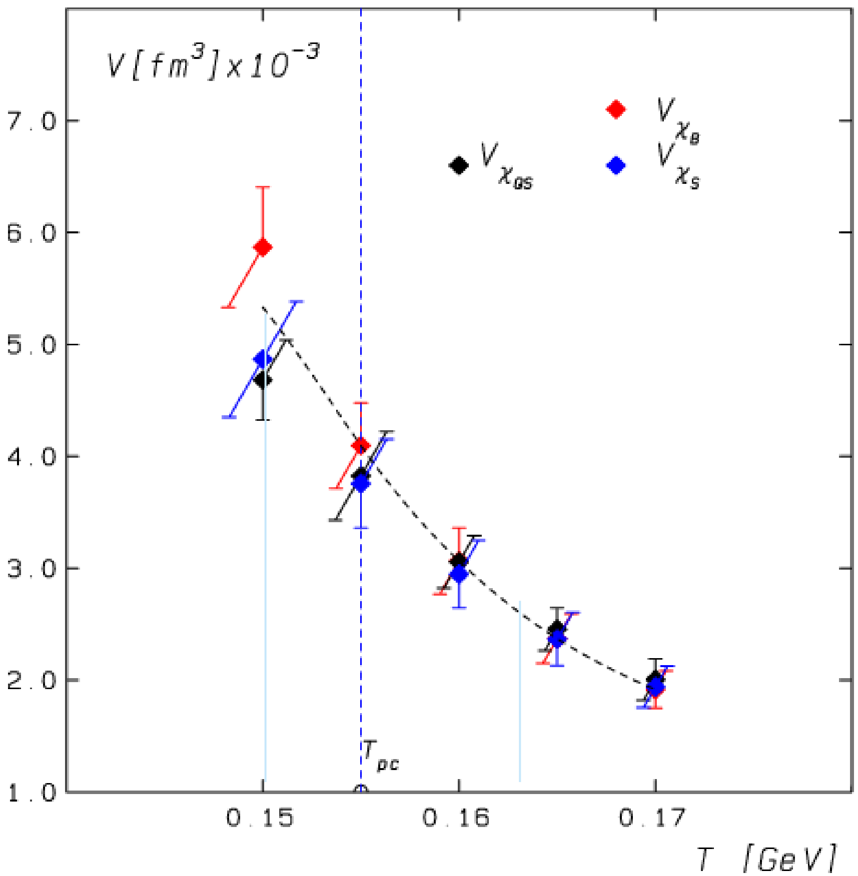
Consider T-dependent LQCD ratios and compare with ALICE data





- The LQCD susceptibilities ratios are weakly T-dependent for $T \geq T_c$
- We can reject $T \leq 0.15\ GeV$ for saturation of χ_B, χ_S *and* χ_{QS} at LHC and fixed to be in the range $0.15 < T \leq 0.21\ GeV$, however
- LQCD => for $T > 0.163\ GeV$ thermodynamics cannot be anymore described by the hadronic degrees of freedom

Extract the volume by comparing data with LQCD



- Since
$$(\chi_N / T^2)_{LQCD} = \frac{(<N^2> - <N>^2)_{LHC}}{V_N T^3}$$

thus

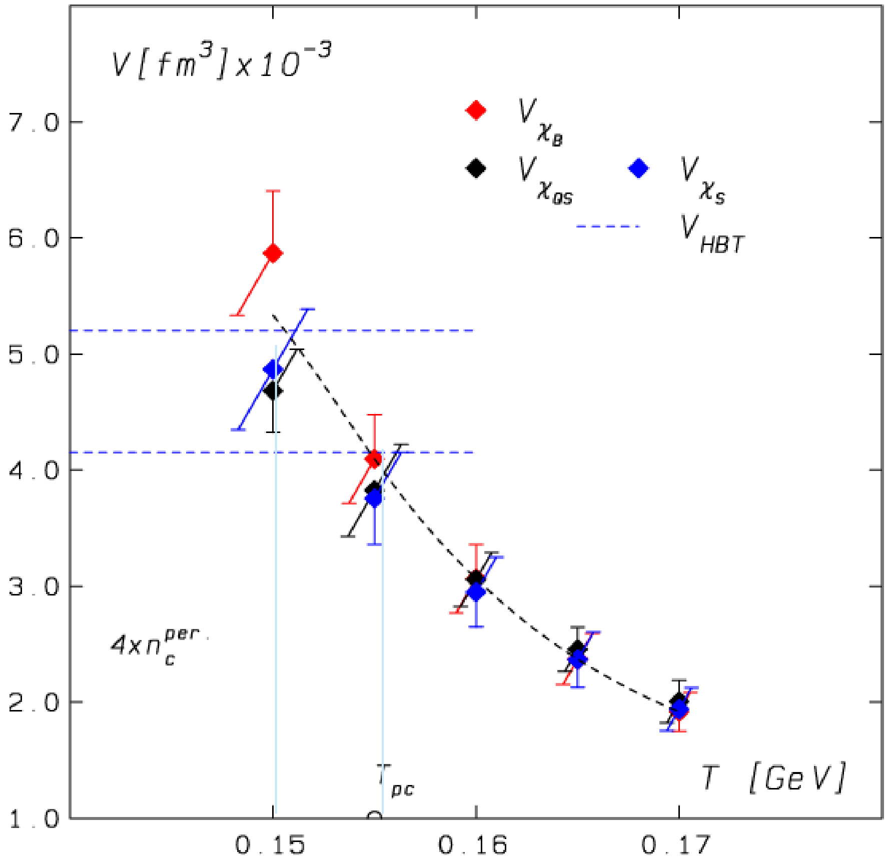
$$V_{\chi_B}(T) = \frac{203.7 \pm 11.4}{T^3 (\chi_B / T^2)_{LQCD}} \qquad V_{\chi_S}(T) = \frac{504.2 \pm 24.2}{T^3 (\chi_B / T^2)_{LQCD}}$$

$$V_{\chi_{QS}}(T) = \frac{191 \pm 12}{T^3 (\chi_B / T^2)_{LQCD}}$$

- All volumes, should be equal at a given temperature if originating from the same source, thus

$$T > 150 \ MeV$$

Constraining the volume from HBT and percolation theory



- Some limitation on volume from Hanbury-Brown–Twiss: HBT volume $V_{HBT} = (2\pi)^{3/2} R_l R_o R_s.$

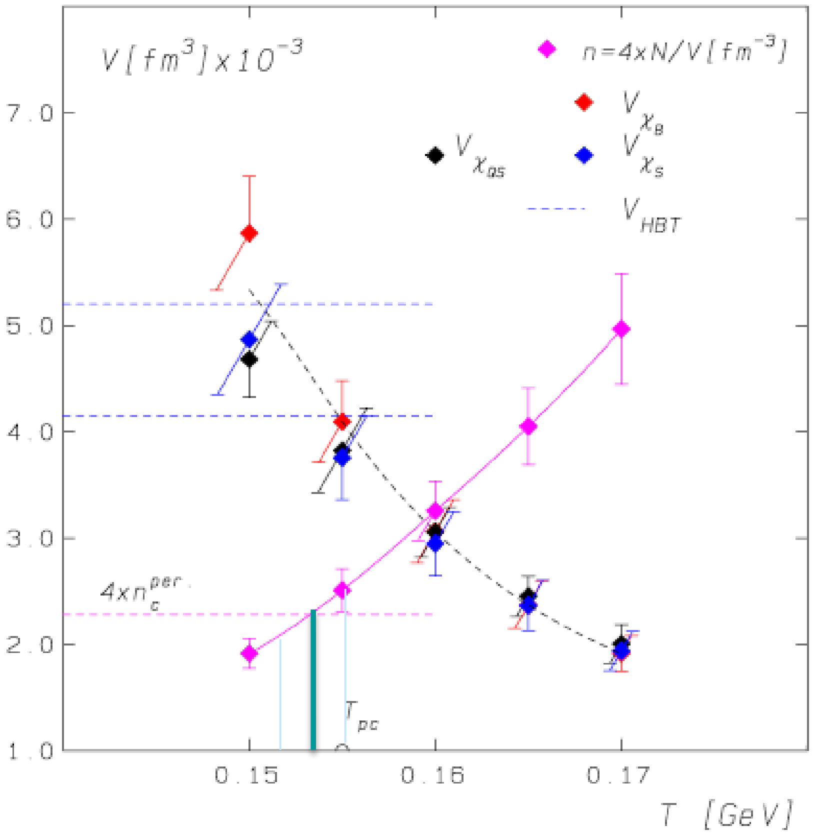
  Take ALICE data from pion interferometry $V_{HBT} = 4800 \pm 640\, fm^{-3}$

If the system would decouple at the chiral crossover, then $V \geq V_{HBT}$

From these results: variance extracted from LHC data and HBT consistent with LQCD for $150 < T \leq 156\ MeV$ and the fireball volume

$$V \approx 4500 \pm 500\ \ fm^3$$

Particle density and percolation theory



- Density of particles at a given volume $n(T) = \dfrac{N^{\exp}_{total}}{V(T)}$

- Total number of particles in HIC at LHC, ALICE

$$\langle N_t \rangle = 3\langle \pi \rangle + 4\langle p \rangle + 4\langle K \rangle + (2 + 4 \times 0.2175)\langle \Lambda_\Sigma \rangle + 4\langle \bar{\Xi} \rangle + 2\langle \bar{\Omega} \rangle,$$

$$\boxed{\langle N_t \rangle = 2486 \pm 146}$$

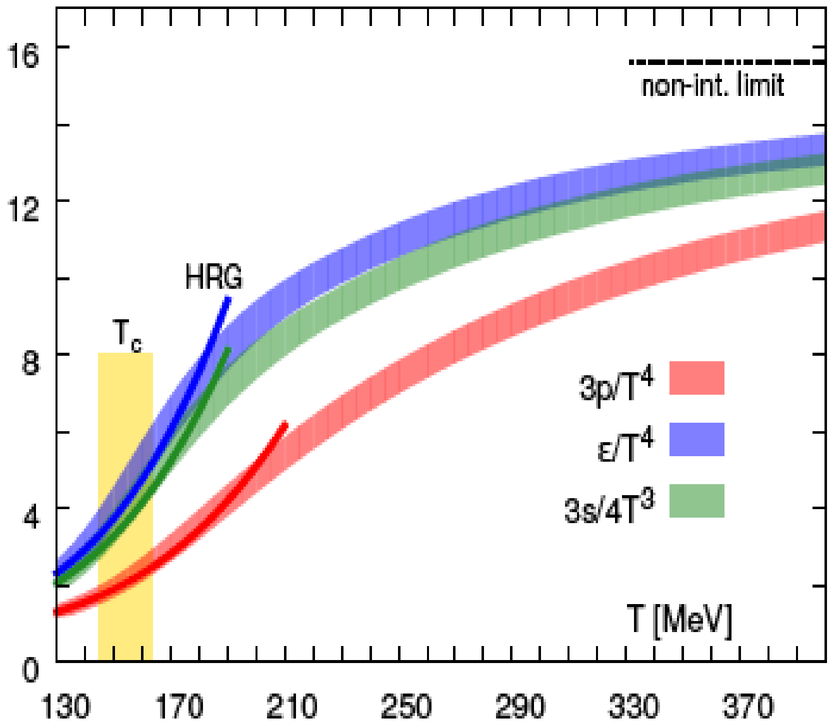
- Percolation theory: 3-dim system of objects of volume $V_0 = 4/3\pi R_0^3$

$$n_c = \frac{1.22}{V_0} \quad \text{take} \quad R_0 \approx 0.8\, fm \Rightarrow n_c \approx 0.57\ [fm^{-3}] \Rightarrow T_c^p \approx 154\ [MeV]$$

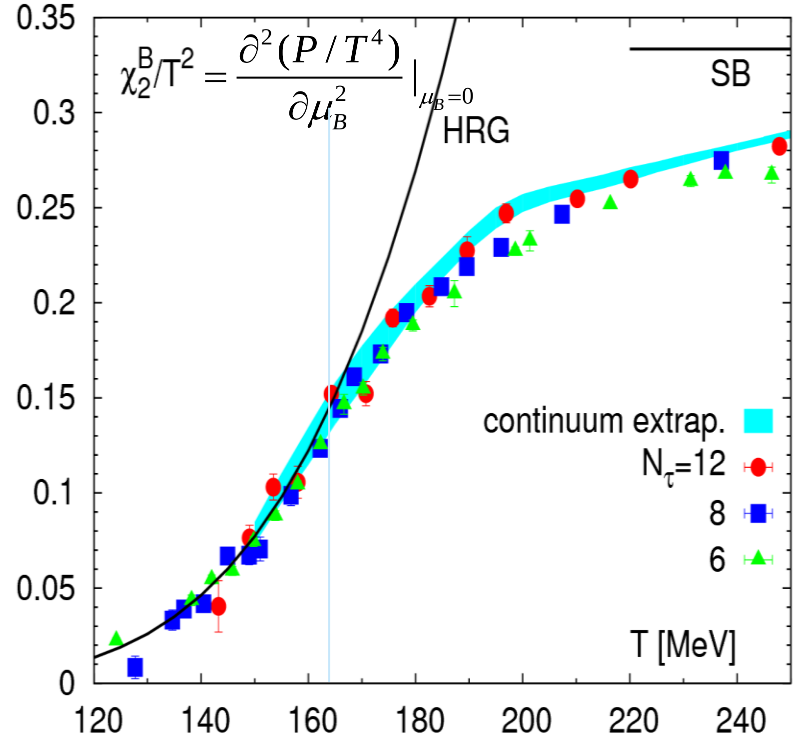
P. Castorina, H. Satz &K.R. Eur.Phys.J. C59 (2009)

Excellent description of the QCD Equation of States by Hadron Resonance Gas

A. Bazavov et al. HotQCD Coll.   July 2014



F. Karsch et al. HotQCD Coll.



- "Uncorrelated" Hadron Gas provides an excellent description of the QCD equation of states in confined phase

- "Uncorrelated" Hadron Gas provides also an excellent description of net baryon number fluctuations

Thermal origin of particle yields with respect to HRG

Rolf Hagedorn => the Hadron Resonace Gas (HRG):

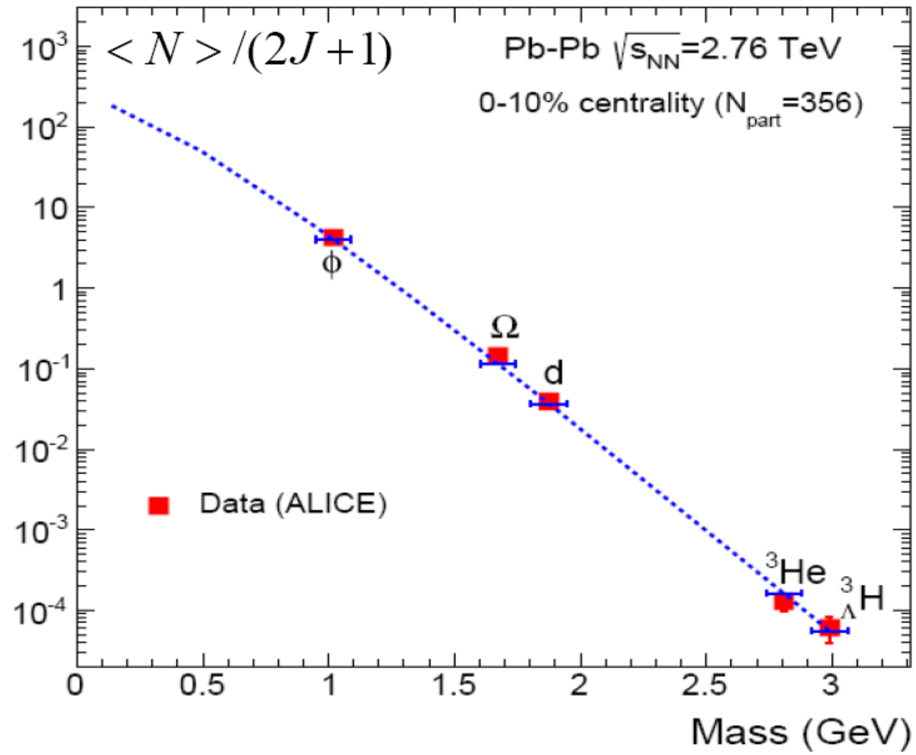
"uncorrelated" gas of hadrons and resonances

$$< N_i >= V \, [n_i^{th}(T, \vec{\mu}) + \sum_K \Gamma_{K \to i} n_i^{th-Res.}(T, \vec{\mu})]$$

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel, et al.

Particle yields with no resonance decay contributions:

$$\frac{1}{2j+1} \frac{dN}{dy} = V(m/T)^2 K_2(m/T)$$



- Measured yields are reproduced with HRG at $T = 156$ MeV

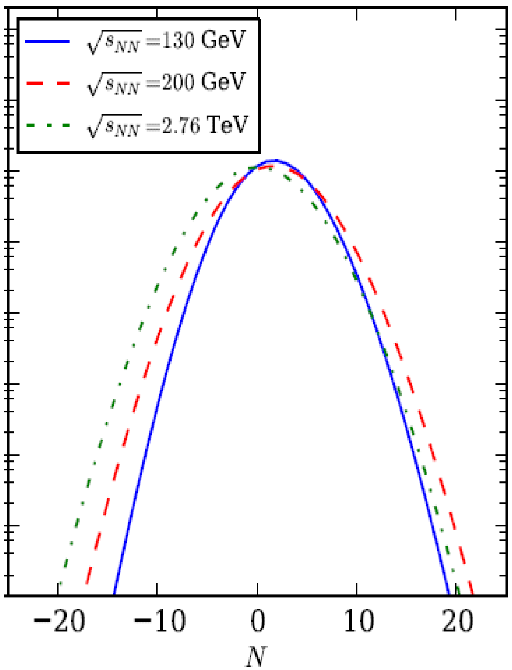
Conclusions:

From a direct comparison of fluctuations constructed from ALICE data, and LQCD results one concludes that:

- there is thermalization in heavy ion collisions at the LHC and the 2nd order charge fluctuations and correlations are saturated at the chiral crossover temperature

Skellam distribution, and its generalization, is a good approximation of the net charge probability distribution P(N) for small N. The chiral criticality sets in at larger N and results in the shrinking of P(N) relative to the Skellam function .

What is the influence of O(4) criticality on P(N)?



- For the net baryon number use the Skellam distribution (HRG baseline)

$$P(N) = \left(\frac{B}{\bar{B}}\right)^{N/2} I_N(2\sqrt{B\bar{B}})\exp[-(B+\bar{B})]$$

as the reference for the non-critical behavior

- Calculate P(N) in an effective chiral model which exhibits O(4) scaling and compare to the Skellam distribtuion

Modelling O(4) transtion: effective Lagrangian and FRG

$$\mathscr{L}_{\rm QM} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U(\sigma,\vec{\pi})$$

Effective potential is obtained by solving *the exact flow equation* (Wetterich eq.) with the approximations resulting in the O(4) critical exponents

J. Berges, D. U. Jungnickel & C. Wetterich; B .J. Schaefer & J. Wambach; B. Stokic, B. Friman & K.R.

$$\partial_k\Omega_k(\sigma) = \frac{Vk^4}{12\pi^2}\left[\sum_{i=\pi,\sigma}\frac{d_i}{E_{i,k}}[1+2n_B(E_{i,k})] - \frac{2\nu_q}{E_{q,k}}[1-n_F(E^+_{q,k})-n_F(E^-_{q,k})]\right]$$

$$E_{\pi,k} = \sqrt{k^2+\Omega'_k}$$

$$E_{\sigma,k} = \sqrt{k^2+\Omega'_k+2\rho\Omega''_k}$$

$$E_{q,k} = \sqrt{k^2+2g^2\rho}$$

$$\Omega'_k \equiv \frac{\partial\Omega_k}{\partial(\sigma^2/2)}$$

Full propagators with k < q < Λ

Γ_Λ=S classical

Integrating from k=Λ to k=0 gives a full quantum effective potential

Put $\Omega_{k=0}(\sigma_{\min})$ into the integral formula for P(N)

Moments obtained from probability distributions

- Moments obtained from probability distribution

$$< N^k >= \sum_N N^k P(N)$$

- Probability quantified by all cumulants

$$P(N) = \frac{1}{2\pi} \int_0^{2\pi} dy \exp[iyN - \chi(iy)]$$

Cumulants generating function: $\chi(y) = \beta V[p(T, y+\mu) - p(T,\mu)] = \sum_k \chi_k y^k$

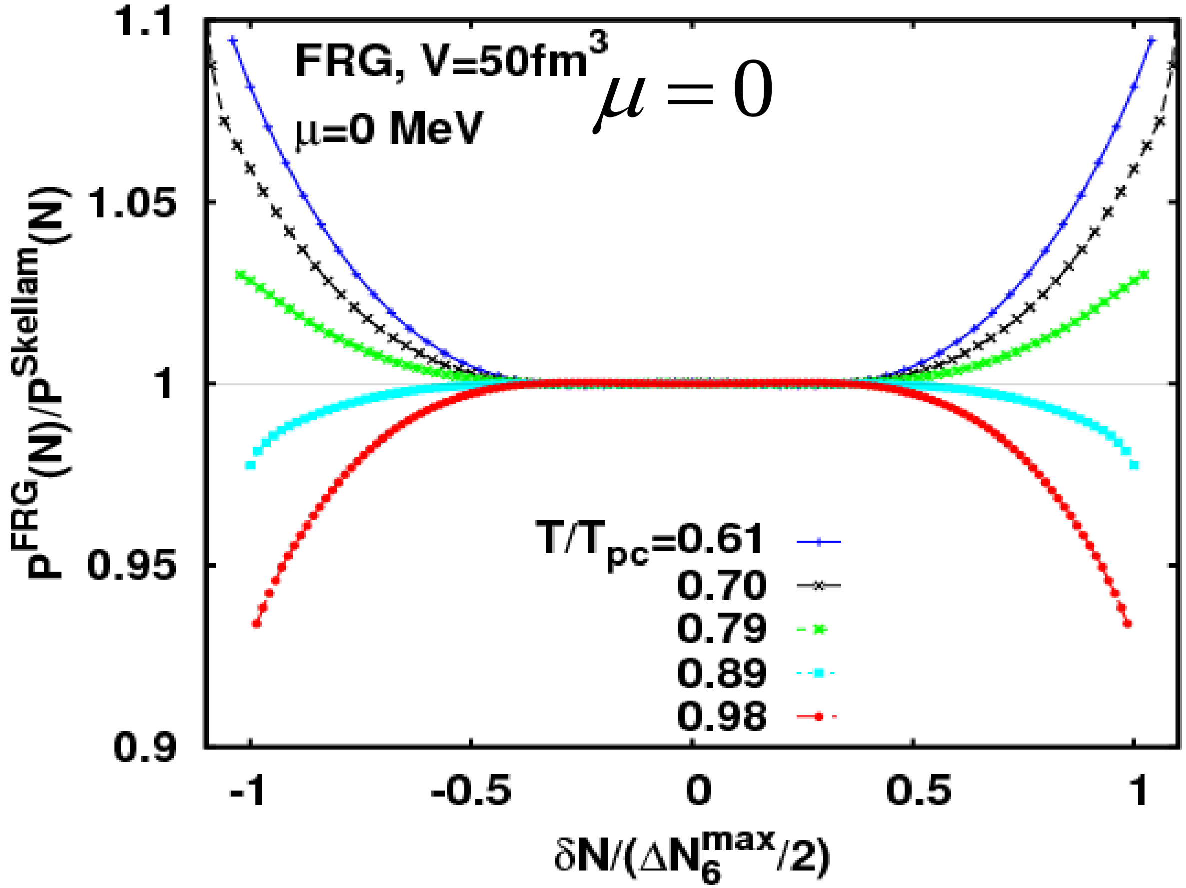
- In statistical physics

$$P(N) = \frac{Z_C(N)}{Z_{GC}} e^{\frac{\mu N}{T}} \quad Z(T,V,N) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta e^{-i\theta N} \mathscr{Z}(T,V,\theta)$$

The influence of O(4) criticality on P(N) for $\mu = 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance at different T/T_{pc}

K. Morita, B. Friman &K.R. (QM model within renormalization group FRG)



Ratio < 1 at larger |N| if $c_6/c_2 < 1$