Inhomogeneous condensation in dense nuclear matter

11-th Polish Workshop on relativistic Heavy-Ion Collisions

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Outline

Part I:

- An hadronic model of QCD: the eLSM
- Study at nonzero density
- Inhomogenoeus condensation: chiral density wave

Part II:

- A lattice-like approach for arbitrary inhomogenoeus condensation
- Applications to 1+1 dimensional models
- Application to the NJL model

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Part I

Fields of the eLSM

- Quark-antiquark mesons: scalar, pseudoscalar, vector and axial-vector quarkonia.

- Additional mesons: The scalar and the pseudoscalar glueballs

- Baryons: nucleon doublet and its partner

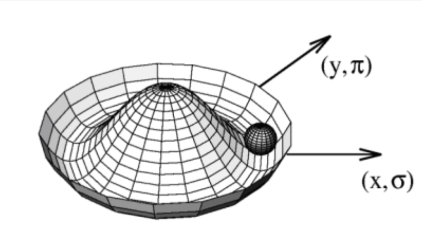
  (in the so-called mirror assignment)

We construct the Lagrangian of the so-called Extended Linear Sigma Model (ELSM) according to

chiral invariance and dilatation symmetry and their explicit breakings.

Model of QCD – eLSM

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right) + \mathrm{Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi)\right] \\
& - m_0^2\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\Phi^\dagger\Phi\right] - \lambda_1(\mathrm{Tr}\left[\Phi^\dagger\Phi\right])^2 - \lambda_2 \mathrm{Tr}\left[(\Phi^\dagger\Phi)^2\right] \\
& + \left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left((L^\mu)^2 + (R^\mu)^2\right)\right] \\
& - \frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] + \mathrm{Tr}\left[H\left(\Phi^\dagger + \Phi\right)\right] \\
& + c_1[\det(\Phi) - \det(\Phi^\dagger)]^2 + \frac{h_1}{2}\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\
& + h_2 \mathrm{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \mathrm{Tr}[\Phi R_\mu \Phi^\dagger L^\mu]\end{aligned}$$



$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

$$\sigma_N \to \sigma_N + \phi$$

$$L^\mu, R^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \pm a_1^0}{\sqrt{2}} & \rho^+ \pm a_1^+ & K^{\star +} \pm K_1^+ \\ \rho^- \pm a_1^- & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \mp a_1^0}{\sqrt{2}} & K^{\star 0} \pm K_1^0 \\ K^{\star -} \pm K_1^- & \bar{K}^{\star 0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S} \end{pmatrix}$$

S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011**)
D. Parganlija, P. Kovacs, G. Wolf , F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011**

Results of the fit (11 parameters, 21 exp. quantities)

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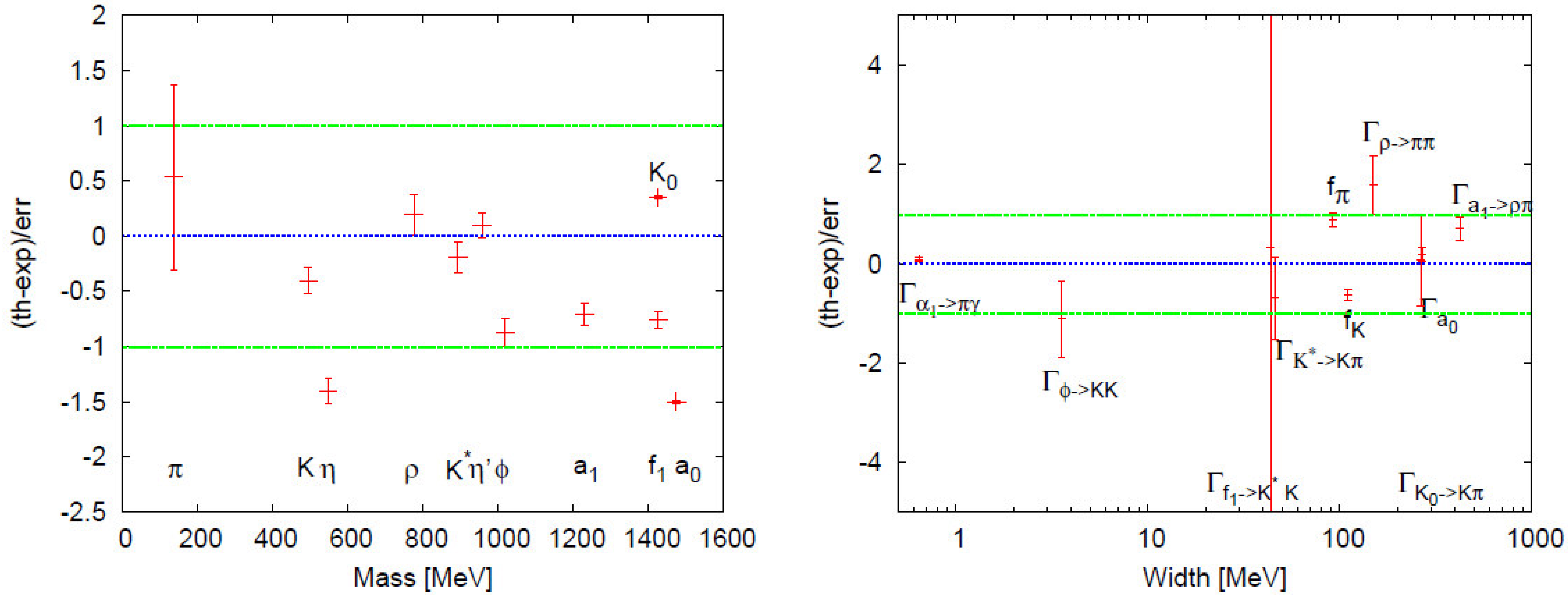
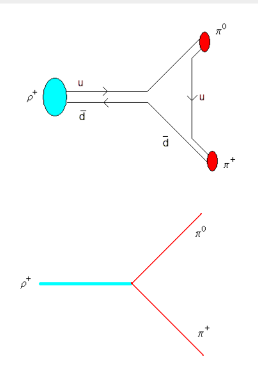


arXiv:1208.0585

Microscpic

eLSM



Overall phenomenology is good. Further quantities calculated afterwards.

Scalar mesons $a_0(1450)$ and $K_0(1430)$ above 1 GeV and are quark-antiquark states. The chiral partner of the pion (the σ) is $f_0(1370)$.

Importance of the (axial-)vector mesons

Baryon sector in the ElSM
(Nf = 2 only)

Nucleon and its chiral partner; chiral symmetry and dilatation invariance

(Axial-)vector mesons are included

Mirror assignment: (C. De Tar and T. Kunihiro, **PRD 39 (1989) 2805**)

$$\Psi_{1,R} \to U_R \Psi_{1,R} \qquad \Psi_{1,L} \to U_L \Psi_{1,L}$$

$$\Psi_{2,R} \to U_L \Psi_{2,R} \qquad \Psi_{2,L} \to U_R \Psi_{2,L}$$

A chirally invariant mass-term is possible!

$$m_0 \left(\overline{\Psi}_{1,L} \Psi_{2,R} - \overline{\Psi}_{1,R} \Psi_{2,L} - \overline{\Psi}_{2,L} \Psi_{1,R} + \overline{\Psi}_{2,R} \Psi_{1,L} \right)$$

$$\mathcal{L}_{mirror} = \overline{\Psi}_{1L} i\gamma_\mu D^\mu_{1L} \Psi_{1L} + \overline{\Psi}_{1R} i\gamma_\mu D^\mu_{1R} \Psi_{1R} + \overline{\Psi}_{2L} i\gamma_\mu D^\mu_{2R} \Psi_{2L} + \overline{\Psi}_{2R} i\gamma_\mu D^\mu_{2L} \Psi_{2R}$$
$$- \widehat{g}_1 \left(\overline{\Psi}_{1L} \Phi \Psi_{1R} + \overline{\Psi}_{1R} \Phi^\dagger \Psi_{1L} \right) - \widehat{g}_2 \left(\overline{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \overline{\Psi}_{2R} \Phi \Psi_{2L} \right) + \mathcal{L}_{\mathrm{mass}}$$

Mass of the nucleon

$$m_{N,N^*} = \sqrt{m_0^2 + \left(\frac{\widehat{g}_1 + \widehat{g}_2}{4}\right)^2 \phi^2} \pm \frac{(\widehat{g}_1 - \widehat{g}_2)\phi}{4}$$

$N = N(940)$

$N^* = N^*(1535)$

If $m_0 = 0 \rightarrow$ only the quark condensate generates the masses. $m_N \sim \phi$

$$m_0 = 460 \pm 136 \text{ MeV}$$

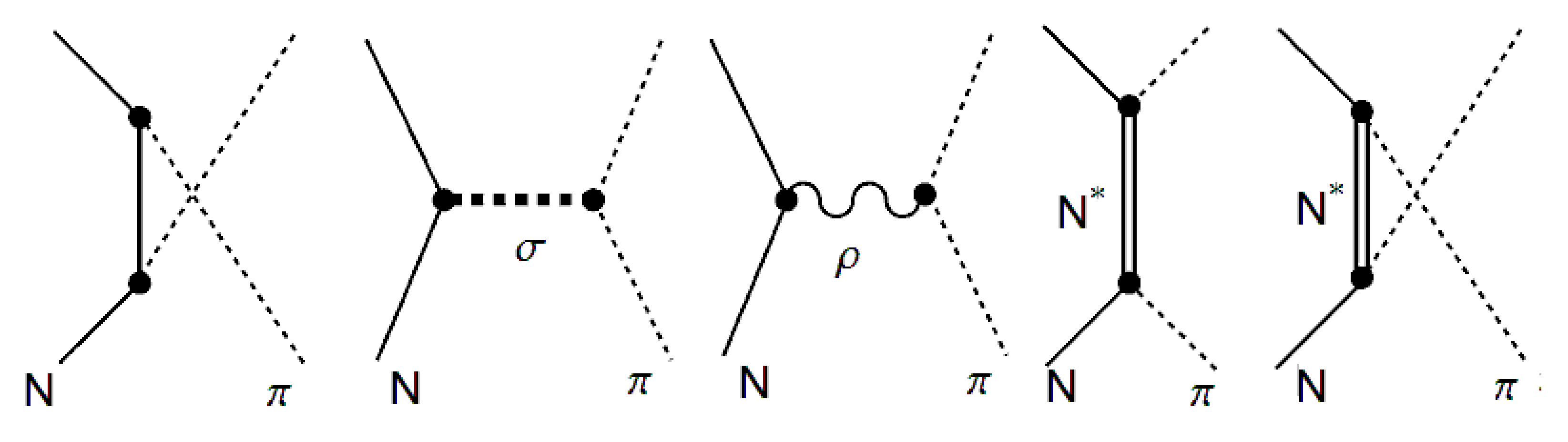
Using $g_A^N = 1.26$ (exp), $g_A^{N^*} \approx 0.2$ (latt) and $\Gamma_{N^* \to N\pi} \approx 67$ MeV

Details in S. Gallas, F. G., D. H. Rischke, **Phys.Rev. D82 (2010) 014004, arXiv:0907.5084**

m_0 parameterizes the contribution which does not stem from the quark condensate

Crucial also at nonzero temperature and density

Test: pion-nucleon scattering lengths



$$a_0^- = (6.04 \pm 0.63) \cdot 10^{-4}\ \text{MeV}^{-1} \qquad a_0^{-(\exp)} = (6.4 \pm 0.1) \cdot 10^{-4}\ \text{MeV}^{-1}$$

$$a_0^+ \approx (\text{from} - 20\ \text{to} + 20 \cdot 10^{-4})\ \text{MeV}^{-1} \qquad a_0^{+(\exp)} = (-8.8 \pm 7.2) \cdot 10^{-4}\ \text{MeV}^{-1}$$

Large theoretical uncertainty due to the scalar-isosocalar sector

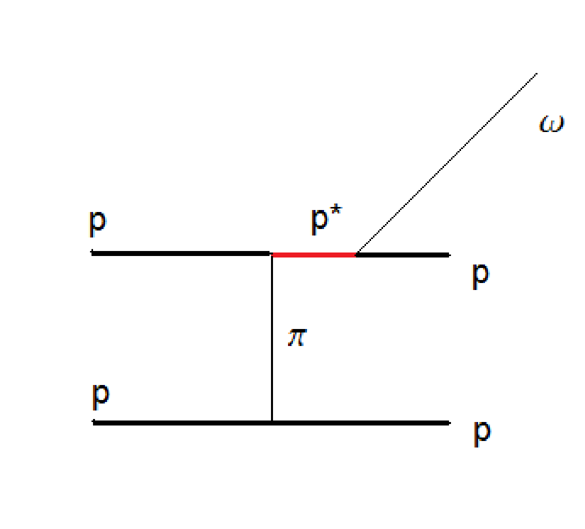
Importance of both vector mesons and mirror assignment in order to get these results

What we are studying rigth now in the vacuum:

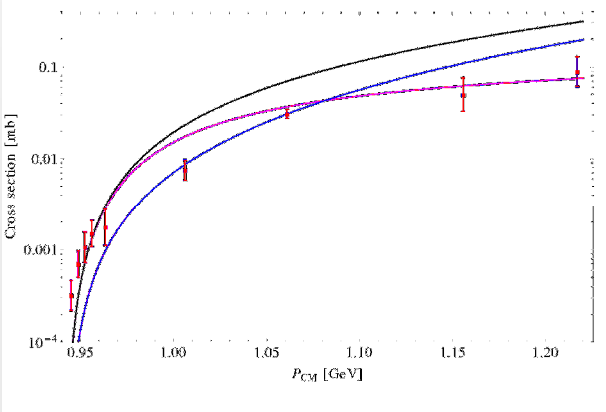
$$p + p \to p + p + X$$

$$X = \omega, \eta, \text{lepton pair}, ...$$

Many diagrams to calculate; the advantage is : chiral symmetry (and also g.i.) built in.



$$p + p \to p + p + \omega$$



Basic considerations for nonzero density

The σ-field of our model corresponds to the resonance $f_0(1370)$
…and not to the lightest scalar resonance $f_0(500)$.

The question is: what is $f_0(500)$ and, more in general, what are the scalar states below 1 GeV?

A good phenomenology (masses and decays) is achieved when interpreting the light scalar states as tetraquarks: $f_0(500) \approx [\bar{u},\bar{d}][u,d]$
(bound states of a diquark and an anti-diquark)

Back to nucleons: where does m_0 comes from?

$$m_0\left(\overline{\Psi}_{1,L}\Psi_{2,R}-\overline{\Psi}_{1,R}\Psi_{2,L}-\overline{\Psi}_{2,L}\Psi_{1,R}+\overline{\Psi}_{2,R}\Psi_{1,L}\right)$$

By requiring dilatation invariance one should modify the mass-term as:

$$a\chi\left(\overline{\Psi}_{1,L}\Psi_{2,R}-\overline{\Psi}_{1,R}\Psi_{2,L}-\overline{\Psi}_{2,L}\Psi_{1,R}+\overline{\Psi}_{2,R}\Psi_{1,L}\right)$$

Tetraquark

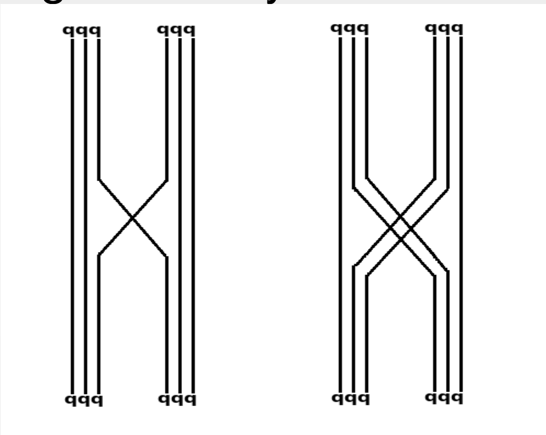
New field: $f_0(500)$

$$\text{By shifting}: \chi \rightarrow \chi_0 + \chi \qquad \text{one has}: \quad m_0 = a\chi_0$$

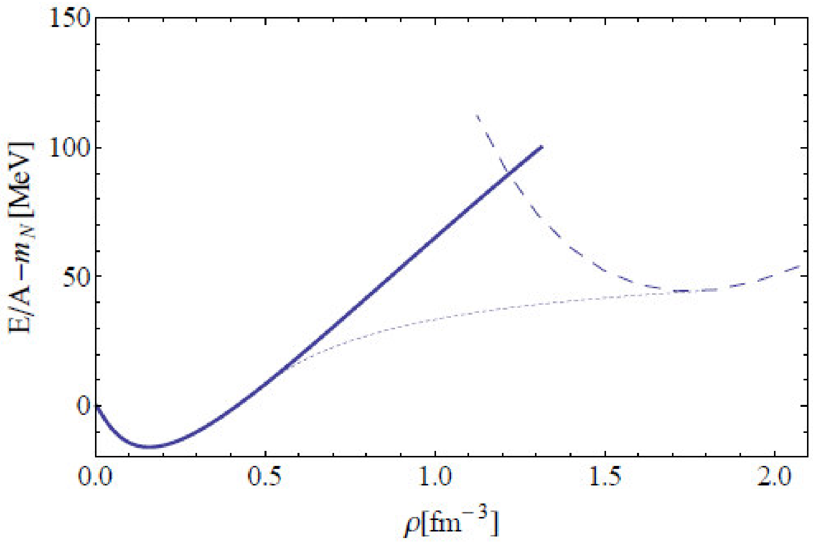
m_0 originates form the tetraquark condensate

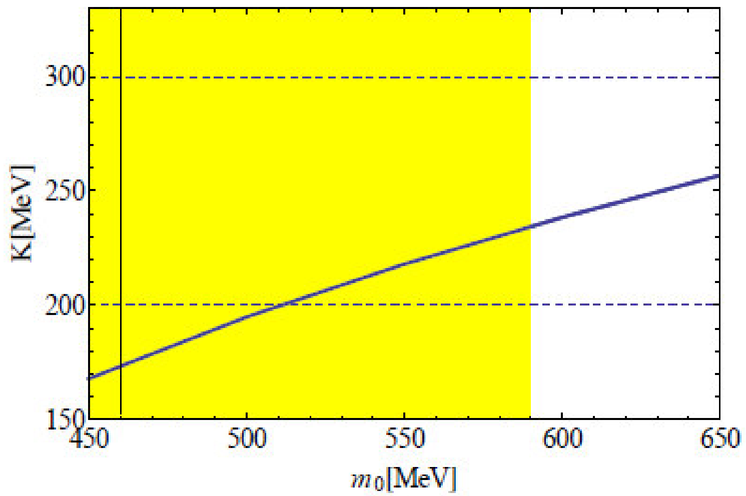
.

Note, also, a tetraquark exchange naturally arises in nucleon-nucleon interactions



Nuclear matter saturation and compressibilty





Details in: S. Gallas, F. G:, G. Pagliara, **Nucl.Phys. A872 (2011) 13-24 arXiv:1105.5003**

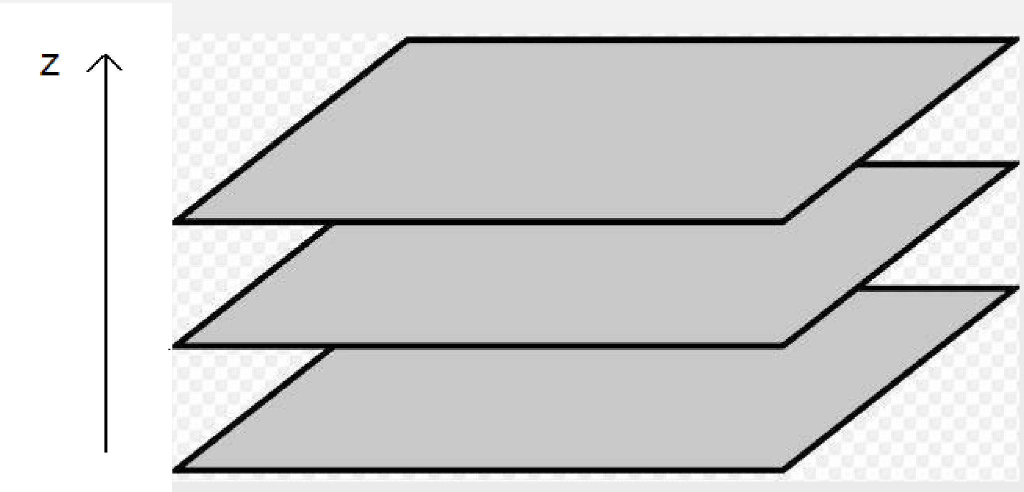
Inhomogeneous condensation at nonzero density

$$\text{Up to now}: \phi = const$$

…but one can have a Chiral Density Wave (**Ansatz**!):

$$\phi(z) = \phi \cos(2fz)$$

$$\langle \pi^0 \rangle = \phi \sin(2fz)/Z$$

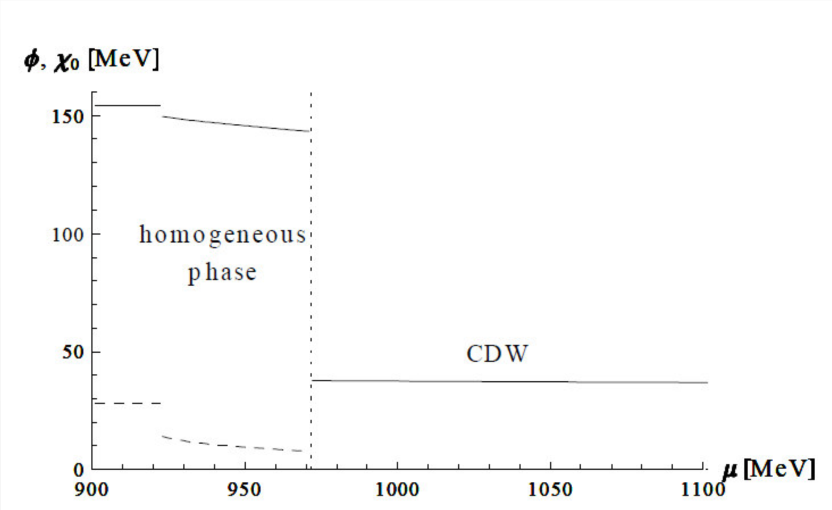


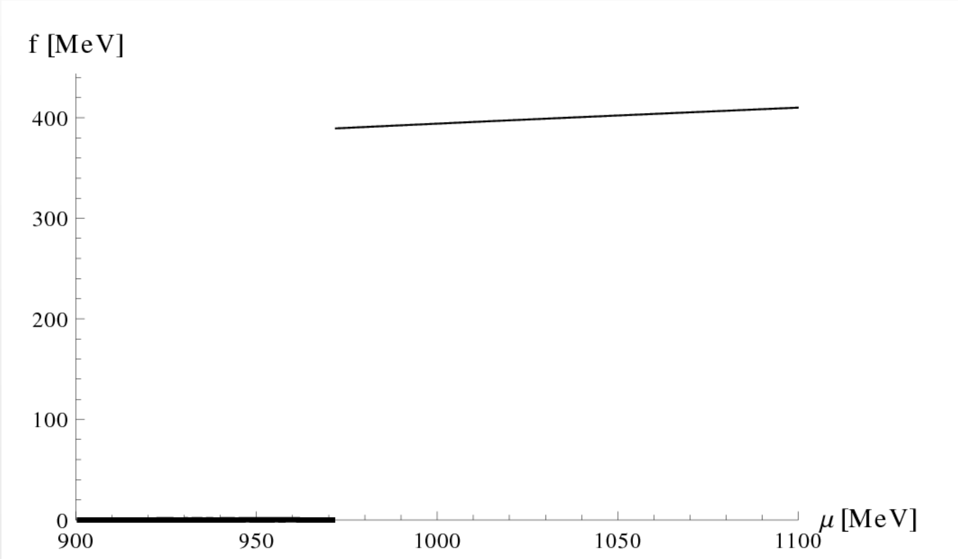
Inhomogeneous condensation/2

$$\phi(z) = \langle \sigma \rangle = \phi \cos(2fz)$$

$$\langle \pi^0 \rangle = \phi \sin(2fz)/Z$$

$m_0 = 460 \text{ MeV}$





$$\rho_{CDW} / \rho_0 = 2.4$$

A. Heinz, F.G., D. H. Rischke, Nucl. Phys. A 933, 34-42, 2015.

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Part II

The finite mode approach

"Wish list" for further calculations

- arbitrary modulations: $\sigma(x)$ is not an input
- several inhomogeneous fields: $\sigma(x)$, $\pi(x)$, $\omega(x)$, ...
- no limitations on the dimension of the model: $1+1$, $1+2$, ...
- higher dimensional modulations: $\sigma(x,y)$, $\sigma(x,y,z)$, ...

New numerical tool: so-called "finite-mode approach"

Lattice inspired method, which allows to access all points of the "wish list".

Not so fast, there are a few limitations:

- numerically complex
- finite size corrections
- optimization and tests

M. Wagner, arXiv:hep-ph/0704.3023v1 [hep-lat]

The Gross-Neveau (GN) model

$$\mathscr{L}_{\mathrm{GN}} = \sum_{i=1}^{N_c} \bar{\psi}_i \left(\gamma_\mu \partial_\mu + \gamma_0 \mu\right) \psi_i + \frac{1}{2} g^2 \left(\sum_{i=1}^{N_c} \bar{\psi}_i \psi_i \right)^2$$

- in $1+1$ dimensions renormalizable
- asymptotic freedom
- discrete chiral symmetry
- in the large N_c limit chiral symmetry is broken

M. Thies, K. Urlichs, [hep-th/0302092]
G. Basar, G. V. Dunne and M. Thies, arXiv:0903.1868 [hep-th]

The finite approach for the GN model

Effective action of the GN model:

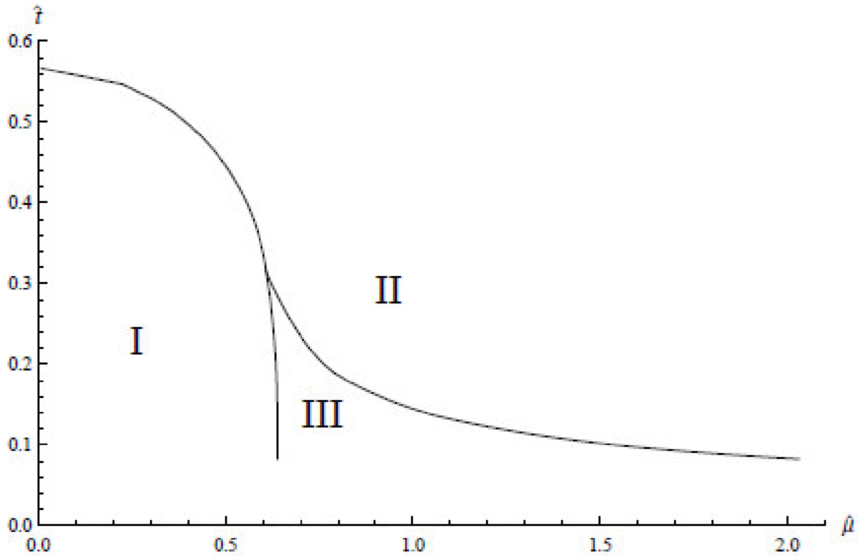
$$S = N_c \left[\frac{1}{2\lambda} \int d^2x \, \sigma(x)^2 - \ln\left(\det Q\right) \right] \; , \quad Q = \gamma_\mu \partial_\mu + \gamma_0 \mu + \sigma(x)$$

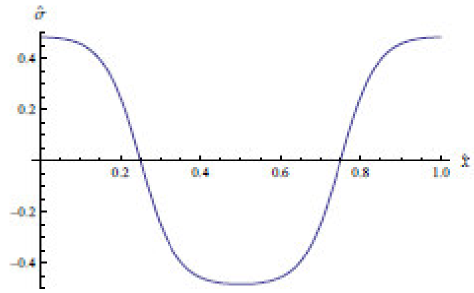
Discretized effective action for a homogeneous condensate $\sigma(x) \equiv \sigma$:

$$\frac{S}{N_c} = L_1 L_0 \frac{\sigma^2}{2\lambda} - \sum_{n_0=0}^{N_0-1} \sum_{n_1=-N_1}^{N_1} \ln\left\{ \left[\left(\frac{2\pi}{L_0} \left(\frac{1}{2} + n_0 \right) \right)^2 + \left(\frac{2\pi}{L_1} n_1 \right)^2 + \sigma^2 - \mu^2 \right]^2 + \left[2\mu \frac{2\pi}{L_0} \left(\frac{1}{2} + n_0 \right) \right]^2 \right\}$$

- finite spacetime volume $L_0 L_1$
- fermion fields as superposition of plane waves
- finite number temporal modes N_0 and space modes N_1

The phase diagram of the GN model





The shape of the condensate $\hat{\sigma}(x)$ close to I / III phase boundary.

- I: homogeneous broken phase
- II: restored phase
- III: inhomogeneous phase

	$\hat{\mu}_3$	$\hat{t}_3$
$N_{00} = N_1 = 192$	0.60938	0.31474
continuum	0.60822	0.31833

$\hat{\sigma} := \sigma/\sigma_0$, $\quad \hat{\mu} := \mu/\sigma_0$, $\quad \hat{t} := T/\sigma_0$

The chiral GN model

$$\mathscr{L}_{\chi\text{GN}} = \sum_{i=1}^{N_c} \bar{\psi}_i \left(\imath\gamma_\mu \partial^\mu + \gamma_0 \mu + \sigma + \imath\gamma_5 \eta \right) \psi_i + \frac{1}{2}\lambda \left(\sigma^2 + \eta^2 \right)$$

discrete chiral symmetry $\to$ continuous chiral symmetry

Analytic solution is known, only two phases are realized

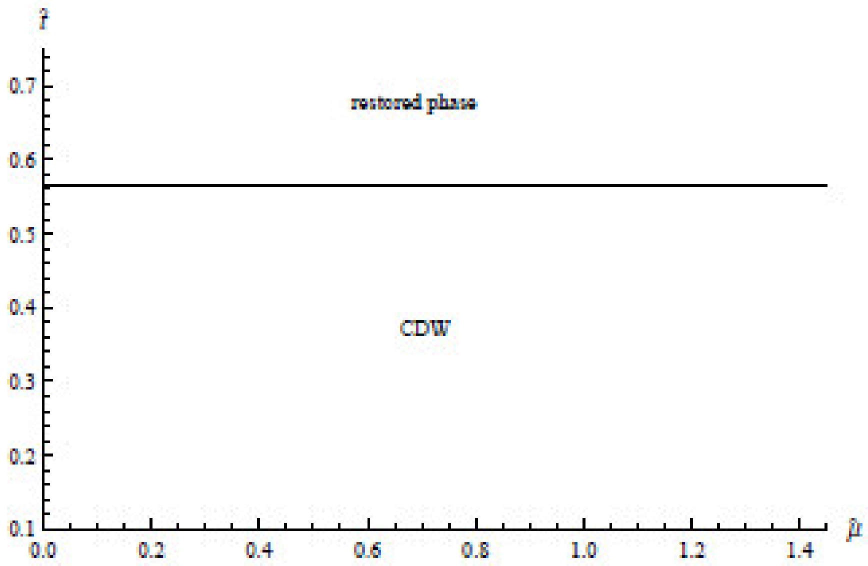
$\hat{t} < \hat{t}_c$, all $\hat{\mu}$: chiral density wave

$$\langle \sigma \rangle \sim \phi \cos(2\ \mu\ x) \text{ and } \langle \eta \rangle \sim \phi \sin(2\ \mu\ x)$$

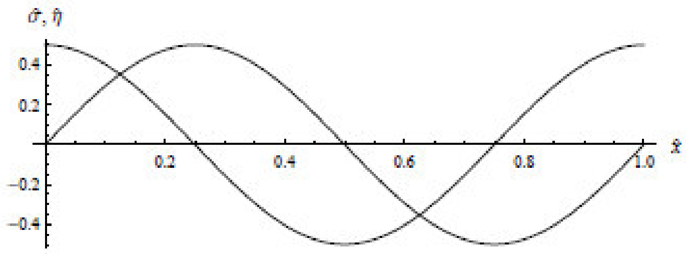
$\hat{t} > \hat{t}_c$, all $\hat{\mu}$: restored chiral symmetry

$$\langle \sigma \rangle = 0 \text{ and } \langle \eta \rangle = 0$$

The phase diagram of the chiral GN model



CDW is **not** an input! It follows from a dynamical minimization of the σ- and the η-condensate.



$\hat{\mu} = 0.295$ at $\hat{t} = 0.0945$

The NJL model

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Lagrangian of the NJL model

$$\mathscr{L}_{NJL} = \sum_{i=0}^{N_c} \bar{\psi}_i \partial\!\!\!/ \psi_i - \frac{G}{N_c}\left[\left(\sum_{i=0}^{N_c} \bar{\psi}_i \psi_i\right)^2 + \left(\sum_{i=0}^{N_c} \bar{\psi}_i \imath\gamma_5 \psi_i\right)^2\right]$$

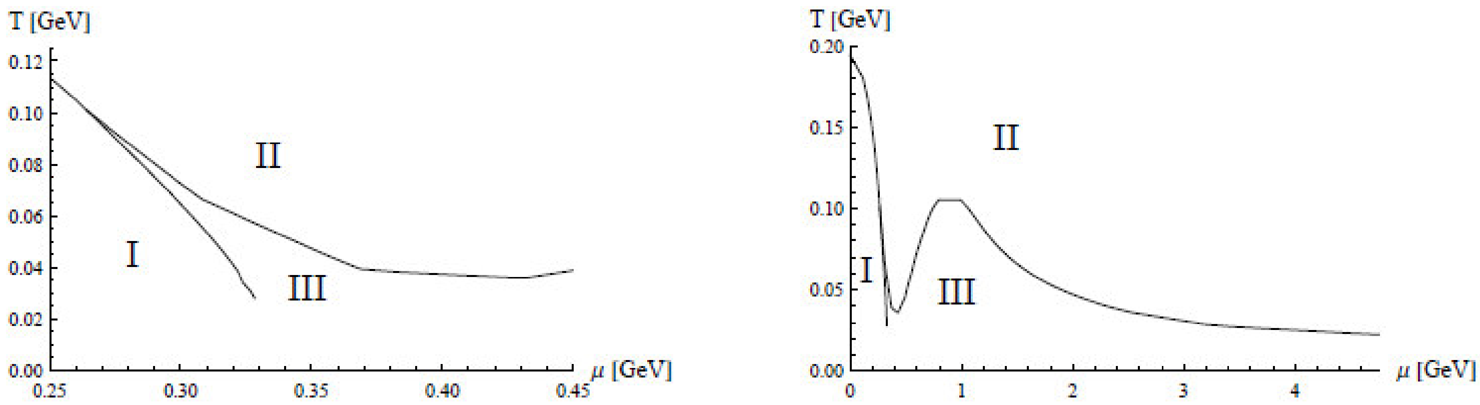
$$\frac{S}{N_c} = \left[\frac{\lambda}{2}\int d^4x\ m^{*2}(x) - \ln\left(\det Q\right)\right] \ , \quad Q = \gamma_\mu \partial_\mu + \gamma_0 \mu + m^*(x)$$

Looks similar but:

- $1+3$ dimensional model
- not renormalizable
- continuous chiral symmetry
- regularization scheme effects phase diagram
- computational effort increases strongly

D. Nickel, arXiv:0906.5295 [hep-ph]   S. Carignano, M. Buballa, arXiv:1111.4400 [hep-ph]

The phase diagram of the NJL model



- I: homogeneous broken phase
- II: restored phase
- III: inhomogeneous region

$m_0 = 350$ MeV, $N_i = 120$, and $N_{00} = 120$

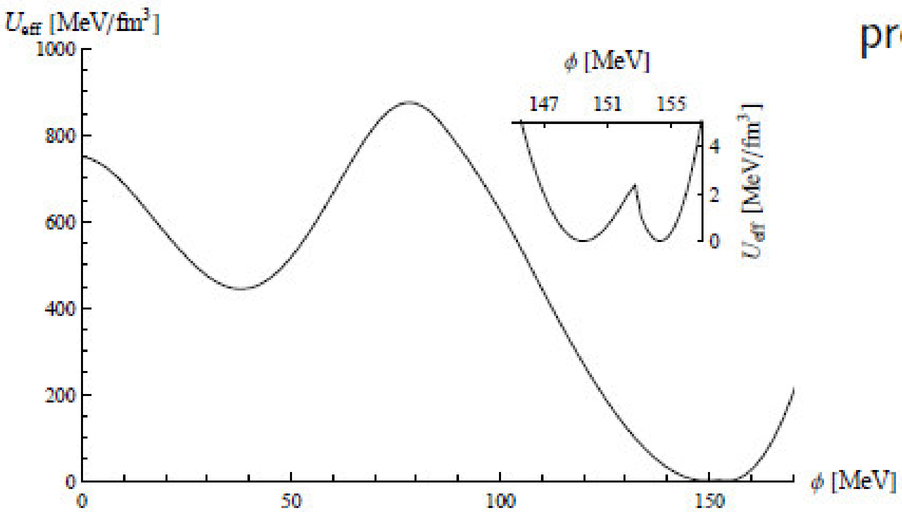
Summary and outlook

- relevance of inhomogeneous phases at moderate densities
- extension to three flavours
- further parity doublets
- Δ resonances
- spatial ω_0 modulations
- ...

- solid approach to calculate inhomogeneous phases
- higher dimensional modulations
- interweaving chiral spirals
- further effective models
- diquark condensation
- ...

Thank You

Inhomogeneous condensation/3

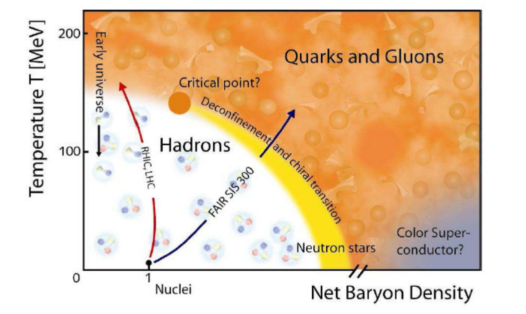


Three distinguished minima are present:

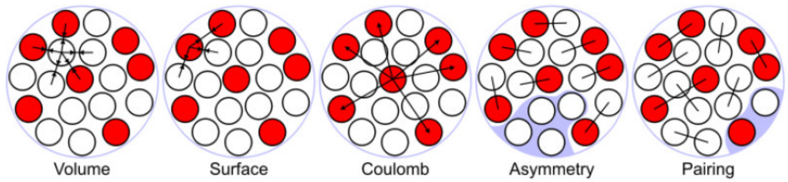
- vacuum ground state ($\rho = 0$) at $\phi = 154.4$ MeV
- homogeneous nuclear matter ground state ($\rho = \rho_0$) at $\phi = 149.5$ MeV
- local minima at $\phi = 38.3$ MeV with $f \neq 0$

ϕ as a function of the extrema of $\bar{\chi}$, $\bar{\omega}_0$ and f.

Description of nuclear matter



Bethe-Weizsäcker formula:



$$E_B = a_V A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A, Z)$$

A: number of nucleons, Z: number of protons

For large systems $A \to \infty$ and neglecting a_c:

$$E_B/A(\rho_0) = E/A(\rho_0) - m_N = -16 \text{ MeV}$$

with $\rho_0 = 0.16 \text{ fm}^{-3}$ and $m_N = 939$ MeV.

Nuclear matter: why does it bind?

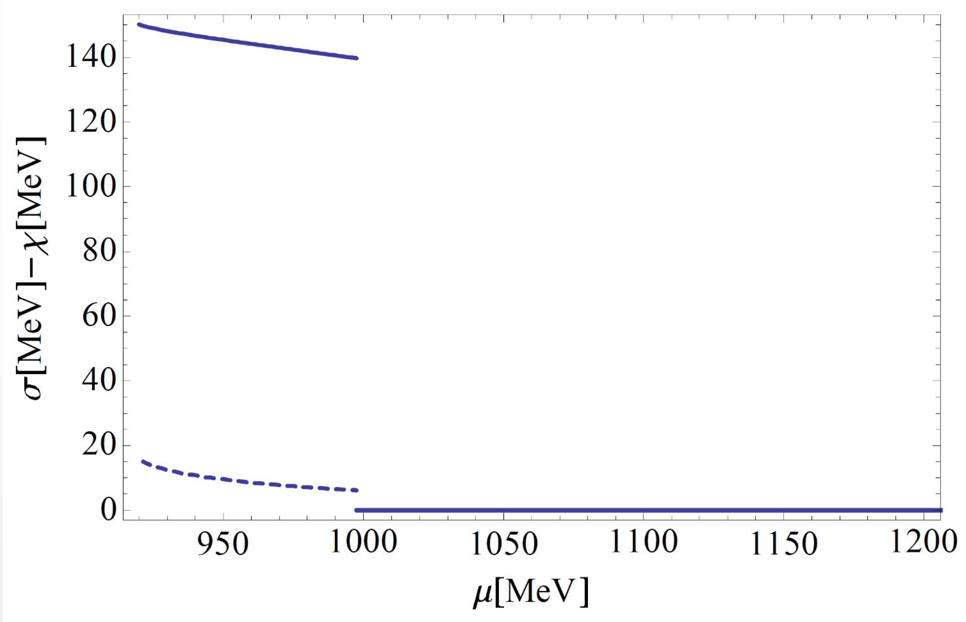
The resonance $f_0(500)$, here interpreted as a tetraquark, plays an important role for the stability of nuclear matter.

Related ‘amusing’ question: does nuclear matter binds at large Nc?

As soon as the lightest scalar $f_0(500)$ is not a quarkonium, nuclear matter ceases to exist already for Nc=4.

Details in: L. Bonanno and F.G., **Nucl.Phys.A859:49-62,2011 arXiv:1102.3367** [hep-ph]

Chiral phase transition



arXiv:1105.5003

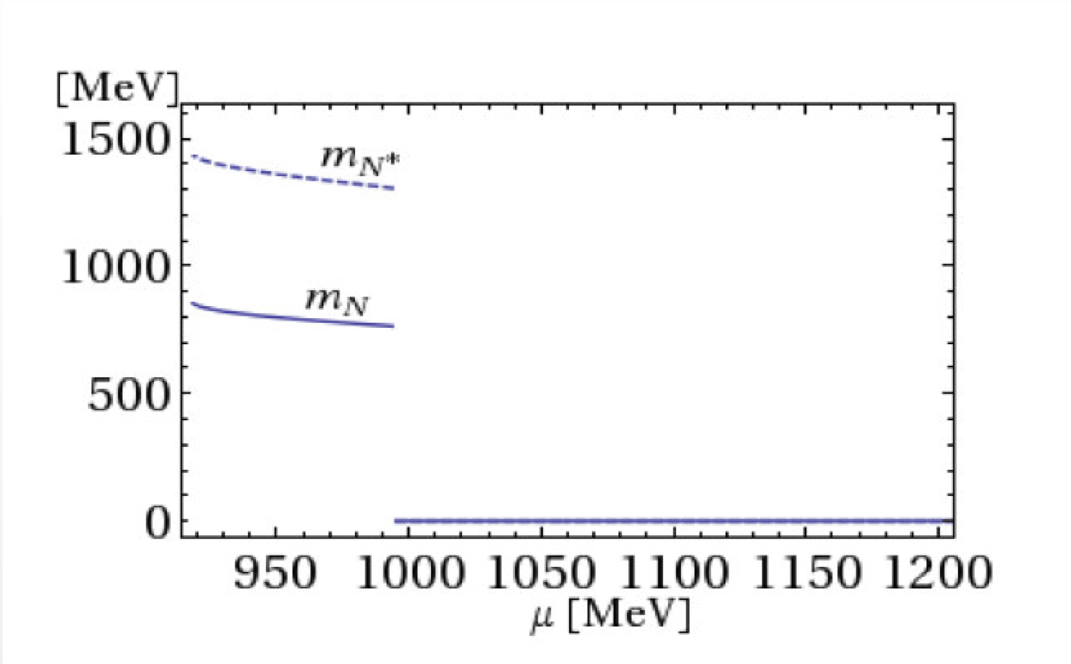
Critical density at the onset of chiral restoration (first order):

$$\rho_{crit} / \rho_0 \approx 2.5$$

(slightly dependent on m_0)

Chiral phase transition/2

Masses



The masses drop almost to zero above the critical value of the chemical potential.

arXiv:1105.5003