Kinetic coefficients in non-conformal viscous hydrodynamics

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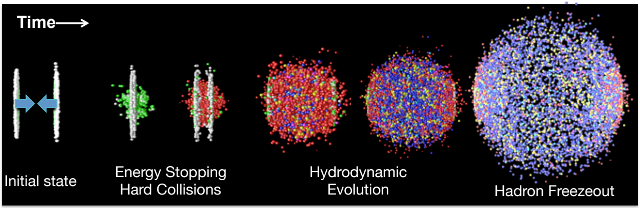
G.S.Denicol, W.Florkowski, R.Ryblewski, M.Strickland, Phys. Rev. **C90**, 044905 (2014)
A.Jaiswal, R.Ryblewski, M.Strickland, Phys. Rev. **C90** 4, 044908 (2014)
C.Chattopadhyay, A.Jaiswal, S.Pal, R.Ryblewski, arXiv:1411.2363

11-th Polish Workshop on Relativistic Heavy-Ion Collisions
Warsaw University of Technology
January 18, 2015

- relativistic hydrodynamics plays an important role in the "standard model" of HIC[1]



- initially perfect fluid hydrodynamics was used
- strongly-coupled $\mathcal{N} = 4$ SYM theory imposes lower bound of the $\eta/\mathcal{S}$[2]
  $\Rightarrow$ dissipative corrections important
  $\Rightarrow$ one should use relativistic viscous hydrodynamics
- models including viscous hydrodynamics describe experimental data very well (Bozek, Schenke, Heinz, ...)

[1] Figure taken from arXiv:1201.4264.
[2] Kovtun, Son, Starinets, Phys. Rev. Lett. **94**, 111601 (2005)

- thermodynamic gradients ⇒ transport phenomena ⇒ transport coefficients
- hydrodynamic evolution is governed by energy and momentum continuity equation (no charge diffusion)

$$\partial_\mu T^{\mu\nu}_{\rm visc} = 0 \qquad T^{\mu\nu}_{\rm visc} = \mathcal{E} u^\mu u^\nu - \Delta^{\mu\nu}(\mathcal{P}_{\rm eq} + \Pi) + \pi^{\mu\nu}$$

- evolution equations for the shear-stress tensor $\pi^{\mu\nu}$ and bulk viscous pressure Π in relaxation-time approximation

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\underline{\pi^{\mu\nu}\sigma_{\mu\nu}} + \ldots$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \underline{\tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma}} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \underline{\lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}} + \ldots$$

$\tau_\Pi\beta_\Pi = \zeta$, $\tau_\pi\beta_\pi = \eta$, relaxation time approximation imposes $\tau_\Pi = \tau_\pi = \tau_{\rm eq}$

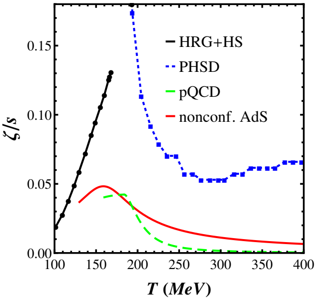
- form of transport coefficients depend on the method employed
- various methods available: Israel-Stewart [3], Grad's 14-moment approximation [4], Chapman-Enskog method [5] , ...

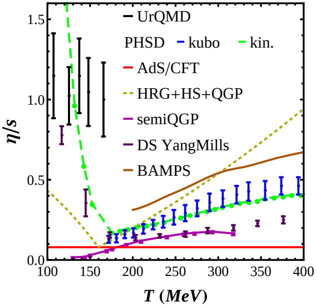
[3] Israel, Stewart, Ann. Phys. (N.Y.) **118**, 341 (1979)

[4] Denicol, Niemi, Molnar, Rischke, Phys. Rev. **D 85**, 114047 (2012)
Denicol, Jeon, Gale, Phys. Rev. **C 90**, 024912 (2014)

[5] Jaiswal, Phys. Rev. **C 87**, 051901 (2013)
Jaiswal, Ryblewski, Strickland, Phys. Rev. **C 90**, 044905 (2014)

- recently the properties of the QGP are studied more precisely (e.g. $\eta/\mathcal{S}(T)$)
- HIC research devoted mainly to the extraction of the $\eta/\mathcal{S}$, a systematic and self-consistent study of the effect of $\zeta/\mathcal{S}$ has not been performed so far
- at large T theory is nearly conformal, the bulk viscosity is expected to be small
- however QCD is non-conformal theory, some estimates suggest that $\zeta/\mathcal{S}$ peaks around T_C [6], and may be comparable with $\eta/\mathcal{S}$ (having minimum around T_C) [7]





[6] Figure taken from S. I. Finazzo, R. Rougemont, H. Marrochio, J. Noronha, arXiv:1412.2968

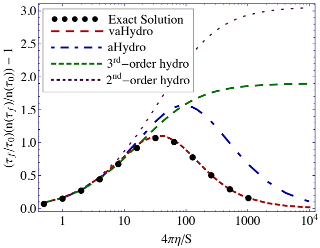
[7] Figure taken from J. Noronha-Hostler, forthcoming

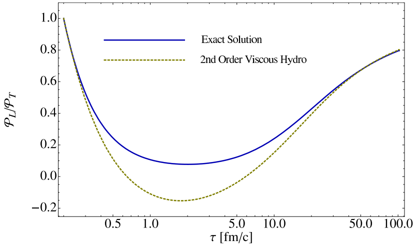
- canonical treatment within viscous hydrodynamics is based on an expansion of the general distribution function around local equilibrium state

$$f(x,p) = \underbrace{f_{\rm iso}\left(\frac{p^\mu u_\mu}{T(x)}\right)}_{\rm LO} + \underbrace{\delta f(x,p)}_{\rm NLO}$$

$\Rightarrow$ early thermalization required

- large anisotropy at early times predicted by microscopic models (AdS/CFT (see J.Jankowski talk), ...)
- studied systems are subject to rapid longitudinal expansion
  $\Rightarrow$ large viscous corrections to the ideal energy-momentum tensor
  $\Rightarrow$ canonical expansion breaks down
  $\Rightarrow$ may cause unphysical results





Tinti, Florkowski, Phys. Rev. **C 89**, 034907 (2014)
Nopoush, Ryblewski, Strickland, Phys. Rev. **C 90**, 014908 (2014)

- anisotropic hydrodynamics → one expands around an anisotropic background, momentum-space anisotropies are built into the LO (see L.Tinti talk)

$$f(x,p) = \underbrace{f_{\rm iso}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}}{\Lambda(x)}\right)}_{\rm LO} + \underbrace{\delta\tilde{f}(x,p)}_{\rm NLO}$$

spheroidal ansatz for $\Xi_{\mu\nu}$ give (LRF) $p^\mu \Xi_{\mu\nu} p^\nu = p_x^2 + p_y^2 + (1+\xi) p_z^2$ (R-S form)

anisotropy tensor decomposition

$$\Xi^{\mu\nu} u^\mu u^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu}\Phi$$

$$u_\mu \xi^{\mu\nu} = 0 \qquad u_\mu \Delta^{\mu\nu} = 0 \qquad \xi^\mu{}_\mu = 0 \qquad \Delta^\mu{}_\mu = 3$$

$$\xi^{\mu\nu} = \mathrm{diag}(0, \boldsymbol{\xi}) \qquad \boldsymbol{\xi} \equiv (\xi_x, \xi_y, \xi_z)$$

Anisotropic hydrodynamics

Tinti, Florkowski, Phys. Rev. **C 89**, 034907 (2014)
Nopoush, Ryblewski, Strickland, Phys. Rev. **C 90**, 014908 (2014)

equations of motion for ξ_z, Φ, λ, T for (0+1)d case are obtained by taking moments of the Boltzmann equation in the relaxation time approximation

$$p^\mu \partial_\mu f = p^\mu \frac{u_\mu}{\tau_{\rm eq}} (f^{\rm eq} - f) \qquad \rightarrow \qquad \partial_{\mu_1} \int dP\, p^{\mu_1} ... p^{\mu_{n+1}} f = u_{\mu_1} \int dP\, p^{\mu_1} ... p^{\mu_n} \frac{1}{\tau_{\rm eq}} (f^{\rm eq} - f)$$

0th moment (1 eq.)

$$\partial_\mu N^\mu = \frac{u_\mu}{\tau_{\rm eq}} (N^\mu_{\rm eq} - N^\mu)$$

1st moment (2 eq.)

$$\begin{aligned} u_\nu \partial_\mu T^{\mu\nu} &= u_\nu \frac{u_\mu}{\tau_{\rm eq}} (T^{\mu\nu}_{\rm eq} - T^{\mu\nu}) \\ u_\mu T^{\mu\nu}_{\rm eq} &= u_\mu T^{\mu\nu} \end{aligned}$$

2nd moment (1 eq.)

$$\begin{aligned} X^i_\mu X^i_\nu \partial_\lambda \Theta^{\lambda\mu\nu} &= X^i_\mu X^i_\nu \frac{u_\lambda}{\tau_{\rm eq}} \left(\Theta^{\lambda\mu\nu}_{\rm eq} - \Theta^{\lambda\mu\nu} \right) \\ i &= 0, 1, 2, 3 \end{aligned}$$

...

- anisotropic hydrodynamics has various appealing features (no negative pressures, reproduced free-streaming limit, kinetic coefficients included implicitly ...)

Quantifying efficacy of various approximation schemes

- goal:
  assess efficacy of various dissipative hydrodynamic approaches by comparing their predictions with exact solutions of the underlying kinetic theory equations
- it is possible using relaxation time approximation for collisional kernel and simple boost-invariant transversely homogeneous symmetry of the system (Bjorken flow)

- Boltzmann equation in the relaxation time approximation

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)] \qquad C[f] = p^\mu u_\mu \, \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

background distribution (Boltzmann statistics)

$$f^{\rm eq} = \frac{g_s}{(2\pi)^3} \exp\left(-\frac{p^\mu u_\mu}{T}\right)$$

- for transversely homogeneous boost-invariant system

$$w = t p_\parallel - z E \qquad v = t E - z p_\parallel \qquad (Bialas, Czyz)$$

$$\frac{\partial f}{\partial \tau} = \frac{f^{\rm eq} - f}{\tau_{\rm eq}}$$

$$f^{\rm eq}(\tau, w, p_\perp) = \frac{g_s}{(2\pi)^3} \exp\left(-\frac{\sqrt{w^2 + (m^2 + p_\perp^2)\tau^2}}{T\tau}\right)$$

- formal solution

$$f(\tau, w, p_\perp) = D(\tau, \tau_0) f_0(w, p_\perp) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} D(\tau, \tau')\, f^{\rm eq}(\tau', w, p_\perp)$$

$$D(\tau_2, \tau_1) = \exp\left[-\int_{\tau_1}^{\tau_2} \frac{d\tau''}{\tau_{\rm eq}(\tau'')}\right]$$

- initial condition (Romatschke-Strickland form)

$$f_0(w, p_\perp) = \frac{g_s}{(2\pi)^3} \exp\left[-\frac{\sqrt{(1+\xi_0) w^2 + (m^2 + p_\perp^2)\tau_0^2}}{\Lambda_0\, \tau_0}\right]$$

$\xi_0 = \xi(\tau_0)$ - initial value of the anisotropy parameter
$\Lambda_0 = \Lambda(\tau_0)$ - initial transverse-momentum scale

- particle density, energy density, transverse and longitudinal pressure

$$
\begin{aligned}
n(\tau) &= g_0 \int dP \, \frac{v}{\tau} \; f(\tau, w, p_\perp) \\
\mathcal{E}(\tau) &= g_0 \int dP \, \frac{v^2}{\tau^2} \, f(\tau, w, p_\perp) \\
\mathcal{P}_T(\tau) &= g_0 \int dP \, \frac{p_T^2}{2} \, f(\tau, w, p_\perp) \\
\mathcal{P}_L(\tau) &= g_0 \int dP \, \frac{w^2}{\tau^2} \, f(\tau, w, p_\perp)
\end{aligned}
$$

- determination of effective temperature (Landau matching)

$$
\begin{aligned}
u_\mu T^{\mu\nu} &= u_\mu T^{\mu\nu}_{\rm eq} \\
\mathcal{E}(\tau) &= \mathcal{E}^{\rm eq}(\tau) \\
&= g_0 \int dP \, \frac{v^2}{\tau^2} \, f^{\rm eq}(\tau, w, p_\perp) \\
&= \frac{g_0 T m^2}{\pi^2} \left[3 T K_2\left(\frac{m}{T}\right) + m K_1\left(\frac{m}{T}\right) \right]
\end{aligned}
$$

$$
\begin{aligned}
T^{\mu\nu} &= (\mathcal{E} + \mathcal{P}_T) u^\mu u^\nu - \mathcal{P}_T g^{\mu\nu} + (\mathcal{P}_L - \mathcal{P}_T) z^\mu z^\nu \\
T^{\mu\nu}_{\rm eq} &= (\mathcal{E}_{\rm eq} + \mathcal{P}_{\rm eq}) u^\mu u^\nu - \mathcal{P}_{\rm eq} g^{\mu\nu}
\end{aligned}
$$

$$
\begin{aligned}
T^{\mu\nu}_{\rm LRF} &= {\rm diag}(\mathcal{E}, \mathcal{P}_T, \mathcal{P}_T, \mathcal{P}_L) \\
T^{\mu\nu}_{\rm eq;LRF} &= {\rm diag}(\mathcal{E}_{\rm eq}, \mathcal{P}_{\rm eq}, \mathcal{P}_{\rm eq}, \mathcal{P}_{\rm eq})
\end{aligned}
$$

- evolution equation for the temperature profile[8]

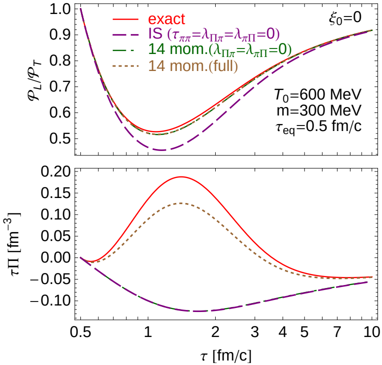
$$T m^2 \left[3 T K_2\left(\frac{m}{T}\right) + m K_1\left(\frac{m}{T}\right) \right] = \frac{g_s}{4} \left[D(\tau,\tau_0) \Lambda_0^4 \widetilde{\mathcal{H}}_2\left(\frac{\tau_0}{\tau\sqrt{1+\xi_0}}, \frac{m}{\Lambda_0} \right) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} D(\tau,\tau') T'^4 \widetilde{\mathcal{H}}_2\left(\frac{\tau'}{\tau}, \frac{m}{T'} \right) \right]$$

- numerical (iterative) method
  1) use a trial function $T' = T(\tau')$ on the RHS of the dynamic equation
  2) the LHS of the dynamic equation determines the new $T = T(\tau)$
  3) use the new $T(\tau)$ as the trial one
  4) repeat steps 1-3 until the stable $T(\tau)$ is found

[8] Florkowski, Ryblewski, Strickland, Phys. Rev. **C 88**, 024903 (2013)
Florkowski, Maksymiuk, Ryblewski, Strickland, Phys. Rev. **C 89**, 054908 (2014)

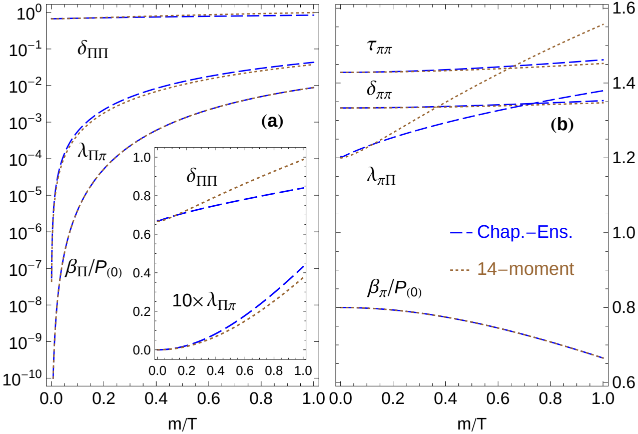
$$T^{\mu}{}_{\mu\,\mathrm{LRF}} = T^{\mu}{}_{\mu\,\mathrm{visc;LRF}}$$
$$\downarrow$$
$$\Pi = \frac{1}{3}\left[\mathcal{P}_{\|}(\tau) + 2\mathcal{P}_{\perp}(\tau) - 3\mathcal{P}_{\mathrm{eq}}(\tau)\right]$$

Denicol, Florkowski, Ryblewski, Strickland, Phys. Rev. **C 90**, 044905 (2014)



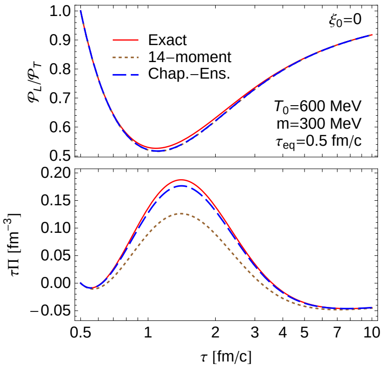
- $\tau_{\pi\pi}$ is extremely important for correct description of shear stress corrections (20% discrepancy)
- shear–bulk couplings ($\lambda_{\Pi\pi}$ and $\lambda_{\pi\Pi}$) are crucial for correct description of the bulk viscous correction

Jaiswal, Ryblewski, Strickland, Phys. Rev. **C 90**, 044905 (2014)



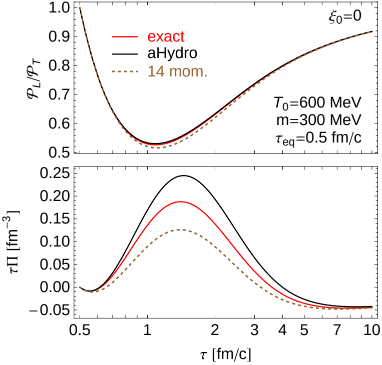
- Chapman-Enskog method and 14-moment approximation provide slightly different form of second-order kinetic coefficients

A. Jaiswal, R. Ryblewski, M. Strickland, Phys. Rev. **C 90**, 044905 (2014)



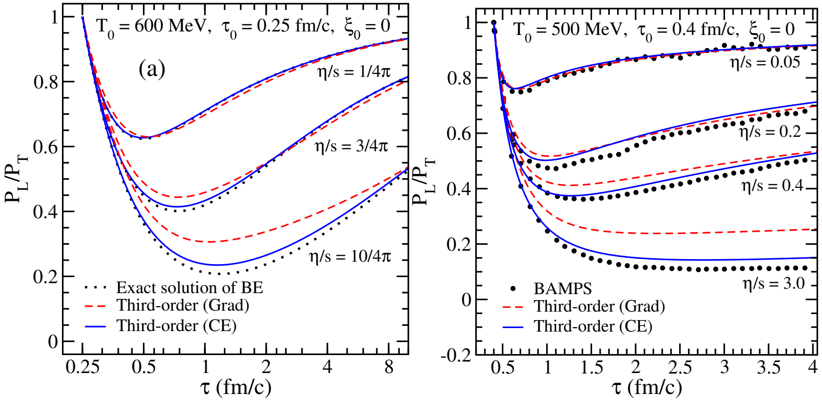
- kinetic coefficients obtained within Chapman-Enskog method provide even better description of bulk pressure evolution than 14-moment approximation

Nopoush, Ryblewski, Strickland, Phys. Rev. **C 90**, 014908 (2014)
Denicol, Florkowski, Ryblewski, Strickland, Phys. Rev. **C 90**, 044905 (2014)



- anisotropic hydrodynamics better captures the $\mathcal{P}_L/\mathcal{P}_T$ behavior and describes bulk correction as good as 14-moment formulation of second-order viscous hydrodynamics
- kinetic coefficients are implicitly included in anisotropic hydrodynamics

C.Chattopadhyay, A.Jaiswal, S.Pal, R.Ryblewski, arXiv:1411.2363



- Chapman-Enskog method was applied to derive second-order viscous hydrodynamic equations and the associated transport coefficients for a massive gas in RTA.
- Exact solution of RTA Boltzmann kinetic equation was applied for testing various hyrodynamic approximation schemes
- It was found that:
  - commonly used Israel-Stewart formulation of second-order viscous hydrodynamics equations do not describe early-time evolution of bulk viscous pressure and shear stress correctly (shear–bulk couplings: $\lambda_{\Pi\pi}$, $\lambda_{\pi\Pi}$ and $\tau_{\pi\pi}$ obtained within 14-moment approximation are crucial)
  - Chapman-Enskog method provides equations which give the best overall agreement with exact solutions of kinetic theory equations
  - anisotropic hydrodynamics provides the best description of $\mathcal{P}_L/\mathcal{P}_T$ evolution thus better captures the anisotropy in the system
- NOTE:
  there are new exact solutions of the RTA Boltzmann equation now available for conformal systems employing so-called Gubser symmetry

Denicol, Heinz, Martinez, Noronha, Strickland, Phys. Rev. Lett. **113**, 202301 (2014)

Thank you for your attention!

ACKNOWLEDGMENTS

This talk was supported by Polish National Science Center Grants:
DEC-2012/07/D/ST2/02125 and **DEC-2011/01/B/ST2/03915**