High-Energy Parton in Unstable Quark-Gluon Plasma

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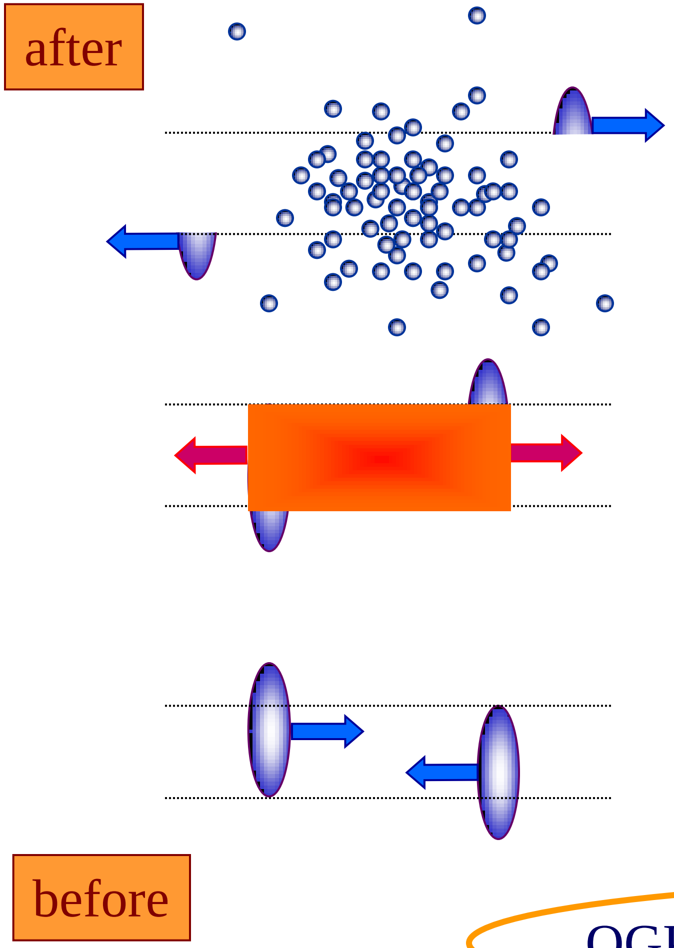
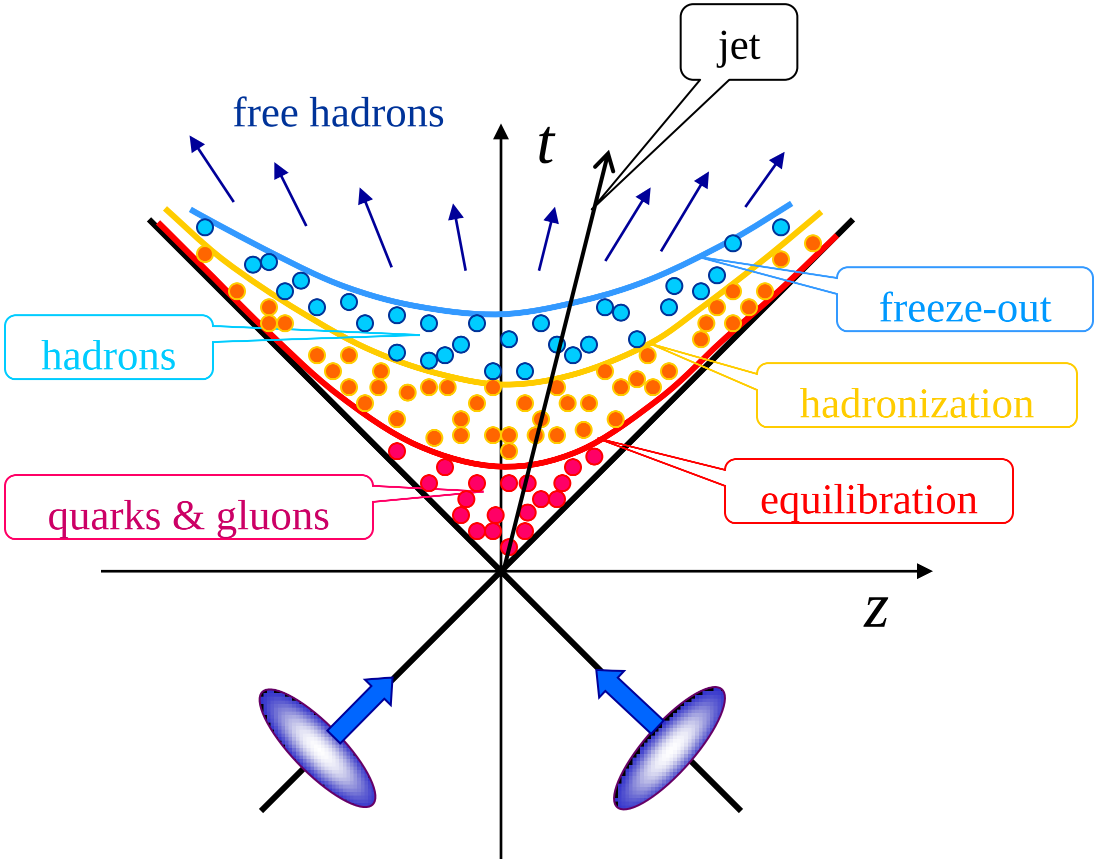
Scenario of relativistic heavy-ion collisions



after

before



jet

free hadrons

t

hadrons

freeze-out

hadronization

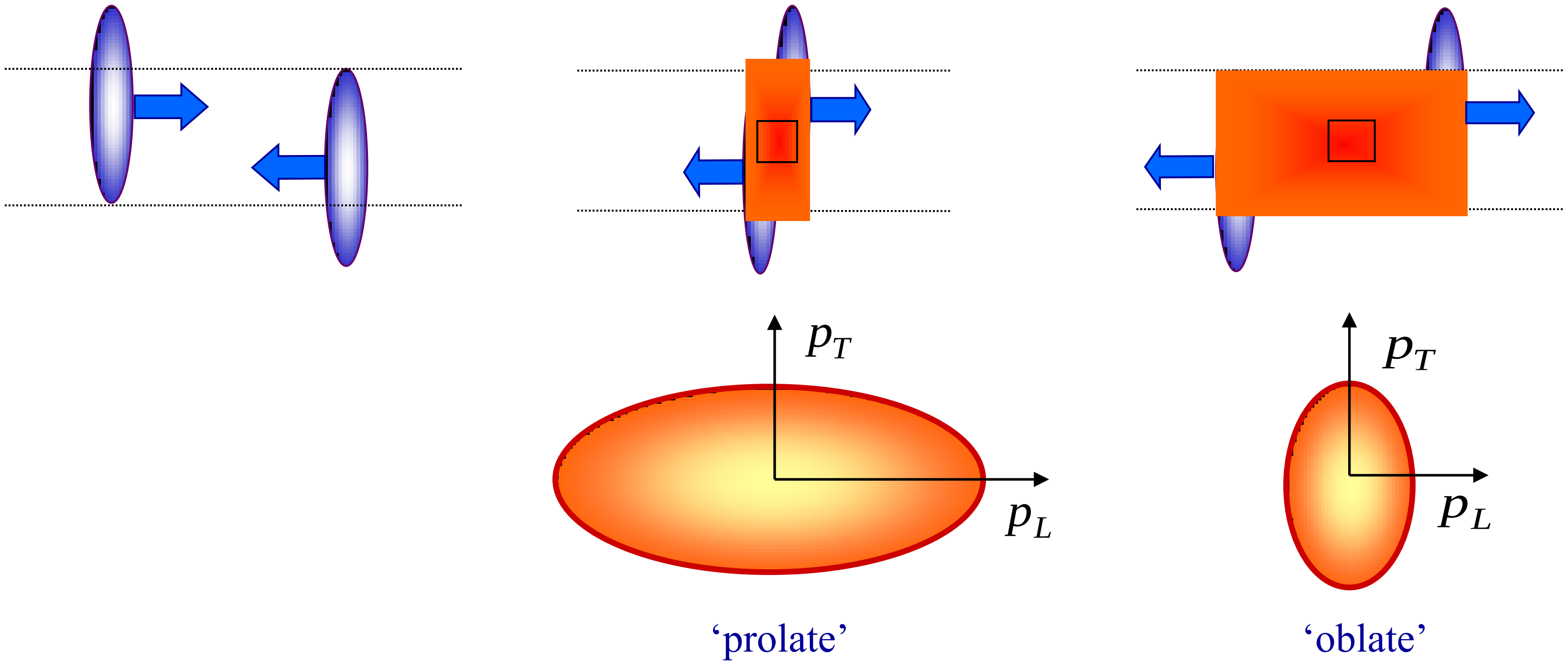
quarks & gluons

equilibration

z

QGP is out of equilibrium at the collision early stage

Anisotropic QGP



Anisotropic QGP is unstable due to magnetic plasma modes

Questions

- What happens to a high-energy parton when it is traversing an unstable QGP?
- Does the parton loose or gains an energy?

A test parton in QGP

Wong's equation of motion (Hard Loop Approximation)

$$\begin{cases} \dfrac{dx^{\mu}(\tau)}{d\tau} = u^{\mu}(\tau) \\ \dfrac{dp^{\mu}(\tau)}{d\tau} = gQ_a(\tau)\,F_a^{\mu\nu}(x(\tau))\,u_{\nu}(\tau) \\ \dfrac{dQ_a(\tau)}{d\tau} = -gf^{abc}p_{\mu}(\tau)\,A_b^{\mu}(x(\tau))\,Q_c(\tau) \end{cases}$$

Simplifications

Gauge condition: $p_{\mu}(\tau)\,A_b^{\mu}\big(x(\tau)\big)=0 \quad \Rightarrow \quad Q_a(\tau)=\mathrm{const}$

Parton travels with constant velocity: $u^{\mu} = (\gamma, \gamma\mathbf{v}) = \mathrm{const}$

Evolution of parton’s energy

$$\frac{dE(t)}{dt} = gQ_a\,\mathbf{E}_a(t,\mathbf{r}(t))\cdot\mathbf{v}$$

induced & spontaneously generated chromoelectric field

parton’s current: $\mathbf{j}_a(t,\mathbf{r}) = gQ_a\mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t)$

$$\boxed{\frac{dE(t)}{dt} = \int d^3r\,\mathbf{E}_a(t,\mathbf{r})\cdot\mathbf{j}_a(t,\mathbf{r})}$$

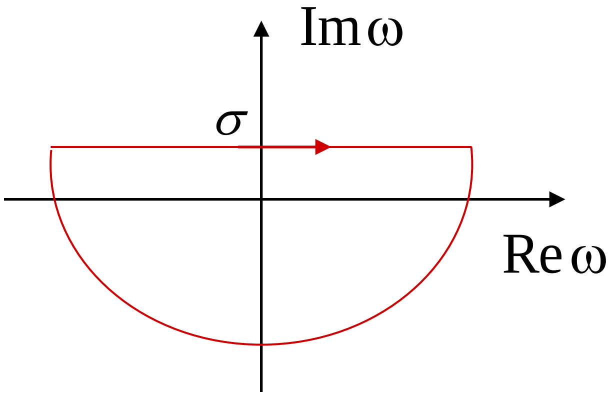
Analog of collisional energy loss

Initial value problem

One-sided Fourier transformation

$$
\begin{cases}
f(\omega,\mathbf{k}) = \int\limits_0^\infty dt \int d^3r\, e^{i(\omega t-\mathbf{kr})} f(t,\mathbf{r}) \\
f(t,\mathbf{r}) = \int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega t-\mathbf{kr})} f(\omega,\mathbf{k})
\end{cases}
$$

$$0<\sigma\in R$$



$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a\mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t) \quad \Rightarrow \quad \mathbf{j}_a(\omega,\mathbf{k}) = \frac{igQ_a\mathbf{v}}{\omega-\mathbf{k}\cdot\mathbf{v}}$$

$$\boxed{\frac{dE(t)}{dt} = gQ_a \int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega-\mathbf{kv})t}\mathbf{E}_a(\omega,\mathbf{k})\cdot\mathbf{v}}$$

Induced Electric Field

Linearized Yang-Mills (Maxwell) equations (Hard Loop Approximation)

$$\boxed{\begin{aligned} &i\mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k}) = \rho(\omega,\mathbf{k}), \qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0, \\ &i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}), \\ &i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mathbf{j}(\omega,\mathbf{k}) - i\omega\mathbf{E}(\omega,\mathbf{k}) - \mathbf{D}_0(\mathbf{k}) \end{aligned}}$$

$$D^i(\omega,\mathbf{k}) = \varepsilon^{ij}(\omega,\mathbf{k})\,E^j(\omega,\mathbf{k})$$

Chromodielectric tensor

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{kv}+i0^+}\frac{\partial f(\mathbf{p})}{\partial p^l}\Big[\Big(1-\frac{\mathbf{kv}}{\omega}\Big)\delta^{lj} + \frac{k^l v^j}{\omega}\Big]$$

dynamical information

$$\boxed{E^i(\omega,\mathbf{k}) = -i(\Sigma^{-1})^{ij}(\omega,\mathbf{k})[\omega\mathbf{j}(\omega,\mathbf{k}) + \mathbf{k}\times\mathbf{B}_0(\mathbf{k}) - \omega\mathbf{D}_0(\mathbf{k})]^j}$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

Formula of evolution of parton’s energy

$$\frac{dE(t)}{dt} = gQ_a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i}\int\frac{d^3k}{(2\pi)^3}\, e^{-i(\omega-\overline{\varpi})t} \times (\Sigma^{-1})^{ij}(\omega,\mathbf{k})\left[\frac{igQ_a\omega\,\mathbf{v}}{\omega-\overline{\varpi}} + \underbrace{\mathbf{k}\times\mathbf{B}_0(\mathbf{k})-\omega\mathbf{D}_0(\mathbf{k})}_{\text{Initial values of the fields}}\right]^j$$

$\overline{\varpi} \equiv \mathbf{k}\cdot\mathbf{v}$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

Dispersion equation

$$\boxed{\det[\Sigma(\omega,\mathbf{k})] = 0}$$

How to choose the field initial values?

1) The initial fields vanish: $\mathbf{D}_0(\mathbf{k}) = \mathbf{B}_0(\mathbf{k}) = 0$

2) The initial fields are independent of the parton's current.

1) is equivalent to 2)

The effect of the initial fields cancels out after an averaging over parton's colors.

$$\int dQ\, Q_a = 0, \qquad \int dQ\, Q_a\, Q_b = C_2 \delta^{ab}, \qquad C_2 \equiv \begin{cases} \dfrac{1}{2} & \text{for} \quad \text{quark} \\ N_c & \text{for} \quad \text{gluon} \end{cases}$$

How to choose the field initial values?

3) The initial fields are induced by the parton’s current.

$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a\mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t), \quad t\in(-\infty,\infty)$$

Maxwell equations

Two-side Fourier transformation

Initial values:

$$D_0^i(\mathbf{k}) = -i\cos\varphi\, gQ_a\overline{\varpi}\varepsilon^{ij}(\overline{\varpi},\mathbf{k})(\Sigma^{-1})^{jk}(\overline{\varpi},\mathbf{k})v^k$$

$$B_0^i(\mathbf{k}) = -i\cos\varphi\, gQ_a\varepsilon^{ijk}k^j(\Sigma^{-1})^{kl}(\overline{\varpi},\mathbf{k})v^l$$

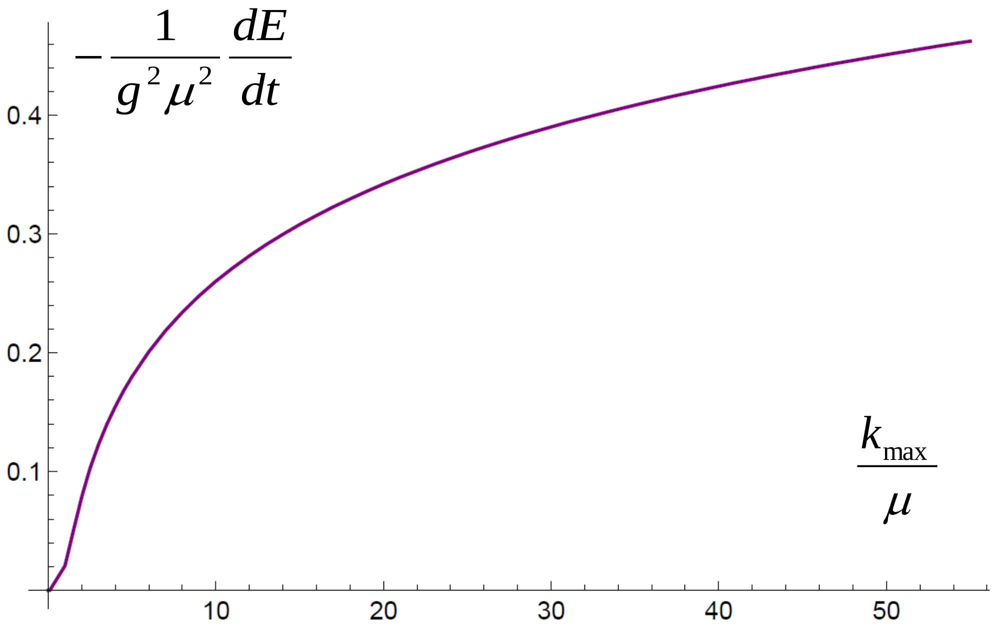
$-1\le\cos\varphi\le 1$ - arbitrary phase factor

Energy loss in equilibrium QGP

The initial conditions are *forgotten*

$$\boxed{\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3k}{(2\pi)^3} \frac{\overline{\omega}}{\mathbf{k}^2}\left[\frac{1}{\varepsilon_L(\overline{\omega},\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \overline{\omega}^2}{\overline{\omega}^2\varepsilon_T(\overline{\omega},\mathbf{k}) - \mathbf{k}^2}\right]}$$

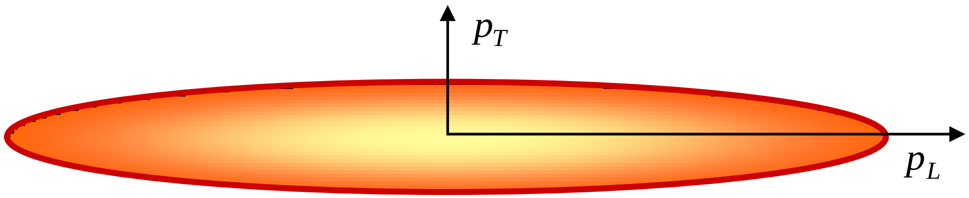
equivalent to the standard result by Braaten & Thoma



Debye mass

$$\boxed{\mu^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3}\frac{f(\mathbf{p})}{|\mathbf{p}|}}$$

Extremely prolate QGP



$$f(\mathbf{p}) \sim \delta(p_T)$$

Collective modes

$$\det[\Sigma^{ij}(\omega,\mathbf{k})] = 0$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{kv}+i0^+}\frac{\partial f(\mathbf{p})}{\partial p^l}\Big[\Big(1-\frac{\mathbf{kv}}{\omega}\Big)\delta^{lj}+\frac{k^l v^j}{\omega}\Big]$$

Spectrum of collective modes

$$\begin{cases} \omega_1(\mathbf{k}) = \mu^2 + \mathbf{k}^2 \\ \omega_2(\mathbf{k}) = \mu^2 + (\mathbf{k}\cdot\mathbf{n})^2 \\ \omega_\pm(\mathbf{k}) = \frac{1}{2}\Big(\mathbf{k}^2 + (\mathbf{k}\cdot\mathbf{n})^2 \pm \sqrt{\mathbf{k}^4 + (\mathbf{k}\cdot\mathbf{n})^4 + 4\mu^2\mathbf{k}^2 - 4\mu^2(\mathbf{k}\cdot\mathbf{n})^2 - 2\mathbf{k}^2(\mathbf{k}\cdot\mathbf{n})^2}\Big) \end{cases}$$

$$\mathbf{n} \equiv (0,0,1)$$

Unstable chromomagnetic mode

stationary state

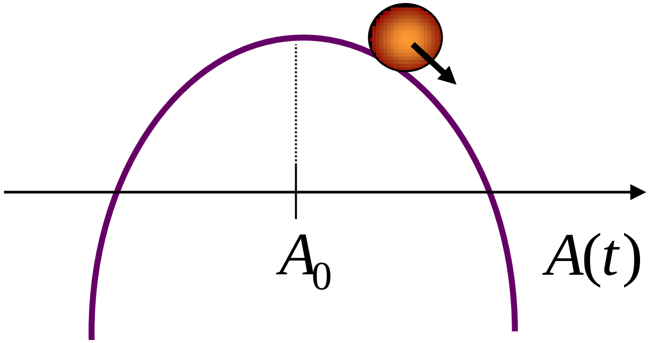
$$\boxed{A(t) = A_0 + \delta A(t)}$$

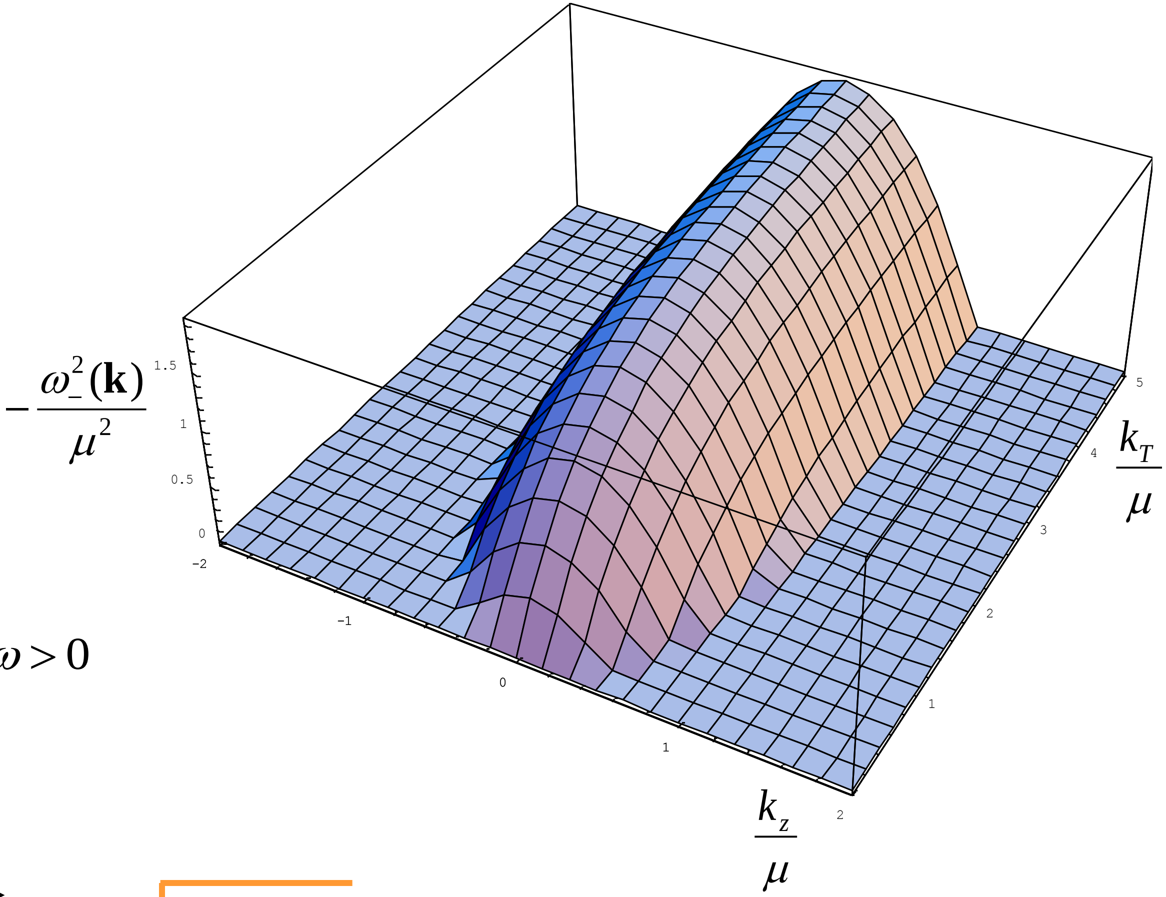
fluctuation

Instability

$$\boxed{\delta A(t) \propto \mathrm{e}^{\mathrm{Im}\,\omega t}}$$

$\mathrm{Im}\,\omega > 0$

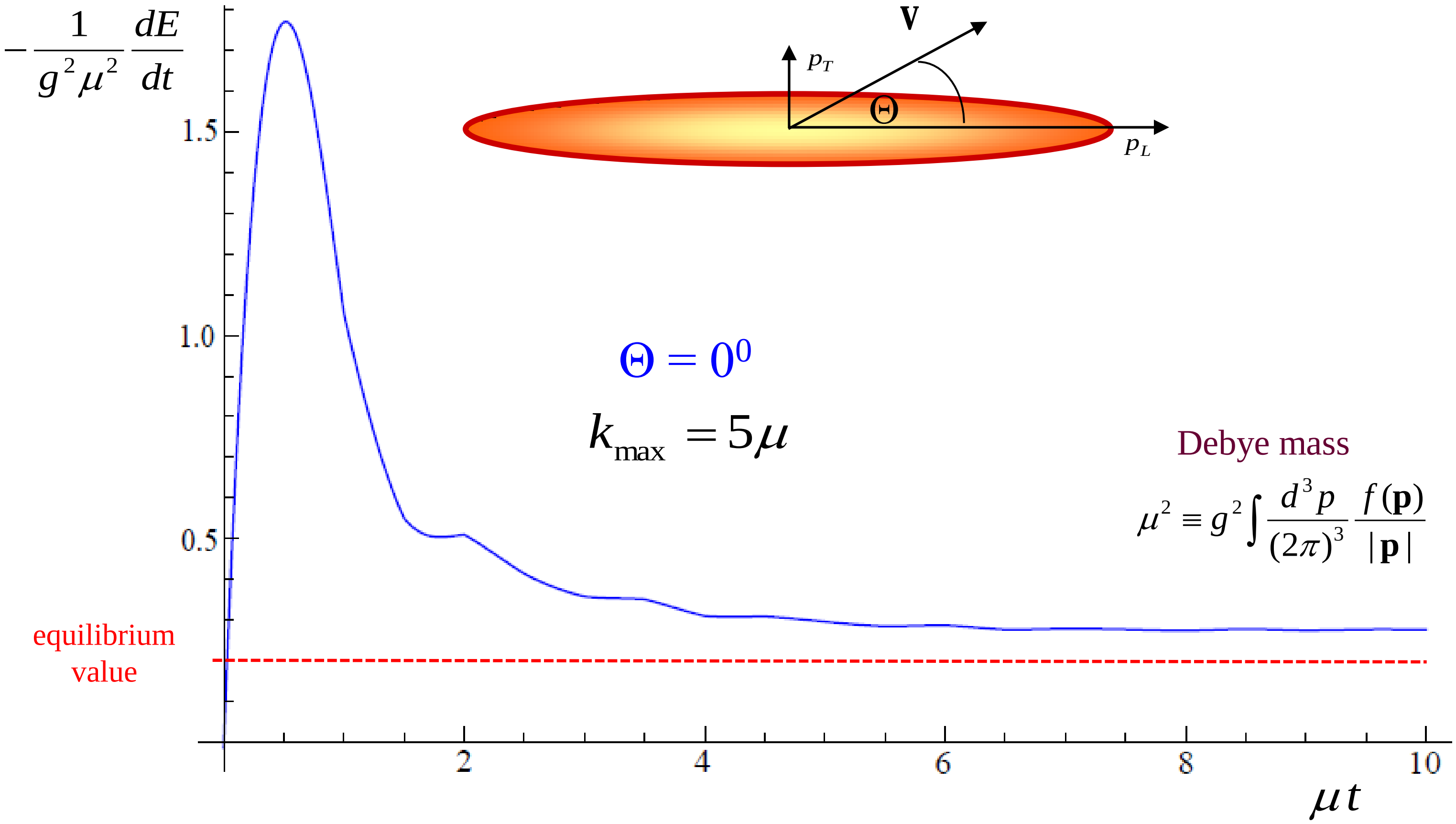




$$\frac{dE(t)}{dt} \sim \int \frac{d^3k}{(2\pi)^3} e^{\mathrm{Im}\,\omega t} \ldots$$

Energy loss in extremely prolate QGP

The initial conditions 1) or 2)



$\Theta = 0^0$

$k_{\max} = 5\mu$

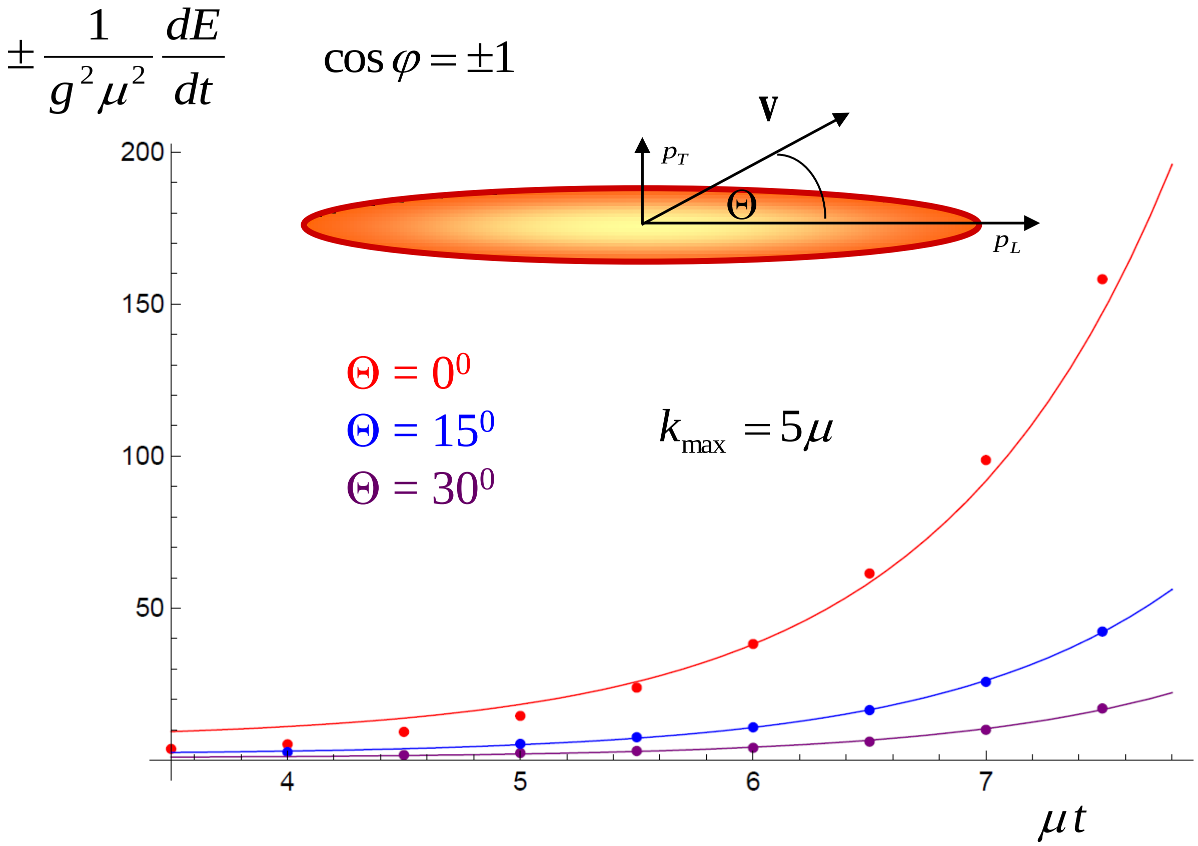
Debye mass

$$\mu^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

Energy change in extremely prolate QGP cont.

The initial condition 3):

- energy gain for cosφ < 0
- energy loss for cosφ > 0



Debye mass

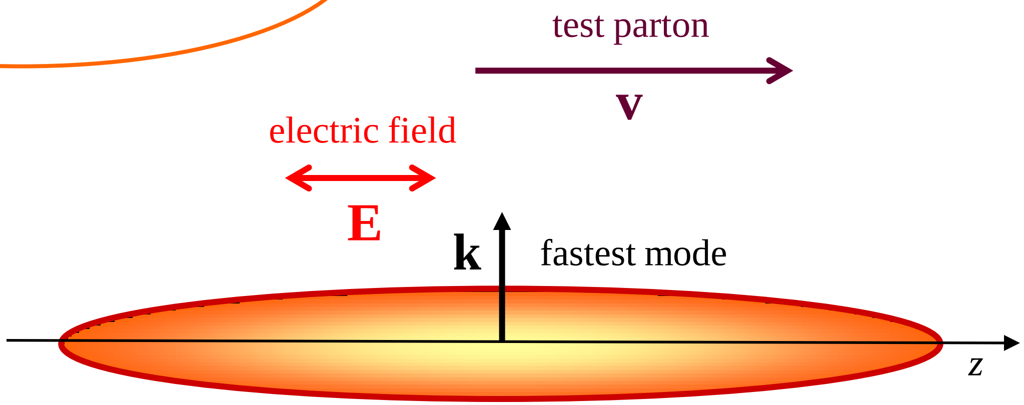
$$\mu^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3}\frac{f(\mathbf{p})}{|\mathbf{p}|}$$

equilibrium value

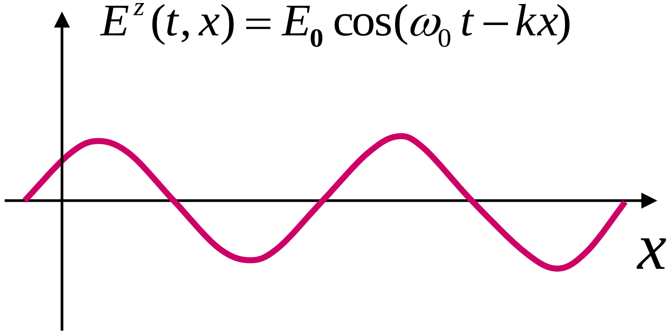
$$\frac{1}{g^2\mu^2}\frac{dE}{dt} \approx 0.2$$

Physical interpretation

$$\frac{dE}{dt} \sim \mathbf{E}\cdot\mathbf{j} \sim \mathbf{E}\cdot\mathbf{v}$$



The largest *dE/dt* for **v** along axis *z* !



$$E^z(t,x) = E_{\mathbf{0}}\cos(\omega_0\, t - kx)$$

Summary and conclusions

Summary

- Parton’s energy is found as a solution of initial value problem.
- Extremely prolate QGP is discussed as an example of unstable system.

Conclusions

- dE/dt crucially depends on initial conditions.
- $dE/dt > 0$ & $dE/dx < 0$
- dE/dt strongly varies with time and direction.
- dE/dt can be much bigger than in equilibrium QGP.