Event-by-Event Fluctuations in High Energy Reactions

Mark I. Gorenstein

Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine

I. Global Conservation Laws. Statistical Ensembles

II. Strongly Intensive Measures of Fluctuations

III. Van der Waals Equation of State in the GCE and Particle Number Fluctuations

I. Global Conservation Laws. Statistical Ensembles

$E \longleftrightarrow T$ $V \longleftrightarrow p$ $Q \longleftrightarrow \mu_Q$

$2^3 = 8$

E, V, Q MCE

T, V, Q CE

T, V, μ_Q GCE

E, V, μ_Q MGCE

E, p, Q T, p, Q

T, p, μ_Q E, p, μ_Q

} Pressure Ensembles

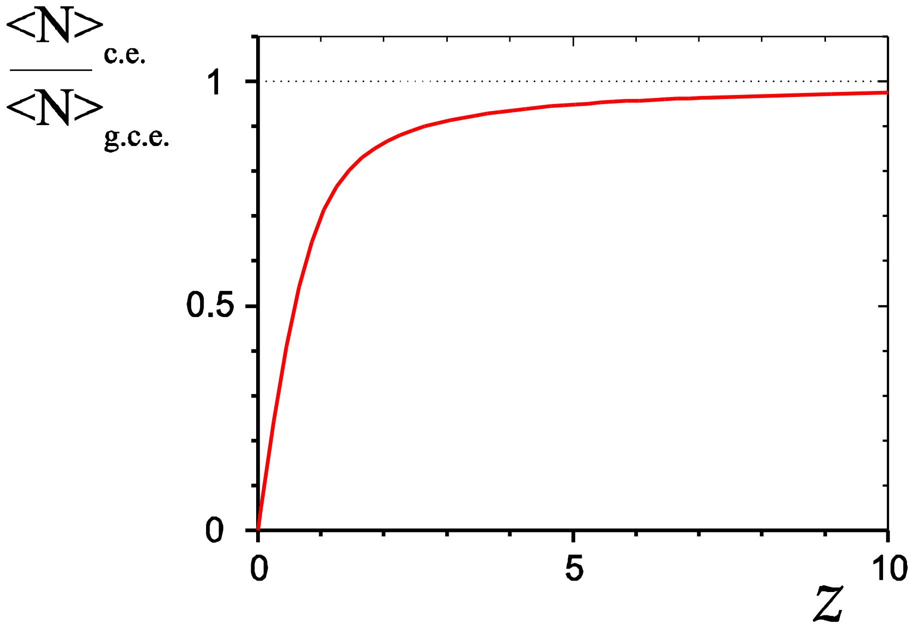
M.I.G. J. Phys. G (2008)

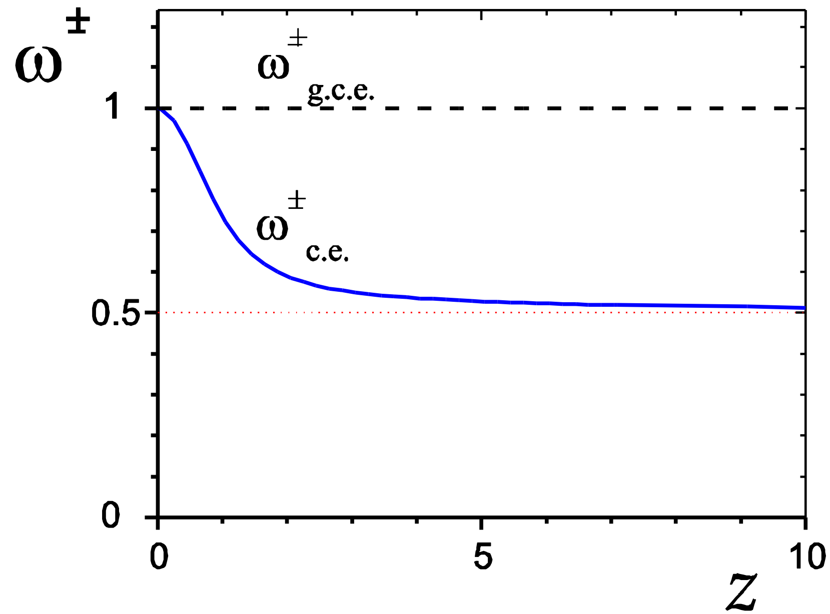
Rafelski, Danos, Phys. Lett. B (1980)
Redlich, Turko, Z. Phys. C (1980)
more than 100 citations per paper

$Q = 0$

GCE: $\mu=0 \rightarrow \langle N_+ \rangle = \langle N_- \rangle$,

CE: $\delta(N_+ - N_-) \rightarrow N_+ = N_-$



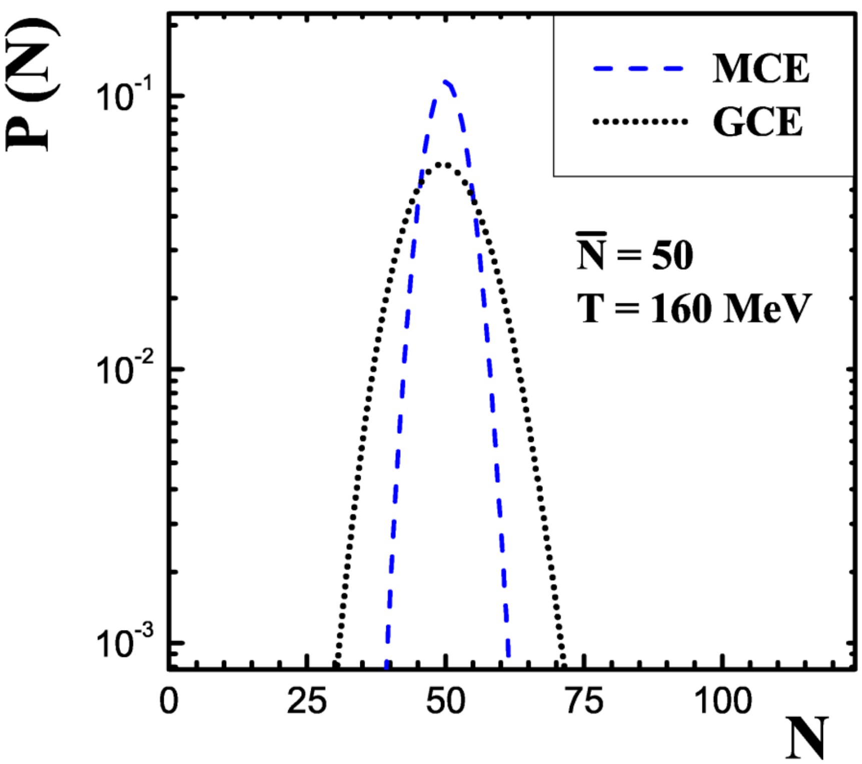


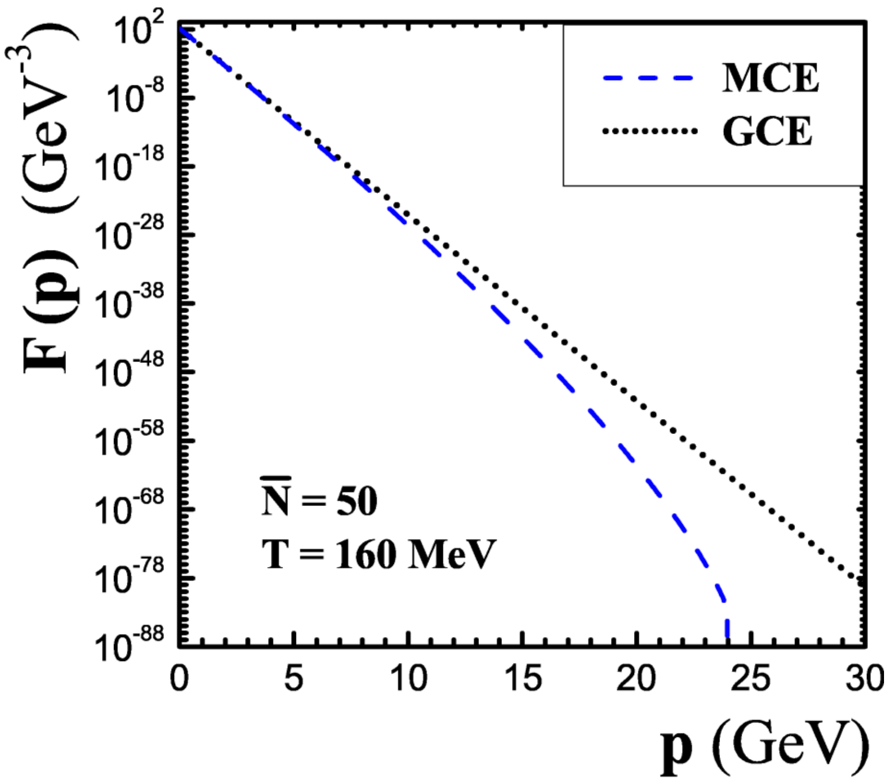
$$z = \frac{V}{2\pi^2} T m^2 K_2(m/T) = \langle N_\pm \rangle_{gce} \; ; \quad \omega[N] = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} ; \quad \omega_{gce}[N_\pm] = 1$$

$$\frac{\langle N_\pm \rangle_{ce}}{\langle N_\pm \rangle_{gce}} = \frac{I_1(2z)}{I_0(2z)} \rightarrow 1 \qquad \omega_{ce}[N_\pm] = 1 - z\left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)}\right] \rightarrow \frac{1}{2}$$

Begun, Gazdzicki, M.I.G., Zozulya, Phys. Rev. C (2004)

Micro Canonical Ensemble of massless neutral particles



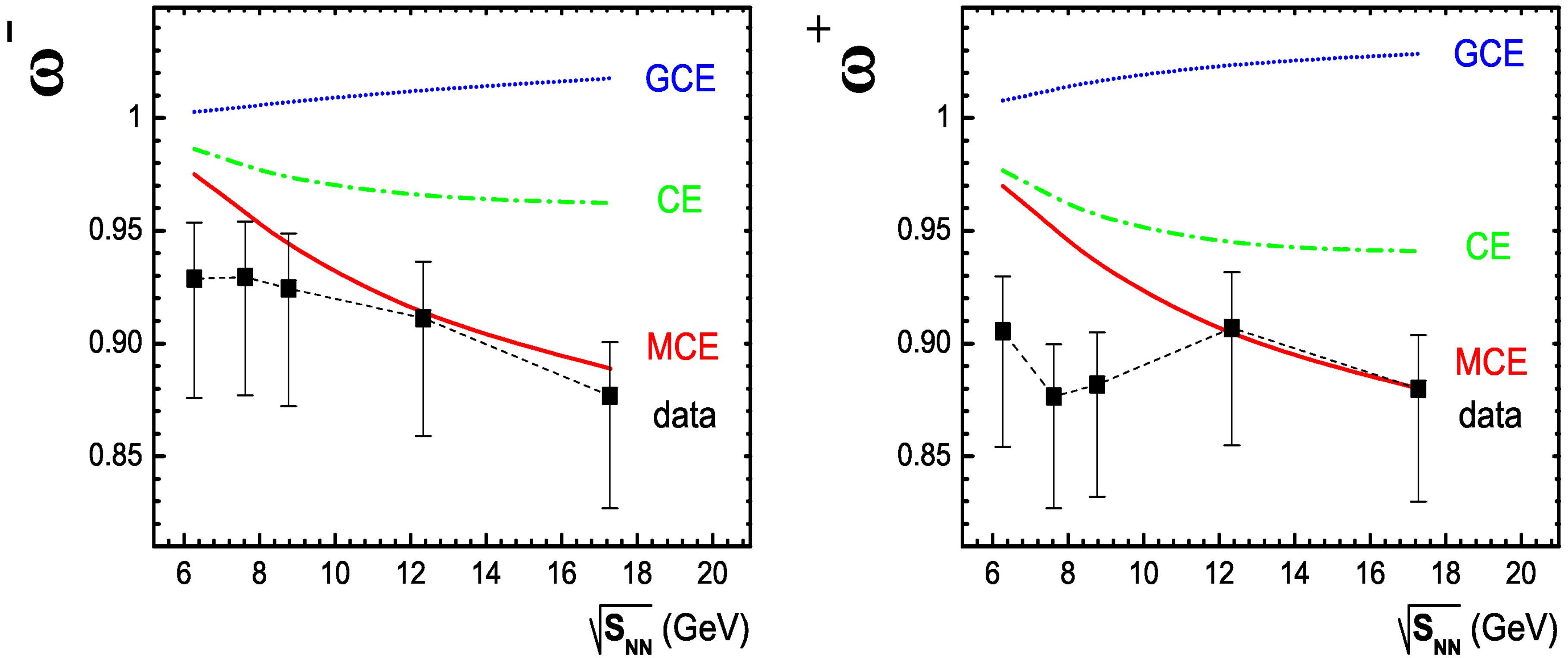


$$\omega_{\mathrm{gce}} = 1, \qquad \omega_{\mathrm{mce}} = \frac{1}{4}$$

Begun, M.I.G., Kostyuk, Zozulya, Phys. Rev. C (2005)

Comparison with the NA49 data

Pb+Pb 1% most central events



Begun, Gazdzicki, M.I.G., Hauer, Konchakovski, Lungwitz, Phys. Rev. C (2007)

Theory (standard statistical ensembles):

Begun, M.I.G., Kostyuk, Zozulya, Phys. Rev. C (2005), J.Phys. G
Begun, M.I.G., Zozulya, Phys. Rev. C (2005)
Becattini, Ferroni, Eur. Phys. J. C(2005) (2007)
Mekjian, Nucl. Phys. A (2005)
Keranen, Becattini, Begun, M.I.G., Zozulya, J. Phys. G (2005)
Cleymans, Redlich, Turko, Phys. Rev. C (2005), J. Phys. G (2005)
Becattini, Keranen, Ferroni, Gabbriellini, Phys. Rev. C (2005)
Torrieri, Jeon, Rafelski, Phys. Rev. C (2006), Nucleonics (2006)
Hauer, Phys. Rev. C (2008); Hauer, Wheaton, Phys. Rev. C (2009)
Turko, Int. J. Mod. Phys. E (2007), Phys. Part. Nucl. (2008)
Becattini, Ferroni, Eur. Phys. J. C (2007)
Torrieri, J. Phys. G (2006, 2008), Eur. Phys. J. C (2007) , (2012),
Yang, Wang, Phys. Rev. C (2011, 2012)
Torrieri, Bellweid, Market, Westfall (2012)

Theory (generalized statistical ensembles):

M.I.G., Hauer, Phys. Rev. C (2008);
Begun, Gazdzicki, M.I.G., Phys. Rev. C (2008, 2009)
Wilk, Wlodarczyk, Physica (2011)
Biro, Barnafoldi, Van, Eur. Phys. J. A (2013), Physica (2014)

Experiment:

Rybczynski et. al, NA49 Collaboration, J. Phys. Conf. Ser. (2005)
Lungwitz (NA49 Collaboration) (2006, 2007)
Alt, et.al. NA49 Collaboration, Phys. Rev C (2007) (2008)
Center, et. al, NA61 Collaboration, Phys. Atom. Nucl. (2012)

II. Strongly Intensive Measures of Fluctuations

$F(V)$ event-by-event volume distribution

$A \sim V, \quad B \sim V$ Extensive Quantities

$\Delta[A, B], \Sigma[A, B]$ are independent of the average volume and of volume fluctuations

M.I.G., Gazdzicki, Phys Rev. C (2011)

Examples:

$$A = P_T = |\mathrm{p}_T^{(1)}| + ... + |p_T^{(N)}|$$

$$B = N$$

$$A = N_1,$$

$$B = N_2$$

Ideal Botzmann gas in the GCE with $F(V)$ volume distribution

$$A = a_1 + a_2 + \ldots + a_N \ , \qquad B = b_1 + b_2 + \ldots + b_N$$

$$< A > = \bar{a} < N > = \bar{a}\ n < V >, \qquad < B > = \bar{b} < N > = \bar{b}\ n < V >$$

$$< A^2 > \ = \ \overline{a^2}\ n < V > \ + \ \bar{a}^2 \left[n^2 < V^2 > \ - \ n < V > \right]$$

$$< B^2 > \ = \ \overline{b^2}\ n < V > \ + \ \bar{b}^2 \left[n^2 < V^2 > \ - \ n < V > \right]$$

$$< AB > \ = \ \overline{ab}\ n < V > \ + \ \bar{a}\ \bar{b} \left[n^2 < V^2 > \ - \ n < V > \right]$$

$$\omega\left[A\right] \ \equiv \frac{< A^2 > - < A >^2}{< A >} = \frac{\overline{a^2} - \bar{a}^2}{\bar{a}} + \bar{a}\ n\ \frac{< V^2 > - < V >^2}{< V >}$$

$$\equiv \omega\left[a\right] + \bar{a}\ n\ \omega\left[V\right]$$

$$< AB > - < A > < B > = (\overline{ab} - \bar{a}\,\bar{b})\, n < V > + \bar{a}\,\bar{b}\, n^2 \left(< V^2 > - < V >^2 \right)$$

$$\Delta[A, B] = \frac{1}{C_\Delta} \left[< B > \omega[A] - < A > \omega[B] \right]$$

$$\Sigma[A, B] = \frac{1}{C_\Sigma} [< B > \omega[A] + < A > \omega[B] - 2\left(< AB > - < A > < B > \right)]$$

$$< C_\Delta >, \; < C_\Sigma > \sim < V >$$

These combinations of second moments $< A^2 >, \; < B^2 >, \; < AB >$ are independent of $< V >$ and $\omega[V]$

Normalization. For the Independent Particle Model: $\Delta\left[A, B\right] = 1$

IB-GCE ; Mixed Event Model $\Sigma\left[A, B\right] = 1$

$$C_{\Delta} = C_{\Sigma} = \omega\left[p_T\right] < N > \qquad \left[A = P_T,\ B = N\right]$$

$$C_{\Delta} = < N_1 > - < N_2 >$$

$$C_{\Sigma} = < N_1 > + < N_2 >$$

$$\left[A = N_1,\ B = N_2\right]$$

Gazdzicki, M.I.G., Mackowiak-Pawlowska, Phys. Rev. C (2013)

$$\Phi_{P_T} = \sqrt{\overline{p_T^2} - \overline{p_T}^2}\left[\sqrt{\Sigma[P_T, N]} - 1\right]$$

The first example of Strongly Intensive Measure

Gazdzicki, Mrowczynski, Z. Phys. C (1992) (176 citations)

Effects of Quantum Statistics
M.I.G., Rybczynski, Phys. Lett. B (2014),

$$\Delta[P_T, N], \ \Sigma[P_T, N]$$

The strongest effect is in $\Delta[P_T, N]$ of pions

p+p at 158 GeV/c : There is a correlation between the inverse slope of m_T -spectrum and hadron multiplicity *N*

$$\Delta[P_T, N] \cong 0.82, \quad \Sigma[P_T, N] \cong 1.01$$

M.I.G., Grebieszkow, Phys. Rev. C (2014)

in a good agreement with (preliminary) NA61/SHINE data in p+p reactions

Experimental results for the Δ and Σ measures:

Rustamov for the NA61/SHINE and NA49 Collaborations, arXiv:1303.5671
Anticic, et.al Phys. Rev. C (2014)
Mackowiak-Pawlowska, PoS CPOD2013 (2013), J. Phys. Conf. Ser. (2014)
Rybczynski, NA61/SHINE Collaboration, PoS EPS-HEP2013, arXiv:1301.3360
Mackowiak-Pawlowska, Wilczek for the NA61 Collaboration (2013)
Grebieszkow, Acta Phys. Pol. (2013), PoS CPOD2013 (2013)
Seyboth, arXiv:1402.4619 (2014)
Stefanek, NA61/SHINE and NA49, arXiv:1411.2396

Resonance Decays

$$R \to \pi^{+}\pi^{-}, \qquad \omega\left[\pi^{+}\right] \cong \omega\left[\pi^{-}\right] \cong \omega\left[R\right] \cong 1,$$

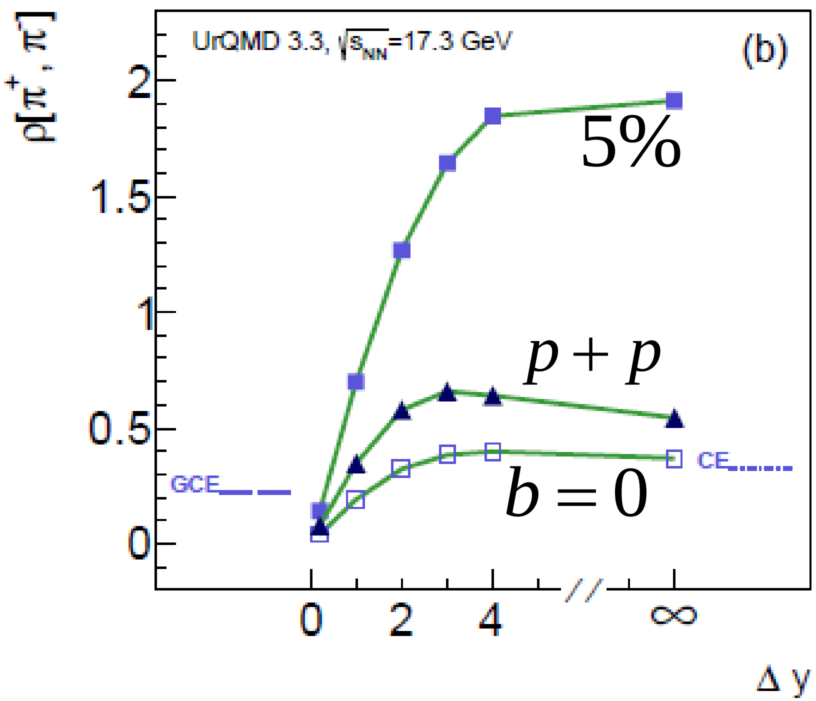
$$\frac{< R >}{< \pi^{+} > + < \pi^{-} >} \cong \rho[\pi^{+},\pi^{-}] \equiv \frac{<\pi^{+}\pi^{-}> - <\pi^{+}><\pi^{-}>}{<\pi^{+}> + <\pi^{-}>}$$

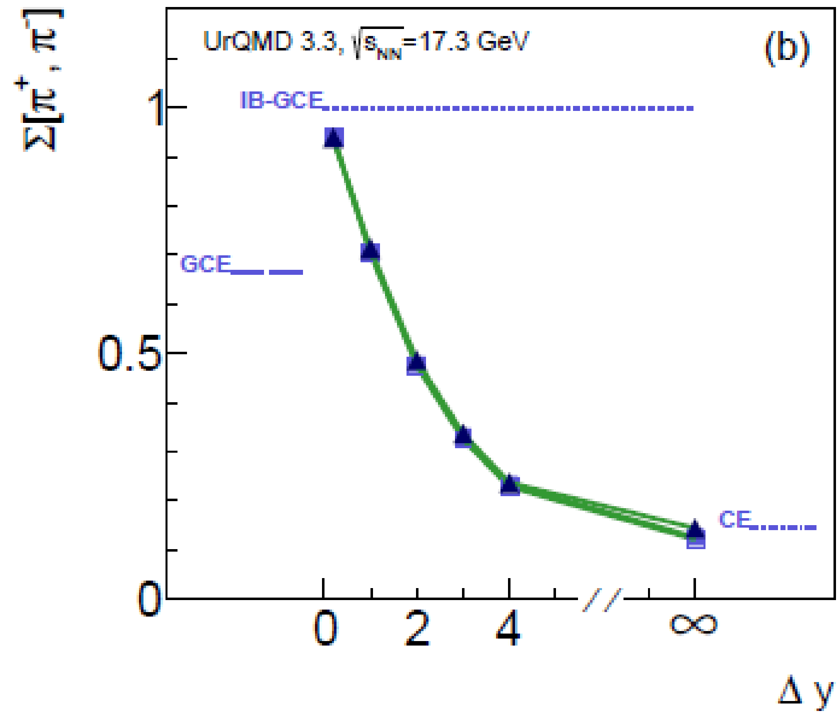
Jeon, Koch, Phys. Rev. Lett. (1999) (132 citations).

$$\frac{< R >}{< \pi^{+} > + < \pi^{-} >} \cong \frac{1 - \Sigma[\pi^{+},\pi^{-}]}{2}$$

UrQMD

Begun, M.I.G., Grebieszkow, arXiv:1409.3032





■ Pb+Pb, 5% most central

□ Pb+Pb, b=0

▲ p+p

$$-\frac{1}{2}\Delta y < y < \frac{1}{2}\Delta y$$

rapidity interval of pion acceptance in the c.m.s.

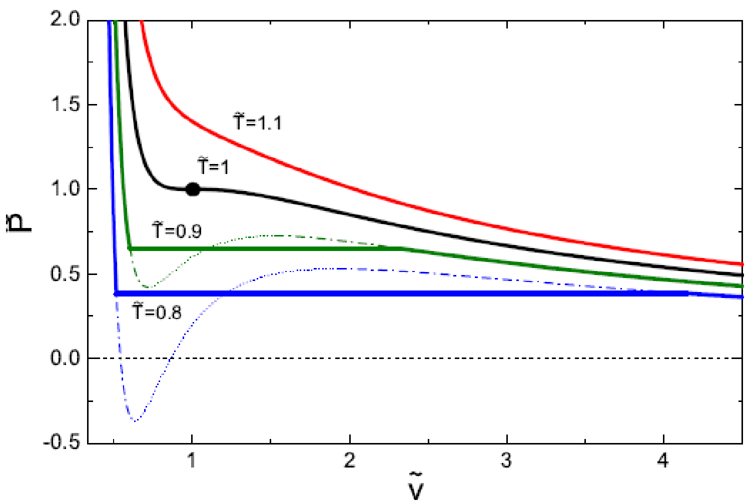
III. Van der Waals Equation of State in the GCE and Particle Number Fluctuations

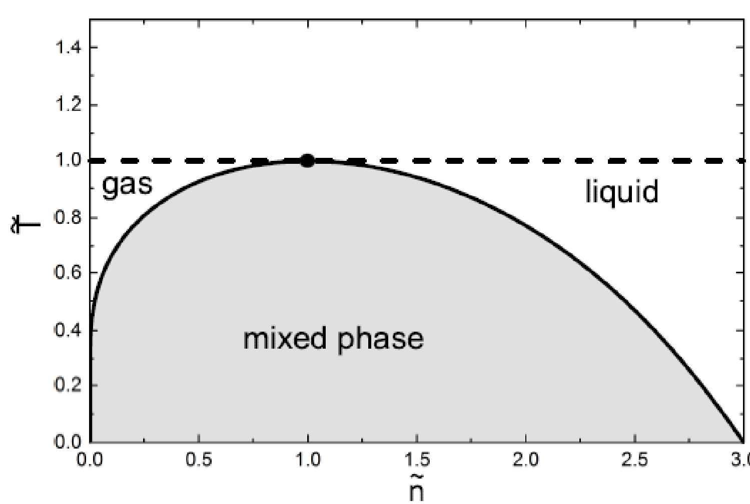
Vovchenko, Anchishkin, M.I.G.
arXiv (2015)

$$p(V,T,N) = \frac{NT}{V-bN} - a\frac{N^2}{V^2} = \frac{nT}{1-bn} - an^2, \quad \text{CE}$$

$$T_c = \frac{8a}{27b}, \quad n_c = \frac{1}{3b}, \quad p_c = \frac{a}{27b^2}$$

$$p = \frac{8Tn}{3-n} - 3n^2, \qquad n = n/n_c, \quad p = p/p_c, \quad T = T/T_c,$$



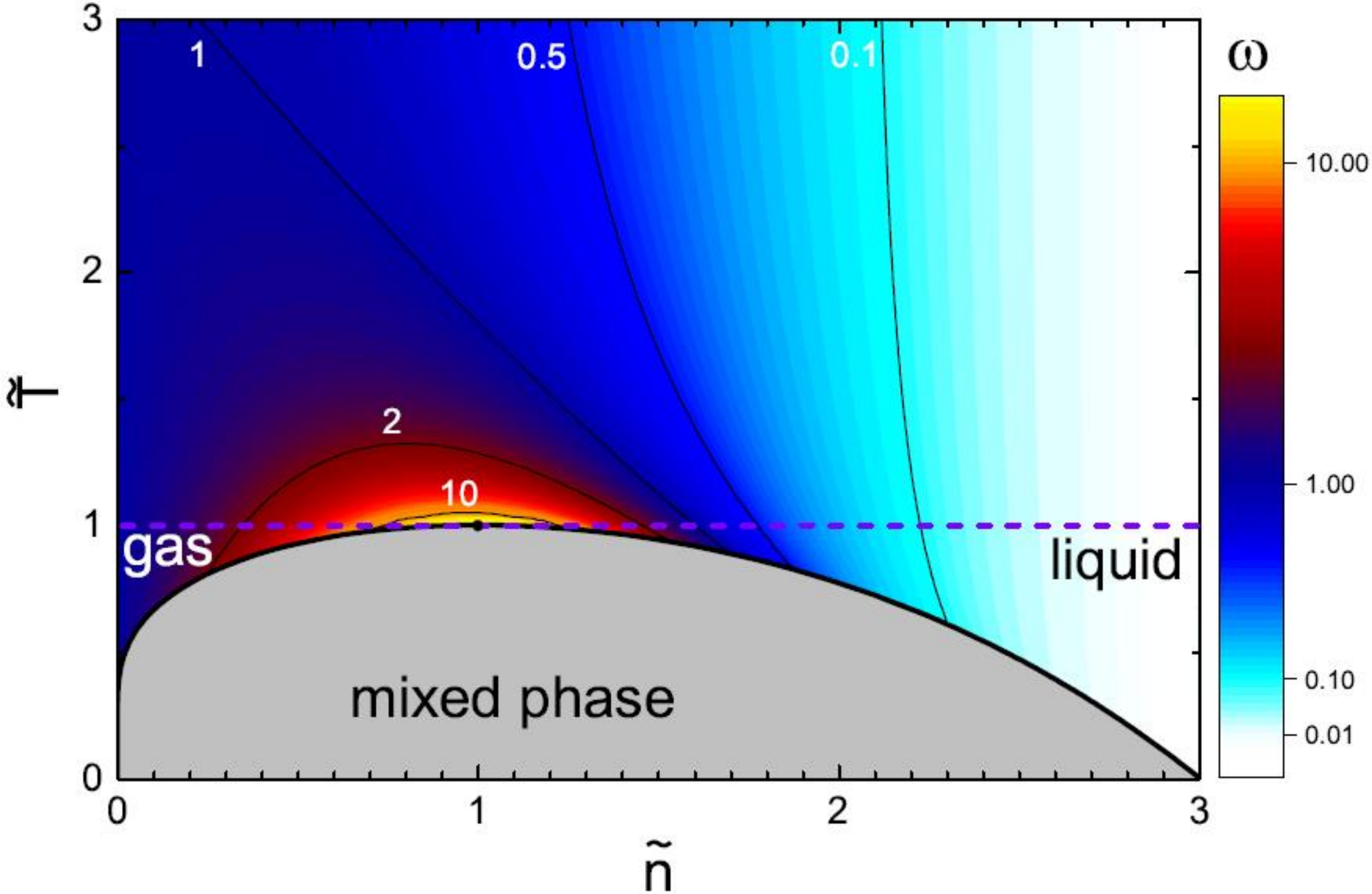


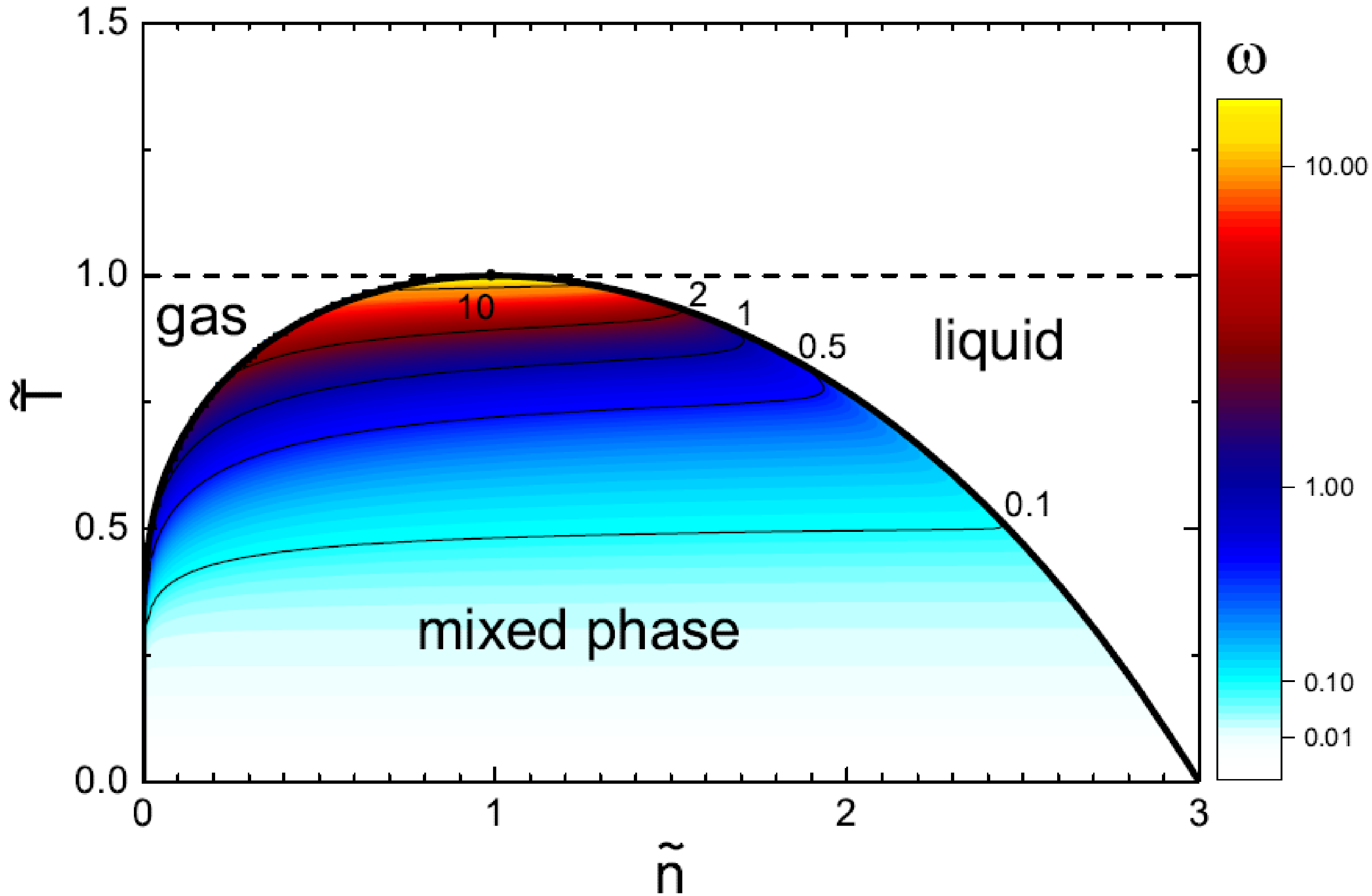
$$p(V,T,N) = \frac{nT}{1-bn} - an^2, \quad \text{CE}$$

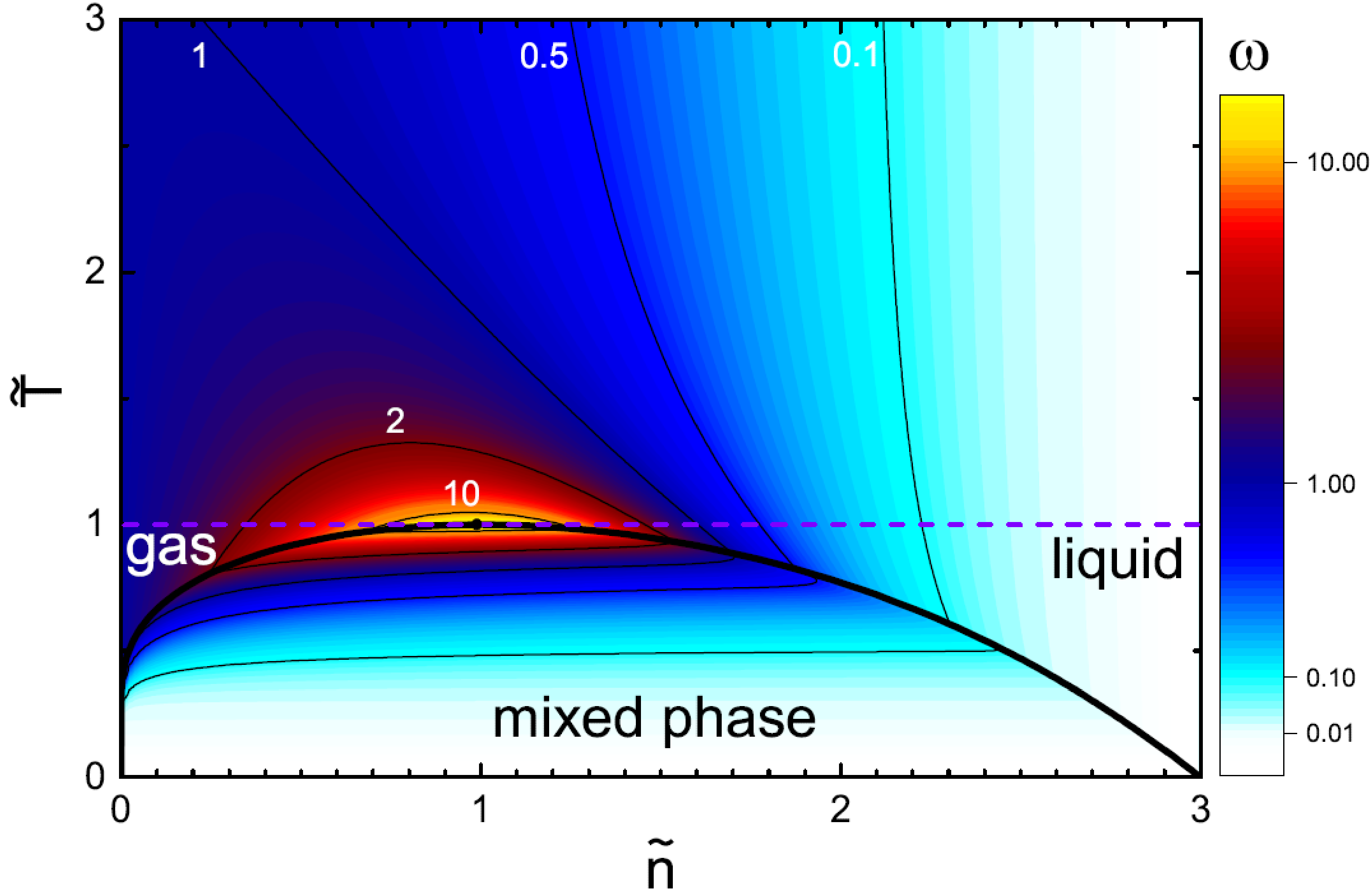
$$n(T,\mu) = \frac{n_{\mathrm{id}}(T,\mu^*)}{1+bn_{\mathrm{id}}(T,\mu^*)}, \qquad \mu^* = \mu - b\frac{nT}{1-bn} + 2an, \qquad \text{GCE}$$

$$\omega[N] \equiv \frac{<N^2> - <N>^2}{<N>} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\right]^{-1}$$

$$\omega[N] = \frac{1}{9}\left[\frac{1}{(3-n)^2} - \frac{n}{4T}\right]^{-1}$$




ω
10.00
1.00
0.10
0.01
1.5
1.0
0.5
0.0
T̃
0
1
2
3
ñ
gas
liquid
mixed phase
10
2
1
0.5
0.1



1
0.5
0.1
2
10
gas
liquid
mixed phase
ω
10.00
1.00
0.10
0.01
$\tilde{T}$
$\tilde{n}$
0
1
2
3

Summary

1. Global Conservation Laws

   suppress particle number fluctuations and introduce interparticle correlations. This is valid also in the thermodynamic limit $V \to \infty$.

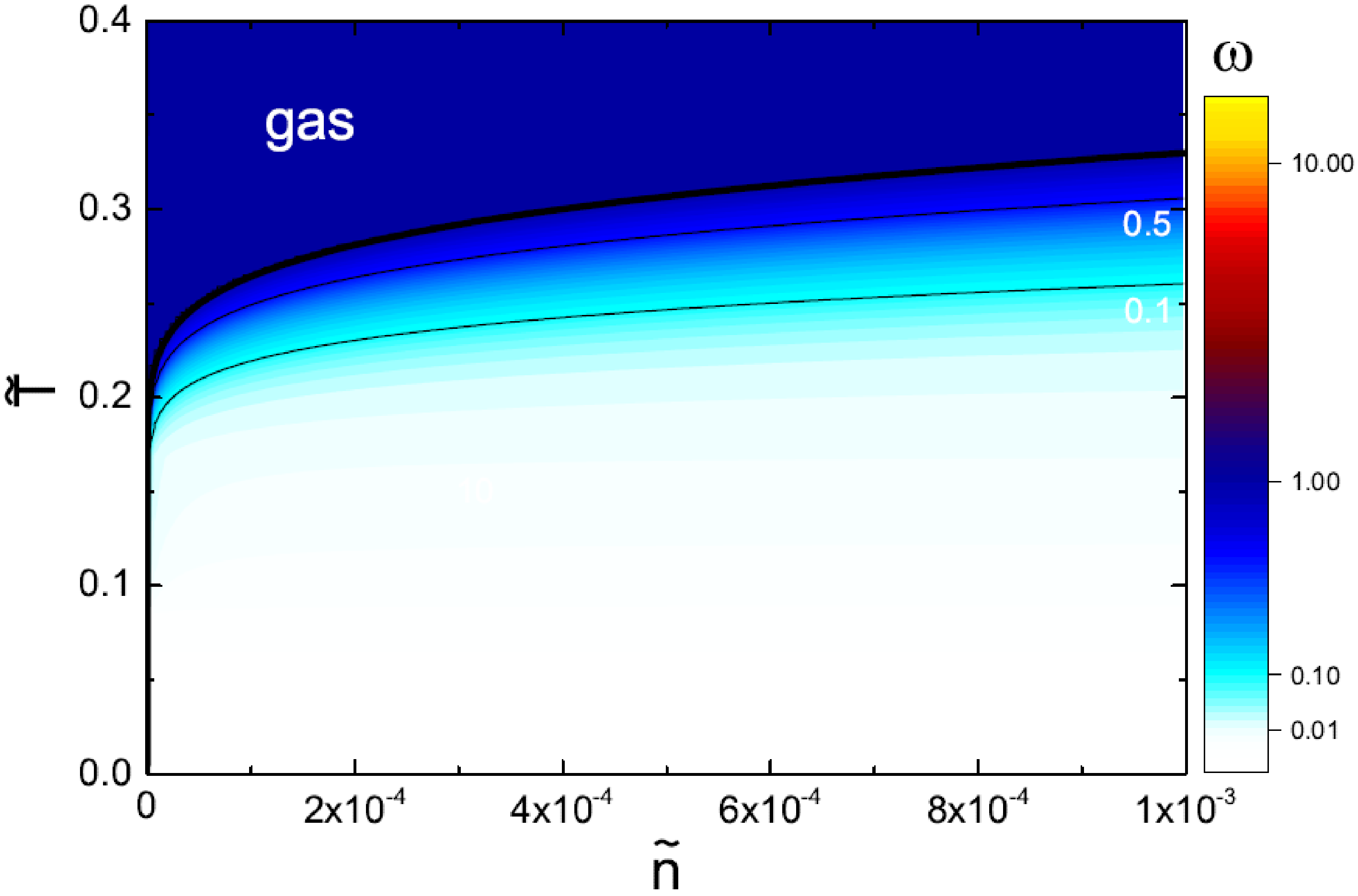
2. Strongly Intensive Measures $\Delta[A, B]$ and $\Sigma[A, B]$

   are independent of the average size of the system and of the fluctuations of the size.
   Examples: GCE, Model of Independent Sources, Mixed Events,….

3. Van der Waals Equation of State and Particle Number Fluctuations

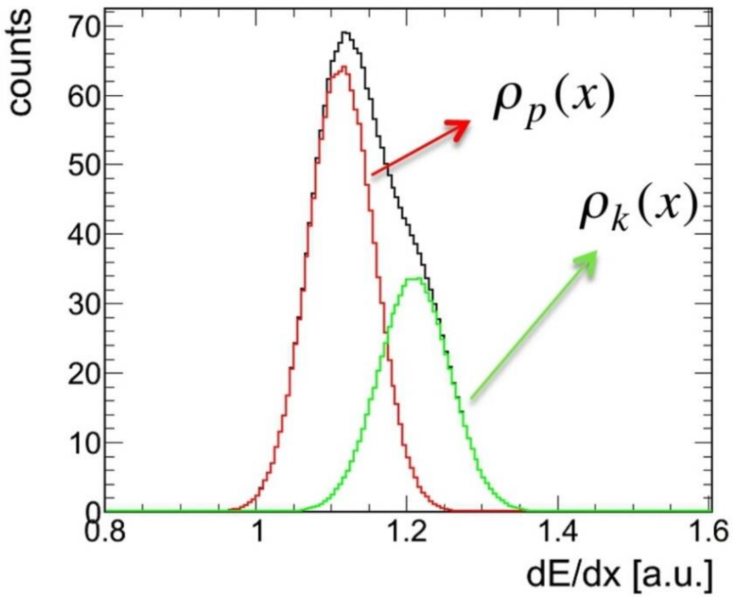
   provides an analytical example of the fluctuations in systems with 1st order liquid-gas phase transition and critical point.

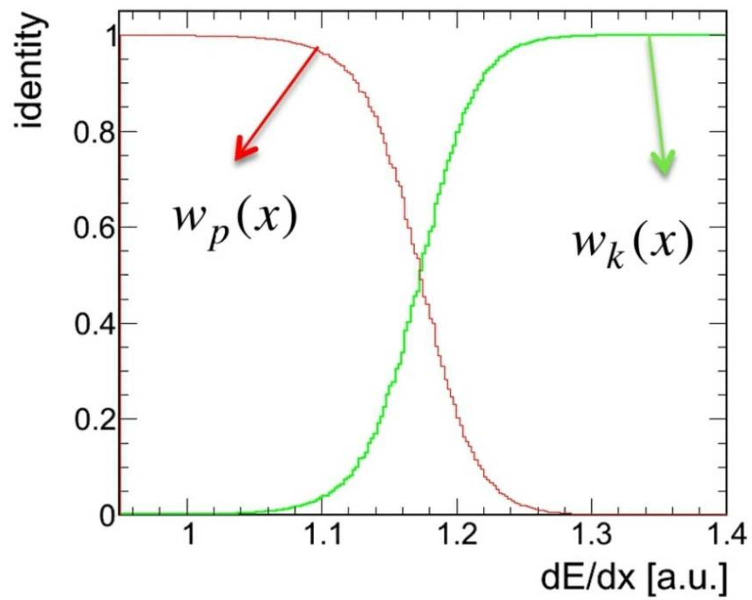
Thank you!



gas
0.5
0.1
0.4
0.3
0.2
0.1
0.0
$\tilde{T}$
0
2x10⁻⁴
4x10⁻⁴
6x10⁻⁴
8x10⁻⁴
1x10⁻³
$\tilde{n}$
ω
10.00
1.00
0.10
0.01

III. Fluctuations with Incomplete Particle Identifications





$$\int dm \rho_j(m) = \langle N_j \rangle,$$

$$\rho(m) = \sum_{i=1}^{k} \rho_i(m)$$

identity variable $$w_j(m) = \frac{\rho_j(m)}{\rho(m)} \subset [0,1]$$

Complete identification:

$\rho_j(m)$ do not overlap

Gazdzicki, Grebieszkow, Mackowiak, Mrowczynski, Phys. Rev. C (2011)
M.I.G., Phys. Rev. C (2011); Rustamov, M.I.G., Phys. Rev. C (2012)

$$W_j = \sum_{i=1}^{N(n)} w_j(m_i), \qquad W_j^2 = [\sum_{i=1}^{N(n)} w_j(m_i)]^2$$

$$W_p W_q = [\sum_{i=1}^{N(n)} w_p(m_i)][\sum_{i=1}^{N(n)} w_q(m_i)]$$

$N(n) = N_1(n) + ... + N_k(n)$ total multiplicity in n-th event

$$< W_j^2 >= \frac{1}{N_{ev}} \sum_{n=1}^{N_{ev}} W_j^2, \qquad < W_p W_q >= \frac{1}{N_{ev}} \sum_{n=1}^{N_{ev}} W_p W_q$$

N_{ev} is the number of events

In the case of complete identifications: $W_j = N_j$,

$$< W_j^2 > \ = \ < N_j^2 >, \qquad < W_p W_q > \ = \ < N_p N_q >$$

$$\sum_{i=1}^{k} < N_i^2 > u_{ji}^2 + 2 \sum_{1\le i<l\le k} < N_i N_l > u_{ji} u_{jl} = b_j$$

$$\sum_{i=1}^{k} < N_i^2 > u_{pi} u_{qi} + \sum_{1\le i<l\le k} < N_i N_l > (u_{pi} u_{ql} + u_{pl} u_{qi}) = b_{pq}$$

$$u_{ji}^s \equiv \frac{1}{< \mathrm{N}_i >} \int dm\, w_j^s, \quad u_{pqi} \equiv \frac{1}{< \mathrm{N}_i >} \int dm\, w_p\ w_q\ \rho_i(m)$$

$$b_j \equiv < W_j^2 > - \sum_{i=1}^{k} < N_i > [u_{ji}^2 - (u_{ij})^2],$$

$$b_{pq} \equiv < W_p W_q > - \sum_{i=1}^{k} < N_i > [u_{pqi} - u_{pi} u_{qi}]$$

$$< N_j^2 >, \quad j=1,\ldots,k \qquad < N_p N_q >, \quad 1 \le p < q \le k$$

The system of k+k(k-1)/2 linear equations

$$< N_1^2 > \; = \; \frac{b_1 u_{22}^2 + b_2 u_{12}^2 - 2 b_{12} u_{12} u_{22}}{(u_{11} u_{22} - u_{12} u_{21})^2}$$

$$< N_2^2 > \; = \; \frac{b_2 u_{11}^2 + b_1 u_{21}^2 - 2 b_{12} u_{21} u_{11}}{(u_{11} u_{22} - u_{12} u_{21})^2}$$

$$< N_1 N_2 > \; = \; \frac{b_{12} (u_{11} u_{22} + u_{12} u_{21}) - b_1 u_{22} u_{21} - b_2 u_{11} u_{22}}{(u_{11} u_{22} - u_{12} u_{21})^2}$$

Experimental results for fluctuations with the identity method:

Rustamov for the NA61/SHINE and NA49, J. Phys. Ser. and arXiv:1303.5671
Anticic, et.al, Phys. Rev. C (2013, 2014)
Mackowiak-Pawlowska, PoS CPOD2013 (2013), J. Phys. Conf. Ser. (2014)
Grebieszkow, Acta Phys. Pol. (2013), PoS CPOD2013 (2013)
Seyboth, arXiv:1402.4619
Stefanek, NA61/SHINE and NA49, arXiv:1411.2396

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2. Strongly Intensive Measures $\Delta[A, B]$ and $\Sigma[A, B]$

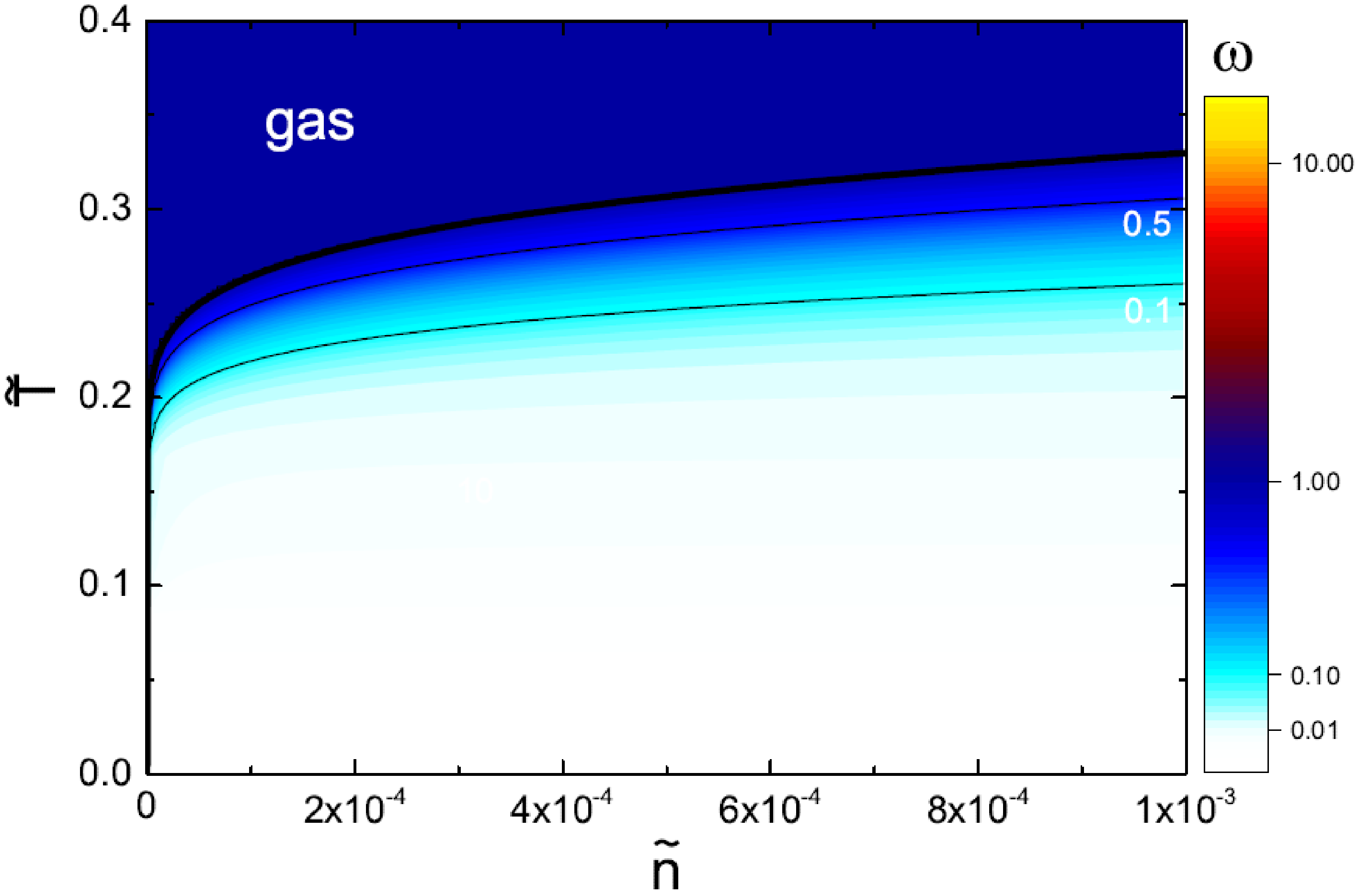
   are independent of the average size of the system and of the fluctuations of the size.
   Examples: GCE, Model of Independent Sources, Mixed Events,….

3. Identity Method

   provides the values of all the second and higher moments of identified particle number distributions in a model independent way for the case of incomplete particle identifications.

Thank you!

Additional Slides



gas
0.5
0.1
ω
10.00
1.00
0.10
0.01
0.4
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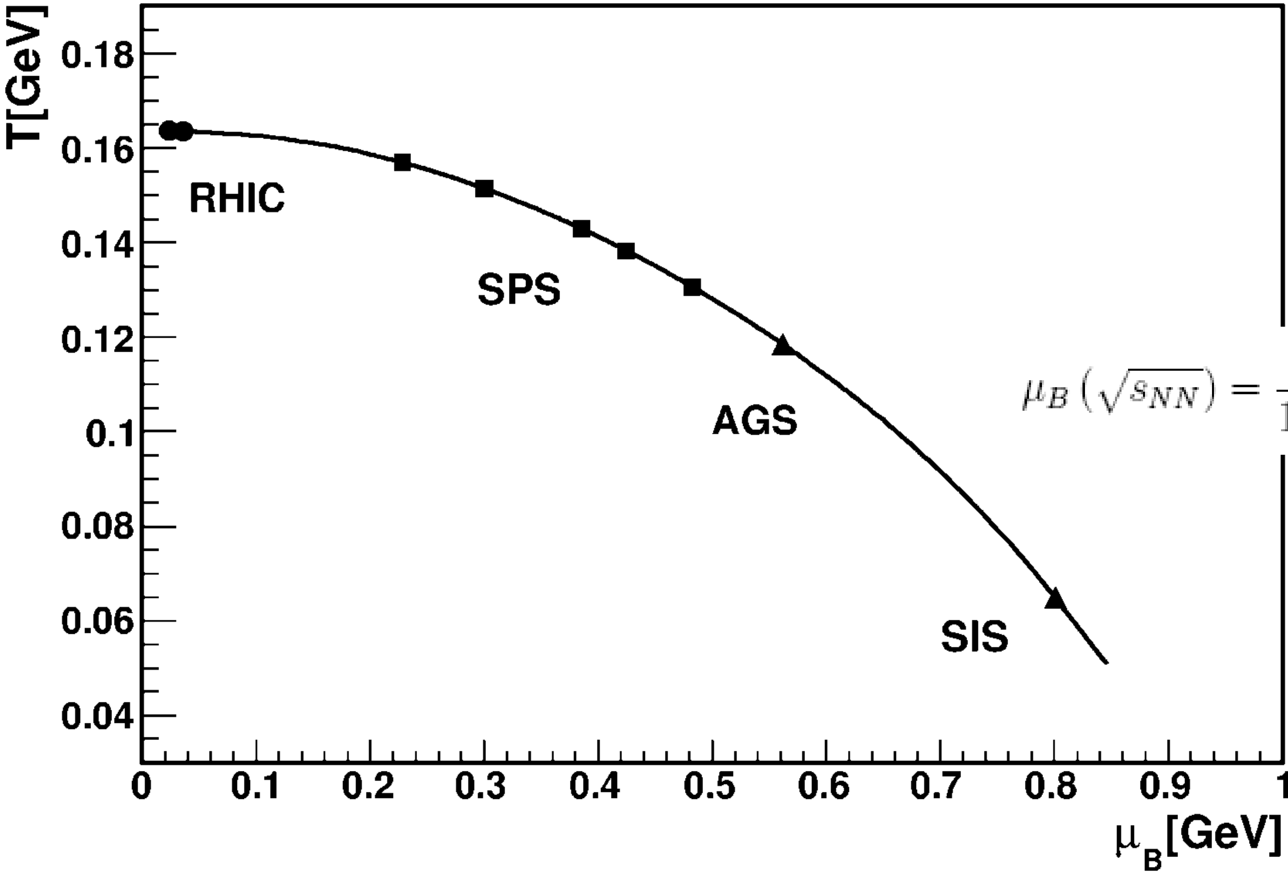
0.2
0.1
0.0
T̃
0
2x10^-4
4x10^-4
6x10^-4
8x10^-4
1x10^-3
ñ

Line of the chemical freeze-out

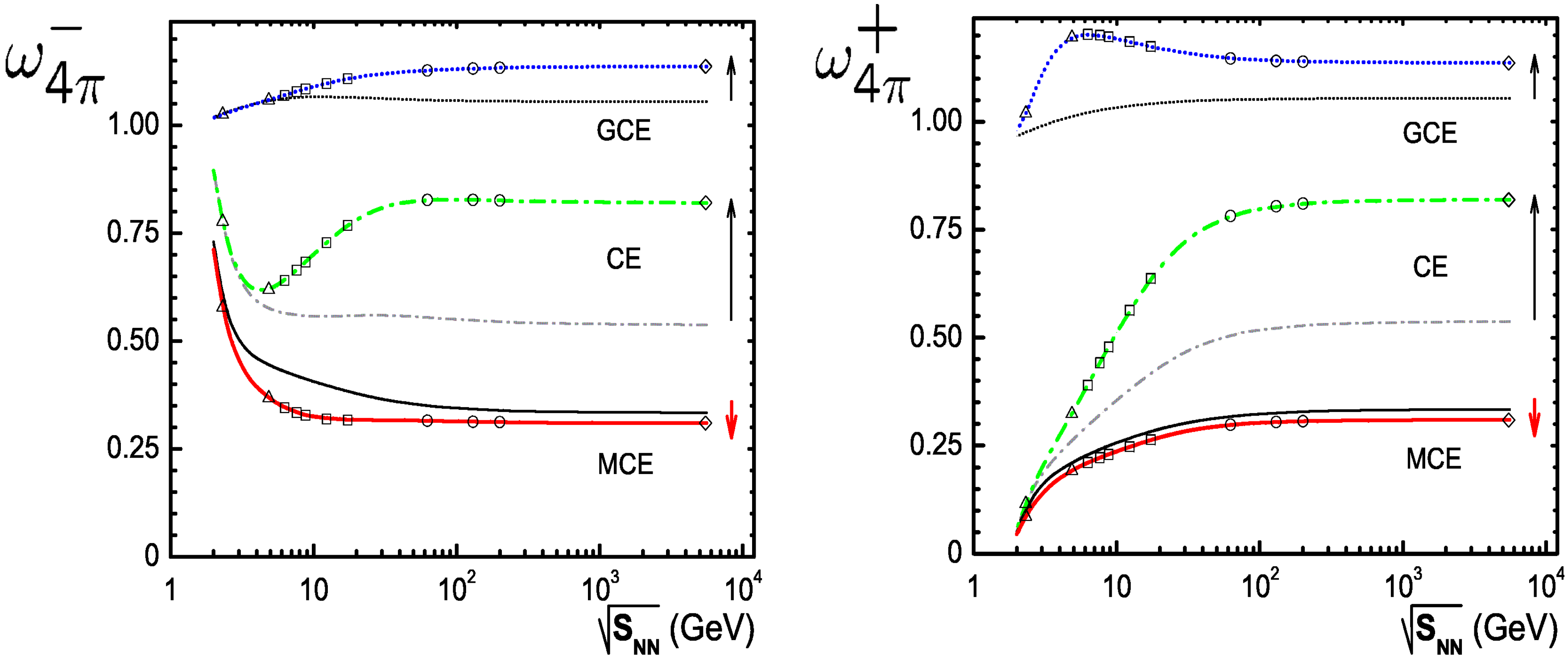


$$\mu_B\left(\sqrt{s_{NN}}\right) = \frac{1.308\ \mathrm{GeV}}{1 + 0.273\ \sqrt{s_{NN}}}\ ,$$

E/N = 1 GeV

Cleymans and Redlich, PRL 81 (1998)

Hadron-Resonance Gas



$$\omega^{\pm}_{4\pi} \equiv \frac{\langle (\Delta N_{\pm})^2 \rangle}{\langle N_{\pm} \rangle} \qquad \omega^{\pm}_{acc} = 1 - q + q\,\omega^{\pm}_{4\pi}$$

Begun, Gazdzicki, M.I.G., Hauer, Konchakovsi, Lungwitz, Phys. Rev. (2007)

Micro Canonical Ensemble with scaling Volume Fluctuations (MCE/sVF)

$$\mathbf{P}_\alpha(\mathbf{X};\mathbf{E}) \;=\; \int_0^\infty \mathbf{dV}\;\mathbf{P}_\alpha(\mathbf{V})\;\mathbf{P}_{\mathbf{mce}}(\mathbf{X};\mathbf{E},\mathbf{V})$$

Begun, Gazdzicki, M.I.G., Phys. Rev. C (2008) and (2009)

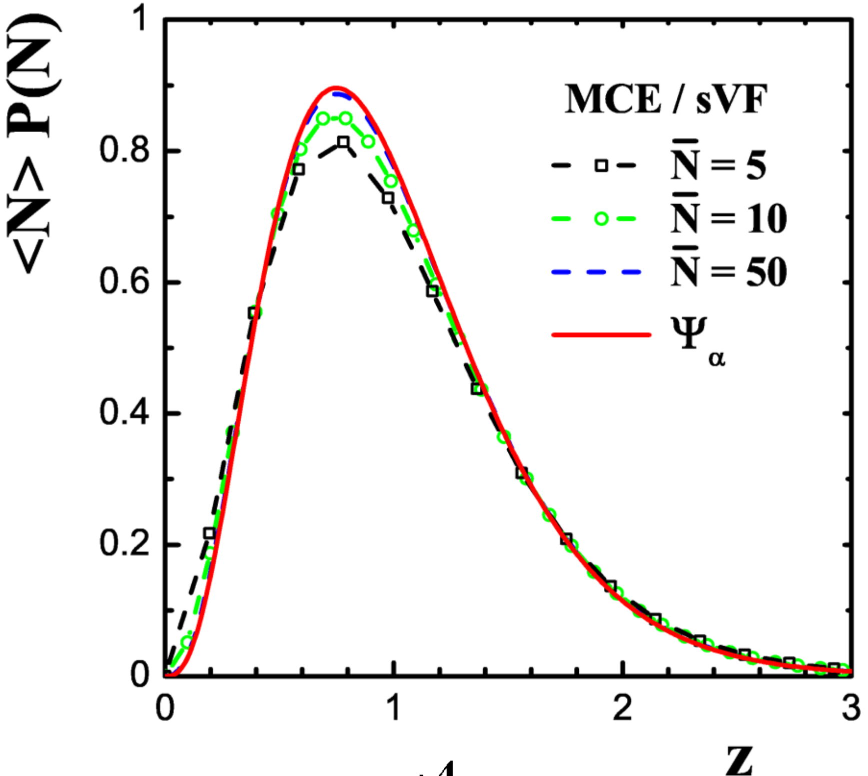
$$\mathbf{P}_\alpha(\mathbf{V}) \;=\; \frac{1}{\overline{\overline{\mathbf{V}}}}\,\Phi_\alpha(\mathbf{V}/\overline{\mathbf{V}})$$

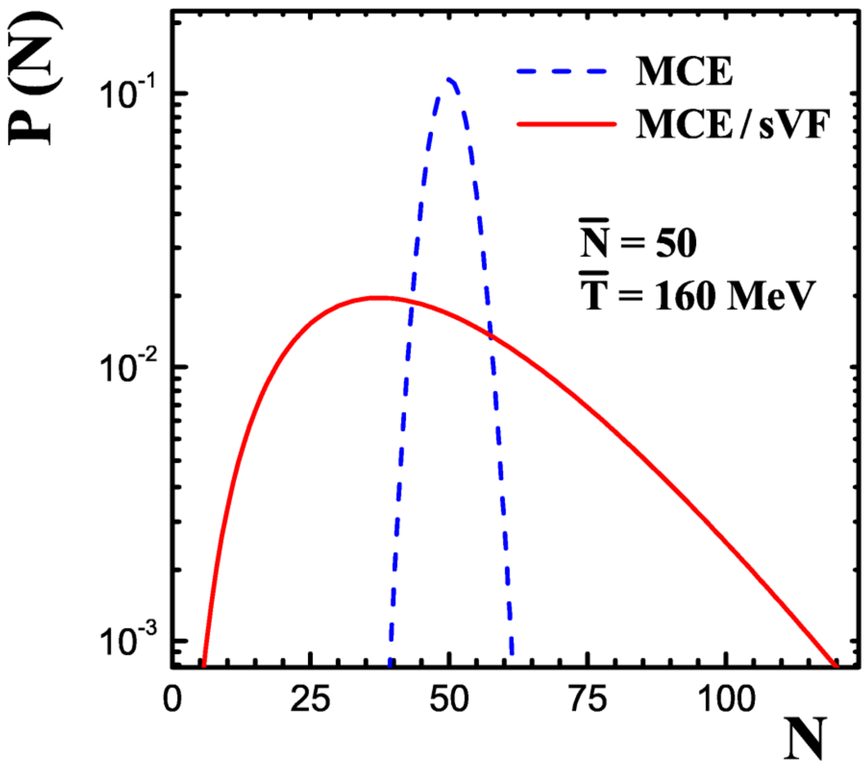
Scaling volume fluctuations selected to fit experimental multiplicity distribution

1) Describes the KNO-scaling of multiplicity distribution

2) Gives the Power Law of (transverse) momentum spectra

Multiplicity distribution in the MCE/sVF



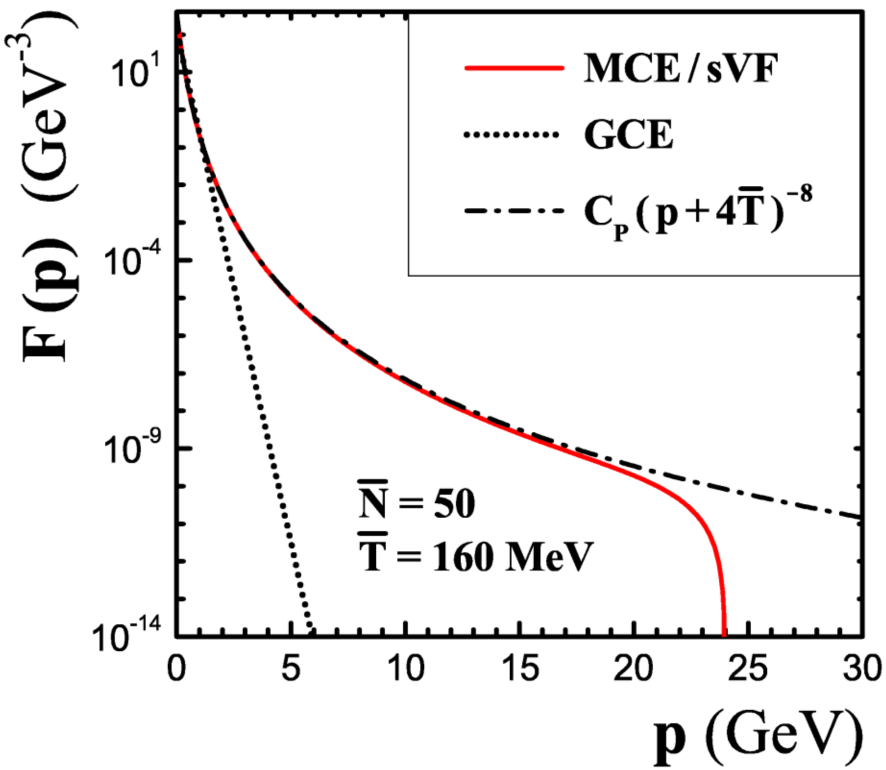


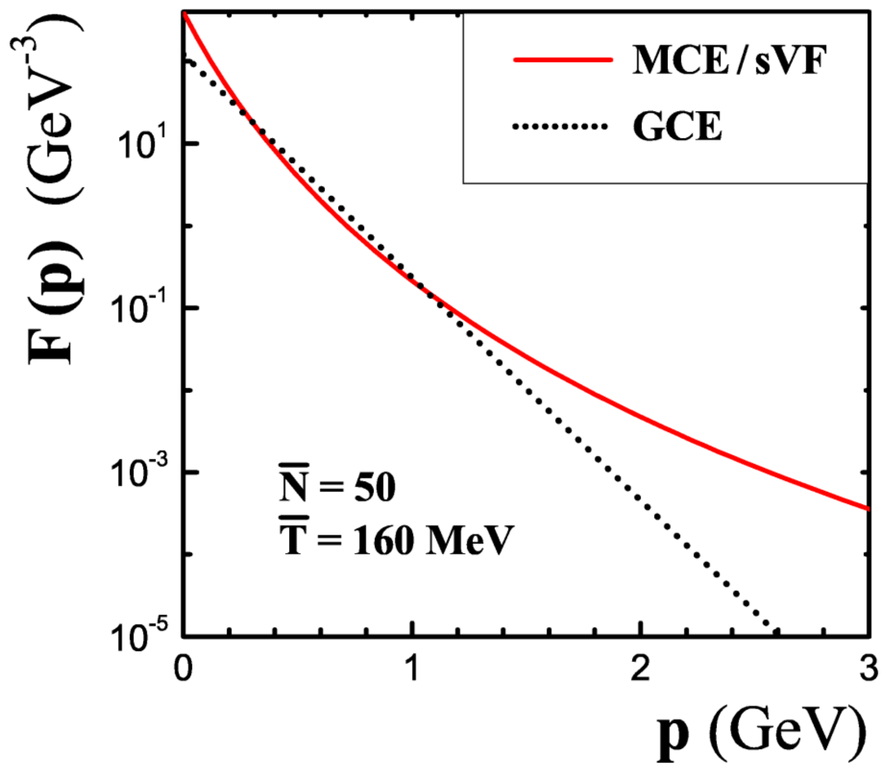
$$\Psi_\alpha(y) = \frac{4^4}{3!}\, y^3 \, \exp(-4\,y)$$

$$z = N/\langle N\rangle_\alpha \,, \quad y = (V/\overline{V})^{1/4}$$

$$\omega_\alpha = \frac{\langle N^2\rangle_\alpha - \langle N\rangle_\alpha^2}{\langle N\rangle_\alpha} \cong \frac{1}{4}\,\langle N\rangle_\alpha$$

Power law in momentum spectrum





$$F_\alpha(p) = \frac{1}{\langle N \rangle_\alpha} \left\langle \frac{dN}{p^2 dp} \right\rangle_\alpha$$

Relation to Other Measures

$$\Phi = \frac{\sqrt{\langle A\rangle\langle B\rangle}}{\langle \mathbf{A}\rangle + \langle \mathbf{B}\rangle}\left[\left(\Sigma^{\mathrm{AB}}\right)^{1/2} - 1\right]$$

Gazdzicki, Mrowczynski (1992)

$$\nu^{\mathrm{AB}}_{\mathrm{dyn}} = \frac{\langle \mathbf{A}(\mathbf{A}-1)\rangle}{\langle \mathbf{A}\rangle^2} + \frac{\langle \mathbf{B}(\mathbf{B}-1)\rangle}{\langle \mathbf{B}\rangle^2} - 2\frac{\langle \mathbf{AB}\rangle}{\langle \mathbf{A}\rangle\langle \mathbf{B}\rangle}$$

Pruneau, Gavin, Voloshin (2002)

$$\nu_{dyn}[A,B] = \frac{<A+B>}{<A><B>}[\Sigma[A,B]-1]$$

$\sqrt{s_{NN}}$	T	μ_B	$\rho[\pi^+,\pi^-]$	$\omega[\pi^+]$	$\omega[\pi^-]$	$\Sigma[\pi^+,\pi^-]$	$\langle R_{\pi\pi}\rangle/(\langle\pi^-\rangle+\langle\pi^+\rangle)$		
[GeV]	[MeV]	[MeV]	GCE	GCE	GCE	GCE	Eq.(47)	Eq.(41)	Eq. (42)
6.27	130.8	482.4	0.132	1.063	1.074	0.804	0.11	0.14	0.10
7.62	139.2	424.6	0.155	1.072	1.083	0.768	0.13	0.16	0.12
8.77	144.2	385.4	0.170	1.079	1.089	0.744	0.14	0.17	0.13
12.3	153.0	300.2	0.199	1.092	1.101	0.699	0.15	0.20	0.15
17.3	158.6	228.6	0.219	1.102	1.109	0.668	0.16	0.22	0.17
200	165.9	23.5	0.246	1.116	1.117	0.624	0.17	0.25	0.19
5500	166.0	0.87	0.246	1.117	1.117	0.624	0.17	0.25	0.19

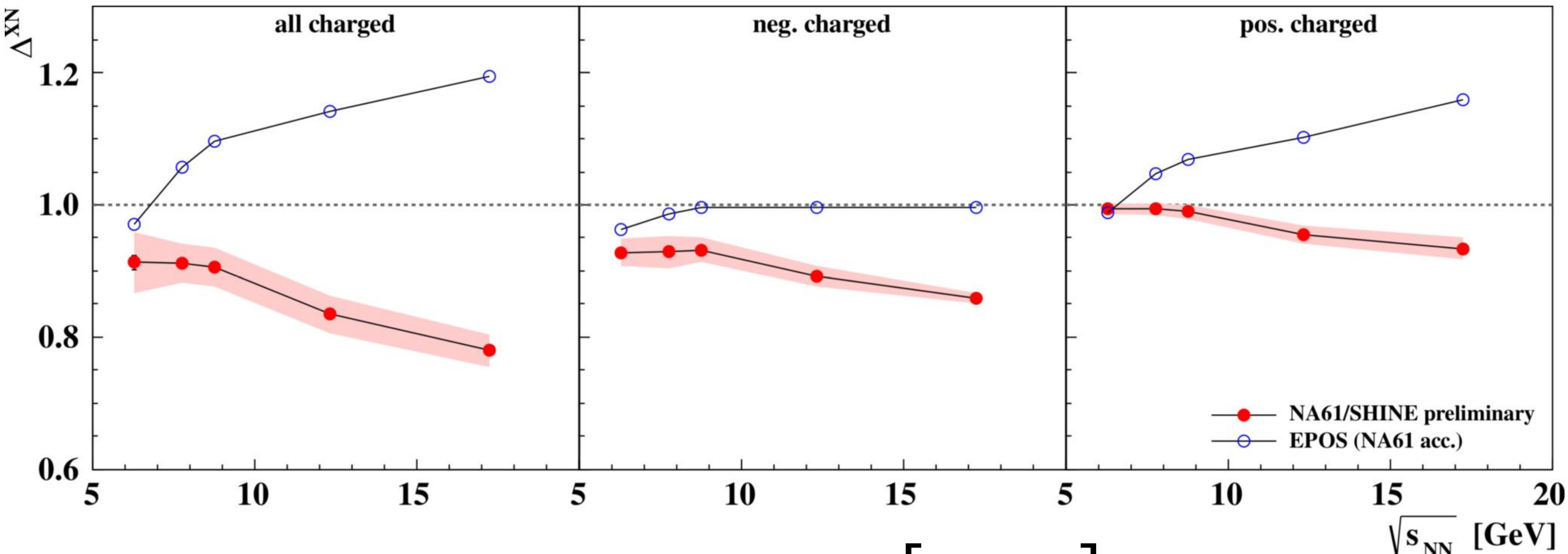
$$\Sigma\left[P_T, N\right] \cong 1.01,$$

p+p at 158 GeV/c

$$\Delta\left[P_T, N\right] \cong 0.82$$

M.I.G., Grebieszkow,
Phys. Rev.C (2014)

NA61/SHINE data in p+p reactions



Data: $\Sigma\left[P_T, N\right] \cong 1$