Statistics of thermalization in Bjorken Flow

Jakub Jankowski

with M. Spaliński, G. Plewa

JHEP 1412 (2014) 105

Institute of Physics
Jagiellonian University

Hydrodynamization process

- At RHIC and LHC applicability of hydrodynamics is at time

$$\tau_0 \sim 0.5 - 1 \text{ fm/c}$$

- It is *fast* in a sense that

$$\tau_0 < \frac{1}{T_0}$$

- Dynamics appears to be strongly coupled, so new theoretical techniques are required
- **How generic is this behaviour?**
- **Are there any universal characteristics?**

SYM vs. QCD plasma

- SUSY Yang-Mills = gluons + 6 scalars + 4 fermions
- **Similarities**
  $\rightarrow$ deconfined phase
  $\rightarrow$ strongly coupled
  $\rightarrow$ no SUSY at finite T
  $\rightarrow$ at weak coupling similar to pQCD plasma
  A. Czajka, S. Mrówczyński, Phys. Rev. D **86**, 025017 (2012)
- **Differences**
  $\rightarrow$ no running coupling
  $\rightarrow$ no confinement-deconfinement phase transition
  $\rightarrow$ exactly conformal EoS
- **Perspective**
  first principle calculation of real time dynamics in a specific strongly coupled gauge theory

- Flow is invariant under longitudinal boosts and does not depend on the transverse coordinates Bjorken '83
- The coordinates are in turn defined by

$$t = \tau \cosh y, \quad z = \tau \sinh y$$

and everything is assumed to be y-independent

- It is strict in the limit of an infinite energy collision of infinitely large nuclei

Boost invariant setup

- Energy momentum tensor is τ-dependent

$$T_{\mu\nu} = \mathrm{Diag}\{\epsilon(\tau), p_L(\tau), p_T(\tau), p_T(\tau)\}$$

and $\epsilon(\tau) = p_L(\tau) + 2p_T(\tau)$

- Energy density *defines* local effective temperature

$$\epsilon(\tau) = \frac{3}{8} N_c^2 \pi^2 T(\tau)^4$$

- In the late times hydrodynamics applies

$$T(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}} \Bigg\{ 1 - \frac{1}{6\pi\,(\Lambda\tau)^{2/3}} + \frac{-1+\log 2}{36\pi^2\,(\Lambda\tau)^{4/3}} + \\ + \frac{-21 + 2\pi^2 + 51\log 2 - 24\log^2 2}{1944\pi^3\,(\Lambda\tau)^2} \Bigg\}$$

A few words about AdS/CFT

- States in 4d field theory $\leftrightarrow$ dual geometries in 5d

- Equilibrium state in field theory $\leftrightarrow$ black hole in the bulk
- Non-equilibrium states $\leftrightarrow$ geometries with *horizons*

- Out-of-equilibrium entropy is defined by

$$S = a_{AH}/\pi$$

where a_{AH} - is the apparent horizon area

Booth, I *et al.* Phys.Rev. D80 (2009) 126013

- S is non-decreasing and agrees with hydrodynamic entropy for late times

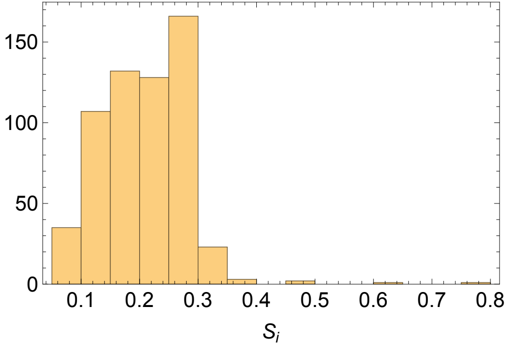
Technical aspects

- Numerically solve 5d Einstein equations to get $\epsilon(\tau)$ and $S(\tau)$
- Initial geometries are subjected to one constraint equation
- In previous studies 29 solutions to this equation have been found and whence 29 evolutions have been analyzed

  M.P. Heller *et al.* Phys.Rev.Lett. 108 (2012) 201602
- By formulating the problem in a different gauge we are able to find generic solutions to the constraint equation and in turn analyze **arbitrary** number of configurations

Initial conditions

- Initial configurations are specified by *initial entropies*
- We assume $\epsilon(\tau = 0) > 0$ and normalize $T(0) = 1/\pi$
- We have analyzed about 600 different configurations
- Relation to HIC phenomenology is still unclear

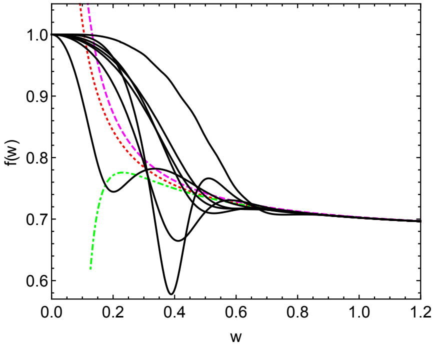


Boost invariant hydrodynamization

- Define $w = \tau T(\tau)$ with the logarithmic derivative

$$\frac{\tau}{w}\frac{dw}{d\tau} = f(w)$$

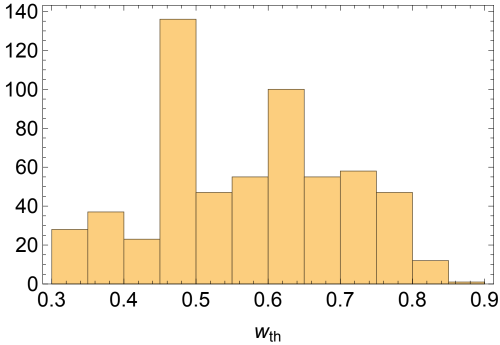
- In hydrodynamics regime there is an universal $f_{\text{hydro}}(w)$
- Different time profiles show presence of nonhydrodynamical dof
- The same as in Heller *et al.* Phys.Rev.Lett. 108 (2012) 201602



- Hydrodynamization condition

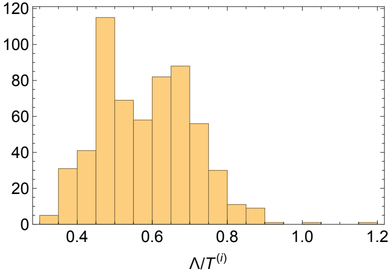
$$\left|\frac{F_{\rm hydro}(w)}{F(w)} - 1\right| < 0.05$$

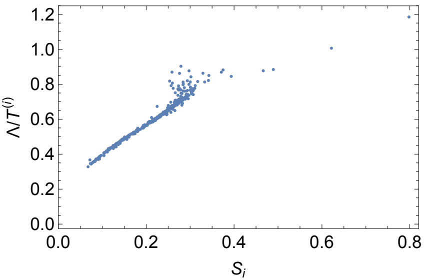
- $w_{av} = 0.57$ compare to $T = 500$ MeV, $\tau = 0.25$ fm $w = 0.63$
- $w_{\rm th} < 1 \Rightarrow \tau_{\rm th} < 1/T_{\rm th} \Rightarrow$ fast hydrodynamization



Starting condition for hydrodynamics

- Late time dynamics is determined by a single energy scale Λ
- S_i appears to essentially determine Λ for some range of S_i
- For higher entropies the relation is lost $\rightarrow$ „chaotic phase"



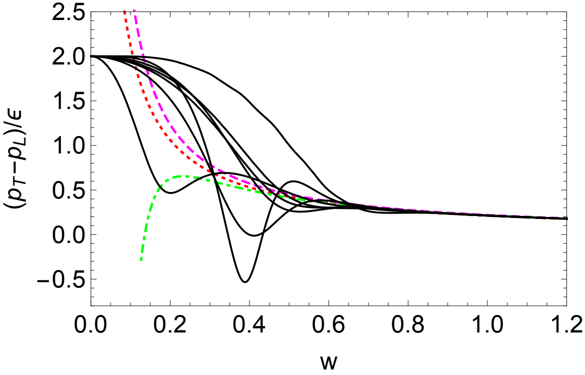


Anisotropies

- At the transition to hydrodynamics the anisotropy of $T_{\mu\nu}$ is sizable

$$\Delta p := \frac{p_L - p_T}{\epsilon} = 6f(w) - 4 \sim 0.35$$

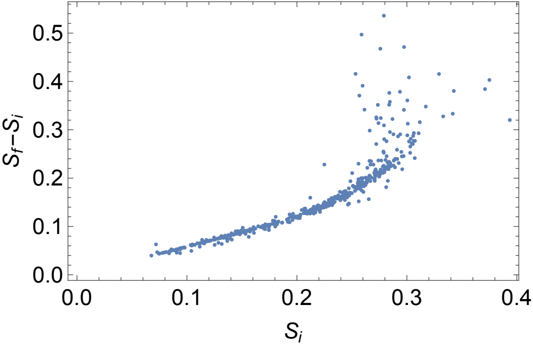
- Thermalization $\neq$ Hydrodynamization



Entropy production

- Final entropy is determined by the scale Λ

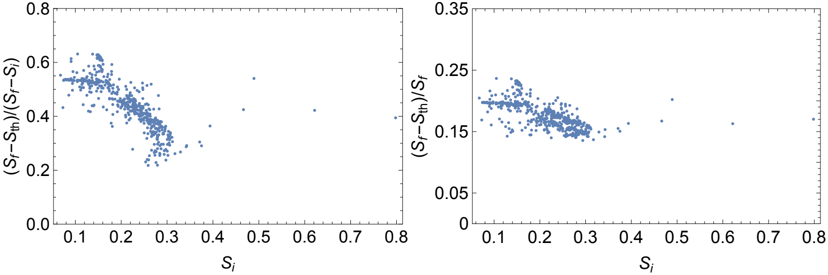
$$S_f = \Lambda^2/(T^{(i)})^2$$



- For higher entropies the relation is lost $\rightarrow$ „chaotic phase"

Entropy production by dissipative effects

- About 15-25% of the total entropy is produced in the hydro phase
- For states with small S_i oftem more than half of the entropy produced comes from the hydro

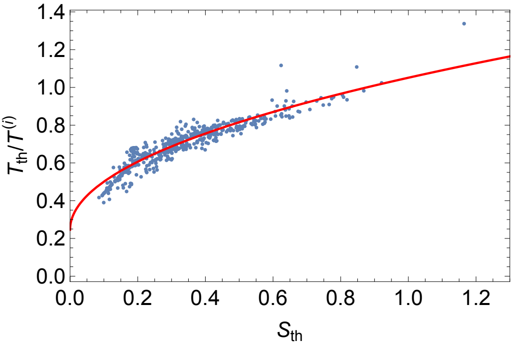


Hydrodynamization temperature correlation

- Apparent square root correlation

$$T_{\mathrm{th}} = a\sqrt{S_{\mathrm{th}}} + b$$

- This correlation is seen in the wide range of initial conditions $\rightarrow$ universal feature of hydrodynamization



Summary

- We have analyzed dynamics of 600 different configurations
- We find significant **evidence** supporting the findings of Heller *et al.* Phys.Rev.Lett. 108 (2012) 201602

  → Hydrodynamization is a fast process

  → hydrodynamization is *different* from thermalization
- Our simulations **suggest** that

  → Hydrodynamisation has a universal characterization by $T_{\rm th} - S_{\rm th}$ correlation

  → There exists a linear correlation $\Lambda - S_i$ for intermediate S_i

Future directions

- Detailed studies of observed $T_{\rm th} - S_{\rm th}$ and $\Lambda - S_i$ correlations
- Non-local observables $\rightarrow$ entanglement entropy, Wilson lines

  J. F. Pedraza, Phys. Rev. D **90**, 046010 (2014)
- Initial conditions with $\epsilon(\tau = 0) = 0$ $\rightarrow$ shock wave collisions

  D. Grumiller, P. Romatschke, JHEP **0808**, 027 (2008)