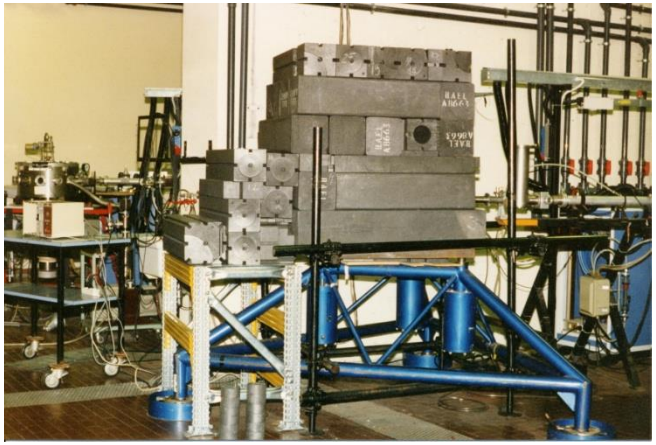
TRAINING COURSE ON
NEUTRON DOSIMETRY, RADIOBIOLOGY AND INSTRUMENTATION:
Neutron Sources

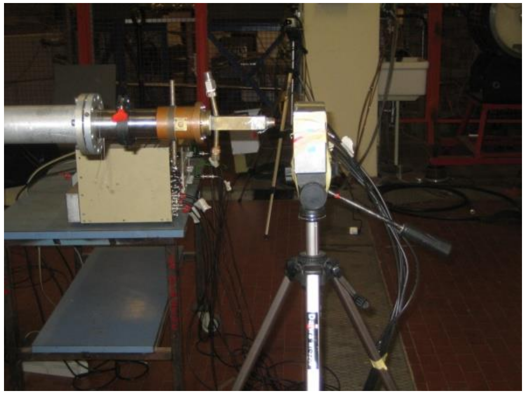
Stefano AGOSTEO, *POLIMI*

Wed. 24/06/2015, 9:00 – 10:00 am

NEUTRON SOURCES

- Spontaneous fission;
- Isotopic sources based on (α,n) and (γ,n) reactions;
- Accelerator-based;
- Reactor-based (fission);
- Photoneutrons.





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SPONTANEOUS FISSION: ^{252}Cf

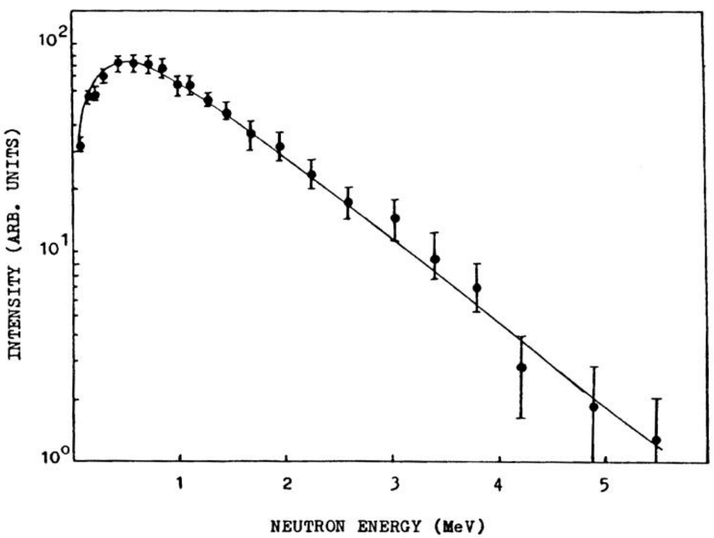
- ^{252}Cf spontaneous fission.
- $T_{1/2}$ = 2.65 y;
- Mean number of neutrons per fission: 3.8;
- α-decay is predominant (32 times more than spontaneous fission);
- Neutron yield: 0.116 s^{-1} Bq^{-1} (activity sf+α) → 2.30×10^{6} s^{-1} μg^{-1};
- Generated from plutonium or curium neutron irradiation (Oak Ridge National Labs and Research Institute of Atomic Research, Russia);
- Average annual production: about 0.5 g (0.27 g in 2003);
- Very small sources (a few µg) → point source approximation @ low distance.

SPONTANEOUS FISSION: ^{252}Cf

- Fission spectrum (bare source):

$$\frac{dN}{dE} = \sqrt{E} \times e^{-E/T}$$

  - where T = 1.3 MeV for ^{252}Cf spontaneous fission.



From G. Knoll, Radfiation Detection and Measurements, Wiley

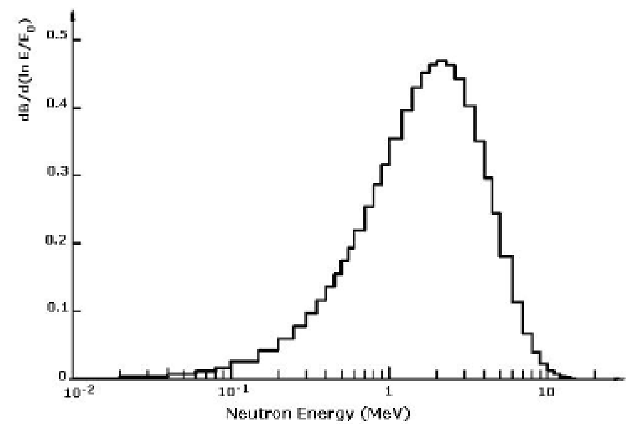


Figure 4. Equilethargic representation of the neutron spectrum from a ^{252}Cf spontaneous fission source (ISO 8529-1). The area subtended by the curve is proportional to the neutron fluence

From CONRAD

SPONTANEOUS FISSION: ^{252}Cf

- ISO standard: ^{252}Cf neutrons moderated in a D_2O sphere 30 cm in diameter (stainless steel container+ Cd).

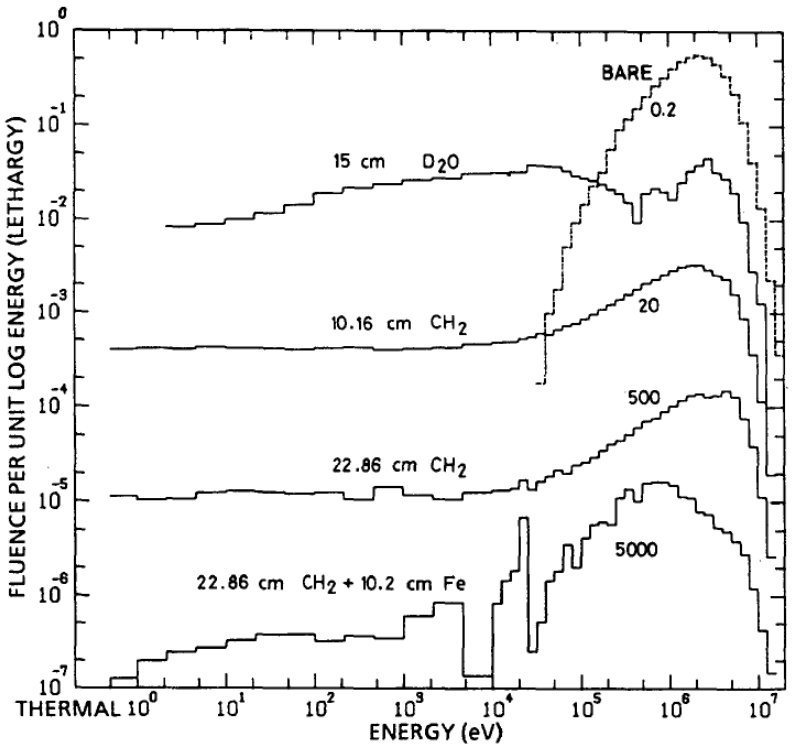


FIG. 4-1. Bare and moderated Cf-252 spectra.

From IAEA Tech. Report Series n. 318

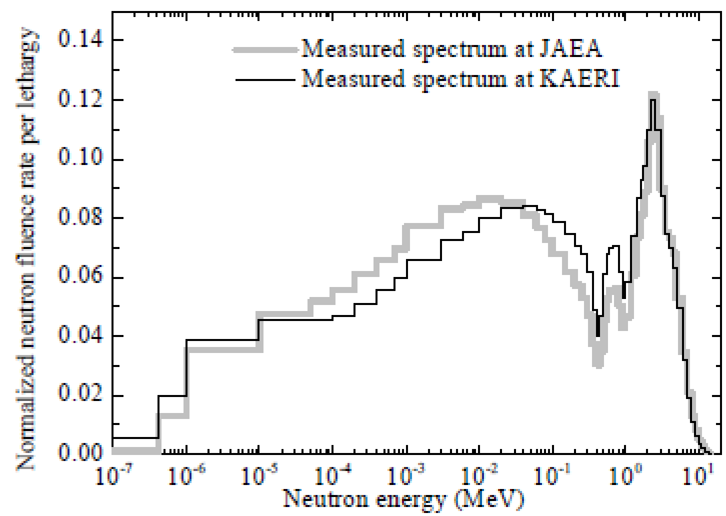
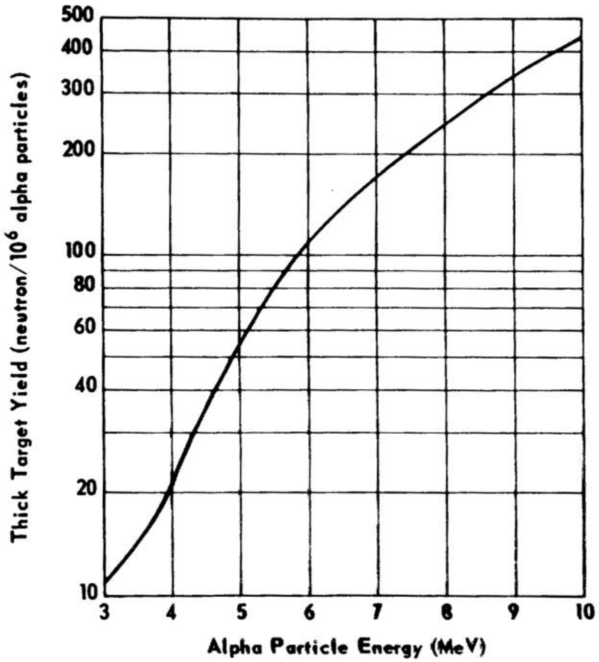


Fig. 1 Comparison of the measured scatter-free neutron spectra for D_2O-Cf source between KAERI and JAEA

From Kowatari et al, J. Nucl. Sci. Tech. 5 (2008) 217-220.

(α,n) ISOTOPIC SOURCES

- Mixture of an α-emitter and a target material;
- The maximum neutron yield is obtained with beryllium as a target:
  - $\alpha + {}^{9}Be \rightarrow {}^{12}C + n$ Q-value = +5.71 MeV



From G. Knoll, Radfiation Detection and Measurements, Wiley

$^{9}Be(\alpha,n)$ ISOTOPIC SOURCES

			Neutron yield			
Source	Half-life	E_α (MeV)	Calculated (s^{-1} Bq^{-1})	Experimental (s^{-1} Bq^{-1})	Calculated (s^{-1} Ci^{-1})	Gamma dose @ 1 m for 10^6 neutrons per second (mGy)
^{239}Pu-Be	24000 y	5.14	6.5×10^{-5}	5.7×10^{-5}	2.41×10^{6}	<0.01
^{210}Po-Be	138 d	5.30	7.3×10^{-5}	6.9×10^{-5}	2.55×10^{6}	<0.001
^{238}Pu-Be	87.4 y	5.48	7.6×10^{-5}	-	2.92×10^{6}	<0.01
^{241}Am-Be	433 y	5.48	8.2×10^{-5}	7.0×10^{-5}	3.03×10^{6}	0.01
^{244}Cm-Be	18 y	5.79	1.0×10^{-4}	-	3.70×10^{6}	<0.01
^{242}Cm-Be	162 d	6.10	1.18×10^{-4}	1.06×10^{-4}		<0.01
^{226}Ra-Be + daughters	1602 y	Multiple	5.02×10^{-4}	-	1.86×10^{7}	0.5
^{227}Ac-Be + daughters	21.6 y	Multiple	7.02×10^{-4}	-	2.59×10^{7}	0.07

$^{9}Be(\alpha,n)$ ISOTOPIC SOURCES: spectra

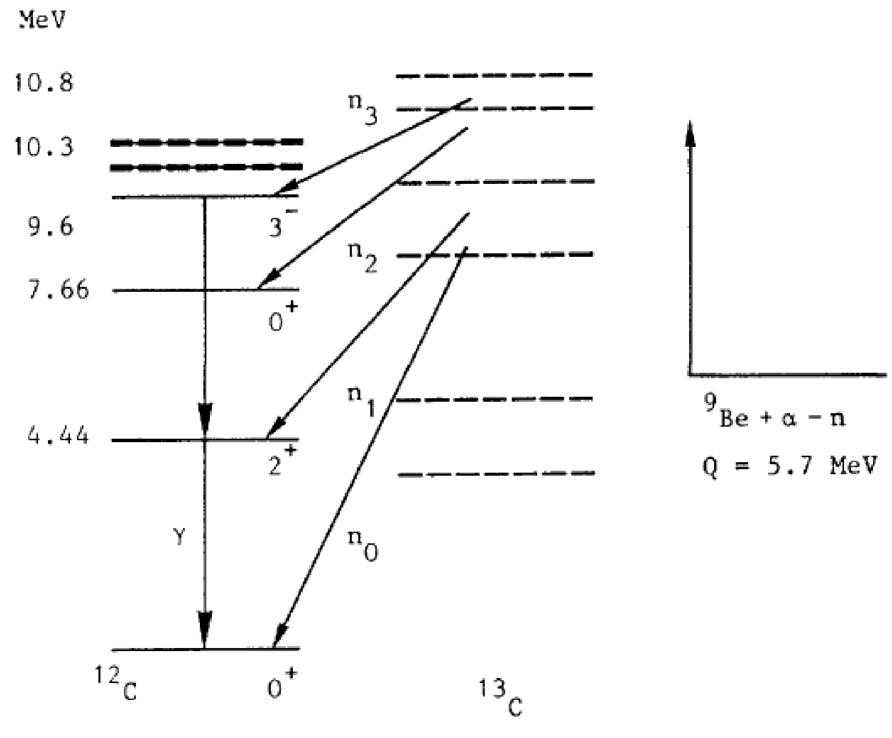
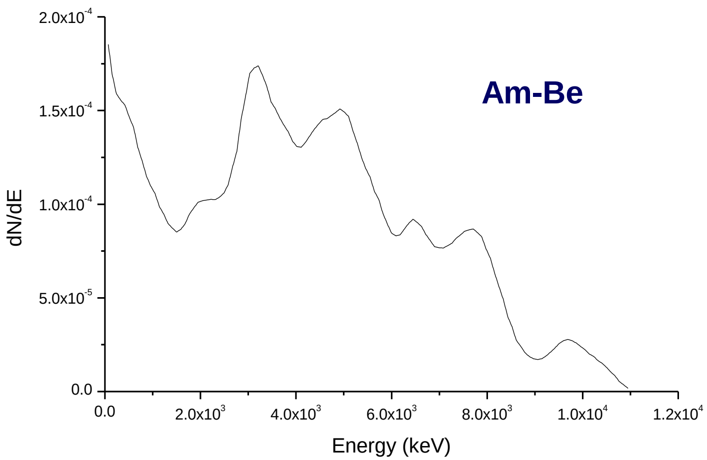


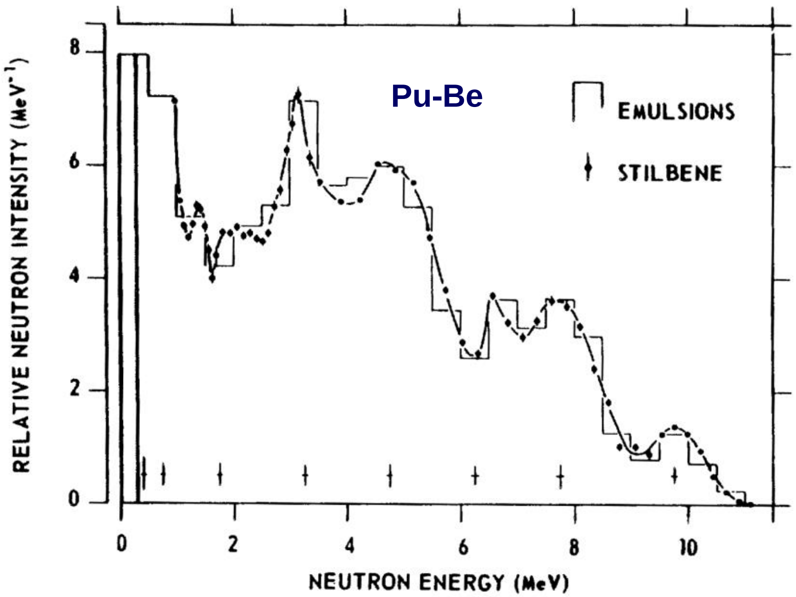
Fig. 1.1 : The $^{9}Be(\alpha,n)^{12}C$ reaction

From IAEA-TECDOC-465 (1988)

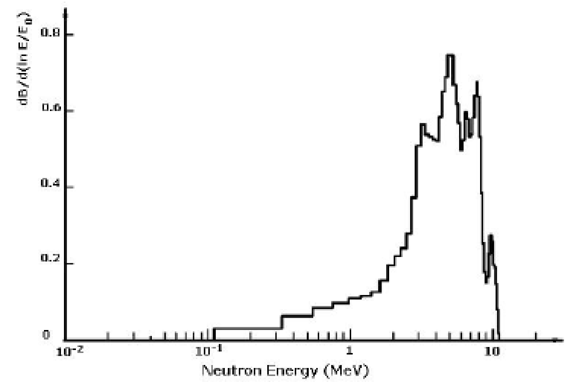
- The energy continuum below 3.5 MeV is due to:
  - broad levels in ^{12}C above 9.6 MeV;
  - mainly to $^{9}Be(\alpha,\alpha')^{9}Be^{*}\rightarrow{}^{8}Be+n$ involving levels @ 1.67, 2.43 and 3.06 MeV of ^{9}Be.



$^{9}Be(\alpha,n)$ ISOTOPIC SOURCES: spectra

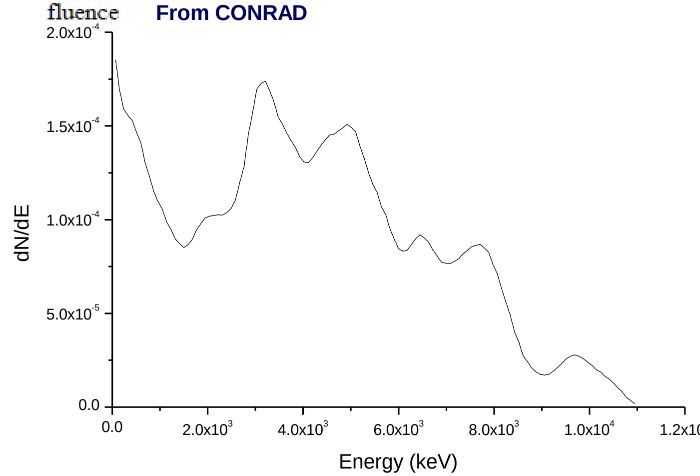


From G. Knoll, Radfiation Detection and Measurements, Wiley



Am-Be

Figure 3. Equilethargic representation of the neutron spectrum from a ^{241}Am-Be(α,n) source (ISO 8529-1). The area subtended by the curve is proportional to the neutron fluence **From CONRAD**



^{9}Be(α,n) ISOTOPIC SOURCES: spectra

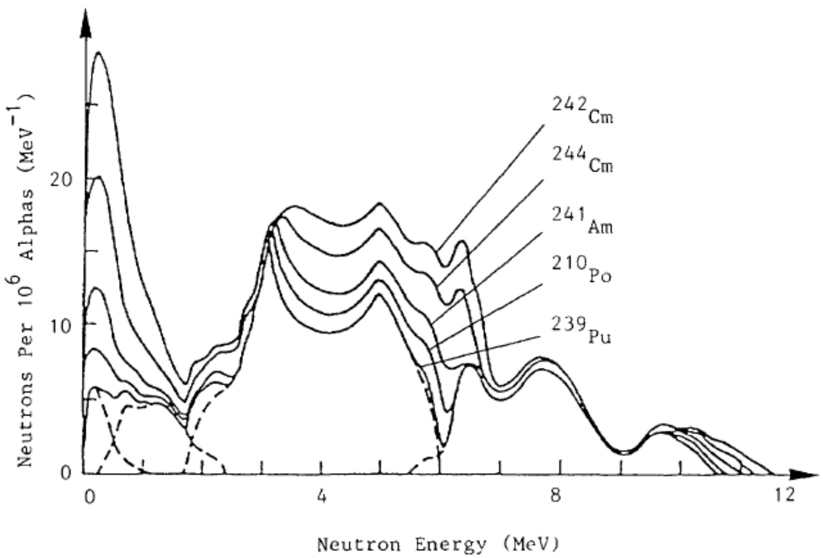


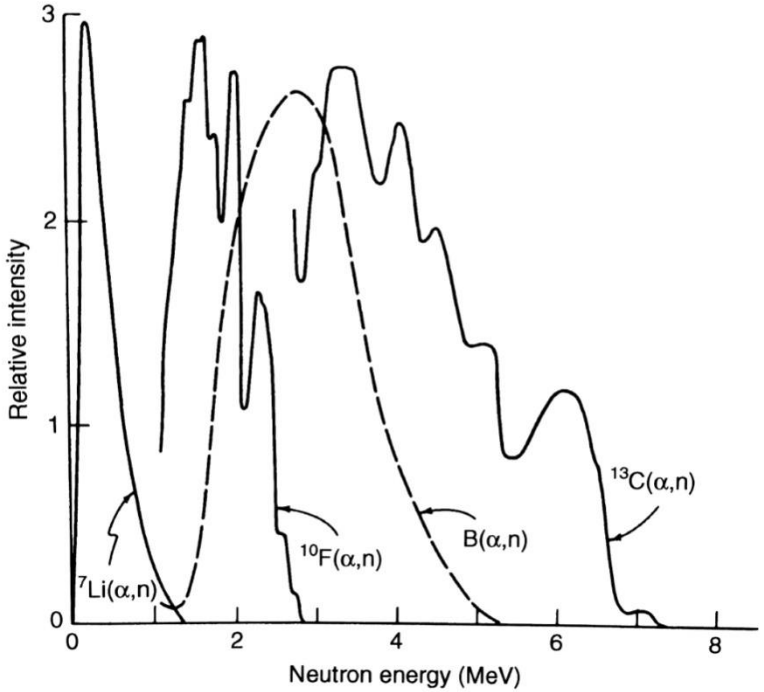
Fig. 1.3 : Neutron spectra of isotopic Be(α,n) sources.

From IAEA-TECDOC-465 (1988)

- The following issues can modify the neutron spectrum and yield from an ideal distribution:
  - elastic and inelastic neutron scattering in the source;
  - neutron-induced fission within the α-emitter;
  - ^{9}Be(n,2n) reaction;
  - ^{10}Be(γ,n) reaction;
  - α-particle slowing-down inside the source mixture;
  - daughter equilibrium of α-emitters.

ALTERNATIVE (α,n) ISOTOPIC SOURCES

Target	Reaction	Q-value (MeV)	Neutron yield (s^{-1} Bq^{-1})
Natural B	$^{10}B(\alpha,n)$	+1.07	1.3×10^{-5}
	$^{11}B(\alpha,n)$	+0.158	-
F	$^{19}F(\alpha,n)$	-1.93	4.1×10^{-5}
^{13}C (isotopically separated)	$^{13}C(\alpha,n)$	+2.2	1.1×10^{-5}
Natural Li	$^{7}Li(\alpha,n)$	-2.79	2.6×10^{-6}
Be	$^{9}Be(\alpha,n)$	+5.71	7.0×10^{-5}



From G. Knoll, Radfiation Detection and Measurements, Wiley

(γ,n) ISOTOPIC SOURCES

- The following reactions are of practical significance for neutron producton:
  - $\gamma + {}^{9}Be \rightarrow {}^{8}Be + n$ Q-value = -1.666 MeV;
  - $\gamma + {}^{2}H \rightarrow {}^{1}H + n$ Q-value = -2.226 MeV.

- The corresponding neutron energy is given by:

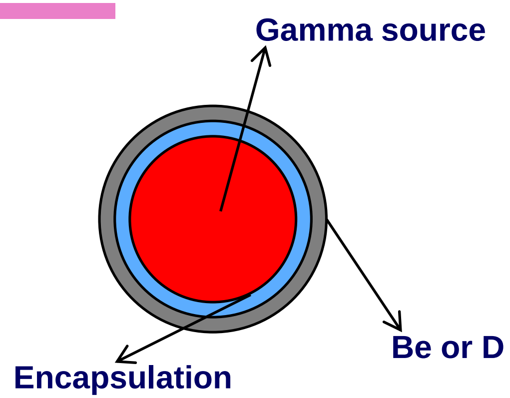
$$E_n(\theta) = \frac{M(E_\gamma + Q)}{m+M} + \frac{E_\gamma\sqrt{(2mM)(m+M)(E_\gamma + Q)}}{(m+M)^2}\cos\theta$$

- where:
  - θ is the angle between the gamma and the neutron directions;
  - E_γ is the photon energy ($E_\gamma << mc^2$);
  - M is the target nucleus mass $\times c^{2;}$
  - m is the neutron mass $\times c^2$.

- For monoenergetic photons the neutron energy spread by varying θ between 0° and π is only a few percent. Therefore neutrons are nearly monoenergetic.

(γ,n) ISOTOPIC SOURCES

Source	$T_{1/2}$	E_γ (MeV)	Neutron yield (s^{-1} Bq^{-1})	Neutron energy (keV)
^{24}Na-Be	15.0 h	2.7541	3.4×10^{-5}	967
^{24}Na-D	15.0 h	2.7541	3.3×10^{-5}	263
^{72}Ga-Be	14.1 h	1.8611 2.2016 2.5077	6.5×10^{-6} (from all gammas)	174 476 748
^{124}Sb-Be	60.2 d	1.6910	2.1×10^{-5}	23



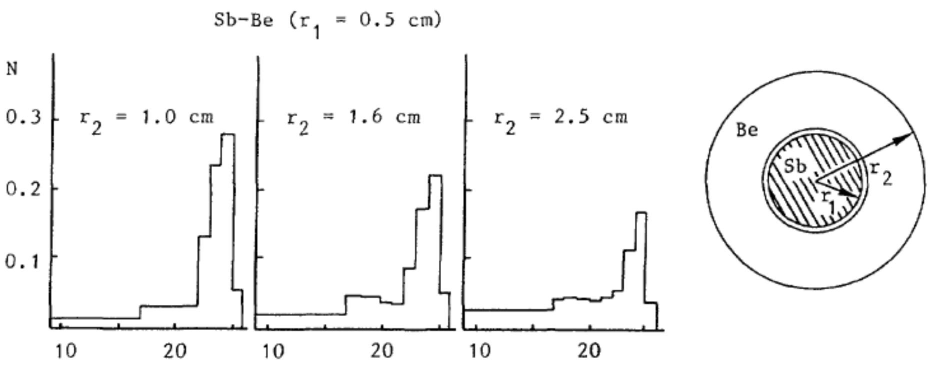
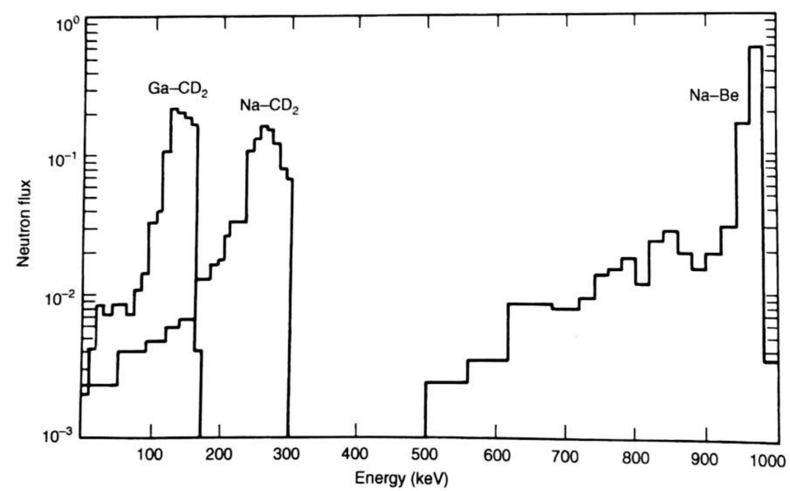


Fig. 1.12 : A Be(γ,n) neutron source.

From IAEA-TECDOC-465 (1988)



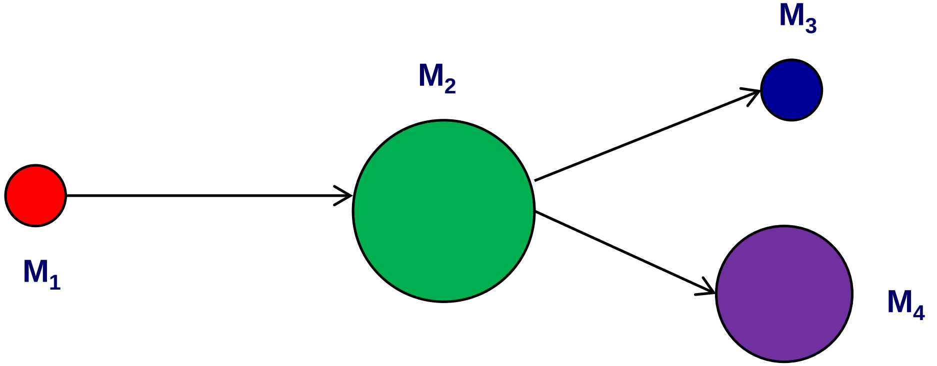
From G. Knoll, Radfiation Detection and Measurements, Wiley

ACCELERATOR-BASED SOURCES: mononergetic neutrons

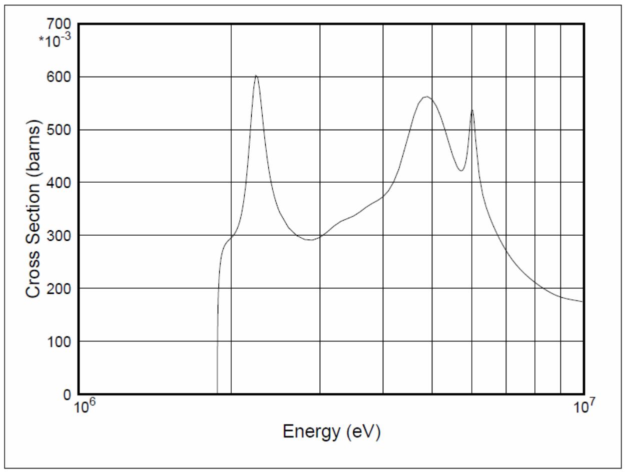
- Main reactions:
  - $^{45}Sc(p,n)^{45}Ti$ Q=-2.844 MeV E_{th}=2.908 MeV
  - $^{7}Li(p,n)^{7}Be$ Q=-1.644 MeV E_{th}=1.879 MeV
  - $^{2}H(d,n)^{3}He$ Q=+3.269 MeV
  - $^{3}H(d,n)^{4}He$ Q=+17.589 MeV

- For threshold reactions (approximation for $M_2c^2 >> Q$):

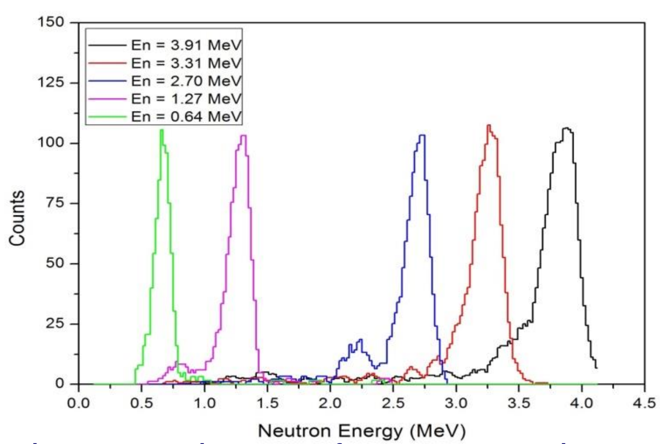
$$E_{th} = -Q\left(\frac{M_3+M_4}{M_3+M_4-M_1}\right) \cong -Q\frac{M_1+M_2}{M_2}$$



ACCELERATOR-BASED SOURCES: mononergetic neutrons

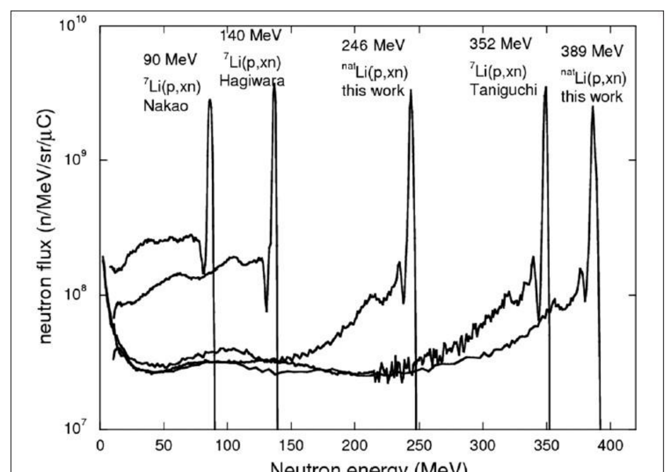


^{7}Li(p,n) cross section



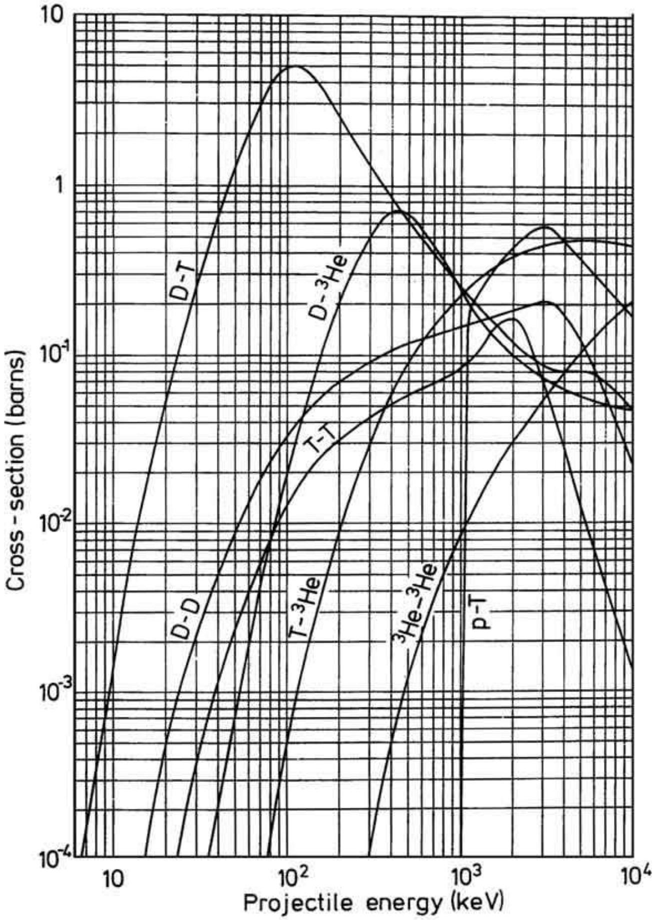
Quasi-monoenergetic neutrons from protons on a LiF target @LNL

- ^{7}Li(p,n)
  - thin targets (usually LiF deposited on C);
  - ^{7}Li(p,n)^{7}Be* opens at 2.37 MeV (^{7}Be excitation level @ 0.43 MeV).
  - for a given proton energy, the neutron energy is angular dependent;
  - quasi-monoenergetic neutrons up to projectile energies of a few MeV;
  - for higher energies other (p,xn) channels open and a broad continuum appears at low energies .

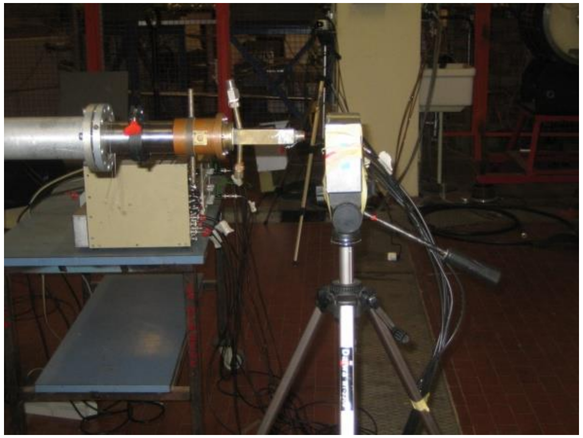


From T. Nakamura, Rad. Prot. Env. 35 (2012) 111-125

ACCELERATOR-BASED SOURCES: mononergetic neutrons



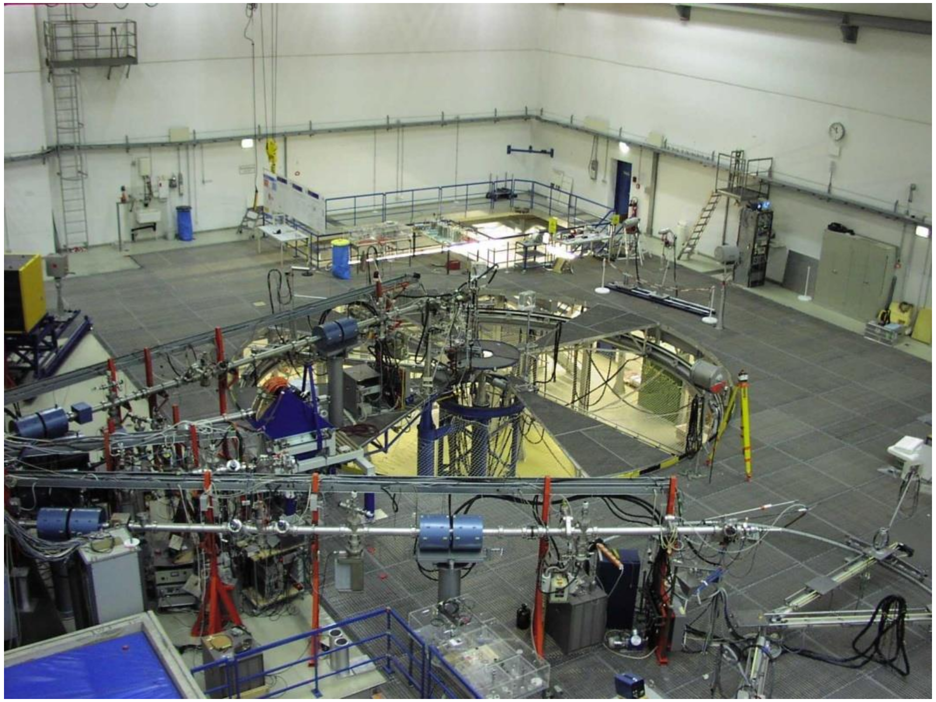
- $^2H(d,n)$
  - Gas target;
  - 2.45 MeV neutrons near the Coulomb barrier energy .
- $^3H(d,n)$
  - tritium adsorbed on titanium
  - 14.08 MeV neutrons near the Coulomb barrier energy;



ACCELERATOR-BASED SOURCES: mononergetic neutrons

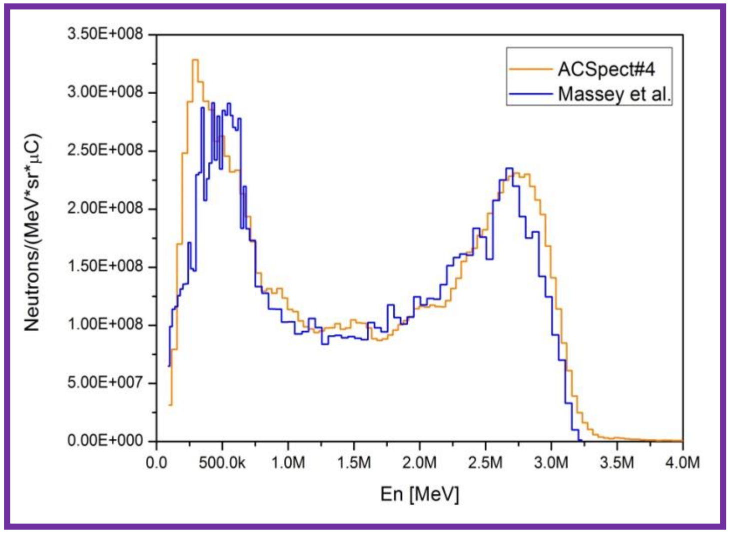
- Example: monoenergetic neutrons @ PTB, ISO beams
  - **^{45}Sc(p,n)^{45}Ti: 24.5 keV**;
  - **^{7}Li(p,n)^{7}Be: 144, 250, 565 keV**;
  - **T(p,n)^{3}He: 1.2, 2.5 MeV;**
  - **D(d,n)^{3}He: 5.0, 8.0 MeV;**
  - **T(d,n)^{4}He: 14.8, 19.0 MeV.**

  - Free-scattering room.

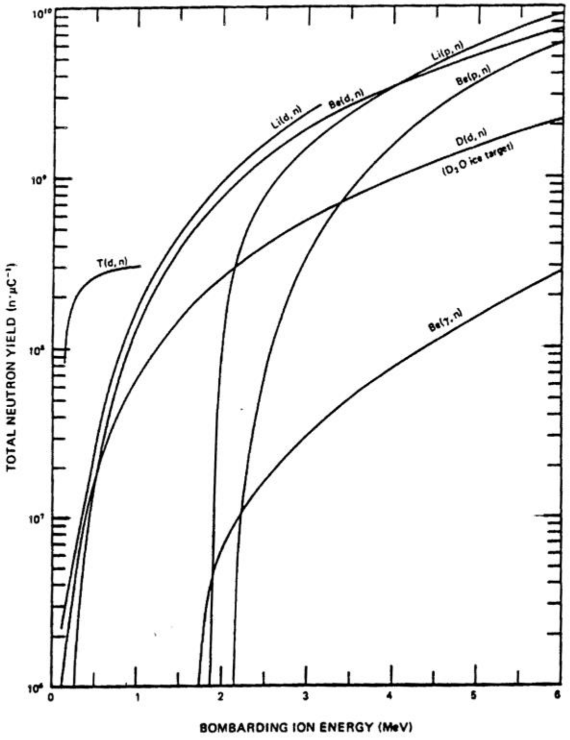


ACCELERATOR-BASED SOURCES: thick targets

- Continuous spectra and intense fields.
  - the beam power (W=ΔV×I) is deposited in the target which has to be cooled;
  - liquid targets may be employed (e.g. liquid Li targets, melting temperature 180.54 °C);
  - alternatively materials with a higher melting temperature should be employed (e.g. beryllium, melting temperature 1287 °C).



Energy distribution of neutrons generated at 0° by 5 MeV protons on a thick Be target



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ACCELERATOR-BASED SOURCES: spallation

Table 2.2 Past, existing, and future spallation source and their respective parameters

Country	United States	United States	United States	U.K.	Switzerland	China	Europe	Japan	Japan
Neutron source	IPNS	LANSCE	SNS	ISIS	SINQ	CSNS	ESS	KENS	JSNS
Organization	Argonne National Laboratory	Los Alamos National Laboratory	Oak Ridge National Laboratory	Rutherford Appleton Laboratory	Paul Scherrer Institute	Institute of High Energy Physics	Undecided	High Energy Accelerator Research Organization	Japan Atomic Energy Agency
Proton energy (MeV)/ Current (μA)	450/15	800/70	1000/1400	800/200	590/1500	1600	1333/7500	500/9	3000/333
Proton beam power	7 kW	56 kW	1.4 MW	160 kW	1 MW	100 kW	5 MW	4.5 kW	1 MW
Repetition rate (Hz)	30	20	60	50/10 (2 targets)	Continuous	25	16 (long pulse)	20	25
Target material	Depleted Uranium	Tungsten	Mercury	Tantalum	Zircaloy	Tungsten	Mercury	Tungsten	Mercury
Moderator	$S\text{-}CH_4/L\text{-}CH_4$	$L\text{-}H_2/H_2O$	$L\text{-}H_2/H_2O$	$L\text{-}H_2/L\text{-}CH_4/H_2O$	$L\text{-}D_2/D_2O$	$H_2OL\text{-}CH_4L\text{-}H_2$	$L\text{-}H_2$	$S\text{-}CH_4/H_2O$	$L\text{-}H_2$
Number of instruments	12	7	24 (beam ports)	22 (TS1) 7 (TS2)	15		20 (beam ports)	15	23 (beam ports)
Existing neutron imaging instrument					NEUTRA [30] and ICON [31]				
Facility operating since or planned to operate in	1981 (closed 2008)	1983	2006	1985 (TS1) 2008 (TS2)	1996	2014	Under planning	1980 (closed 2005)	2008

IPNS: Intense Pulsed Neutron Source [32]; LANSCE: Los Alamos Neutron Science Center [33]; SNS: Spallation Neutron Source [8, 9]; ISIS: [34, 35]; SINQ: Swiss Spallation Neutron Source [36, 37]; CSNS: Chinese Spallation Neutron Source [10, 11]; ESS: European Spallation Source [38, 39]; KENS: Koh-Energy-ken Neutron Source [40, 41]; JSNS: Japanese Spallation Neutron Source [8, 9]. Consult the websites for these facilities to obtain additional information and current details.

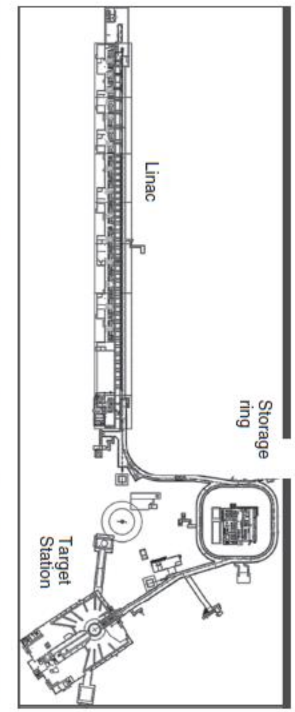
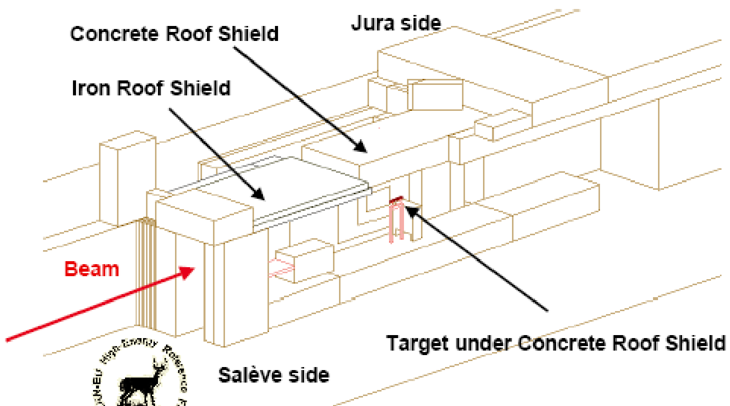


Fig. 2.5 A typical short-pulse spallation neutron source, the SNS facility. (Oak Ridge National Laboratory, United States) [8, 9]

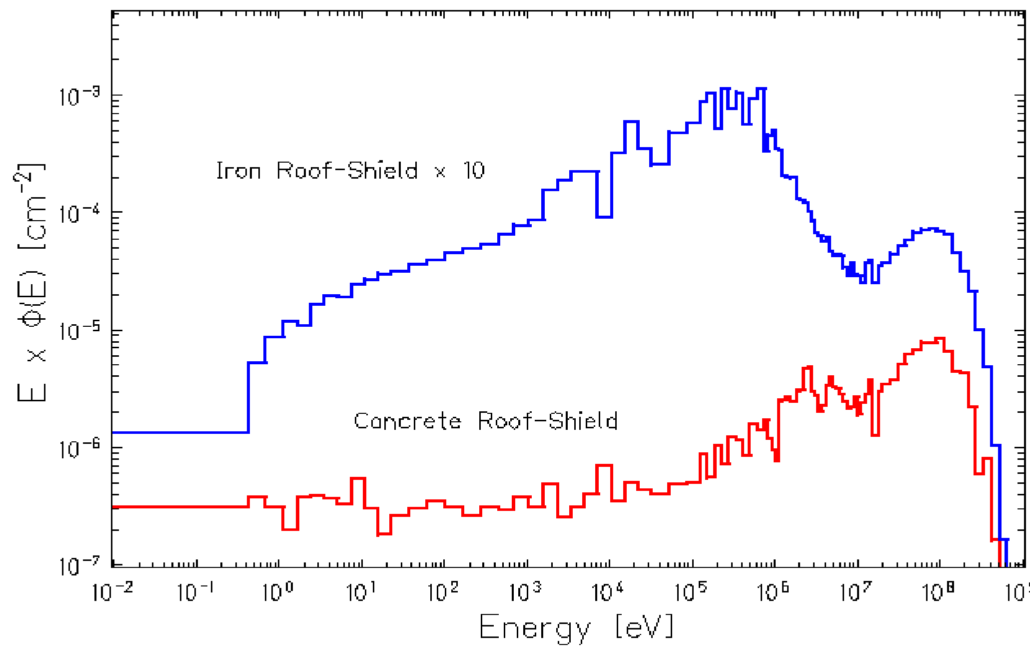
From M. Arai and K. Crawford, Neutron Imaging and Applications, Chapter 2- Neutron Sources and Facilities, Springer

ACCELERATOR-BASED SOURCES: CERF @ CERN



Mitaroff, A., Silari, M., 2002. Radiat. Prot. Dosimetry 102, 7-22.

Neutron spectral fluence outside a 80 cm thick concrete shield and a 40 cm thick iron shield

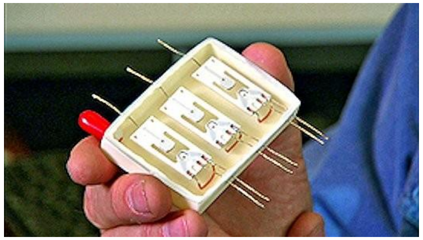


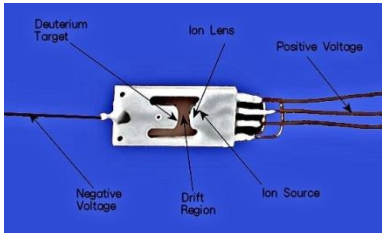
ACCELERATOR-BASED SOURCES: thick targets

Reaction	Example	Neutron yield
d-t	400 keV deuterons on T (adsorbed on Ti)	4×10^{-5} neutrons per deuteron
(d,n)	35 MeV deuterons on liquid Li	2.5×10^{-5} neutrons per deuteron
Photoneutrons (bremsstrahlung X-rays)	100 MeV electrons on ^{238}U	5×10^{-2} neutrons per electron
Spallation	800 MeV protons on ^{238}U	30 neutrons per proton

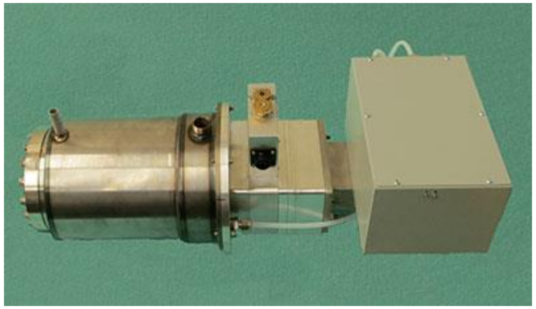
NEUTRON GENERATORS

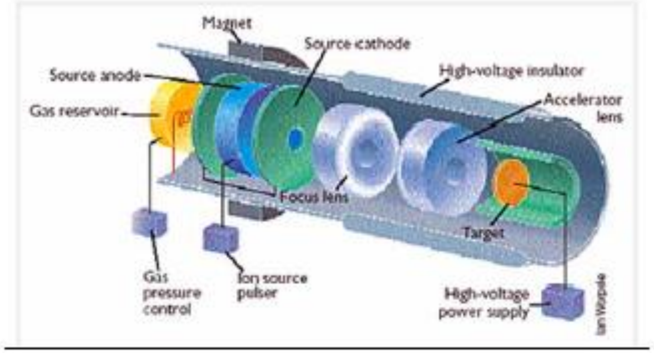
- Based on the D-D and on the D-T reaction.
  - Beam + target;
  - Plasma;
  - D-D neutron yield up to about 10^8 s^{-1};
  - D-T neutron yield up to about 10^{10} s^{-1}.
  - Applications:
  - NAA and PGNAA;
  - homeland security;
  - Illicit material and explosive detection;
  - land mine detection;
  - cargo screening.





Gizmag – the world smallest neutron generator, www.gizmag.com

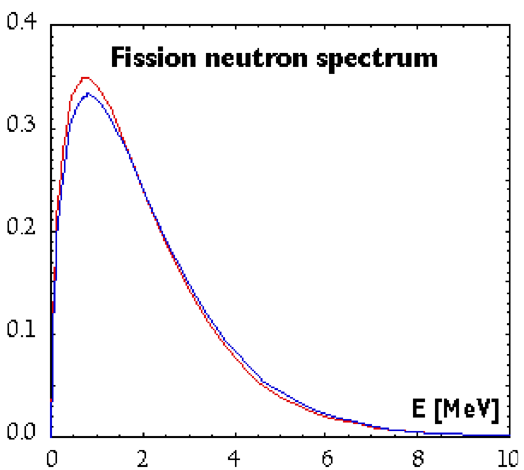


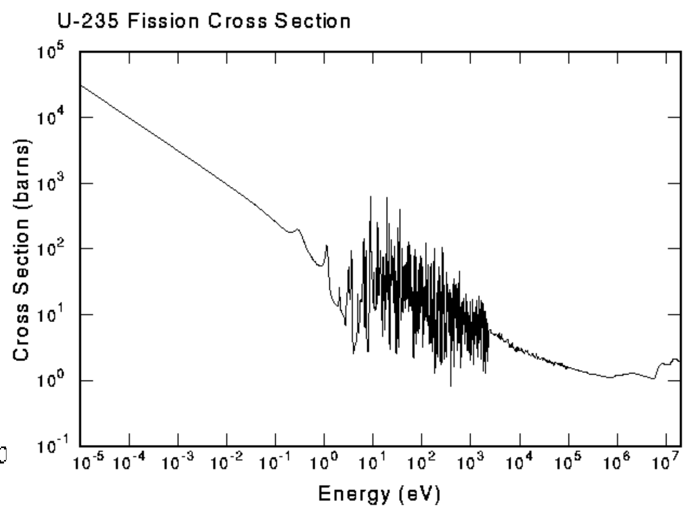


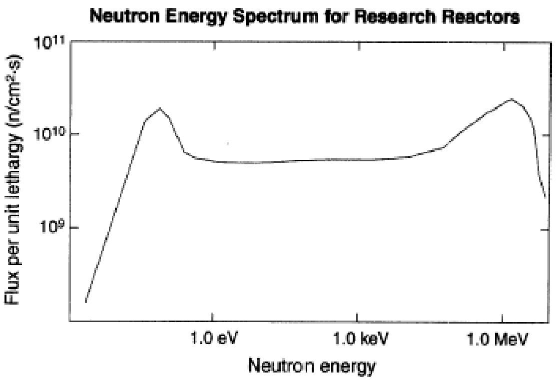
From IAEA Technology report series n. 1 (2012)

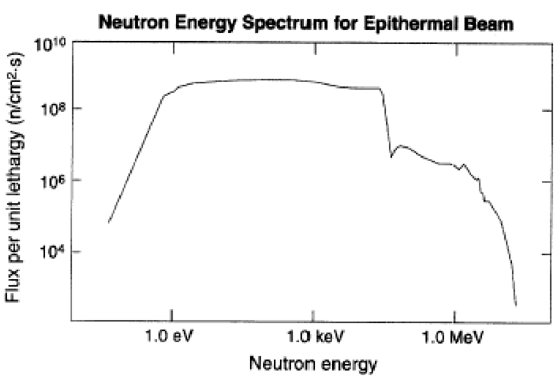
RESEARCH REACTOR-BASED SOURCES

- Neutron from fission;
  - $^{235}U + n \rightarrow X + Y + 2.47$ neutrons (mean number)
- The fission spectrum is modified by moderators/filters;
- Watt spectrum with T = 1 MeV for ^{235}U;



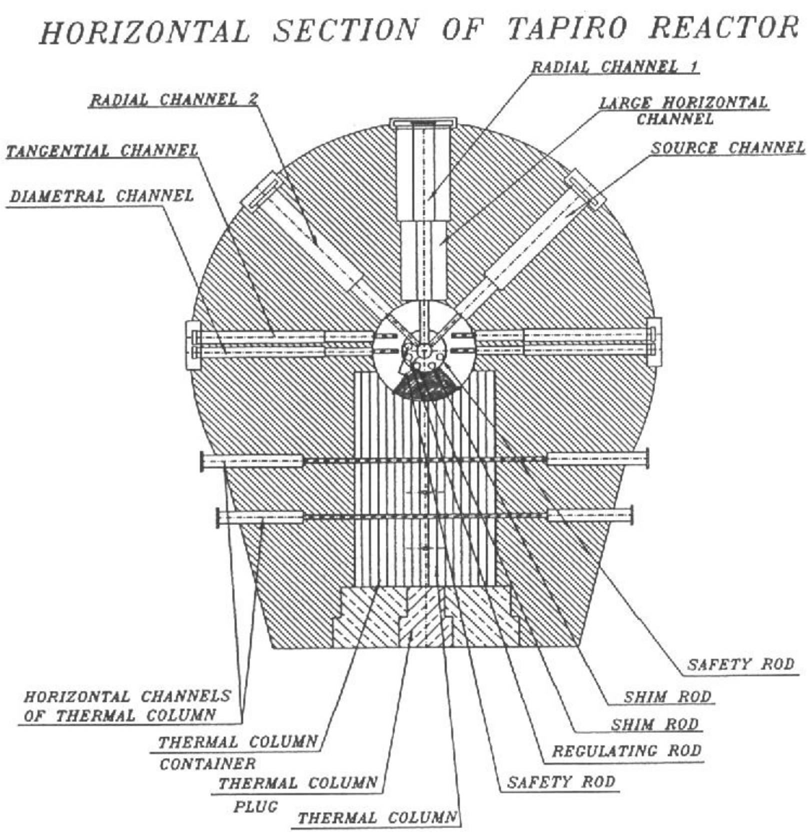






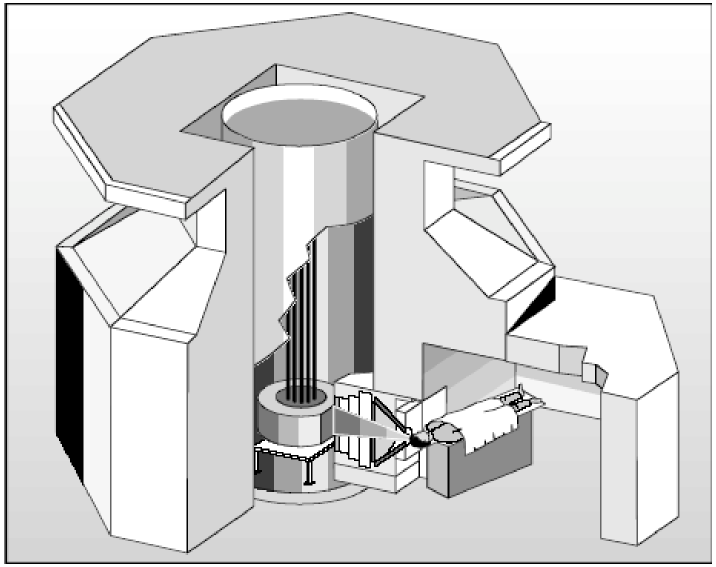
Red U-235, Blue Pu-239

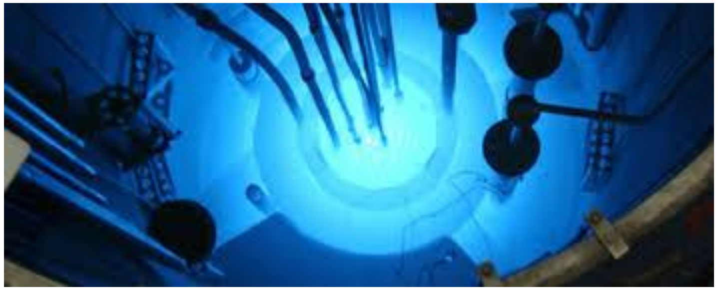
RESEARCH REACTOR-BASED SOURCES



- The TAPIRO reactor (ENEA Rome).
  - Fast reactor;
  - Thermal power 5 kW;
  - Cylindrical core (10.87 cm and 12.58 cm in height and diameter) ^{235}U enriched (93.5%);
  - 22.2 kg of U contained in the reactor core;
  - Copper reflector

RESEARCH REACTOR-BASED SOURCES





- The TRIGA reactor.
  - Pool reactor moderated with light water;
  - Thermal power 250 kW (LENA, Pavia);
  - Intrinsic safe;
  - Fuel/moderator: matrix of U and ZrH_2;
  - ZrH_2 shows a tetrahedron structure and behaves as an harmonic oscillator with discrete energy levels $(N+3/2)h\nu$ ($3/2h\nu=0.13$ eV);
  - The ZrH_2 moderating properties are inhibited below 0.13 eV (neutron thermalization is given by the light-water moderator;
  - the density of ZrH_2 excited levels increases with temperature leading to an increase of neutron energy and a decrease of fission probability;
  - Therefore reactivity (and power) decreases with temperature.

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REACTOR-BASED INTENSE SOURCES

Table 2.1 Existing medium- and high-flux reactor sources and their respective parameters

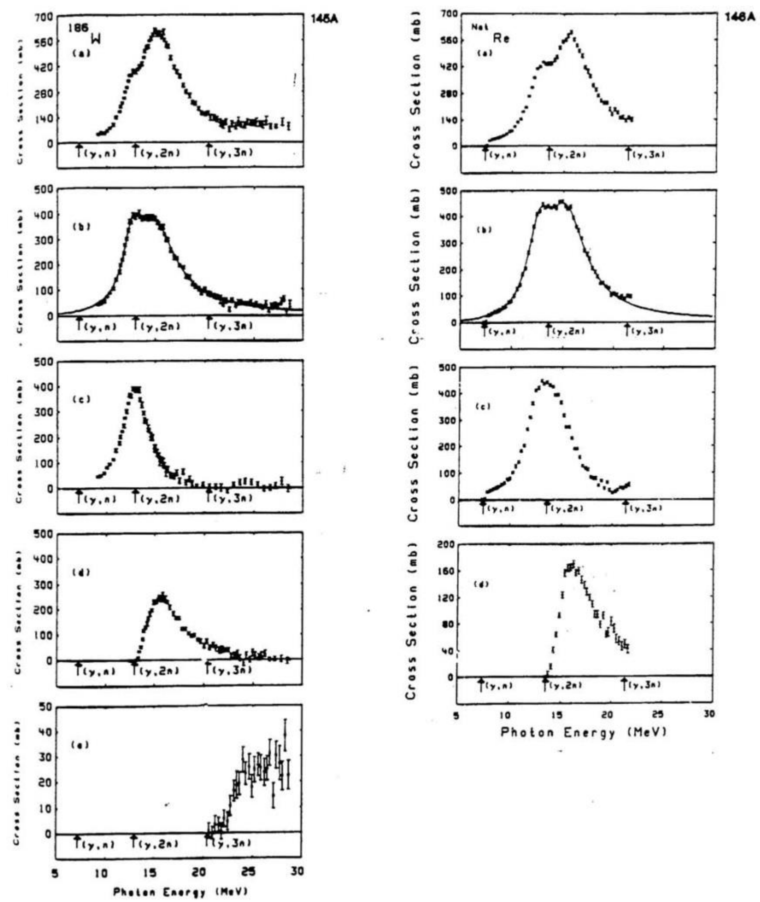
Country	United States	United States	Canada	France	France	Germany	Germany	Australia	Korea	Japan
Neutron source	HFIR	NBSR	NRU	HFR	ORPHEE	BENSC	FRM-II	OPAL	HANARO	JRR-3 M
Organization	Oak Ridge National Laboratory	National Institute of Standards and Technology	Atomic Energy of Canada Limited	Institut Laue-Langevin	Laboratoire Léon Brillouin	Helmholtz-Zentrum Berlin	Technische Universitat Munchen	Australian Nuclear Science and Technology Organization	Korea Atomic Energy Research Institute	Japan Atomic Energy Agency
Power (MW)	85	20	120	58	14	10	20	20	24 (present) 30 (designed)	20
Flux ($n \cdot cm^{-2} \cdot s^{-1}$)	1.5×10^{15}	3×10^{14}	3×10^{14}	1.5×10^{15}	3×10^{14}	2×10^{14}	8×10^{14}	3×10^{14}	2×10^{14}	3×10^{14}
Number of cold/hot sources	1/0	1/0	0/0	2/1	1/1	1/0	1/1	1/0	1(planned)/0	1/0
Number of instruments	9(present) + 6 (planned by 2012)	24	5	26	22	22	20 (present) + 10 (under construction)	6	6	24
Existing neutron imaging instrument		BT-2 [14]			[15]	CONRAD [16]	ANTARES [17]		NR-port [18]	TNRF [19] And TNRF-2 [20]
Facility operating since	1967	1970	1957	1972 (refurbished 1993)	1980	1973	2004	2006	1997	1990

HFIR: High-Flux Isotope Reactor [21]; NBSR: National Bureau of Standards Reactor [22]; NRU: National Research Universal Reactor, Chalk River, Canada [23]; HFR: High-Flux Reactor at ILL [1, 2]; ORPHEE: reactor at LLB [24]; BENSC: Berlin Neutron Scattering Centre [25]; FRM-II: Forschungsneutronenquelle Heinz Maier-Leibnitz [5]; OPAL: Open Pool Australian Light-water Reactor [6]; HANARO: High-flux Advanced Application Reactor [4]; JRR-3 M: Japan Research Reactor No. 3 Modified [3]. Consult the web sites for these facilities to obtain additional information and current details. A number of smaller research reactors, primarily at universities, are not listed here.

From M. Arai and K. Crawford, Neutron Imaging and Applications, Chapter 2- Neutron Sources and Facilities, Springer

PHOTONEUTRONS

- Photonucleon emission occurs through an energy transfer of the photon electric field to the nucleus;
- This energy transfer causes a nucleus oscillation of a spherical system composed by protons opposing to another spherical system composed by neutrons;
- Threshold reactions: for photoneutrons the threshold energy is the binding energy of the less bound neutron.



GRAPHS 1-174A. Photoneutron Cross Sections
See page 207 for Explanation of Graphs

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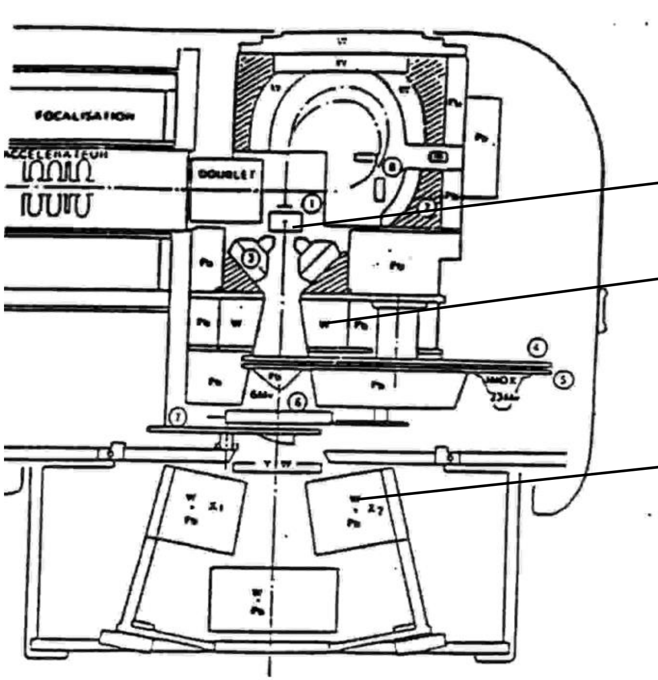
PHOTONEUTRON YIELD

- For an electron beam of energy E impinging on a semi-infinite target (500<E<1000 MeV):

$$Y(s^{-1}kW^{-1}) = 1.21 \times 10^{11} Z^{0.66}$$

- where Z is the atomic number of the bremsstrahlung target

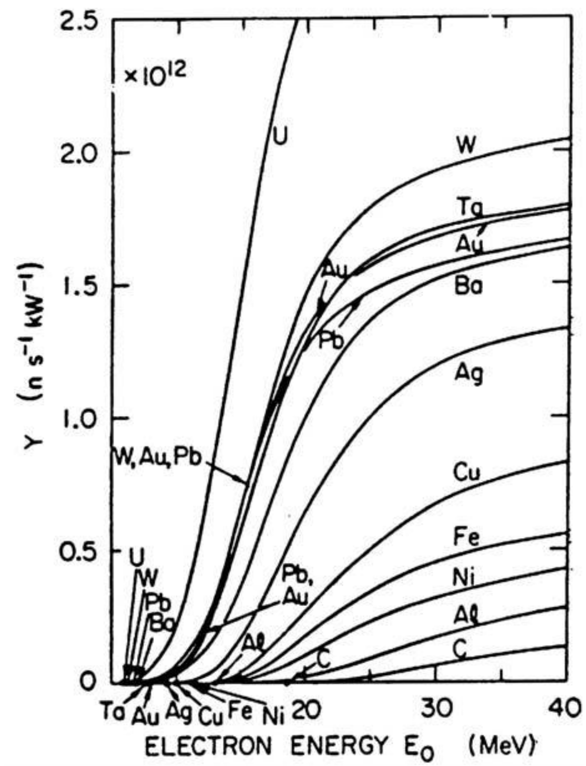


1

1.26

0.4 (40×40 cm²)
1.0 (closed jaws)

Contributions to the photoneutron yield from different structural parts of a Varian Clinac 2500 (18 MV) (La Riviere, Med. Phys. 12 (1985) 806-809).

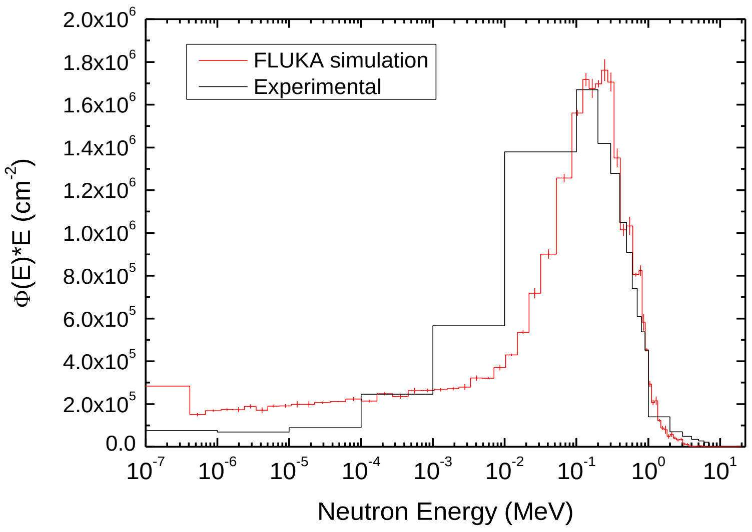


PHOTONEUTRON SPECTRUM

- Direct + statistical component.

$$\frac{dN}{dT} = \frac{E}{T^2} e^{-E/T}$$

- where T is the nuclear temperature.



Photoneutron spectral fluence @ 40 cm from the Isocentre of a 15 MV linac (measured with BSS)

PHOTONEUTRON YIELD FROM MEDICAL e^- LINACS

Accelerator	Photoneutron yield (forward) per prescribed X ray dose ($Gy^{-1}\ sr^{-1}$)
15 MV Varian Clinac DHX-HP	8.43×10^{10}
CGR Saturno 43 (15 MV)	$6.0 \text{x} 10^{10}$
18 MV Saturne 43	1.19×10^{11}
Varian Clinac 18C (18 MV)	$2.16 \text{x} 10^{11}$
CGR Saturno 43 (18 MV)	$1.6 \text{x} 10^{11}$
CGR Saturno 43 (25 MV)	$3.5 \text{x} 10^{11}$

PHOTONEUTRON DOSE FROM MEDICAL e^- LINACS

Author	Accelerator	Voltage	Distance from isocentre	Dose eq. (mSv Gy^{-1})	Detector
This work	Saturne 43	18 MV	50 cm	2.53 (H*(10))	Bonner spheres
This work	Saturne 43	18 MV	100 cm	1.76 (H*(10))	Bonner spheres
LaRiviere (Med. Phys. 12(6) (1985) 806-809)	Clinac 2500	24 MV	100 cm	1.4	Moderated activation
d'Errico et al. (Health. Phys. 80 (2001) 4-11)	Siemens KD2	10 MV	1 cm	1.4	SDD
d'Errico et al.	Siemens KD2	15 MV	1 cm	1.9	SDD
d'Errico et al.	Siemens KD2	18 MV	1 cm	5.5	SDD

PHOTONEUTRON DOSE FROM MEDICAL e$^-$ LINACS

Author	Accelerator	Voltage	Distance from the isocentre	Dose eq. (mSv Gy^{-1})	Detector
Tosi et al. (Med. Phys. 18(1) (1991) 54-60)	Saturne 43	15 MV	5 cm	1.5 (H*(10))	Moderated activation
Tosi et al.	Saturne 43	15 MV	50 cm	0.6 (H*(10))	Moderated activation
Tosi et al.	Saturno II	21 MV	40 cm	1.8 (H*(10))	Moderated activation
Tosi et al.	Scanditronix MM22	21 MV	25 cm	2.6 (H*(10))	Moderated activation
Tosi et al.	Saturne 43	25 MV	5 cm	7.2 (H*(10))	Moderated activation
Tosi et al.	Saturne 43	25 MV	50 cm	2.6 (H*(10))	Moderated activation

Additional Slides

Lethargy plots

- Conservative in terms of area for semi-logarithmic plots

$$\int_{E1}^{E2} f(E)dE = \int_{E1}^{E2} E\, f(E)dE/E = \int_{E1}^{E2} E\, f(E)d(\ln E) = \ln 10 \int_{E1}^{E2} E\, f(E)\, d(\log E)$$

- Therefore:

$$f(E)dE = Ef(E)d(\ln E) \quad \text{and:}\ Ef(E) = \frac{f(E)dE}{d(\ln E)}$$

- Histogram:

$$E_i f_i(E) = \frac{f_i(E)\times(E_{i+1}-E_i)}{\ln E_{i+1} - \ln E_i} = \frac{f_i(E)\Delta E}{\ln(E_{i+1}/E_i)}$$

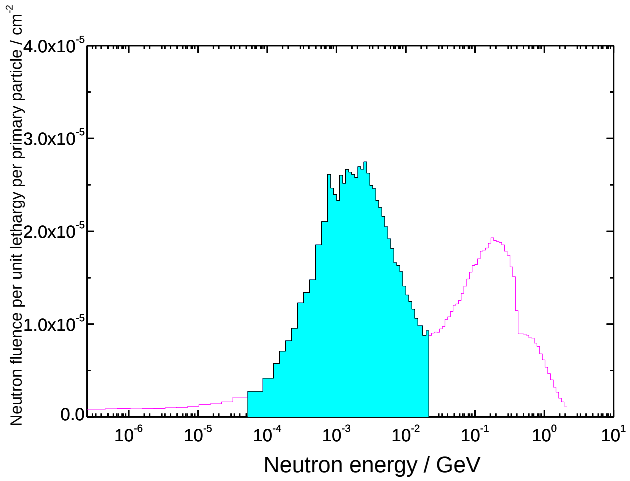
- Lethargy (definition):

$$u = \ln\frac{E_0}{E} = \ln E_0 - \ln E$$

$$du = -\frac{dE}{E}$$

$$F(u)du = -F(E)dE$$

$$F(u) = EF(E)$$



PARTICLE FLUENCE: COSINE-WEIGHTED BOUNDARY CROSSING

- The spectral distribution of particle radiance is defined as:

$$p_E = \frac{d^4N}{da\,d\Omega\,dE\,dt} = v\,n(\vec{r},\Omega,E)$$

✓ v=particle velocity;

✓ n=particle density (number of particles N per unit volume).

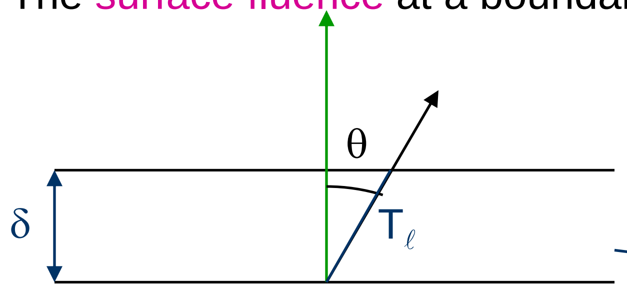
- The particle fluence averaged over a region of volume V can be estimated as:

$$\Phi = \int_V\int_\Omega\int_E\int_t v\,n(\vec{r},\Omega,E)\,dt\,dE\,d\Omega\frac{dV}{V} = \frac{\int\int nv\,dt\,dV}{V} = \frac{\int nds\,dV}{V} = \frac{T_\ell}{V}$$

✓ nds is a "track-length density";

✓ T_ℓ sum of track lengths.

- The surface fluence at a boundary crossing is, for one particle of weight w:



$$\Phi_S = \lim_{\delta\to 0} w\frac{T_\ell}{V} = w\frac{\delta/|\cos\theta|}{S\delta} = \frac{w}{S|\cos\theta|}$$

Infinitely thin region of volume Sδ