Effect of magnet errors within the alpha bucket regime in SIS-100 during proton extraction

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High-γ_{tr} proton cycle in SIS-100

Proton operation in SIS-100 with $2 \cdot 10^{13}$ particles.

- Injection of 4 bunches at $E = 4$ GeV.
- Merge bunches to single bunch, accelerate and extract it at $E = 29$ GeV.
- Goals:
  1. Avoid transition crossing. Therefore, extraction with $\gamma_{tr} = 45.5$, while $\gamma = 31.9$.
  2. Creation of single bunch with final area $A = 5 \cdot 2.4$ eVs $= 12$ eVs and small bunch length $t_{bu} \approx 50$ ns corresponding to bunch length $l_{bu} \approx 15$ m $\ll C = 1083.6$ m.
- Short bunch can be created by alpha bucket, requires small zero order phase slip factor, η_0, and momentum compaction factor, $\alpha_0 = 1/\gamma_{tr}^2$ and, therefore, large γ_{tr}.

Present study: Tracking simulations to estimate beam loss at extraction energy with errors in dipole and quadrupole magnet and without space charge. Use MAD-X.

Alpha bucket

- Alpha bucket requires small zero order momentum compaction factor α_0 and phase slip factor

$$\eta_0 = \alpha_0 - \frac{1}{\gamma^2}.$$

Actually $\eta_0 = -0.0005$.

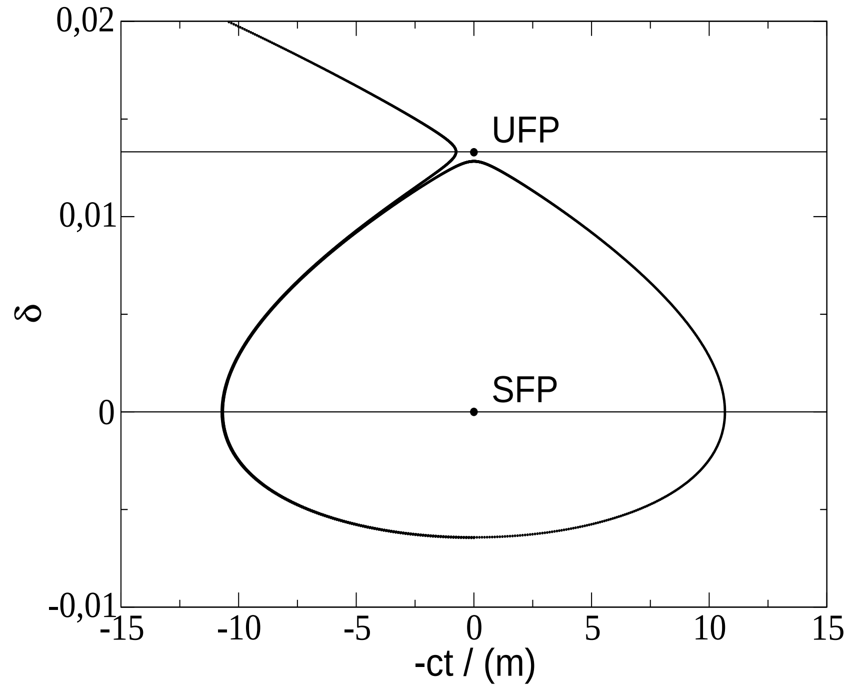
- Significant influence of higher order contributions to change of revolution time. Regard up to δ^2:

$$\frac{\Delta T}{T_0} = \eta_0 \delta + \eta_1 \delta^2,$$

- Set $\Delta T / T_0 = 0$ $\rightarrow$ Unstable fixed point (UFP)

$$\delta_{UFP} = -\frac{\eta_0}{\eta_1},$$

no symmetry in $\pm\delta$.



Alpha bucket for unperturbed SIS-100 lattice

Small momentum compaction factor

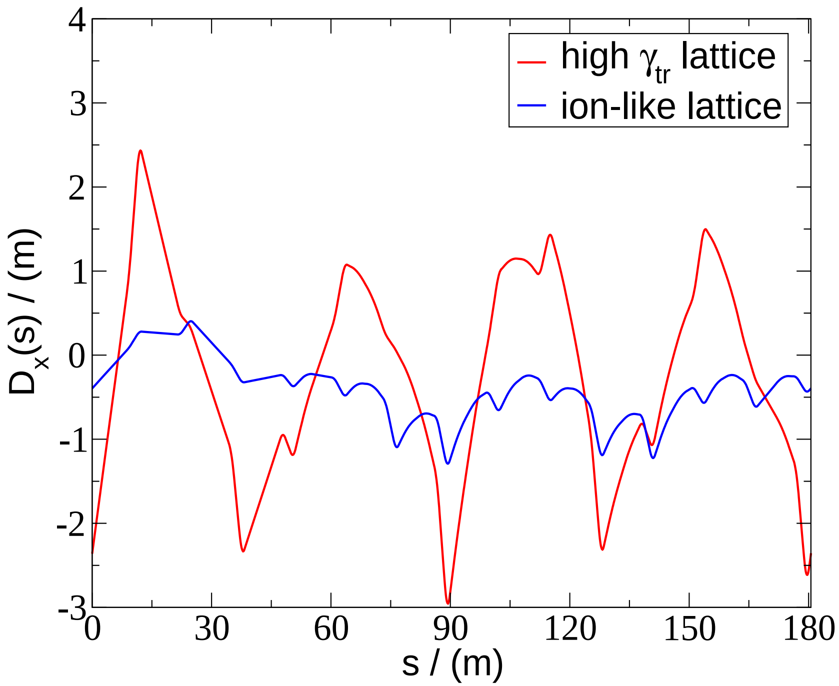
- Zero order momentum compaction factor:

$$\alpha_0 = \frac{1}{\gamma_{tr}^2} = \frac{1}{C} \oint \frac{D(s)\ \mathrm{d}s}{\rho(s)}$$

- Small α_0 due to strongly oscillating dispersion
  - large positive and negative contributions which compensate each other.
  - Use three independent quadrupole families
- Generally, lattice functions with extreme values much larger than those of "ion-like" lattice.

Dispersion function in SIS-100 period.

$$Q_x = 21.8, Q_y = 17.7$$



General Lattice Properties

Lattice without errors at working point $Q_x = 21.8, Q_y = 17.7$

	high γ_{tr} lattice	"ion-like" lattice
γ_{tr}	45.5	18.4
Number of quadrupole families	3	2
Maximum dispersion function $D_{x,max}/(\mathrm{m})$	2.9	1.3
Maximum beta functions, $(\beta_{x,max}, \beta_{y,max})/(\mathrm{m})$	$(71,\ 29)$	$(19,\ 21)$
Natural normalised chromaticities, $(\xi_{x,nat}, \xi_{y,nat})$	$(-2.4, -1.4)$	$(-1.2, -1.3)$

Consequences of high-γ_{tr} lattice:

1. Larger beam width so that particles reach transverse regions of larger magnetic field errors
2. Larger chromaticities leading to larger chromatic tune spread

Chromatic tune spread

Total horizontal chromatic tune spread nearly 0.5 and crosses several resonances.

- Desired bunch area and small bunch length lead to large maximum momentum deviation: Initial bunch area: $A = 2.4$ eVs. Dilution factor: 5

$$\delta_{max} = \pm 0.0043.$$

(O. Chorniy using ESME)

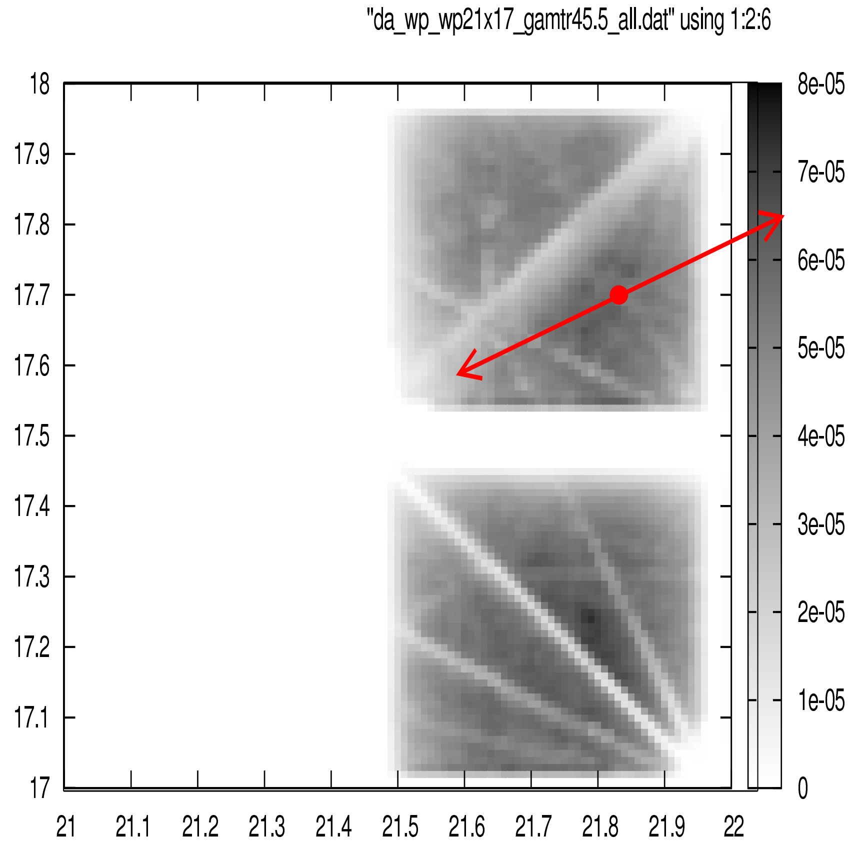
- Resulting maximum chromatic tune deviation:

$$\Delta Q_{x,max} = \xi_x Q_x \delta_{max} = \pm 0.23.$$

$\Downarrow$

Need for chromaticity reduction.

$Q_x = 21.8, Q_y = 17.7$



Chromaticity correction

- Goal: Correct chromaticity with sextupoles to reach certain chromatic tune spreads.

  Found in simulations: $\Delta Q_{x,max} = \Delta Q_{y,max} = \pm 0.05$ is suitable.

- There are 8 sextupoles per period to set only two variables, ξ_x and ξ_y.

  Provides space to optimise sextupole settings. Follow rule: sextupole should be:

  weak, where β_x large to minimise non-linear impact, and

  strong, where D is large to have strong impact to chromaticity

  (Yu. Senichev, private communication),

$$\sum_n (k_2 L)_n^2 \left(\beta_{x,n}^2 - D_n^2\right) \to \text{minimum}$$

and apply correction goal as constraints

$$\xi_{z,nat} \pm \frac{1}{4\pi Q_z} \sum_n (k_2 L)_n \beta_{z,n} D_n - \xi_{z,desired} = 0, \quad z = x, y,$$

where $``+/-''$ $\leftrightarrow$ horizontal / vertical.

Bucket area

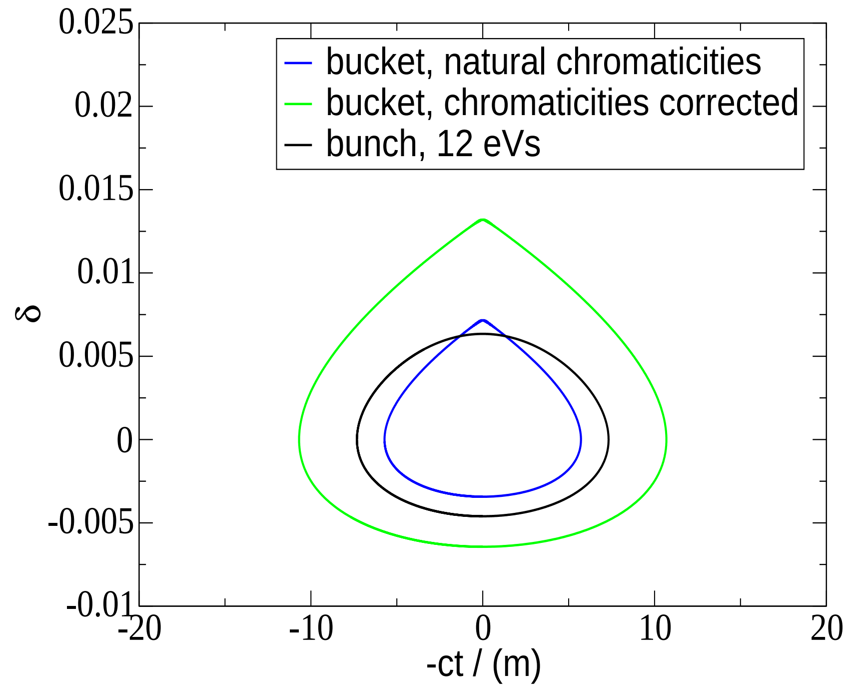
- Sextupoles alter second order phase slip factor (D. Robin, E. Forest, PRE 48, 2149 (1993).)

$$\Delta\eta_1 = -\frac{1}{3C}\sum_n (k_2 L)_n \left(D_{x,n}^3 - 3D_{x,n}D_{y,n}^2\right)$$

and bucket area.

- Sufficient bucket area reached only with chromaticity correction.

- Applied only scheme described at previous page, explicit usage of equation above not necessary.



Sextupole settings require that every sextupole in a period can be independently powered.

Magnet imperfections

- Magnet imperfections: field errors and misalignments.
- Field errors represented by magnet field multipoles and yield resonance excitation and reduction of dynamic aperture.
- Misalignments yield closed orbit distortions (COD) and shifts alpha bucket.
- Both affect momentum compaction factor and change bucket shape.

Magnet imperfections

Field errors

- Field imperfections in dipoles and quadrupoles.
- Transverse field given by series with normal and skew multipole coefficients, k_n and $k_{s,n}$.
- Distinguish between field in magnet centres and ends $\rightarrow$ stray fields.
- Systematic and random multipole coefficients:
  - Systematic multipoles due to magnet design. Present study: from field simulations.

    Multipoles up to order $n = 15$, in quadrupole ends up to $n = 19$.
  - Random multipoles due to random deviations from design.

    Ansatz: Gaussian distribution truncated at 2σ with $\sigma_{k_n} = \sigma_{k_{s,n}} = 0.3 \cdot k_{n,syst}$.

    Assumption on random contributions questionable. At least conservative?

Magnet imperfections

Misalignments

- Misalignments random,

  Gaussian distributions truncated at 2σ.

- Quadrupoles: transverse shifts:

$$d_{rms} \equiv \sigma_{\Delta x} = \sigma_{\Delta y} = 1 \text{ mm}$$

- Dipoles: rotations around z axis (tilt), $\Delta\psi$,

  $\sigma_{\Delta\psi} = d_{rms} \;/\; (\sqrt{2} \cdot w_{1/2}) = 4.0 \text{ mrad},$

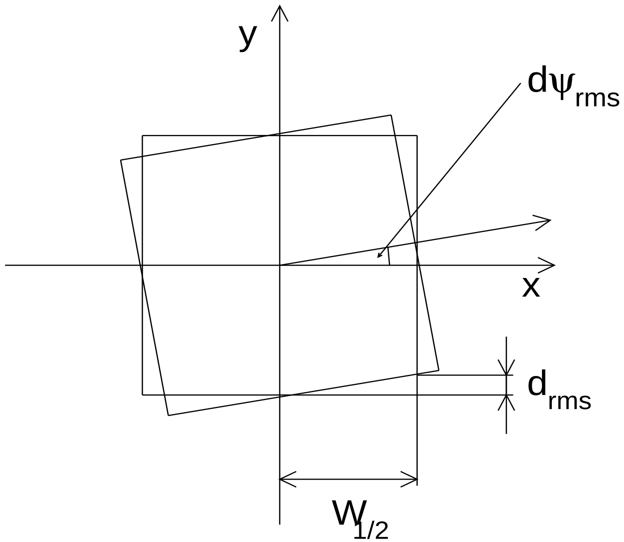
  and deviations in total bending angle, $\Delta\alpha$:

  $\sigma_{\Delta\alpha}/\alpha = 0.004$

- Residual COD (rms):

  $x_{co} = 1.3 \text{ mm},\; y_{co} = 0.5 \text{ mm}$

Dipole tilt:



$$d_{rms} = 1 \text{ mm}, w_{1/2} = 0.175 \text{ m}$$

Alpha bucket with COD and betatron motion

- Relative path length and revolution time altered by COD and betatron motion.

$$\chi = \chi_{co} + \chi_\beta$$

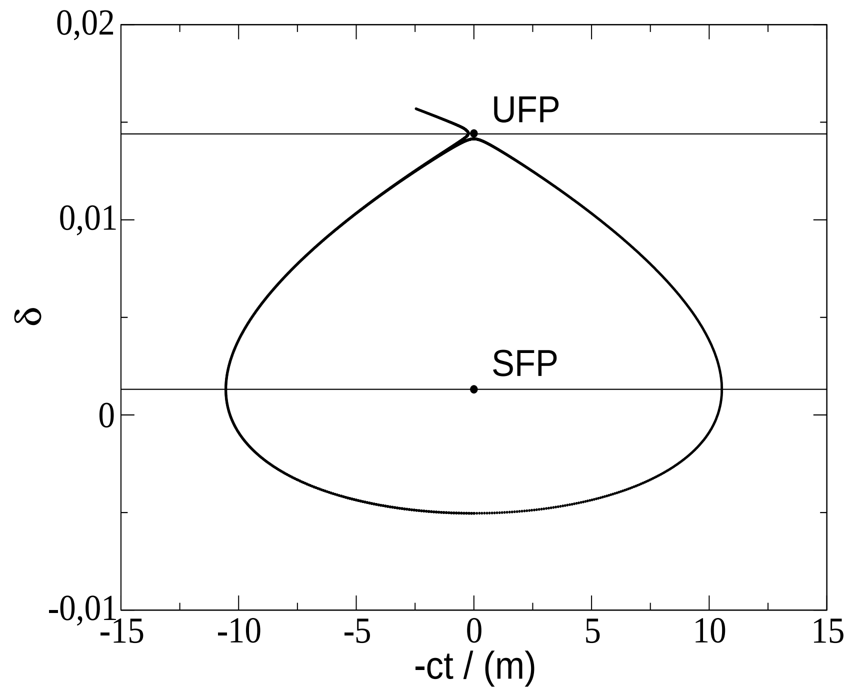
Contribution momentum independent.

- Change of single particle's revolution time

$$\frac{\Delta T}{T_0} = \chi + \eta_0 \delta + \eta_1 \delta^2,$$

- Unstable and stable fixed points at

$$\delta_{UFP,SFP} = -\frac{\eta_0}{2\eta_1}\left(1 \pm \sqrt{1 - \frac{4\chi\eta_1}{\eta_0^2}}\right).$$



Alpha bucket for SIS-100 lattice with misalignment errors $\rightarrow \delta_{SFP} \neq 0$.

Initial longitudinal particle distribution

Need of matched longitudinal particle distribution to avoid artificial particle loss.

- Ansatz:

$$f(\phi,\delta) \propto \mathrm{e}^{-H(\phi,\delta)/H_{max}}\, \Theta\left[|H_{max}| - |H(\phi,\delta)|\right]$$

  - Heaviside step function: Θ
  - Hamiltonian in short bunch approximation without acceleration

$$\begin{aligned} H(\phi,\delta) &= h\left(\chi\delta + \frac{\eta_0}{2}\delta^2 + \frac{\eta_1}{3}\delta^3\right) - \frac{eV_{rf}}{4\pi\beta E}\phi^2 \\ &\quad -h\left(\chi\delta_{SFP} + \frac{\eta_0}{2}\delta_{SFP}^2 + \frac{\eta_1}{3}\delta_{SFP}^3\right) \end{aligned}$$

  with harmonic number $h = 5$.

- Desire: Usage of Gaussian random generator of MAD-X, but δ^3 term in H.

Initial longitudinal particle distribution

- Therefore, rewrite Hamiltonian

$$H(\phi,\delta) \to H(\phi,\tilde{\delta}) = h\left(\frac{\tilde{\eta}_0}{2}\tilde{\delta}^2 + \frac{\tilde{\eta}_1}{3}\tilde{\delta}^3\right) - \frac{eV_{rf}}{4\pi\beta E}\phi^2$$

with $\tilde{\delta} = \delta - \delta_{SFP}$, $\tilde{\eta}_0 = \eta_0 + 2\eta_1\delta_{SFP}$, and $\tilde{\eta}_1 = \eta_1$.

- Define new variable Δ by

$$\frac{\eta_0}{2}\Delta^2 \equiv \frac{\tilde{\eta}_0}{2}\tilde{\delta}^2 + \frac{\tilde{\eta}_1}{3}\tilde{\delta}^3,$$

1. Distribution of Δ can be determined with Gaussian random generator
2. Calculate $\tilde{\delta}$ by solving 3rd order equation

$$\tilde{\delta}^3 + \frac{3}{2}\frac{\tilde{\eta}_0}{\tilde{\eta}_1}\tilde{\delta}^2 - \frac{\eta_0}{\tilde{\eta}_1}\Delta^2 = 0$$

   using Cardan's formula.

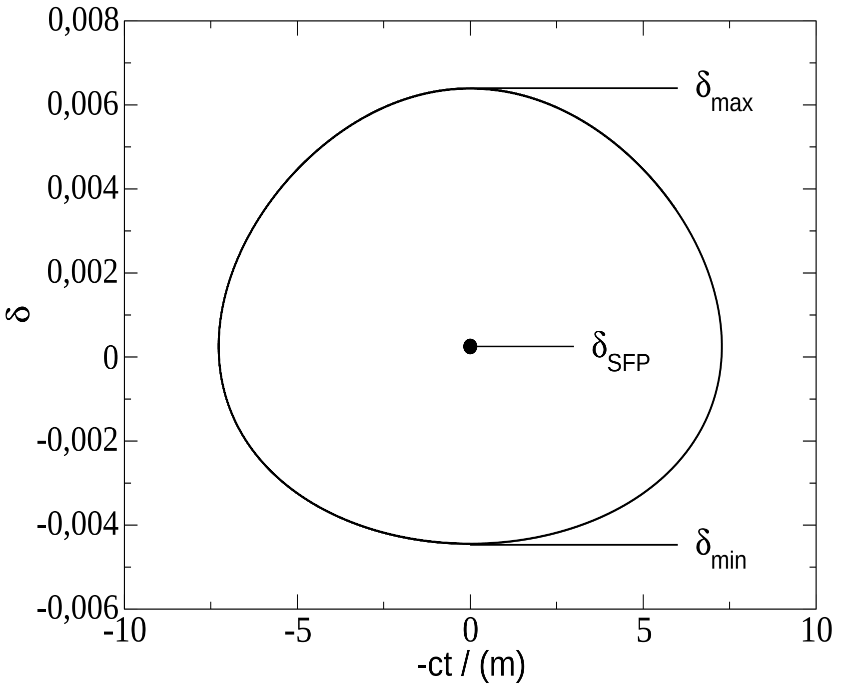
- Bucket parameters χ, η_0, η_1 affected by magnet errors.

  $\rightarrow$ Have to be determined for the actual error sample.

Initial longitudinal particle distribution

Contribution due to COD χ_{co}

- Parameters known from MAD-X output: η_0, δ_{SFP}
- Unknown parameters: χ_{co}, η_1:
  - Determination via few particle tracking with $\epsilon_x = \epsilon_y = 0$ using $H = \text{const}$.
  - System of three linear equations:



$$h\left(\delta_{min} - \delta_{SFP}\right)\chi_{co} + \frac{h}{3}\left(\delta_{min}^3 - \delta_{SFP}^3\right)\eta_1 - H = -h\frac{\eta_0}{2}\left(\delta_{min}^2 - \delta_{SFP}^2\right)$$

$$h\left(\delta_{max} - \delta_{SFP}\right)\chi_{co} + \frac{h}{3}\left(\delta_{max}^3 - \delta_{SFP}^3\right)\eta_1 - H = -h\frac{\eta_0}{2}\left(\delta_{max}^2 - \delta_{SFP}^2\right)$$

$$\chi_{co} + \delta_{SFP}^2\eta_1 = -\eta_0\delta_{SFP}$$

Initial longitudinal particle distribution

Contribution due to betatron motion χ_β

- For $\delta = 0$:

$$\frac{\Delta T}{T} = \frac{\Delta C}{C} = \chi_{co} + \chi_\beta.$$

- Contribution to path length

$$\Delta C = \frac{1}{2} \oint \mathrm{d}s \left(x'^2_{co} + y'^2_{co} + x'^2_\beta + y'^2_\beta \right)$$

- In linear optics is

$$\chi_\beta = \frac{1}{4C} \oint \mathrm{d}s \, (\epsilon_x \gamma_x + \epsilon_y \gamma_y) = \frac{\pi}{C} (\epsilon_x \xi_{x,nat} Q_x + \epsilon_y \xi_{y,nat} Q_y)$$

- If chromaticity corrected, use corrected chromaticies to calculate χ_β, see Y. Shoji, PRSTAB **8**, 094001 (2005).

- Fixed points of synchrotron motion depend on single particle emittances.

  $\rightarrow$ Fixed points for each particle separately calculated.

Tracking simulation

- 5000 test particles tracked during 33000 turns corresponding to about two synchrotron periods.
- Finally, chose working point $Q_x = 21.83, Q_y = 17.75$.
- Applied 5 samples of random misalignments and field error multipoles.
- Found beam loss:

sample	1	2	3	4	5	averaged
P_{loss}	$0.14\ \%$	0	$1.8\ \%$	$0.04\ \%$	0	$0.396\ \%$

- Averaged beam loss smaller than beam loss budget of $P_{loss,max} \approx 1\ \%$.
- Beam loss depends on sample of random magnet errors.

Tracking simulation

Loss mechanism: slow resonance crossing

- Working point $Q_x = 21.83, Q_y = 17.75$,

  chosen to fit tune spread into mesh of resonances

  of order ≤ 3.

- Particle loss near resonances:

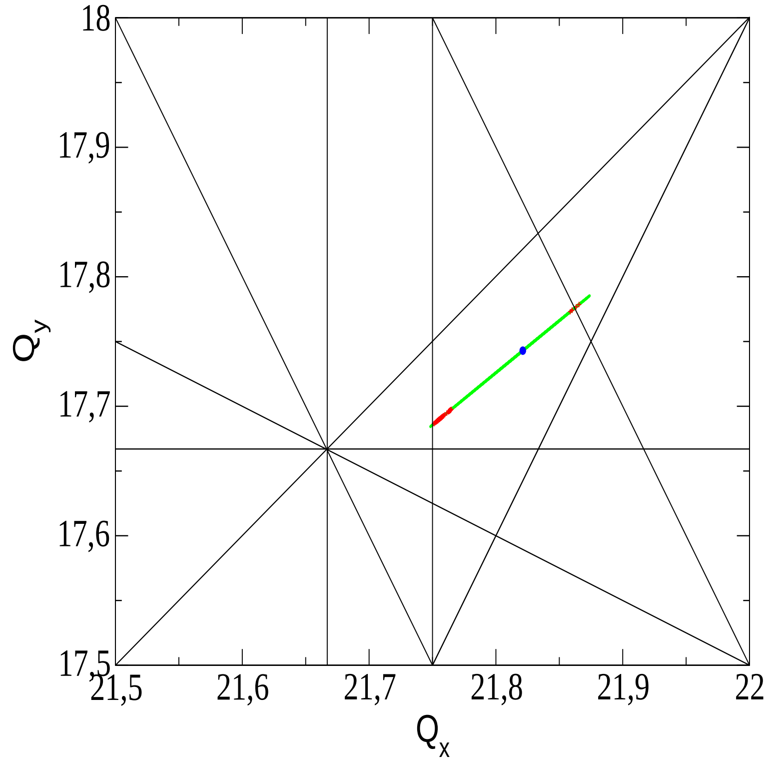
  order 4: $4 \cdot Q_x = 87$

  order 6: $4 \cdot Q_x + 2 \cdot Q_y = 123$,

- Importance of knowing magnet field errors.

  – Strengths of resonances

  – δ_{SFP} $\rightarrow$ tune spread position in tune diagram.



Initial tunes of all particles (green),

final tunes of lost particles (red),

and tunes of δ_{SFP} (blue dot).

Summary

Aim: Estimate particle loss at SIS-100 proton extraction energy with particle tracking.

- Alpha bucket generated by large γ_{tr} optics to create short bunch.
  - Large natural chromaticities and chromatic tune spread, need to be reduced.
  - In addition, chromaticity correction increases bucket to sufficient size.

    $\rightarrow$ Independently powered sextupoles necessary.
- Regard misalignment and field errors with systematic and random contributions in dipole and quadrupole magnets $\rightarrow$ COD, resonance excitation, change of alpha bucket shape.
  - Matched initial particle distribution $\rightarrow$ find alpha bucket parameters χ, η_0, η_1.
  - Particle loss in simulations driven by particles slowly crossing resonances.

Magnet errors determine impact of resonances in two ways: resonances strengths and which resonances are crossed by chromatic tune spread.

Thank you for your attention

Initial longitudinal particle distribution

To calculate initial particle coordinates, χ_{co} and η_1 have to be determined

for actual sample of systematic and random magnet errors.

- Variables known from MAD-X output: η_0, δ_{SFP}

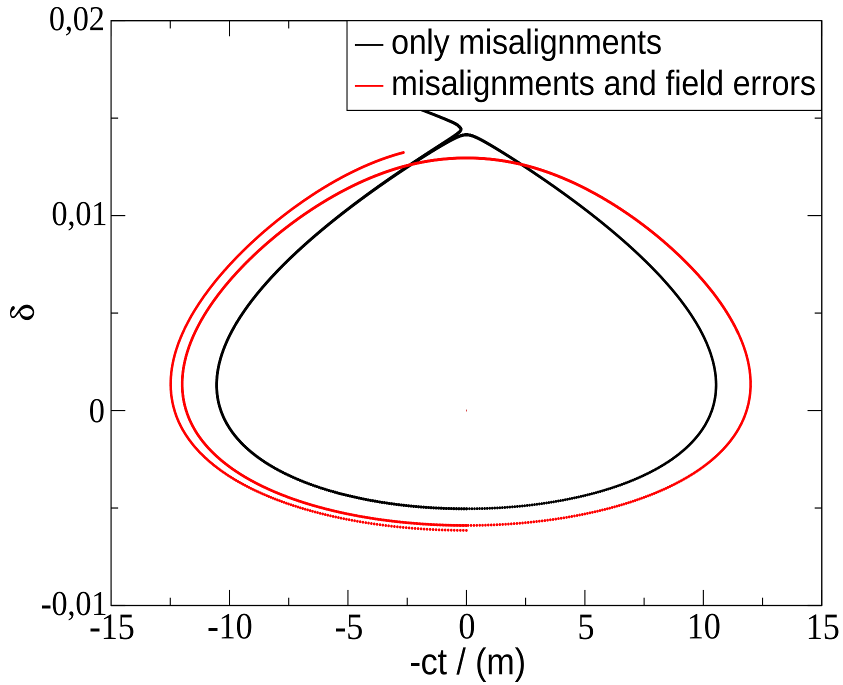
- Unknown variables: χ_{co}, η_1.

  They can be determined from few particle tracking results with $\epsilon_x = \epsilon_y = 0$.

- Simple way: determine χ_{co}, η_1 from fixed points

$$\delta_{UFP,SFP} = -\frac{\eta_0}{2\eta_1}\left(1 \pm \sqrt{1 - \frac{4\chi\eta_1}{\eta_0^2}}\right),$$

  is not possible because tracked particles do not reach UFP.



Initial longitudinal particle distribution

Contribution due to betatron motion χ_β

Difference of path length from design orbit

(see e.g. H. Wiedemann, “Particle Accelerator Physics”)

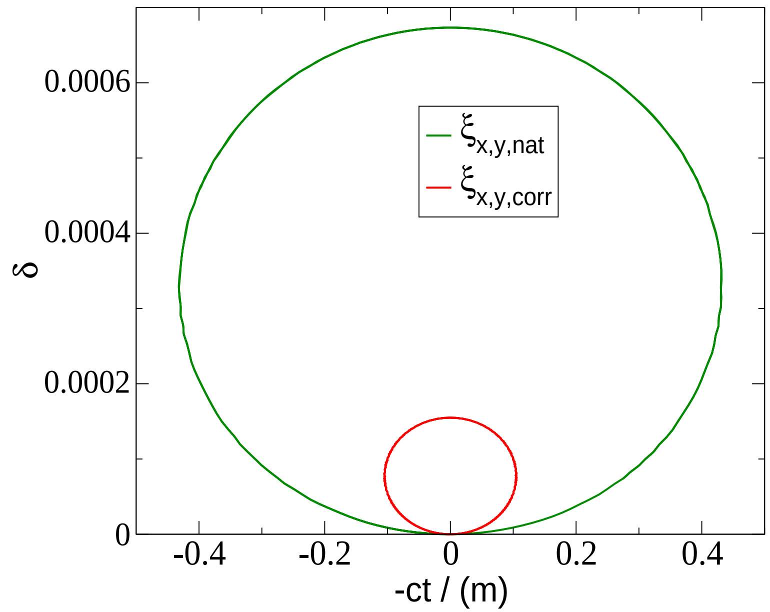
$$\begin{aligned}
\Delta C &= \oint \mathrm{d}s \left[\sqrt{\left(1+\frac{x}{\rho}\right)^2 + x'^2 + y'^2} - 1 \right] \\
&= \oint \mathrm{d}s \left[1 + \frac{1}{2}\left(2\frac{x}{\rho} + \frac{x^2}{\rho^2} + x'^2 + y'^2 \right) - \frac{1}{4}\left(2\frac{x}{\rho} + \frac{x^2}{\rho^2} + x'^2 + y'^2 \right)^2 \ldots - 1 \right] \\
&\approx \frac{1}{2} \oint \mathrm{d}s \left(x'^2 + y'^2 \right) .
\end{aligned}$$

Influence of betatron motion

Simulated synchrotron orbit with finite horizontal emittance without magnet errors

- Emittance dependent contribution:

$$\chi_\beta = \frac{\pi}{C}\left(\epsilon_x \xi_x Q_x + \epsilon_y \xi_y Q_y\right)$$



- Track single particle with
  - $\epsilon_x = 1\ \mu\text{m}$, $\epsilon_y = 0$, and $(\phi, \delta) = (0, 0)$
  - natural and corrected chromaticities.

- Compare ratios of momenta in the centres of circles, δ_{cent}, and horizontal chromaticities:

corrected: $\xi_{x,corr} = -0.534,\ \delta_{cent,corr} = 7.7 \cdot 10^{-5}$

natural: $\xi_{x,nat} = -2.44,\ \delta_{cent,nat} = 3.4 \cdot 10^{-4}$

$$\left.\right\} \frac{\xi_{x,corr}}{\xi_{x,nat}} = 0.219 \approx \frac{\delta_{cent,corr}}{\delta_{cent,nat}} = 0.226$$

$\rightarrow$ SFP momentum approximately proportional to chromaticity, as expected.