Advanced algorithms for the LHC optics

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Outline

- Description of optics measurement method (β-function from BPM turn by turn data)
- Improvements
  - Random/statistical errors
  - Systematic errors
  - ➡ Better estimate of β-function
- Application on 2012 measurement data
- Improvements for betatron coupling control

Motivation

- Tight tolerances for β-beat due to
  - Available mechanical aperture

> „For the LHC the total tolerance for the β-beat has been specified as 14 %.“

–LHC Report 501

- In 2012 a β-beat of up to 100% was observed before local corrections

Motivation

- Run II at 6.5 TeV —> allows for smaller beta*

➡ Enhances optics errors of triplet magnets

- More quadrupole magnets in saturation regime

- Higher damage potential at 6.5 TeV

➡ Limits maximum excitation amplitude and total beam charge

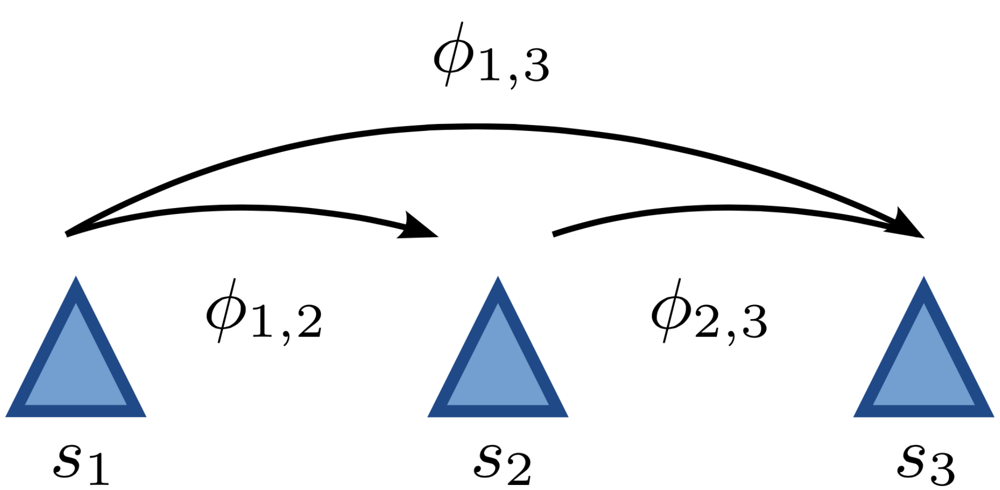
➡ Reduced signal to noise ratio for optics measurement

Optics measurement

- Measurement of BPM turn-by-turn data x_i
- Harmonic analysis $C(w) = \sum_{i=0}^{N-1} x_i \cos(w\, i),\, S(w) = \sum_{i=0}^{N-1} x_i \sin(w\, i)$

$$\phi(w) = -\arctan\left(\frac{S(w)}{C(w)}\right)$$

➡ Phase advance of betatron oscillation between BPM



$$\beta_i = \frac{\epsilon_{ijk}\cot(\phi_{i,j}) + \epsilon_{ikj}\cot(\phi_{i,k})}{\epsilon_{ijk}\frac{M_{11(i,j)}}{M_{12(i,j)}} + \epsilon_{ikj}\frac{M_{11(i,k)}}{M_{12(i,k)}}}$$

Optics measurement

$$\beta_i = \frac{\epsilon_{ijk}\cot(\phi_{i,j}) + \epsilon_{ikj}\cot(\phi_{i,k})}{\epsilon_{ijk}\frac{M_{11(i,j)}}{M_{12(i,j)}} + \epsilon_{ikj}\frac{M_{11(i,k)}}{M_{12(i,k)}}}$$

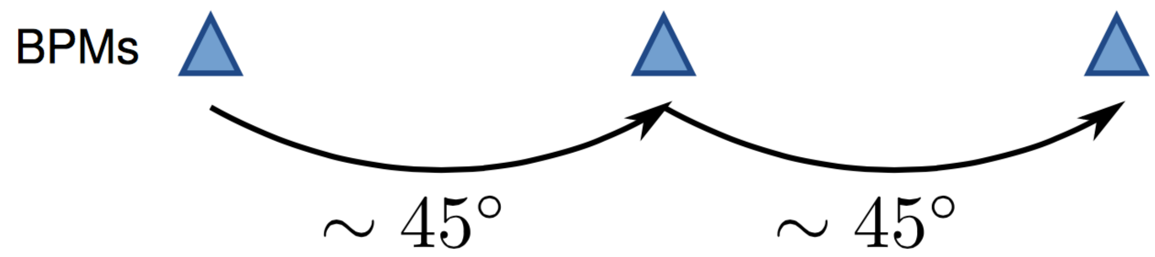
- Optimum phase advances

$$\phi_{i,j} = \frac{\pi}{4} + n_1\frac{\pi}{2},$$
$$\phi_{i,k} = \frac{\pi}{4} + (2n_2 + 1 - n_1)\frac{\pi}{2},$$

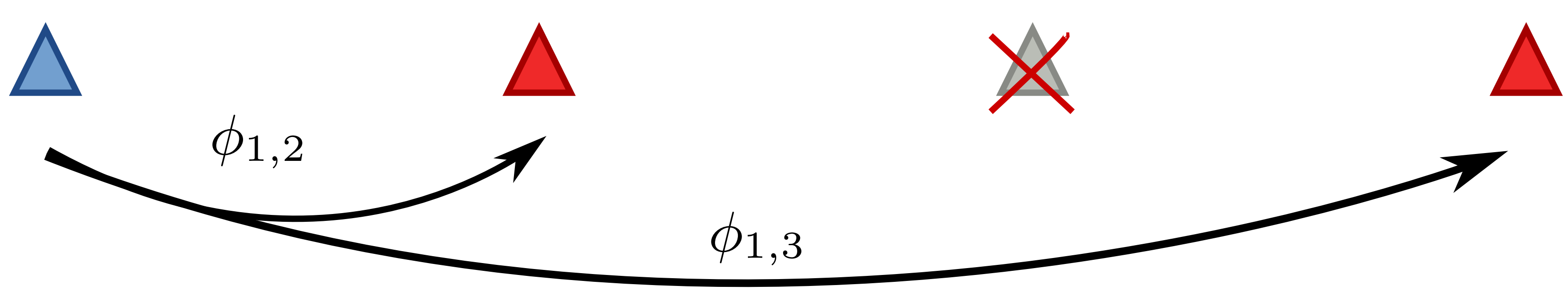
$$n_1, n_2 \in \mathbb{Z}.$$

- Multiples of π should be avoided as the cotangent becomes infinite

Situation in the arcs



- Phase advances between consecutive BPMs not always suited for measurement
- Previous implementation used only **neighboring BPMs**
- Optimum if probed BPM in the middle
- If probed BPM right/left of other BPMs the optimum is to skip one BPM

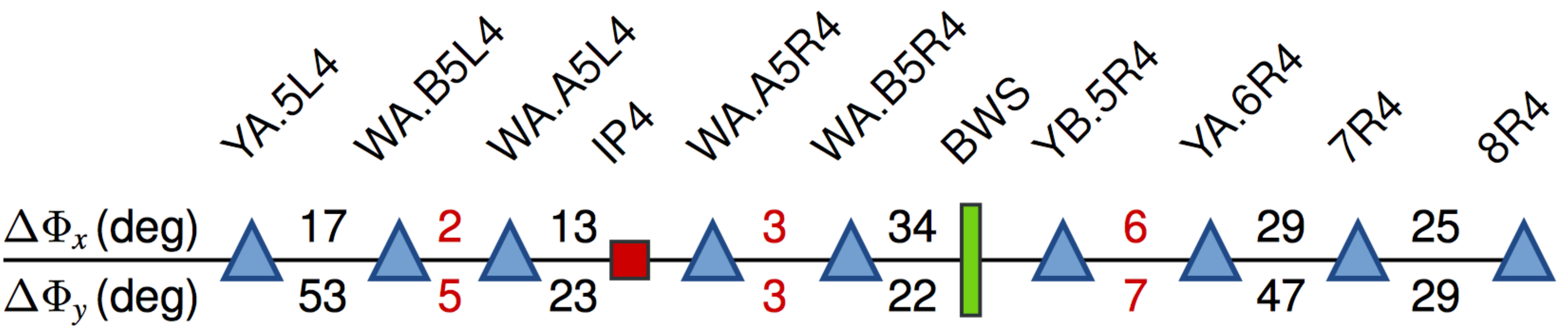


Situation in the interaction regions (IRs)

- Phase advances are irregular and can be very small

- Using neighboring BPM will result in large uncertainties

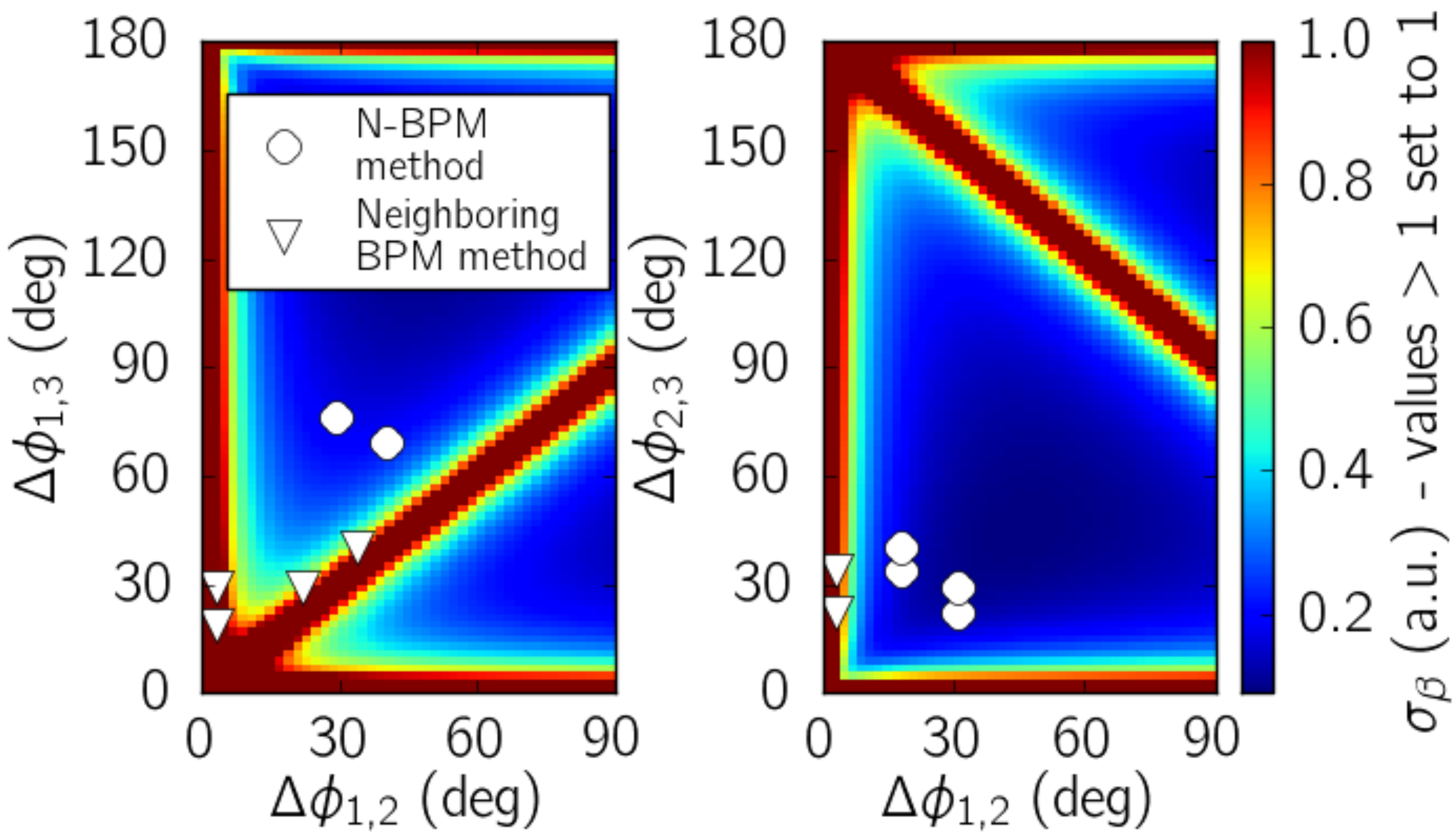
Sketch of phase advances in IR4



Improvements

- Increase the range from which BPM combinations are chosen

- Choose BPM combinations with good phase advances



N-BPM method

- Consider more β_i from different BPM combinations
- Minimize $S(\beta)$ (least squares)

$$S(\beta) = \sum_{i=1}^{N}\sum_{j=1}^{N}(\beta_i - \beta)V_{ij}^{-1}(\beta_j - \beta)$$

V_{ij}, covariance matrix

$$\text{Min}(S(\beta)) = S(\hat{\beta})$$

$$\hat{\beta} = \sum_{i=1}^{N} w_i \beta_i$$

$$w_i = \frac{\sum_{k=1}^{N} V_{ik}^{-1}}{\sum_{k=1}^{N}\sum_{j=1}^{N} V_{jk}^{-1}}.$$

Problem: Good knowledge of V_{ij}

Statistical error

- Uncertainty of the phase advance derived as standard deviation of n measurement files

$$\sigma_{\phi_{i,j}} = t(n)\sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(\overline{\phi_{i,j}} - \phi_{i,j,(k)}\right)^2}$$

- $t(n)$ is a correction for small sample sizes from Student t-distribution

- Amount of measurements is always limited due to beam time

Number of measurements	$t(n)$
2	1.84
3	1.32
4	1.20
5	1.15
10	1.06

Statistical error

- All phase advances that share one BPM are correlated

$$\rho(\phi_{i,j}, \phi_{i,k}) = \frac{\partial \phi_{i,j}}{\partial \phi_i} \frac{\partial \phi_{i,k}}{\partial \phi_i} \frac{\sigma_{\phi_i}^2}{\sigma_{\phi_{i,j}} \sigma_{\phi_{i,k}}}.$$

- Uncertainty of phase advance from standard deviation of all measurement files

- Not possible for single phase uncertainty since the value is arbitrary and may vary from measurement to measurement

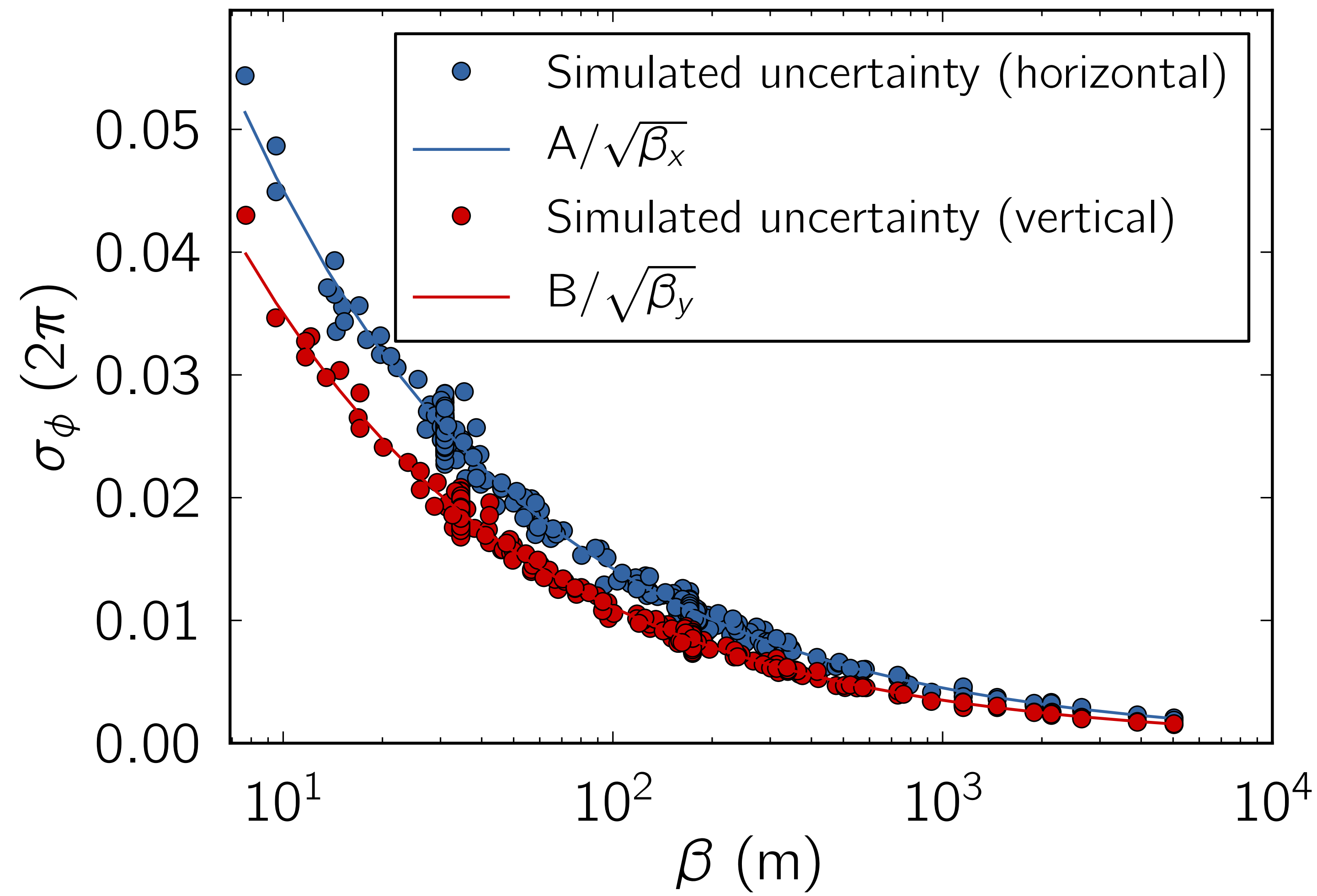
Statistical error

- Ansatz for single phase uncertainty

$$\sigma_\phi \sim \beta^{-\frac{1}{2}}$$

- Therefore

$$\sigma^2_{\phi_{i,j}} = \sigma^2_{\phi_i}\left(1 + \frac{\beta_i}{\beta_j}\right)$$



Statistical error

- For a probed BPM with ϕ_1 the covariance matrix is

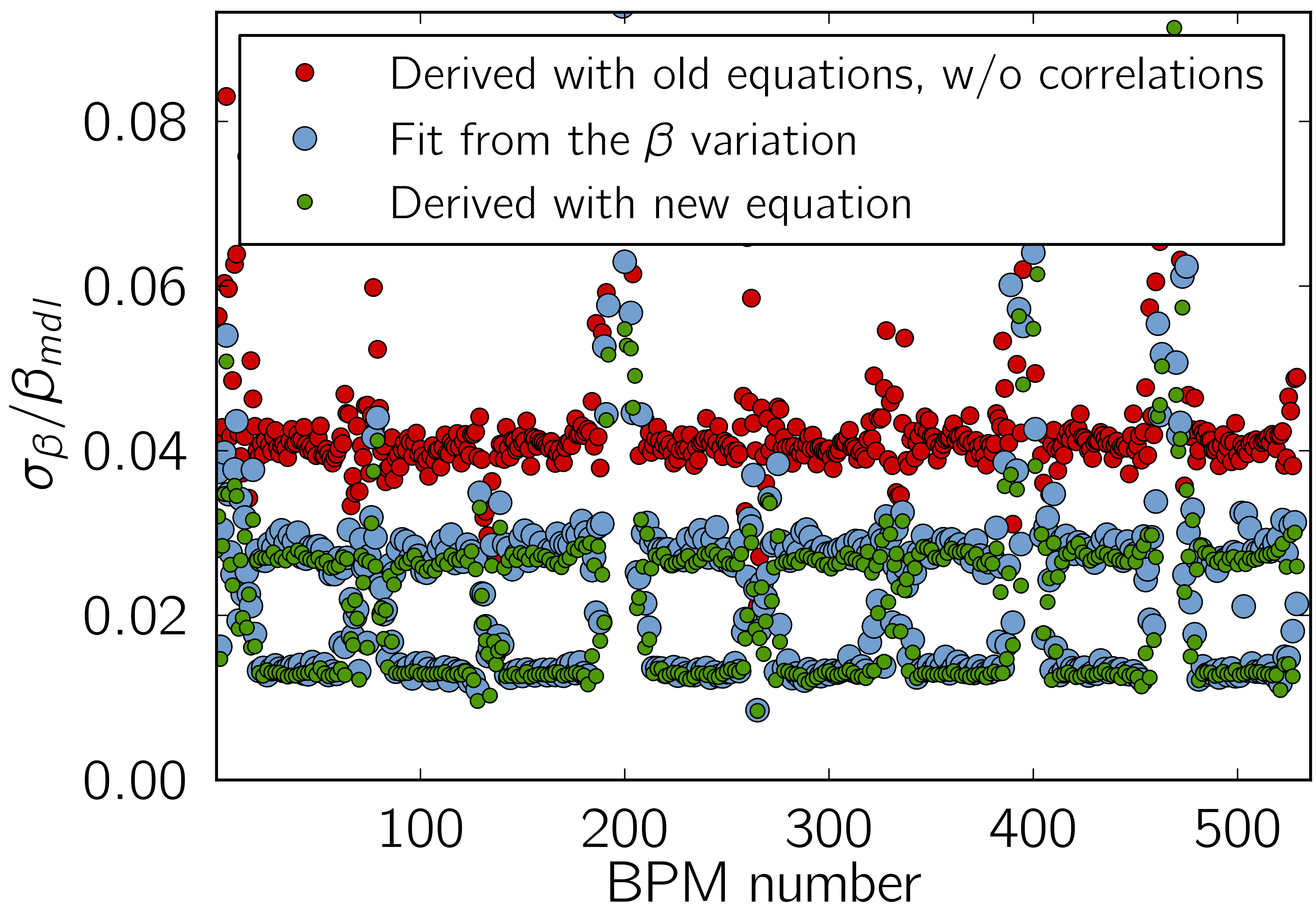
$$C_{i-1,j-1} = \rho(\phi_{1,i}, \phi_{1,j})\sigma_{\phi_{1,i}}\sigma_{\phi_{1,j}}, \quad i \geq 2, j \geq 2$$

- This can be transformed to a covariance matrix for the different β-functions

$$T = \begin{pmatrix} \frac{\partial\beta_1}{\partial\phi_{1,2}} & \cdots & \frac{\partial\beta_N}{\partial\phi_{1,2}} \\ \vdots & \ddots & \vdots \\ \frac{\partial\beta_1}{\partial\phi_{1,n}} & \cdots & \frac{\partial\beta_N}{\partial\phi_{1,n}} \end{pmatrix} \qquad V_{stat} = T^T C T$$

Statistical error

- Test of error bars in a simulation show good agreement

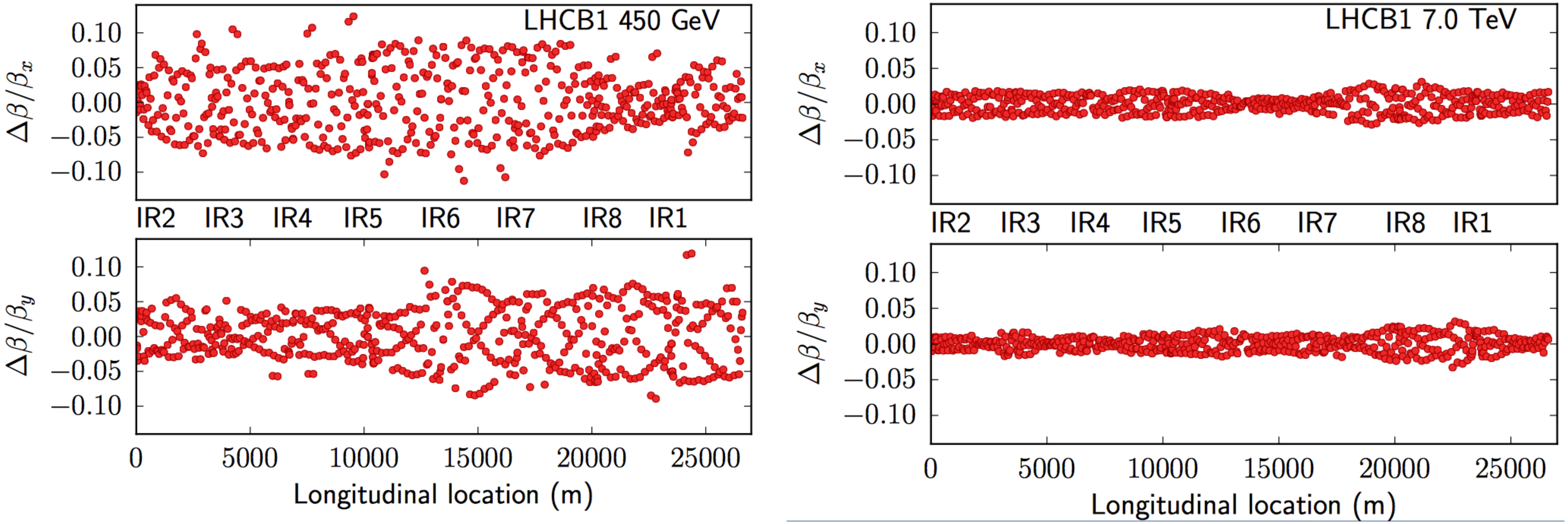


Systematic errors

$$\beta_i = \frac{\epsilon_{ijk}\cot(\phi_{i,j}) + \epsilon_{ikj}\cot(\phi_{i,k})}{\epsilon_{ijk}\frac{M_{11(i,j)}}{M_{12(i,j)}} + \epsilon_{ikj}\frac{M_{11(i,k)}}{M_{12(i,k)}}}$$

- Improve the accuracy of the optics model

➡ Include measured dipole b2 errors



Systematic errors

$$\beta_i = \frac{\epsilon_{ijk}\cot(\phi_{i,j}) + \epsilon_{ikj}\cot(\phi_{i,k})}{\epsilon_{ijk}\frac{M_{11(i,j)}}{M_{12(i,j)}} + \epsilon_{ikj}\frac{M_{11(i,k)}}{M_{12(i,k)}}}$$

- We consider the following perturbations of the optics model
  - Uncertainty of dipole b2 errors
  - Quadrupole gradient uncertainty
  - Longitudinal displacement of quadrupoles
  - Transverse displacement of sextupoles

Systematic errors

- Monte-Carlo Simulation using MADX for deriving the covariance matrix

➡ V_{syst}

BPM combination (probed, used, unused)	Systematic error (%)
unused, used, probed, used, unused	0.3
used, used, probed, unused, unused	0.4
used, unused, probed, used, unused	1.0
used, unused, probed, unused, used	7.1
used, unused, used, probed, unused, unused, unused	1.1
used, unused, unused, probed, unused, unused, used	1.4
used, used, unused, probed, unused, unused, unused	1.7
unused, used, unused, probed, unused, unused, used	1.8
used, unused, unused, probed, used, unused, unused	7.9
used, used, unused, unused, unused, probed, unused, unused, unused, unused, unused	22.3
used, unused, used, unused, unused, probed, unused, unused, unused, unused, unused	1.3
used, unused, unused, used, unused, probed, unused, unused, unused, unused, unused	1.9
used, unused, unused, unused, used, probed, unused, unused, unused, unused, unused	6.1
used, unused, unused, unused, unused, probed, used, unused, unused, unused, unused	1.0
used, unused, unused, unused, unused, probed, unused, used, unused, unused, unused	3.0
used, unused, unused, unused, unused, probed, unused, unused, used, unused, unused	4.5
used, unused, unused, unused, unused, probed, unused, unused, unused, used, unused	5.2
used, unused, unused, unused, unused, probed, unused, unused, unused, unused, used	1.6

Systematic errors

- Final covariance matrix $V_{ij} = V_{ij,stat} + V_{ij,syst}$

$$S(\beta) = \sum_{i=1}^{N} \sum_{j=1}^{N} (\beta_i - \beta) V_{ij}^{-1} (\beta_j - \beta)$$

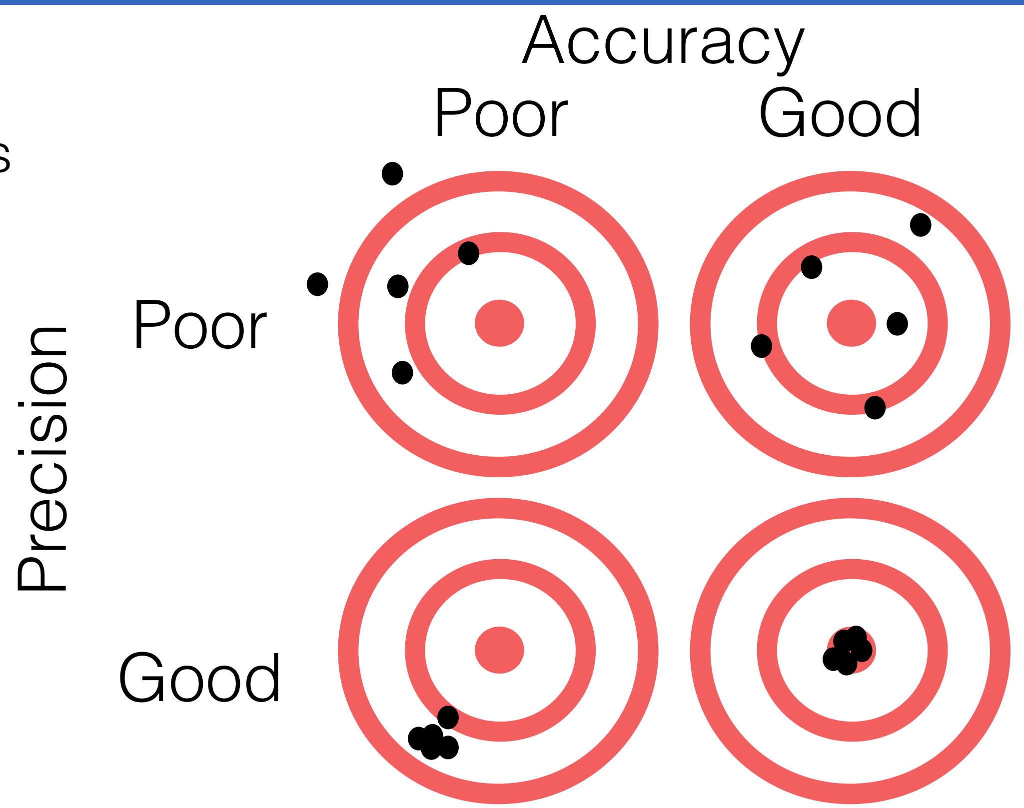
$$\beta = \sum_{i=1}^{N} w_i \beta_i \qquad w_i = \frac{\sum_{k=1}^{N} V_{ik}^{-1}}{\sum_{k=1}^{N} \sum_{j=1}^{N} V_{jk}^{-1}}.$$

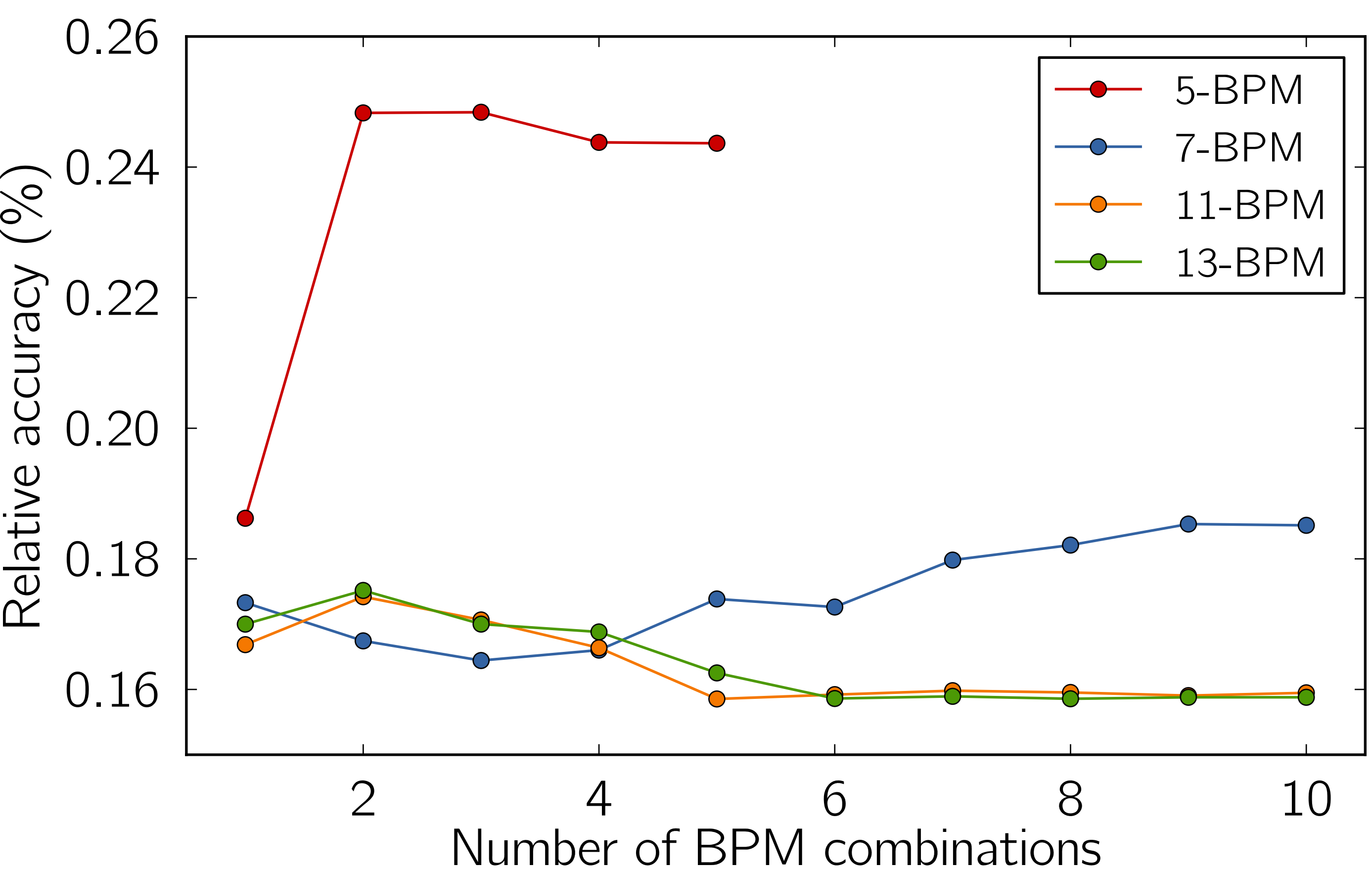
- Computation of systematic covariance matrix time consuming for large ranges of BPMs

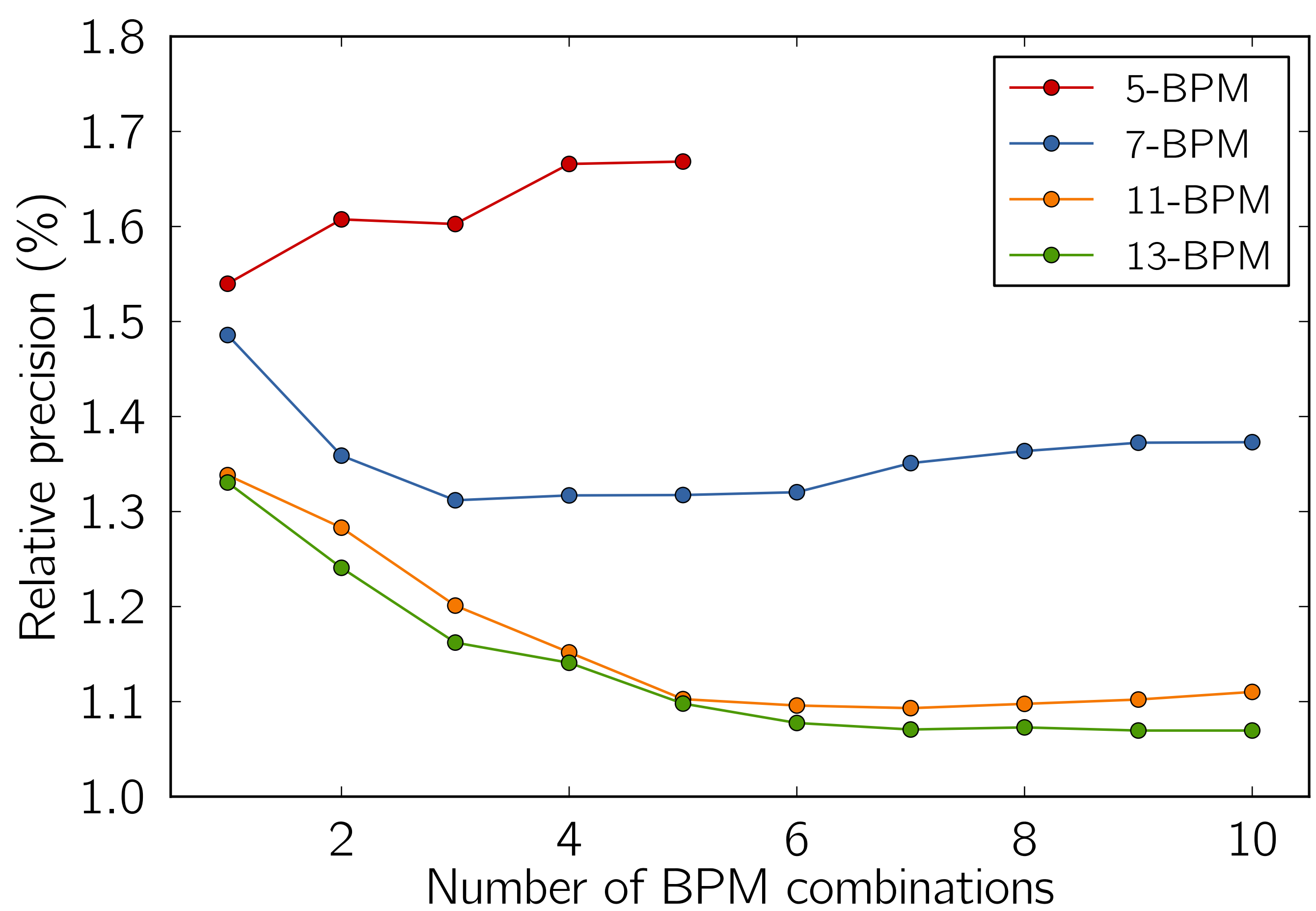
- How many BPM combinations should be regarded?

Uncertainty from simulated measurement

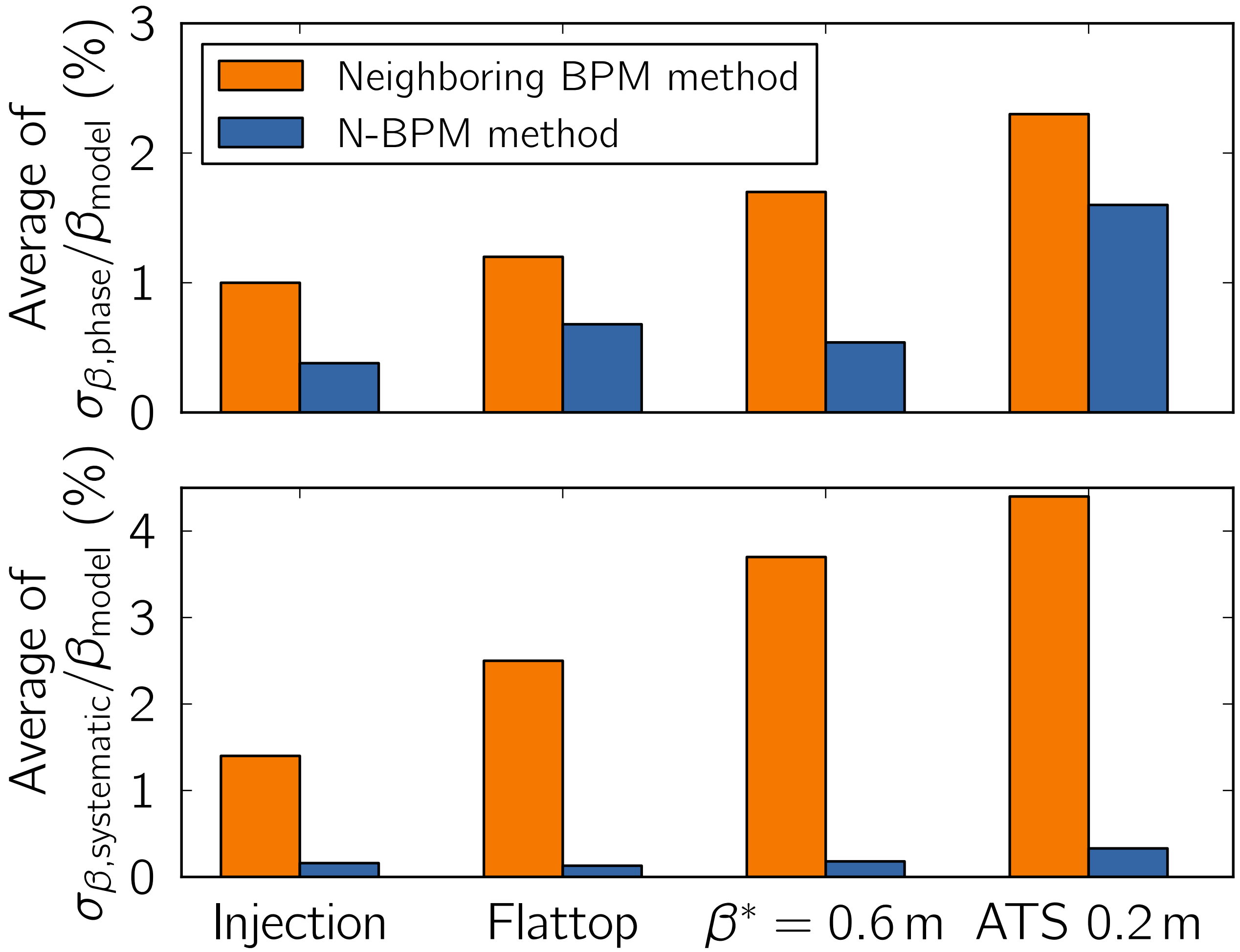
- Simulation of optics measurement under realistic conditions
- Scan of using different amount of BPM combinations which are chosen from different range of BPMs
- Accuracy: average relative shift from true value
- Precision: average relative spread

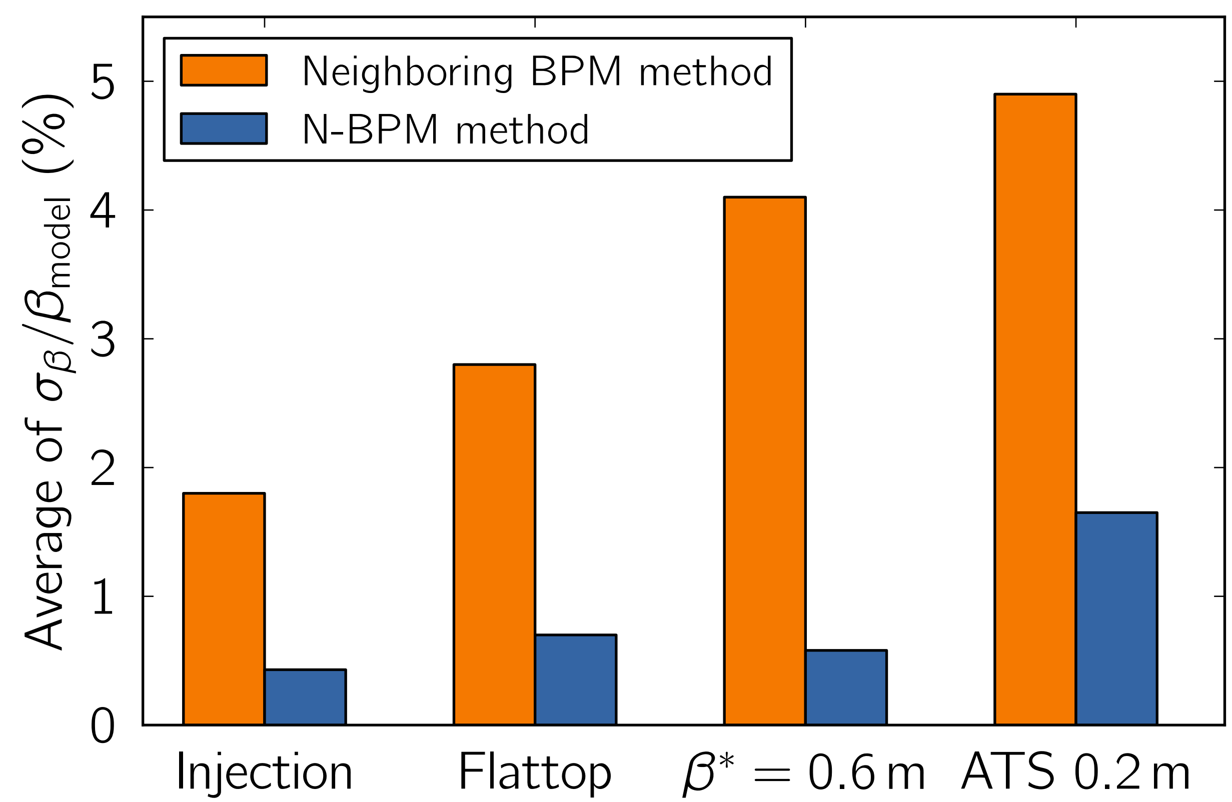






2012 measurement re-analysis



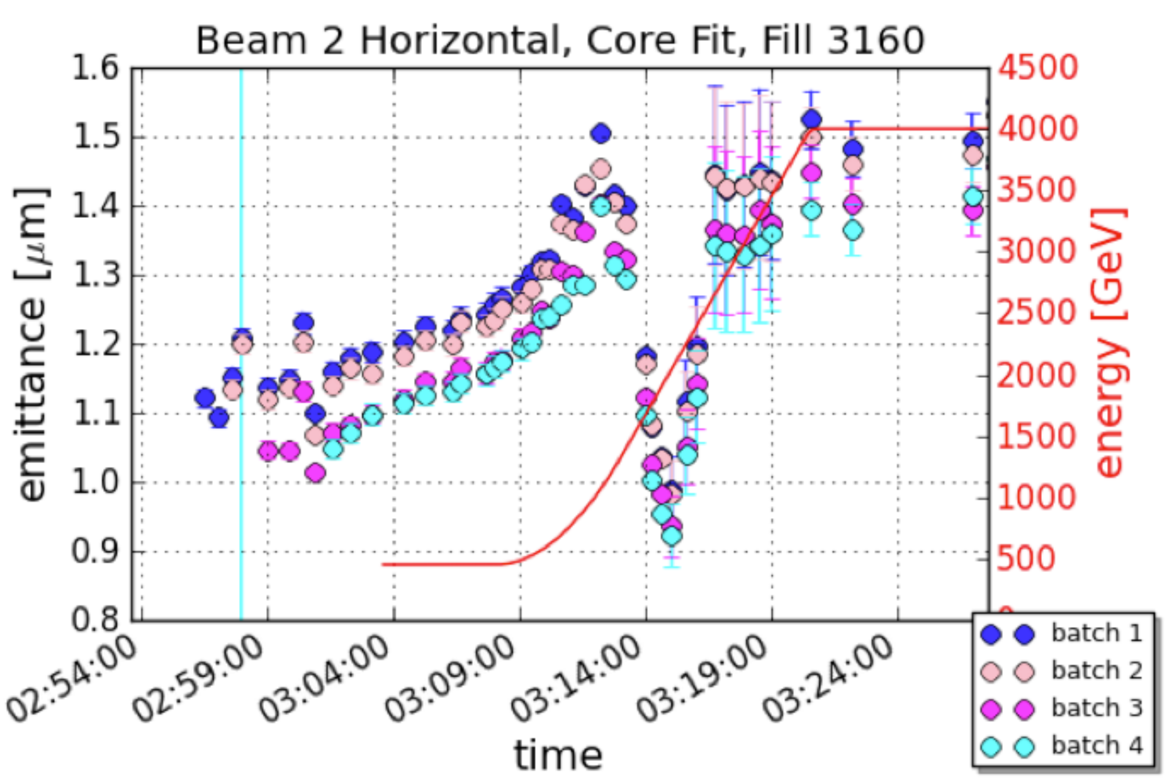


- Significant improvement in the error bars
- Especially systematic errors were overestimated in the past

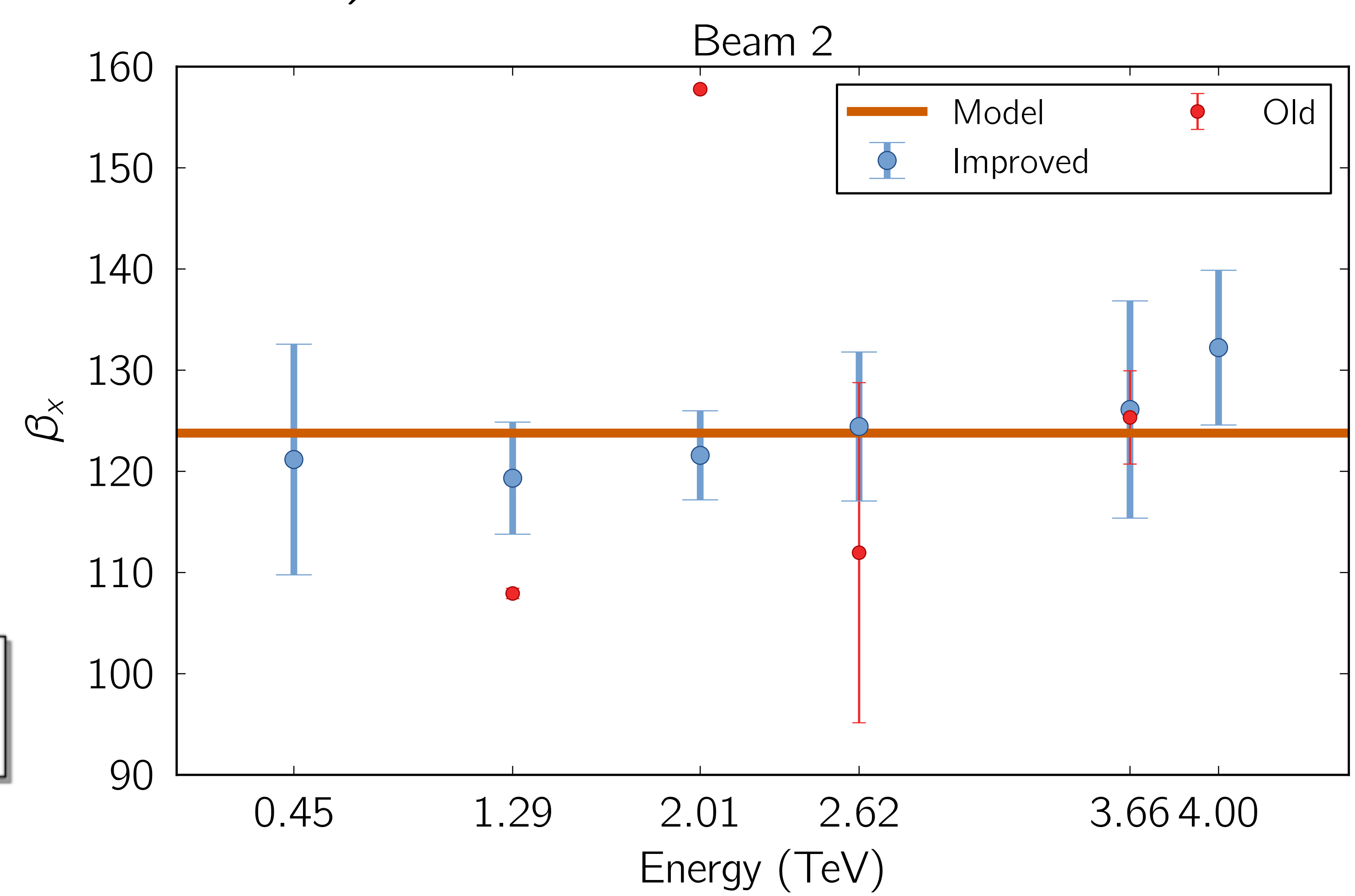
Beta-function during the ramp

- Motivated by emittance study during the ramp
- Propagation of optical functions to in this case beam wire scanners
- New analytic equations for error propagation

$$\sigma^2_{\beta_s} = \left(\beta_s \sin(2\phi)\frac{\alpha_0}{\beta_0} + \beta_s \cos(2\phi)\frac{1}{\beta_0}\right)^2 \sigma^2_{\beta_0} + \left(\beta_s \sin(2\phi)\right)^2 \sigma^2_{\alpha_0}$$



Courtesy Maria Kuhn

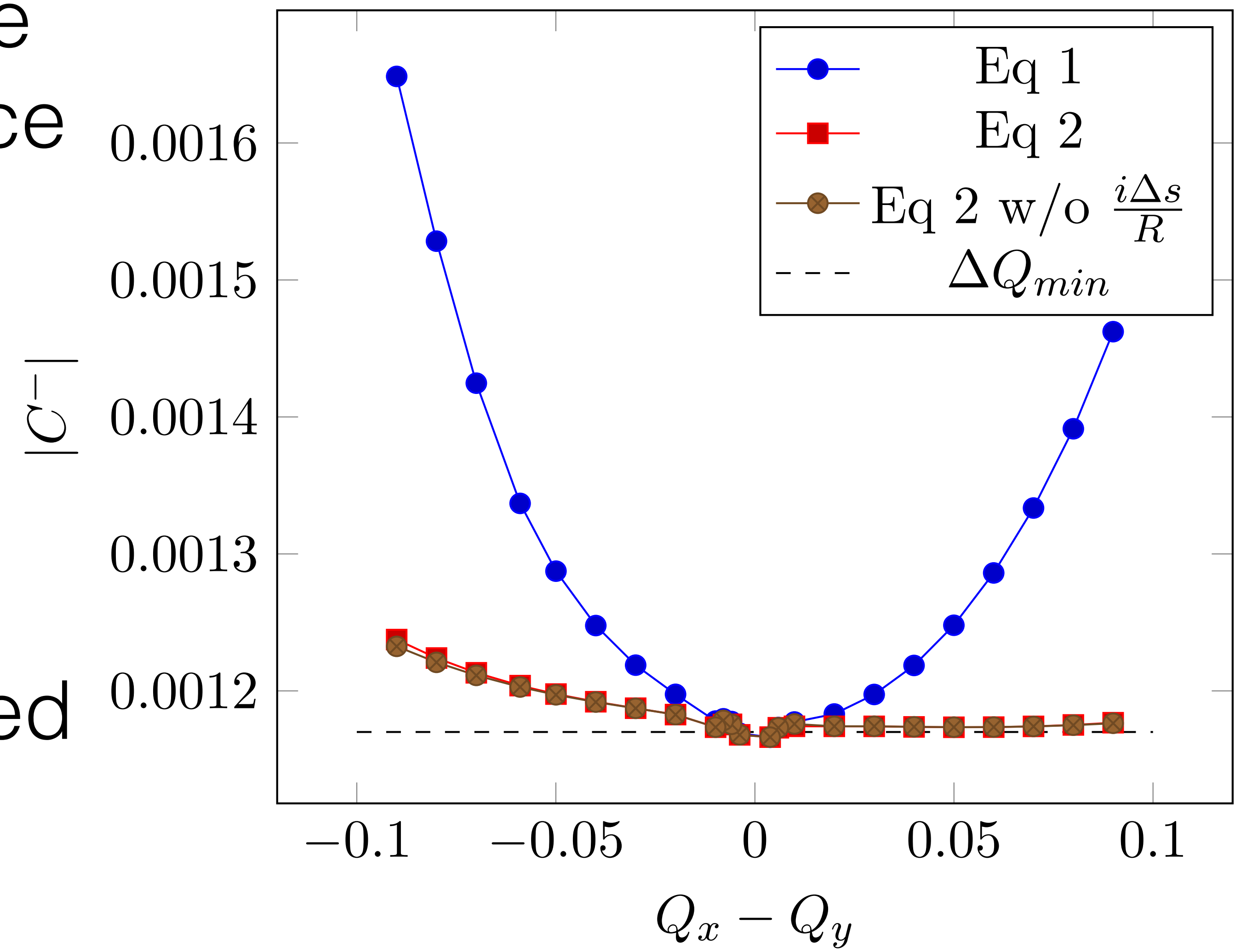


Betatron coupling control

- An approximative $|C^-|$ can be calculated from the resonance driving terms

$$\Delta Q_{min} = |C^-| \approx 4\Delta \frac{1}{N}\sum_{i=1}^{N} |f_{1001\,i}| \quad (1)$$

- A new improved formula (Eq(2)) has been implemented which better relates the f_{1001} to the $|C^-|$

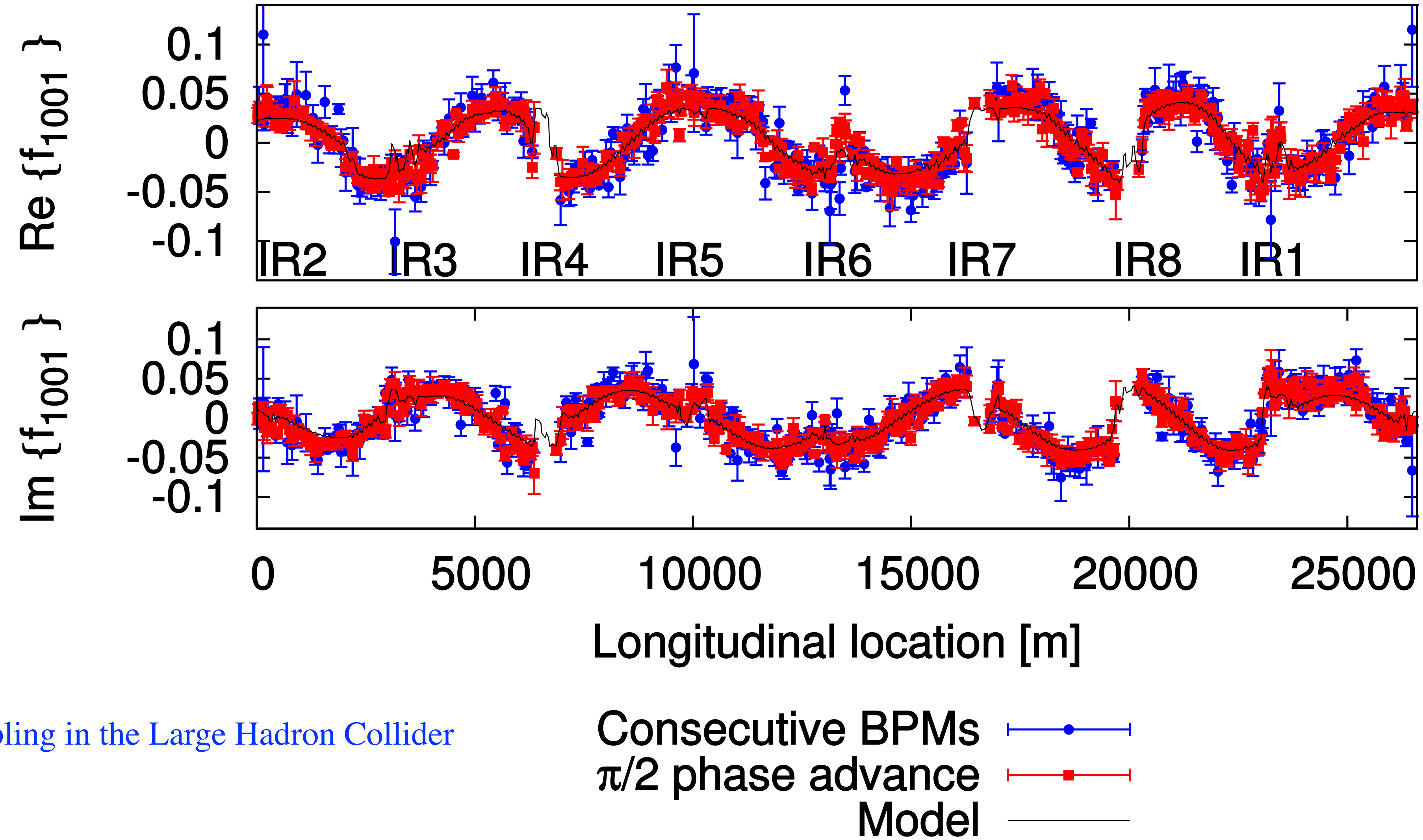


$$\Delta Q_{min} = |C^-| = \left| \frac{4\Delta}{2\pi R} \oint ds f_{1001} e^{i(\phi_x - \phi_y) + is\Delta/R} \right| (2)$$

Improved control of the betatron coupling in the Large Hadron Collider
T. Persson and R. Tomás
Phys. Rev. ST Accel. Beams 17, 051004

Betatron coupling control

- Two BPMs are used for deriving the resonance driving terms f_{1001} and f_{1010}
- Error propagation shows an optimum phase advance between both BPMs of $\pi/2$
- Pairing BPMs with optimal phase advance improves resolution by a factor of 3 for the LHC



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Summary

- LHC run at 6.5 TeV requires more precise optics measurements and corrections
- Detailed error analysis for systematic and random errors for β-measurement
- Re-analyzing 2012 data demonstrates better resolution
- Significant better resolution in coupling measurement

Thank you for your attention!