

Higgs-mass predictions in SUSY: MSSM vs. EFT and Implications for Theoretical Uncertainties

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with *G. Weiglein*

- Why we need a precise M_h^{MSSM} prediction
- MSSM diagrammatic/resummed calculations (FeynHiggs)
- EFT approach (SusyHD)
- Implications for theory uncertainties

Importance of precise M_h^{MSSM} predictions

The Higgs mass accuracy: experiment vs. theory:

Experiment:

$$\text{ATLAS:} \quad M_h^{\text{exp}} = 125.36 \pm 0.37 \pm 0.18 \text{ GeV}$$

$$\text{CMS:} \quad M_h^{\text{exp}} = 125.03 \pm 0.27 \pm 0.15 \text{ GeV}$$

$$\text{combined:} \quad M_h^{\text{exp}} = 125.09 \pm 0.21 \pm 0.11 \text{ GeV}$$

MSSM theory:

LHCHSWG adopted [FeynHiggs](#) for the prediction of MSSM Higgs boson masses and mixings (considered to be the code containing the most complete implementation of higher-order corrections)

$$\text{FeynHiggs:} \quad \delta M_h^{\text{theo}} \sim 3 \text{ GeV}$$

→ rough estimate, FeynHiggs contains algorithm to evaluate uncertainty, depending on parameter point

SusyHD: Higgs mass Determination in Supersymmetry

Javier Pardo Vega, Giovanni Villadoro

(Submitted on 20 Apr 2015)

We present the state-of-the-art of the effective field theory computation of the MSSM Higgs mass, improving the existing ones by including extra threshold corrections. We show that, with this approach, the theoretical uncertainty is within 1 GeV in most of the relevant parameter space. We confirm the smaller value of the Higgs mass found in the EFT computations, which implies a slightly heavier SUSY scale. We study the large $\tan(\beta)$ region, finding that sbottom thresholds might relax the upper bound on the scale of SUSY. We present SusyHD, a fast computer code that computes the Higgs mass and its uncertainty for any SUSY scale, from the TeV to the Planck scale, even in Split SUSY, both in the DRbar and in the on-shell schemes. Finally, we apply our results to derive bounds on some well motivated SUSY models, in particular we show how the value of the Higgs mass allows to determine the complete spectrum in minimal gauge mediation.

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⇒ We will try to give some answers: full diagrammatic vs. EFT, uncertainty estimates, ...

FeynHiggs approach (simplified): “from below”

Propagator / mass matrix with higher-order corrections

$$M_{hH}^2(q^2) = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ ($i, j = h, H$) : renormalized Higgs self-energies

⇒ complex roots of $\det(M_{hH}^2(q^2))$: $\mathcal{M}_{h_i}^2$ ($i = 1, 2$): $\mathcal{M}^2 = M^2 - iM\Gamma$

⇒ Feynman-diagrammatic approach

- diagrammatic calculation up to the **two-loop level**
- all MSSM particles contribute
main contribution: t/\tilde{t} sector
- all (possibly different) mass scales taken into account explicitly
- self-energies as building blocks for further evaluations

⇒ FeynHiggs provides consistent predictions for Higgs masses, Higgs couplings, Higgs BRs, ...

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To capture effects of large stop mass scales:

Resummation of leading and next-to-leading Logs from t/\tilde{t} sector:

$$L := \log\left(\frac{m_{\tilde{t}}}{m_t}\right) \Rightarrow \text{at } n\text{-loop order} : \sim \sum_{k=1}^n \alpha_s^k \alpha_t^{n-k} \times L^n, L^{n-1}$$

Assumes that all SUSY mass scales are high at $m_{\tilde{t}}$

\Rightarrow added consistently to the diagrammatic result

EFT approach (simplified / SusyHD): “from above”

- Assume all SUSY mass scales at one high scale M_S
Below M_S : only SM (“heavy particles integrated out”)
⇒ mass gap between EW scale and SUSY scale required
- EW scale input: SM parameter: $h_t(m_t), g_s(m_t), \dots$ at the 2-loop level
- SUSY enters only via threshold corrections at M_S at the 2-loop level
- Between EW scale m_t and SUSY scale M_S :
SM RGEs at the 3-loop level

$$\lambda(m_t), h_t(m_t), g_s(m_t), \dots \leftrightarrow \lambda(M_S)h_t(M_S), g_s(M_S), \dots$$

⇒ captured: logs of type L^n, L^{n-1}, L^{n-2}

- Evaluate running mass: $M_h^2 \sim 2\lambda(m_t)v^2$ ⇒ conversion to pole mass
- Log resummation in t/\tilde{t} sector: SusyHD: 3-loop , FeynHiggs: 2-loop

Ranges of applicability:

1.) SUSY mass scales below ~ 1 TeV require full calculation

2.) Log resum. for t/\tilde{t} (beyond 2L) at M_S

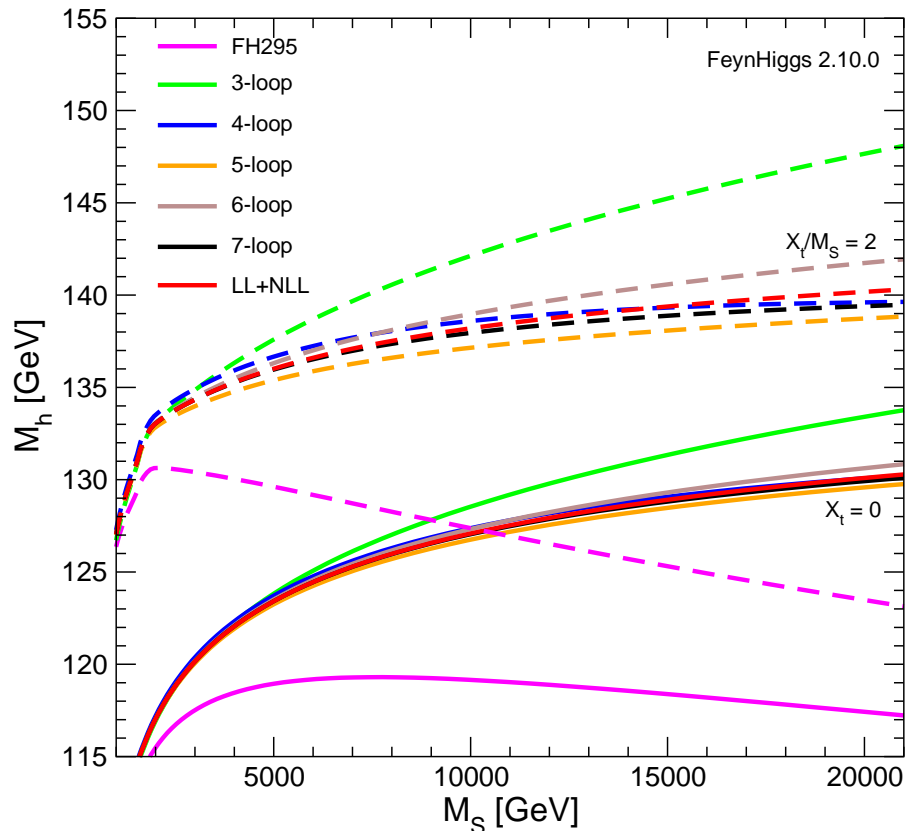
– effects at $M_S = 1$ TeV:

– at $M_S = 2$ TeV:

– at $M_S = 3$ TeV:

Ranges of applicability:

- 1.) SUSY mass scales below ~ 1 TeV require full calculation
- 2.) Log resum. for t/\tilde{t} (beyond 2L) at M_S for $\Delta M_h^{\text{diagrammatic}} \sim 40$ GeV
 - effects at $M_S = 1$ TeV: tiny
 - at $M_S = 2$ TeV: $\Delta M_h^{\text{log-resum}} \sim 2$ GeV
 - at $M_S = 3$ TeV: $\Delta M_h^{\text{log-resum}} \gtrsim 3$ GeV



$M_h(M_S)$ for various approximations:

[FeynHiggs 2.10.0]

magenta: no log-resum for t/\tilde{t}

red: log-resum at 2-loop level

(\rightarrow included in FH)

All other logs less relevant!

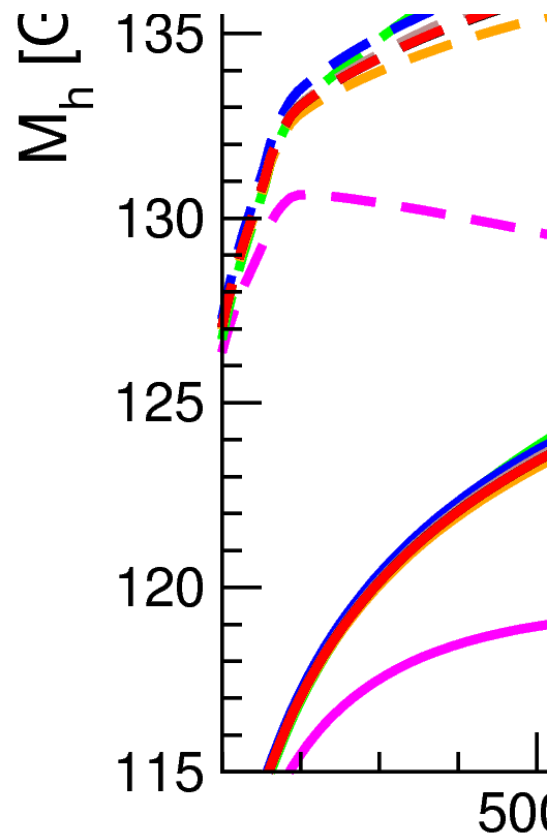
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- ⇒ FeynHiggs gives most reliable predictions for SUSY mass scales below the level of 2 – 3 TeV, where log contributions are not too large i.e. at the scales relevant/interesting for LHC physics (e.g. with light EW SUSY particles in the spectrum)
- ⇒ uncertainty estimate based on diagrammatic calculation reliable
- ⇒ EFT gives most reliable predictions for all SUSY mass scales in the multi-TeV range
- ⇒ intermediate region:
both types of calculations can be used for uncertainty estimate

Uncertainty estimates:

FeynHiggs (diagrammatic + log-resum): linear sum of

- missing 3-loop corrections in t/\tilde{t} sector (change of m_t def.)
 - missing 2-loop corrections in b/\tilde{b} sector (Δ_b resummation)
 - missing 2-loop corrections in EW sector (change of renormalization scale)
- ⇒ reliable estimate up to 2 – 3 TeV or higher

SusyHD (EFT): linear sum of

- SM unc.: missing corrections from matching at m_t and RGE evolution
- MSSM unc.: missing corrections from matching at M_S
- EFT unc.: effects not captured by EFT: $\mathcal{O}(v^2/M_S^2)$ (prefactor 1)

⇒ uncertainty estimate of ~ 1 GeV

⇒ estimate for the multi-TeV range

⇒ unclear to which low scales it can be extrapolated

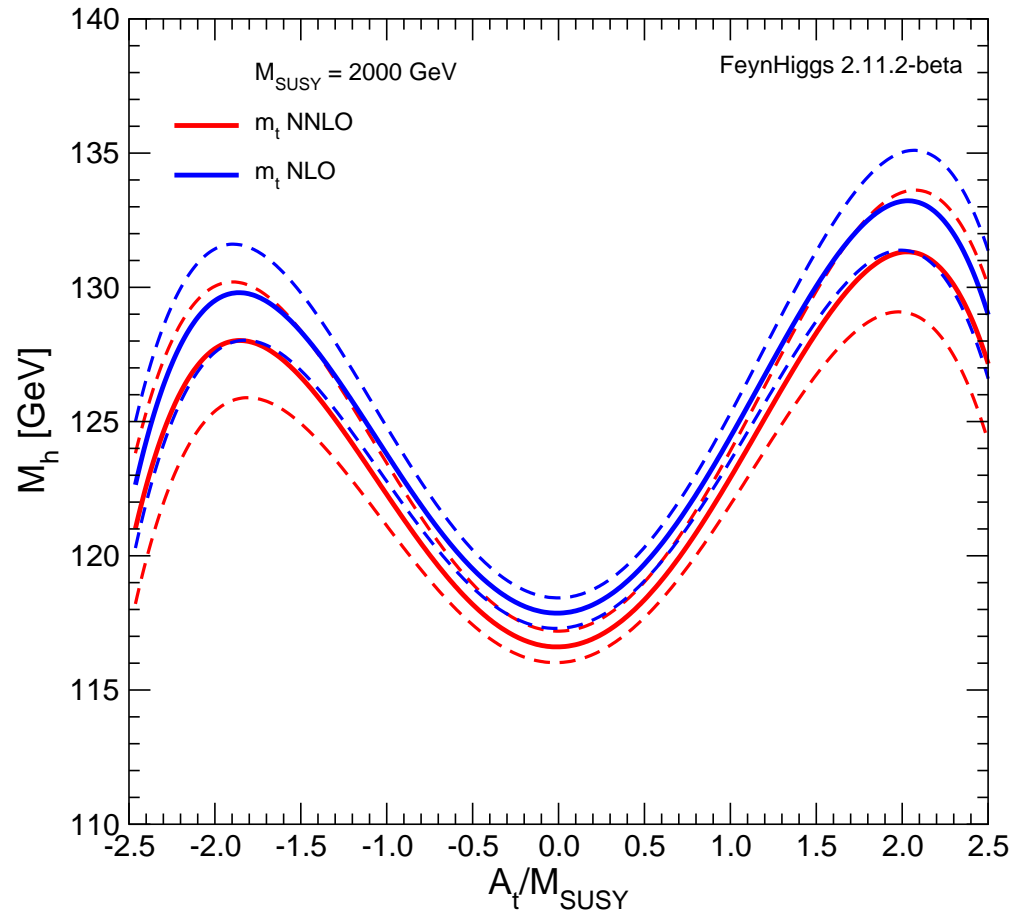
Uncertainty estimate in SusyHD:

- non-log terms via threshold corrections to $\lambda(M_S)$
 - neglects terms of $\mathcal{O}(m_t/M_{\text{SUSY}})$
 - non-log terms can be sizable, $\mathcal{O}(m_t/M_{\text{SUSY}})$ can easily be of $\mathcal{O}(\text{GeV})$
 - theoretical unc. in SusyHD for threshold terms via scale variation \Rightarrow does not capture non-log terms
- Effects of $m_t(m_t)^{\text{NLO}} \rightarrow m_t(m_t)^{\text{NNLO}}$ in SusyHD: 4 GeV
Same effects in [Draper, Lee, Wagner '13]: $\lesssim 2 \text{ GeV}$
Difference should give indication of theory uncertainties $\gtrsim 2 \text{ GeV}$
- Effects of uncertainty of higher-dimensional operators $\mathcal{O}(v^2/M_S^2)$
(prefactor 1) \Rightarrow very optimistic
- Not clear where (e.g. at $M_S = 2 \text{ TeV}$) differences between SusyHD and FeynHiggs originate from

SusyHD claim: large effects from top Yukawa coupling $m_t(m_t)$:

SusyHD: 2-loop, FeynHiggs: 1-loop (consistent choice!) $\Rightarrow \Delta \lesssim 9$ GeV

Shift in $m_t(m_t)$ from NLO to NNLO: 1.8 GeV (for $A_t/M_{\text{SUSY}} \sim 2$)



\Rightarrow not the reason for discrepancy, effects captured by FH unc. estimate

\Rightarrow EFT: effect of missing non-log contributions?

Uncertainty estimate too small ... ?!

Conclusions

- Logarithmic contributions to M_h become larger at $M_S = 2 - 3$ TeV
Largest logs from t/\tilde{t} sector
- Below ~ 2 TeV full MSSM spectrum covered by diagrammatic calculation up to the 2-loop level
- FeynHiggs: diagrammatic (up to 2-loop) plus resummed logs from t/\tilde{t}
 \Rightarrow reliable results (at least) up to 2 – 3 TeV
 \Rightarrow reliable uncertainty estimate
- EFT (one scale) approach better for all masses in the multi-TeV range
Not analyzed: down to which scale reliable
- SusyHD: uncertainty estimate below 2 – 3 TeV questionable

Back-up

\tilde{t} sector of the MSSM:

Stop mass matrices

$$M_{\tilde{t}}^2 = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + DT_{t_1} & m_t X_t \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix}$$

with

$$X_t = A_t - \mu / \tan \beta$$

⇒ mixing important in stop sector!

Simplifying abbreviation:

$$M_{\text{SUSY}} := M_{\tilde{t}_L} = M_{\tilde{t}_R}$$

Structure of higher-order corrections:

One-loop:

$$\Delta M_h^2 \sim m_t^2 \alpha_t [L + L^0] , \quad L := \log \left(\frac{m_{\tilde{t}}}{m_t} \right)$$

Two-loop:

$$\Delta M_h^2 \sim m_t^2 \left\{ \alpha_t \alpha_s [L^2 + L + L^0] + \alpha_t^2 [L^2 + L + L^0] \right\}$$

Three-loop:

$$\Delta M_h^2 \sim m_t^2 \left\{ \begin{aligned} &\alpha_t \alpha_s^2 [L^3 + L^2 + L + L^0] \\ &+ \alpha_t^2 \alpha_s [L^3 + L^2 + L + L^0] \\ &+ \alpha_t^3 [L^3 + L^2 + L + L^0] \end{aligned} \right\}$$

Partial results: [S. Martin '07]

[R. Harlander, P. Kant, L. Mihaila, M. Steinhauser '08] \Rightarrow H3m

H3m adds $\mathcal{O}(\alpha_t \alpha_s^2)$ corrections to FeynHiggs

Large $m_{\tilde{t}}$ \Rightarrow large L \Rightarrow resummation of logs necessary \Rightarrow Method II

Method II: Log resummation via RGE's:

Excellent recent overview paper: [*P. Draper, G. Lee, C. Wagner, arXiv:1312.5743*]

Simple example for log resummation:

SUSY mass scale: $M_{\text{SUSY}} = M_S \sim m_{\tilde{t}}$

Above M_{SUSY} : MSSM

Below M_{SUSY} : SM

Relevant SM parameters: – quartic coupling λ
– top Yukawa coupling h_t ($\alpha_t = h_t^2/(4\pi)$)
– strong coupling constant g_s ($\alpha_s = g_s^2/(4\pi)$)

Procedure (as in FeynHiggs):

1. Take: $h_t(m_t), g_s(m_t)$

SM RGEs for h_t, g_s : $h_t, g_s(m_t) \rightarrow h_t, g_s(M_S)$

2. Take $\lambda(M_S), h_t(M_S), g_s(M_S)$

SM RGEs for λ, h_t, g_s : $\lambda, h_t, g_s(M_S) \rightarrow \lambda, h_t, g_s(m_t)$

3. Evaluate M_h^2

$$M_h^2 \sim 2\lambda(m_t)v^2$$

Resumming Stop-sector Contributions

M_S  we know: $\lambda(M_S)$

m_t  we want: $M_h^2 = 2 \lambda(m_t) v^2$



Resumming Stop-sector Contributions

M_S ← we know: $\lambda(M_S)$

no SUSY particles

assumed here

SM running

m_t ← we want: $M_h^2 = 2 \lambda(m_t) v^2$



Standard Model RGEs

$$g_s^{2'} = \frac{1}{16\pi^2} g_s^4 (G_1 + \frac{1}{16\pi^2} G_2),$$

$$G_1 = \frac{2}{3} N_f - 11,$$

$$G_2 = (\frac{38}{3} N_f - 102) g_s^2 - 2h_t^2,$$

$$h_t^{2'} = \frac{1}{16\pi^2} h_t^2 (H_1 + \frac{1}{16\pi^2} H_2),$$

$$H_1 = \frac{9}{2} h_t^2 - 8g_s^2,$$

$$H_2 = 6h_t^2(6g_s^2 - 2h_t^2 - \lambda) + \frac{3}{2}\lambda^2 + (\frac{40}{9}N_f - \frac{404}{3})g_s^4,$$

$$\lambda' = \frac{1}{16\pi^2} \frac{1}{2} (\Lambda_1 + \frac{1}{16\pi^2} \Lambda_2),$$

$$\Lambda_1 = 12(\lambda^2 - h_t^4 + \lambda h_t^2),$$

$$\Lambda_2 = h_t^2(3h_t^2(20h_t^2 - \lambda) + g_s^2(80\lambda - 64h_t^2) - 72\lambda^2) - 78\lambda^3.$$

Espinosa, Quiros 1991 – Arason et al. 1992



FeynHiggs Update – p.5

Integrating the RGEs

M_S

start from

g_s^2, h_t^2 from 1L RGE (analytic)

$$\lambda(M_S) = \frac{3}{8\pi^2} h_t^4 \left(\frac{X_t}{M_S}\right)^2 \left(1 - \frac{1}{12} \left(\frac{X_t}{M_S}\right)^2\right)$$

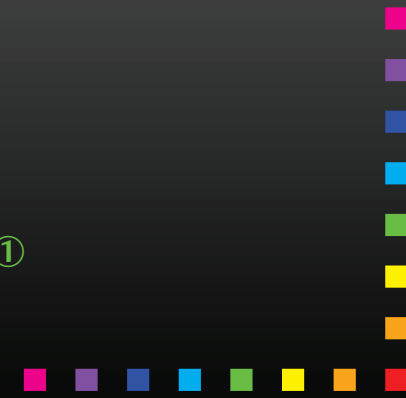
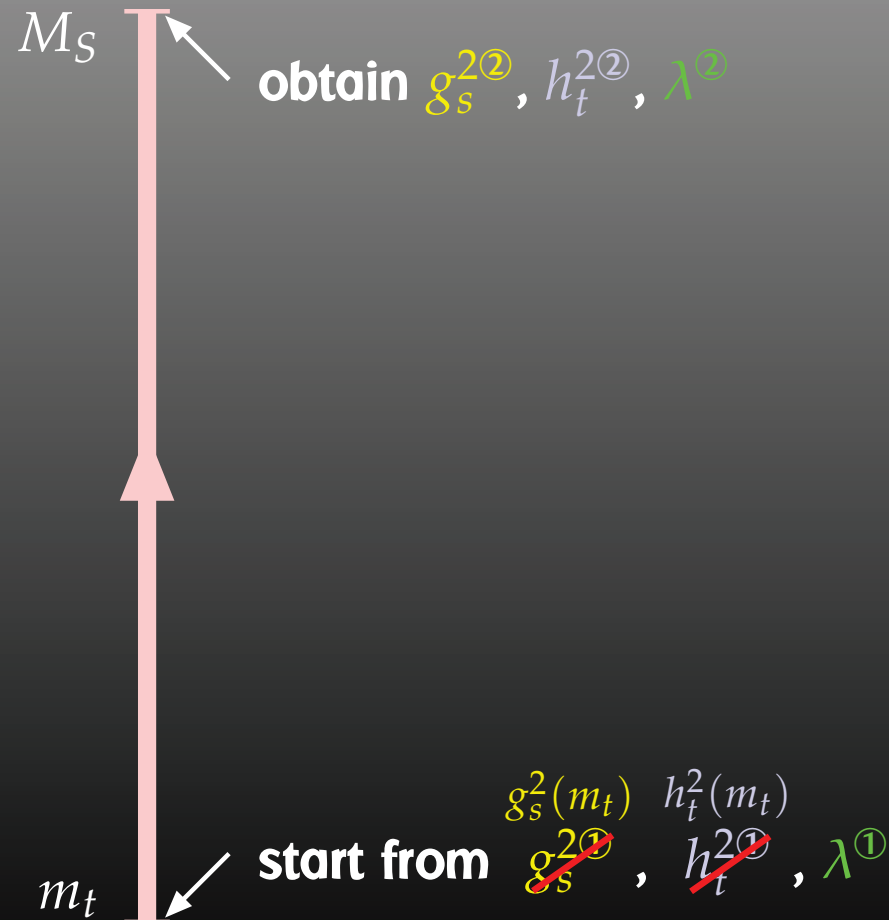
Carena et al. 2000

m_t

obtain $g_s^{2\textcircled{1}}, h_t^{2\textcircled{1}}, \lambda^{\textcircled{1}}$



Integrating the RGEs



Integrating the RGEs

M_S start with $g_s^{2(2)}$, $h_t^{2(2)}$, ~~$\lambda^{2(2)}$~~
 $\lambda(M_S)$

must subtract double counting!

m_t obtain $g_s^{2(3)}$, $h_t^{2(3)}$, $\lambda^{(3)} = \lambda(m_t)$



The best of both worlds:

to get the most precise prediction of M_h :

Combination of FD and RGE result!

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to get the most precise prediction of M_h :

Combination of FD and RGE result!

Problem:

Some terms exist in both calculations!

One-loop:

$$\Delta M_h^2 \sim m_t^2 \alpha_t [L + L^0] , \quad L := \log \left(\frac{m_{\tilde{t}}}{m_t} \right)$$

Two-loop:

$$\Delta M_h^2 \sim m_t^2 \left\{ \alpha_t \alpha_s [L^2 + L] + \alpha_t^2 [L^2 + L] \right\}$$

Combination of FD and RGE result:

- ⇒ to avoid double counting:
subtract leading and subleading logs at one- and two-loop

Problem:

- FD result with $X_t^{\text{OS}}, M_S^{\text{OS}}, \overline{m}_t$
- RGE result with $X_t^{\overline{\text{MS}}}, M_S^{\overline{\text{MS}}}, \overline{m}_t$

$$\overline{m}_t = \frac{m_t^{\text{pole}}}{1 + \frac{4}{3\pi}\alpha_s(m_t^{\text{pole}}) - \frac{1}{2\pi}\alpha_t(m_t^{\text{pole}})}$$

$$X_t^{\overline{\text{MS}}} = X_t^{\text{OS}} \left[1 + 2L \left(\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi} \left(1 - \frac{X_t^2}{M_S^2} \right) \right) \right]$$

$$M_S^{\overline{\text{MS}}} \sim M_S^{\text{OS}} : \text{ no log differences!}$$

Combination of FD and RGE result:

$$\Delta M_h^2 = (\Delta M_h^2)^{\text{RGE}}(X_t^{\overline{\text{MS}}}, M_S^{\overline{\text{MS}}}, \overline{m}_t) - (\Delta M_h^2)^{\text{FD,LL1,LL2}}(X_t^{\text{OS}}, M_S^{\text{OS}}, \overline{m}_t)$$

$$M_h^2 = (M_h^2)^{\text{FD}} + \Delta M_h^2$$

Technical aspect:

$$\begin{aligned} & (\Delta M_h^2)^{\text{FD,LL1,LL2}}(X_t^{\text{OS}}, M_S^{\text{OS}}, \overline{m}_t) \\ & := (\Delta M_h^2)^{\text{FD,LL1,LL2}}(X_t^{\overline{\text{MS}}}, M_S^{\overline{\text{MS}}}, \overline{m}_t) \Big|_{X_t^{\overline{\text{MS}}} \rightarrow X_t^{\text{OS}}, M_S^{\overline{\text{MS}}} = M_S^{\text{OS}}} \end{aligned}$$

⇒ combination of best FD result with

resummed LL, NLL corrections for large $m_{\tilde{t}}$

⇒ most precise M_h prediction for large $m_{\tilde{t}}$

⇒ FeynHiggs 2.10.0

Advantages of Feynman-diagrammatic method:

- all contributions at fixed order are captured
- trivial to include many SUSY scales
- full control over Higgs boson self-energies
 - needed for other quantities (production and decay)

Problems of Feynman-diagrammatic method:

- always only fixed order
- large logs not captured beyond the calculated order

Advantages of RGE log resummation:

- large logs taken into account to all orders
- calculation can easily be extended to very large scales

Problems of RGE log resummation:

- **not all** contributions at fixed order are captured
 - sub-leading logs more difficult
 - momentum dependence
- difficult (impossible?): include many different SUSY scales
- difficult (impossible?): control over Higgs boson self-energies
 - needed for other quantities (production and decay)

Parameters:

$$M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$$

$$M_A = 1000 \text{ GeV}$$

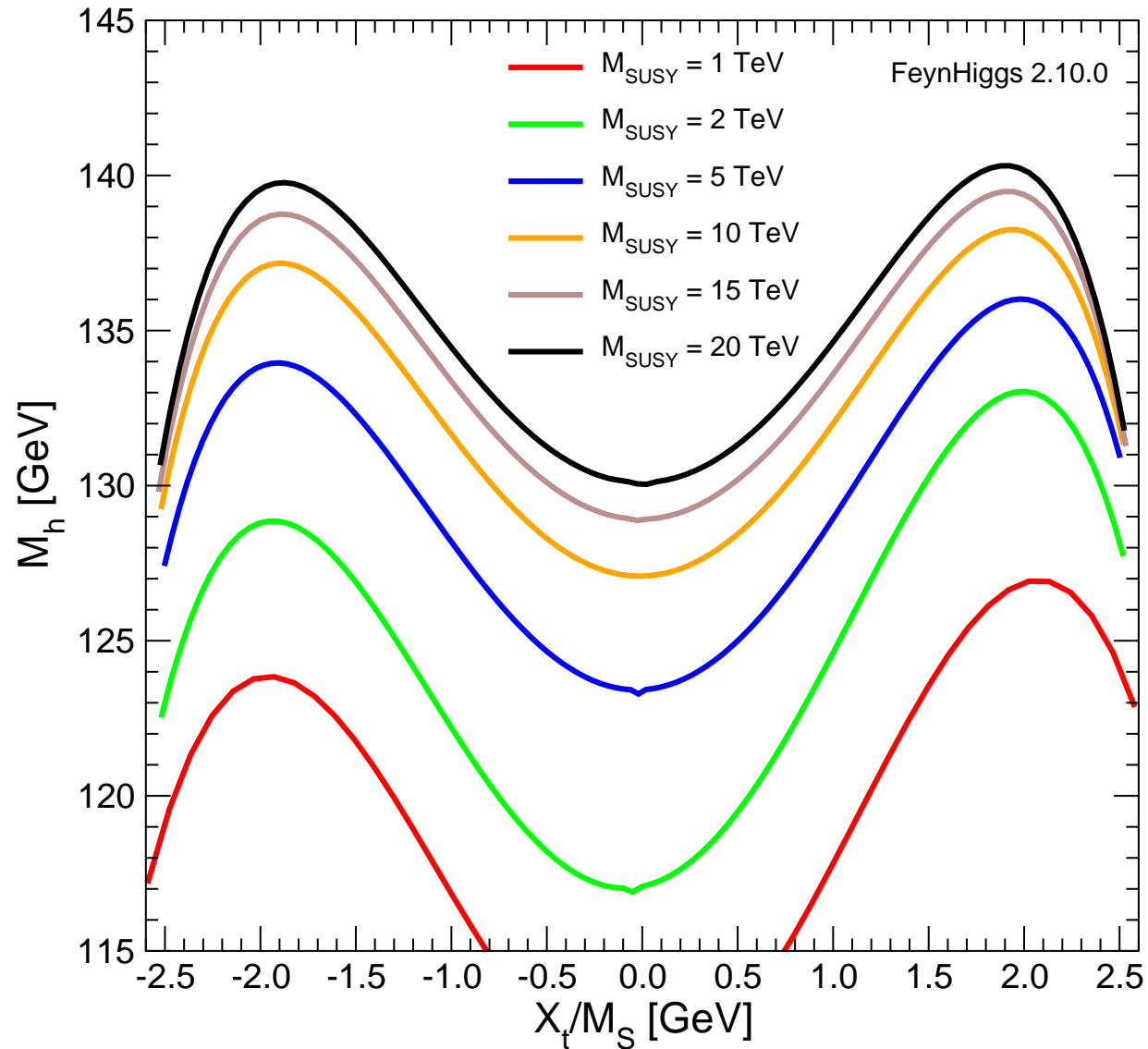
$$\mu = 1000 \text{ GeV}$$

$$M_2 = 1000 \text{ GeV}$$

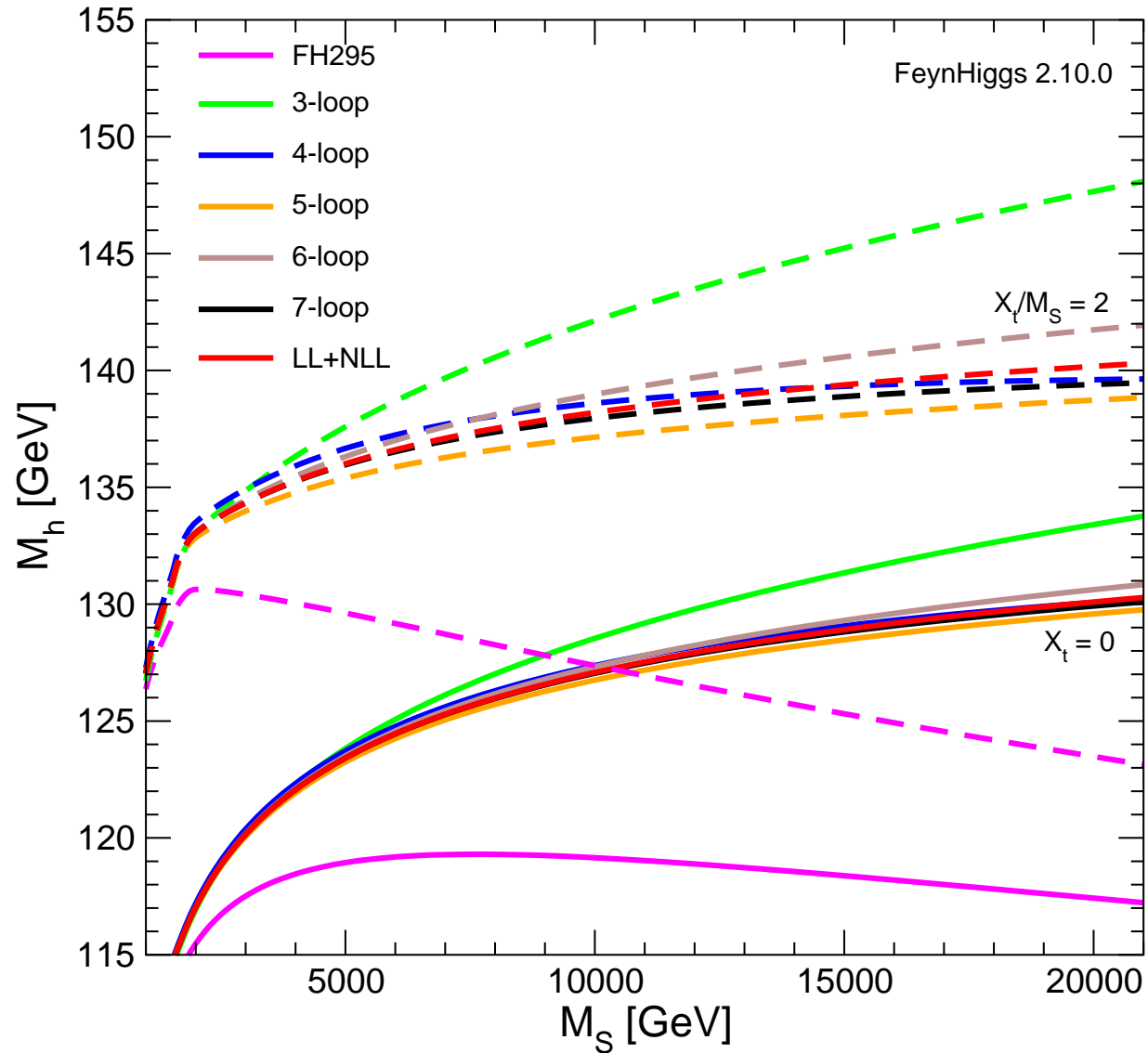
$$m_{\tilde{g}} = 1600 \text{ GeV}$$

$$\tan \beta = 10$$

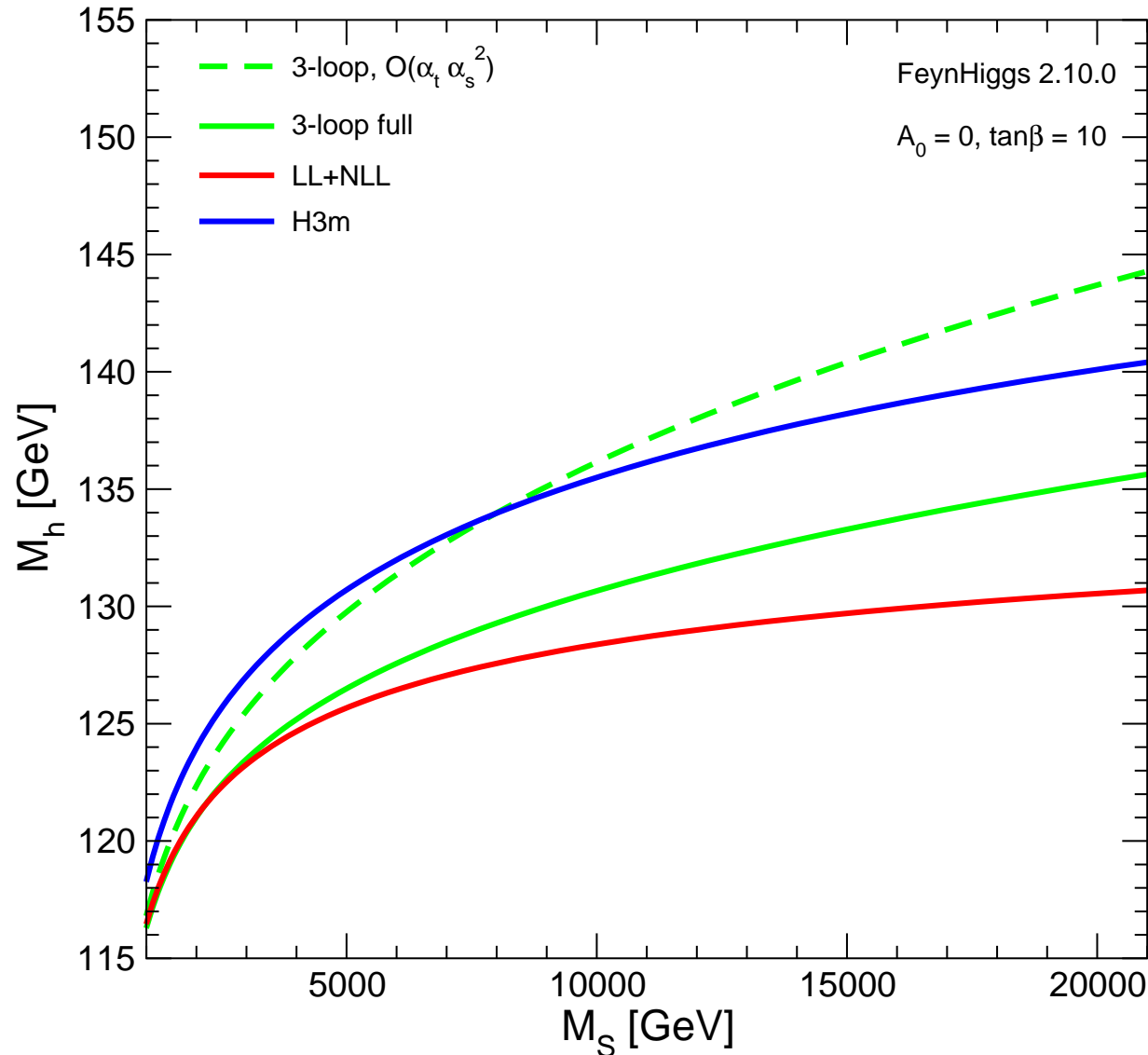
Vary M_S , X_t to analyze effects



\Rightarrow increase with M_S , maxima at $X_t/M_S = \pm 2$



\Rightarrow 3-loop good for $M_S \lesssim 2$ TeV, 7-loop: $\Delta \sim 1$ GeV for $M_S = 20$ TeV



\Rightarrow 3-loop $\mathcal{O}(\alpha_t^2 \alpha_s, \alpha_t^3) \oplus$ beyond 3-loop important for precise M_h prediction!