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LHC Higgs Cross Section Working Group 2 (Higgs Properties)

Higgs Basis: Proposal for an EFT basis choice for LHC HXSWG

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1 Introduction

The LHC Higgs Cross Section Working Group is focused on various steps of the analysis chain:

$$\textbf{Data} \to \textbf{Pseudo-observables} \to \textbf{Model-independent EFT} \to \textbf{BSM Models}\,.$$

This note concerns model-independent interpretations of the data in the framework of effective field theory (EFT) beyond the Standard Model (SM), which is a part of the scope of the Working Group 2. The purpose of this note is to propose a common EFT language and conventions that could be universally used in LHC Higgs analyses and be implemented in numerical tools.

In the EFT approach, the basic assumption is that the mass scale Λ of new particles in the UV theory beyond the SM is larger than the electroweak scale v, $\Lambda \gg v$. If this is the case, physics at energies $E \ll \Lambda$ can be parametrized by the SM Lagrangian supplemented by a set of higher-dimensional operators. These operators are constructed out of the SM fields, and respect the local $SU(3) \times SU(2) \times U(1)$ symmetry of the SM. The coefficients of $d > 4$-dimensional operators in the EFT Lagrangian are of order $1/\Lambda^{d-4}$, and their contribution to amplitudes of physical processes at the energy scale of order v scales[1] as $(v/\Lambda)^{d-4}$. The leading new physics effects are expected from operators with $d = 6$ whose effects scale as $(v/\Lambda)^2$ (all dimension-5 operators violate the lepton number; experimental constraints dictate that their coefficients must be suppressed at the level unobservable at the LHC). Since $(v/\Lambda)^2 < 1$ by construction, EFT is suitable to describe *small* deviations from the SM predictions, except for observables that vanish or are suppressed by small parameters in the SM.

[1] Apart from the scaling with Λ, the effects of higher-dimensional operators also scale with appropriate powers of couplings in the UV theory. The latter may be important to assess the validity range of the EFT description.

An *operator basis* is a complete, non-redundant set of dimension-6 operators. Complete means that any dimension-6 operator is either a part of the basis, or can be obtained from a combination of operators in the basis using equations of motion, integration by parts, field redefinitions, and Fierz transformations. Non-redundant means it is a minimal such set. Any basis leads to the same physical predictions concerning possible new physics effects. Several bases have been proposed in the literature, and they may be convenient for specific applications. In this note we propose a basis that is particularly convenient for LHC Higgs analyses.

Preparing this proposal, we have taken into account the following guidelines:

- The formulation should be simple enough that it can be used by people not acquainted with the nuts and bolts of EFTs.
- The relationship between parameters of the EFT and (pseudo)-observables should be transparent.
- The constraints on EFT parameters from electroweak precision observables should be easy to impose.
- The formalism should be easily implementable in Monte-Carlo codes.
- The formalism should be flexible enough, such that, in the future, the application scope may be extended beyond the original one. In particular, the formalism should be applicable outside Higgs physics and allow one to also combine non-LHC data.
- A connection to the pseudo-observables in the *extended kappa formalism* should be straightforward.
- Limits of the EFT validity range should be easy to define.
- The formalism should be well suited to include higher-order QCD and electroweak corrections.

The salient features of our proposal are the following:

- We restrict ourselves to EFT with dimension-6 operators in the *linear* formulation of electroweak symmetry breaking. This means that, much as the SM, the theory contains the Higgs field H in the doublet representation of the SM SU(2) group. The Lagrangian is invariant under the local $SU(3) \times SU(2) \times U(1)$, and the $SU(2) \times U(1) \to U(1)$ electroweak symmetry breaking is b the vacuum expectation value (VEV) of the field H.
- In the spirit of Ref. [1], we proceed with a classification of the operators that more easily map to independent interaction terms of the SM mass eigenstates, in particular the W, Z, and the Higgs boson. Such interaction terms are invariant under $SU(3) \times U(1)$ color and electromagnetic symmetry, but they do not necessarily correspond to $SU(2)$-invariant operators. However, they allow us to identify a set of *independent couplings* from which a complete basis of $SU(2)$-invariant terms is constructed. We denote the latter the *Higgs basis*. The advantage of this formulation is that the effective couplings are related in a simpler way to quantities observable in experiments, compared to other proposals.

- We choose the independent couplings such that the constraints from the Z and W partial decay widths (measured with a per-mille precision by the LEP experiment) can be easily incorporated. These are among the most stringent constraints on EFT parameters, and they have an important impact on possible signals in Higgs searches. It is unlikely that, at any point in the future, the precision of LHC Higgs searches will be such that the couplings constrained by LEP can be probed by the LHC with a comparable accuracy. Therefore it is recommended that the the electroweak constraints on Z and W boson couplings to fermions are always imposed when analyzing LHC data, especially on Higgs physics. Other precision observables, such as WW production or off-shell fermion scattering, lead to less stringent constraints that are not discussed in this note (see e.g. [2, 3, 4] for a recent discussion).

- The disadvantage of the Higgs basis is that the operator list is cumbersome, being defined by the identification of a set of independent interaction terms after electroweak symmetry breaking. For this reason, we also map the Higgs basis to a set of manifestly $SU(3) \times SU(2) \times U(1)$ invariant operators before electroweak symmetry breaking. For the latter, in this note we use operators in the *Warsaw basis* of Ref. [5] and in the *SILH basis* of Ref. [6], but it is straightforward to work out a map to any other basis used in the literature. Working with $SU(3) \times SU(2) \times U(1)$ invariant operators may be more convenient for certain calculations (for example, when renormalization group running of the Wilson coefficients needs to be calculated).

- We do not demand that the dimension-6 operators are flavor blind. While generic constraints on flavor violation are strong, it is plausible that there is a large hierarchy between the coefficients of dimension-6 operators corresponding to different fermion generations. In particular, many models predict the coefficients of operators involving the 3rd generation to be much larger than those involving the first two generations. Keeping the more general approach will allow us to obtain much more robust constraints on new physics.

- We allow CP violating operators to be present in our basis. In particular, we discuss the most general set of Higgs couplings to matter that include CP violating couplings.

- We assume that dimension-6 operators conserve the baryon and lepton number.

In Section 2, to define our notation and conventions, we write down the Standard Model (SM) Lagrangian. In Section 3 we introduce an effective Lagrangian summarizing the new interactions of the SM mass eigenstates that arise in the presence of dimension-6 operators beyond the SM. The mapping between the couplings in that effective Lagrangian and Wilson coefficients of $SU(3) \times SU(2) \times U(1)$ invariant dimension-6 operators in the Warsaw basis is worked out in Section 4. In Section 5 we define the Higgs basis, which is spanned by a subset of the independent couplings of the effective Lagrangian.

2 Standard Model Lagrangian

The SM Lagrangian in our notation takes the form

$$\begin{aligned}\mathcal{L}^{\rm SM} &= -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} + D_\mu H^\dagger D_\mu H + \mu_H^2 H^\dagger H - \lambda(H^\dagger H)^2 \\ &+ \sum_{f\in q,\ell} i\bar{f}_L\gamma_\mu D_\mu f_L + \sum_{f\in u,d,e} i\bar{f}_R\gamma_\mu D_\mu f_R \\ &- \left[\tilde{H}^\dagger \bar{u}_R y_u q_L + H^\dagger \bar{d}_R y_d V_{\rm CKM}^\dagger q_L + H^\dagger \bar{e}_R y_e \ell_L + {\rm h.c.}\right]. \end{aligned} \tag{2.1}$$

Here, G^a_μ, W^i_μ, and B_μ denote the gauge fields of the $SU(3)\times SU(2)\times U(1)$ local symmetry. The corresponding gauge couplings are denoted by g_s, g, g'; we also define the electromagnetic coupling $e = gg'/\sqrt{g^2+g'^2}$, and the Weinberg angle $s_\theta = g'/\sqrt{g^2+g'^2}$. The field strength tensors are defined as $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^b_\mu G^c_\nu$, $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk}W^j_\mu W^k_\nu$, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The Higgs doublet is denoted as H, and we also define $\tilde{H}_i = \epsilon_{ij}H^*_k$. It acquires the VEV $\langle H^\dagger H\rangle = v^2/2$. In the unitary gauge we have $H = (0, (v+h)/\sqrt{2})$, where h is the Higgs boson field. After electroweak symmetry breaking, the electroweak gauge boson mass eigenstates are defined as $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$, $Z = c_\theta W^3 - s_\theta B$, $A = s_\theta W^3 + c_\theta B$, where $c_\theta = \sqrt{1-s_\theta^2}$. The tree-level masses of W and Z bosons are given by $m_W = gv/2$, $m_Z = \sqrt{g^2+g'^2}v/2$. The left-handed Dirac fermions $q_L = (u_L, V_{\rm CKM}d_L)$ and $\ell_L = (\nu_L, e_L)$ are doublets of the SU(2) gauge group, and the right-handed Dirac fermions u_R, d_R, e_R are SU(2) singlets. All fermions are 3-component vectors in the generation space, and y_f are 3×3 matrices. We work in the basis where the fermion mass matrix is diagonal with real, positive entries. In this basis, y_f are diagonal, and the fermion masses are given by $m_{f_i} = v[y_f]_{ii}/\sqrt{2}$.

For later convenience, we explicitly write down the mass terms:

$$\mathcal{L}^{\rm SM}_{\rm mass} = \frac{g^2v^2}{4}W^+_\mu W^-_\mu + \frac{(g^2+g'^2)v^2}{8}Z_\mu Z_\mu + \sum_{f\in u,d,e} m_f \bar{f}f, \tag{2.2}$$

the gauge boson couplings to fermions:

$$\begin{aligned}\mathcal{L}^{\rm SM}_{vff} &= eA_\mu \sum_{f\in u,d,e} Q_f \bar{f}\gamma_\mu f + g_s G^a_\mu \sum_{f\in u,d} \bar{f}\gamma_\mu T^a f, \\ &+ \frac{g}{\sqrt{2}}\left(W^+_\mu \bar{u}_L\gamma_\mu V_{\rm CKM} d_L + W^+_\mu \bar{\nu}_L\gamma_\mu e_L + {\rm h.c.}\right) \\ &+ \sqrt{g^2+g'^2}Z_\mu \sum_{f\in u,d,e,\nu}\left(T^3_f \bar{f}_L\gamma_\mu f_L - s_\theta^2 Q_f \bar{f}\gamma_\mu f\right), \end{aligned} \tag{2.3}$$

the couplings of a single Higgs boson to gauge bosons and fermions:

$$\mathcal{L}^{\rm SM}_h = \frac{h}{v}\left[\frac{g^2v^2}{2}W^+_\mu W^-_\mu + \frac{(g^2+g'^2)v^2}{4}Z_\mu Z_\mu\right] - \frac{h}{v}\sum_f m_f\bar{f}f \tag{2.4}$$

the couplings involving two or more gauge bosons

$$\mathcal{L}^{\rm SM}_{hh} = \frac{h^2}{2v^2}\left[\frac{g^2v^2}{2}W^+_\mu W^-_\mu + \frac{(g^2+g'^2)v^2}{4}Z_\mu Z_\mu\right] - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4, \tag{2.5}$$

and the triple and quartic self-interactions of the vector bosons:

$$\begin{aligned}\mathcal{L}^{\rm SM}_{\rm tgc} &= ie\left[\left(W^+_{\mu\nu}W^-_\mu - W^-_{\mu\nu}W^+_\mu\right)A_\nu + A_{\mu\nu}\,W^+_\mu W^-_\nu\right]\\ &+ igc_\theta\left[\left(W^+_{\mu\nu}W^-_\mu - W^-_{\mu\nu}W^+_\mu\right)Z_\nu + Z_{\mu\nu}\,W^+_\mu W^-_\nu\right]\\ &- g_s f^{abc}\partial_\mu G^a_\nu G^b_\mu G^c_\nu. \end{aligned} \tag{2.6}$$

$$\begin{aligned}\mathcal{L}^{\rm SM}_{\rm qgc} &= \frac{g^2}{2}\left(W^+_\mu W^+_\mu W^-_\nu W^-_\nu - W^+_\mu W^-_\mu W^+_\nu W^-_\nu\right) + g^2 c^2_\theta\left(W^+_\mu Z_\mu W^-_\nu Z_\nu - W^+_\mu W^-_\mu Z_\nu Z_\nu\right)\\ &+ g^2 s^2_\theta\left(W^+_\mu A_\mu W^-_\nu A_\nu - W^+_\mu W^-_\mu A_\nu A_\nu\right)\\ &+ g^2 c_\theta s_\theta\left(W^+_\mu Z_\mu W^-_\nu A_\nu + W^+_\mu A_\mu W^-_\nu Z_\nu - 2W^+_\mu W^-_\mu Z_\nu A_\nu\right)\\ &- g^2_s f^{abc} f^{ade} G^b_\mu G^c_\nu G^d_\mu G^e_\mu. \end{aligned} \tag{2.7}$$

These couplings depend on just 5 input parameters: g_s, g, g', m_h and v. The Higgs boson mass m_h has been precisely measured at the LHC, while the strong coupling constant is extracted from jet production data. The remaining 3 parameters are customarily derived from the observable Fermi constant G_F (more precisely, from the measured muon lifetime $\tau_\mu = 192\pi^3/G_F^2 m_\mu^5$), Z boson mass m_Z, and the low-energy electromagnetic coupling $\alpha(0)$. The tree-level relations between the input observables and the electroweak parameters are given by:

$$G_F = \frac{1}{\sqrt{2}v^2}, \qquad \alpha = \frac{g_L^2 g_Y^2}{4\pi(g_L^2 + g_Y^2)}, \qquad m_Z = \frac{\sqrt{g_L^2 + g_Y^2}\,v}{2}. \tag{2.8}$$

3 Effective Lagrangian

In this section we introduce an effective Lagrangian describing interactions of Higgs and matter mass eigenstates when the SM is extended by dimension-6 operators. The Lagrangian is of the form

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{\rm SM} + \Delta\mathcal{L}. \tag{3.1}$$

Here, $\mathcal{L}^{\rm SM}$ is the SM Lagrangian introduced in Section 2, $\Delta\mathcal{L}$ contains new interactions beyond the SM. The effect of the new interactions is either to shift the coupling strength away from the SM predictions or to introduce a new tensor structure of interactions that is absent in the SM Lagrangian. In particular, these interactions are relevant to describe new physics effects in precisions tests of the SM and in Higgs searches at the LHC. Each term in $\Delta\mathcal{L}$ Lagrangian may be generated by dimension-6 operators beyond the SM, thus each coupling is $\mathcal{O}(\Lambda^{-2})$ in the EFT expansion. However, at this point, we do not yet define the relations between various couplings that are required by the linearly realized electroweak symmetry at the level of dimension-6 operators. Therefore, the couplings of the effective Lagrangian do *not* span a dimension-6 basis. Later in Section 5 we will write down the relations between different couplings and define a dimension-6 basis. We stress that $\mathcal{L}_{\rm eff}$ is intended to be used in the framework of the dimension-6 EFT Lagrangian; if it is used in a different context, care should be taken to define a consistent expansion (akin to the $1/\Lambda$ expansion in the EFT).

The effective Lagrangian $\mathcal{L}_{\rm eff}$ has the following features:

- All kinetic terms of SM mass eigenstates are canonically normalized. In particular, there is no kinetic mixing between the Z boson and the photon.

- Tree-level relations between the electroweak parameters and input observables are the same as the SM ones in Eq. (2.8). In particular, the photon and the gluon interact with fermions as in Eq. (2.3), and there is no correction to the Z boson mass term.

- Two-derivative self-interactions of the Higgs boson are absent.

- For each fermion pair, the coefficient of the vertex-like Higgs interaction term $\delta g \frac{h}{v} V_\mu \bar{f} \gamma_\mu f$ is equal to the

In general, dimension-6 operators can induce corrections to the Lagrangian that do not respect these features. However, all 4 above features can always be achieved, *without any loss of generality*, by using equations of motion, integrating by parts, and redefining the fields and couplings. The required set of transformation starting from the Warsaw basis will be presented in Section 4.

To facilitate presentation, we split ΔL into the following parts,

$$\Delta\mathcal{L} = \Delta\mathcal{L}_{\rm mass} + \Delta\mathcal{L}_{\rm vertex} + \mathcal{L}_{\rm dipole} + \Delta\mathcal{L}_{\rm tgc} + \Delta\mathcal{L}_{\rm qgc} + \Delta\mathcal{L}_{\rm h} + \mathcal{L}_{hvff} + \mathcal{L}_{hdvff} + \Delta\mathcal{L}_{h^2} + \mathcal{L}_{\rm other}. \tag{3.2}$$

Below we define each term in order of appearance.

3.1 Quadratic terms

By construction, there is no corrections to quadratic terms of the SM mass eigenstates with the exception of the shift of the W boson mass in Eq. (2.2):

$$\Delta\mathcal{L}_{\rm mass} = 2\delta m \frac{g^2 v^2}{4} W_\mu^+ W_\mu^- . \tag{3.3}$$

3.2 Gauge boson interactions with fermions

Two types of corrections to the SM gauge boson interactions with fermions may be introduced by dimension-6 operators. One is the so-called *vertex corrections*, which are shift the W and Z couplings to fermions away from the SM Lagrangian of Eq. (2.3):

$$\begin{aligned}\Delta\mathcal{L}_{\rm vertex} &= \frac{g}{\sqrt{2}}\left(W_\mu^+ \bar{\nu}_L \gamma_\mu \delta g_L^{W\ell} e_L + W_\mu^+ \bar{u}\gamma_\mu \delta g_L^{Wq} V_{\rm CKM} d_L + W_\mu^+ \bar{u}_R \gamma_\mu \delta g_R^{Wq} d_R + \text{h.c.}\right) \\ &+ \sqrt{g^2+g'^2} Z_\mu \left[\sum_{f\in u,d,e,\nu} \bar{f}_L \gamma_\mu \delta g_L^{Zf} f_L + \sum_{f\in u,d,e} \bar{f}_R \gamma_\mu \delta g_R^{Zf} f_R\right], \end{aligned} \tag{3.4}$$

where all the δg are 3×3 Hermitian matrices in the generation space, except for δg_R^{Wq} which is a general 3×3 complex matrix.

The other type are the dipole interactions between the gauge boson and fermions, which are not present in the SM Lagrangian. We parametrize them as follows:

$$\begin{aligned}\mathcal{L}_{\rm dipole} &= -\frac{1}{4v}\left[g_s\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a d_{Gf}fG^a_{\mu\nu}+e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{Af}fA_{\mu\nu}\right.\\ &\left.+\sqrt{g^2+g'^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{Zf}fZ_{\mu\nu}+\sqrt{2}g\left(\bar{d}\sigma_{\mu\nu}d_{Wq}uW^-_{\mu\nu}+{\rm h.c.}\right)\right]\\ &-\frac{1}{4v}\left[g_s\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a \tilde{d}_{Gf}f\widetilde{G}^a_{\mu\nu}+e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{Af}f\widetilde{A}_{\mu\nu}\right.\\ &\left.+\sqrt{g^2+g'^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{Zf}f\widetilde{Z}_{\mu\nu}+\sqrt{2}g\left(\bar{d}\sigma_{\mu\nu}\tilde{d}_{Wq}u\widetilde{W}^-_{\mu\nu}+{\rm h.c.}\right)\right], \quad (3.5)\end{aligned}$$

where $\sigma_{\mu\nu}=i[\gamma_\mu,\gamma_\nu]/2$, and all the d_{Vf} and $\widetilde{d}_{Vf}$ are Hermitian 3×3 matrices.

3.3 Gauge boson self-interactions

These couplings are defined via cubic interactions of gauge bosons, in addition to the SM ones in Eq. (2.6):

$$\begin{aligned}\Delta\mathcal{L}_{\rm tgc} &= ie\left[\delta\kappa_\gamma A_{\mu\nu}\,W^+_\mu W^-_\nu+\tilde{\kappa}_\gamma\tilde{A}_{\mu\nu}\,W^+_\mu W^-_\nu\right]\\ &+ igc_\theta\left[\delta g_{1,z}\left(W^+_{\mu\nu}W^-_\mu-W^-_{\mu\nu}W^+_\mu\right)Z_\nu+\delta\kappa_z\,Z_{\mu\nu}\,W^+_\mu W^-_\nu+\tilde{\kappa}_z\,\tilde{Z}_{\mu\nu}\,W^+_\mu W^-_\nu\right]\\ &+ i\frac{e}{m_W^2}\left[\lambda_\gamma W^+_{\mu\nu}W^-_{\nu\rho}A_{\rho\mu}+\tilde{\lambda}_\gamma W^+_{\mu\nu}W^-_{\nu\rho}\tilde{A}_{\rho\mu}\right]+i\frac{gc_\theta}{m_W^2}\left[\lambda_z W^+_{\mu\nu}W^-_{\nu\rho}Z_{\rho\mu}+\tilde{\lambda}_z W^+_{\mu\nu}W^-_{\nu\rho}\tilde{Z}_{\rho\mu}\right]\\ &+ \frac{c_{3G}}{v^2}g_s^3 f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}+\frac{\tilde{c}_{3G}}{v^2}g_s^3 f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}, \quad (3.6)\end{aligned}$$

The couplings of electroweak gauge bosons follow the customary parametrization of Ref. [9].

$$
\begin{aligned}
\Delta\mathcal{L}_{\rm qgc} &= \delta g_{W^4}\frac{g^2}{2}\left(W^+_\mu W^+_\mu W^-_\nu W^-_\nu - W^+_\mu W^-_\mu W^+_\nu W^-_\nu\right) \\
&+ \delta g_{W^2Z^2} g^2 c_\theta^2 \left(W^+_\mu Z_\mu W^-_\nu Z_\nu - W^+_\mu W^-_\mu Z_\nu Z_\nu\right) \\
&+ \delta g_{W^2Z\gamma} g^2 c_\theta s_\theta \left(W^+_\mu Z_\mu W^-_\nu A_\nu + W^+_\mu A_\mu W^-_\nu Z_\nu - 2W^+_\mu W^-_\mu Z_\nu A_\nu\right) \\
&- \frac{g^2}{2}\frac{\lambda_{W^4}}{m_W^2}\left(W^+_{\mu\nu}W^-_{\nu\rho} - W^-_{\mu\nu}W^+_{\nu\rho}\right)\left(W^+_\mu W^-_\rho - W^-_\mu W^+_\rho\right) \\
&- g^2 c_\theta^2 \frac{\lambda_{W^2Z^2}}{m_W^2}\left[W^+_\mu\left(Z_{\mu\nu}W^-_{\nu\rho} - W^-_{\mu\nu}Z_{\nu\rho}\right)Z_\rho + W^-_\mu\left(Z_{\mu\nu}W^+_{\nu\rho} - W^+_{\mu\nu}Z_{\nu\rho}\right)Z_\rho\right] \\
&- e^2 \frac{\lambda_{W^2A^2}}{m_W^2}\left[W^+_\mu\left(A_{\mu\nu}W^-_{\nu\rho} - W^-_{\mu\nu}A_{\nu\rho}\right)A_\rho + W^-_\mu\left(A_{\mu\nu}W^+_{\nu\rho} - W^+_{\mu\nu}A_{\nu\rho}\right)A_\rho\right] \\
&- egc_\theta \frac{\lambda_{W^2AZ}}{m_W^2}\left[W^+_\mu\left(A_{\mu\nu}W^-_{\nu\rho} - W^-_{\mu\nu}A_{\nu\rho}\right)Z_\rho + W^-_\mu\left(A_{\mu\nu}W^+_{\nu\rho} - W^+_{\mu\nu}A_{\nu\rho}\right)Z_\rho\right] \\
&- egc_\theta \frac{\lambda_{W^2ZA}}{m_W^2}\left[W^+_\mu\left(Z_{\mu\nu}W^-_{\nu\rho} - W^-_{\mu\nu}Z_{\nu\rho}\right)A_\rho + W^-_\mu\left(Z_{\mu\nu}W^+_{\nu\rho} - W^+_{\mu\nu}Z_{\nu\rho}\right)A_\rho\right] \\
&+ 3g_s^3\frac{c_{4G}}{v^2} f^{abc}f^{cde}G^a_{\mu\nu}G^b_{\nu\rho}G^d_\rho G^e_\mu + \text{CP odd},
\end{aligned}
\tag{3.7}
$$

where CP odd stands for analogous terms with $\lambda_z \to \tilde{\lambda}_z$, $c_{4G} \to \tilde{c}_{4G}$, and one of the field strength tensor replaced by the dual one.

3.4 Single Higgs couplings

This part is the most relevant one from the point of view of the LHC Higgs phenomenology. First, we define the following single Higgs boson couplings to a pair of the SM fields:

$$
\begin{aligned}
\Delta\mathcal{L}_{\rm h} &= \frac{h}{v}\left[2\delta c_w m_W^2 W^+_\mu W^-_\mu + \delta c_z m_Z^2 Z_\mu Z_\mu\right. \\
&- \frac{h}{v}\sum_{f\in u,d,e}\sum_{ij}\sqrt{m_{f_i}m_{f_j}}[\delta y_f]_{ij}\left[\cos\phi^f_{ij}\bar{f}_i f_j - i\sin\phi^f_{ij}\bar{f}_i\gamma_5 f_j\right]. \\
&\quad + c_{ww}\frac{g^2}{2}W^+_{\mu\nu}W^-_{\mu\nu} + \tilde{c}_{ww}\frac{g^2}{2}W^+_{\mu\nu}\tilde{W}^-_{\mu\nu} + c_{w\Box}g^2\left(W^-_\mu\partial_\nu W^+_{\mu\nu} + \text{h.c.}\right) \\
&\quad + c_{gg}\frac{g_s^2}{4}G^a_{\mu\nu}G^a_{\mu\nu} + c_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}A_{\mu\nu} + c_{z\gamma}\frac{eg}{2c_\theta}Z_{\mu\nu}A_{\mu\nu} + c_{zz}\frac{g^2}{4c_\theta^2}Z_{\mu\nu}Z_{\mu\nu} \\
&\quad + c_{z\Box}g^2 Z_\mu\partial_\nu Z_{\mu\nu} + c_{\gamma\Box}gg' Z_\mu\partial_\nu A_{\mu\nu} \\
&\quad \left. + \tilde{c}_{gg}\frac{g_s^2}{4}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + \tilde{c}_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{z\gamma}\frac{eg}{2c_\theta}Z_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{zz}\frac{g^2}{4c_\theta^2}Z_{\mu\nu}\tilde{Z}_{\mu\nu}\right].
\end{aligned}
\tag{3.8}
$$

The terms in the first two lines shift the SM couplings in Eq. (2.4), while the remaining terms introduce Higgs couplings to matter with a tensor structure that is absent in the SM Lagrangian. Here $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$, and $\tilde{X}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\partial_\rho X_\sigma$. Note that, using equations of motion, we could get rid of certain 2-derivative interactions between the Higgs and gauge bosons: $hZ_\mu\partial_\nu Z_{\nu\mu}$, $hZ_\mu\partial_\nu A_{\nu\mu}$, and $hW^\pm_\mu\partial_\nu W^\mp_{\nu\mu}$. These interactions would then be traded for contact interactions of the Higgs, gauge bosons and fermions in Eq. (3.9). However, one of the defining features of our effective Lagrangian is that the coefficients of the latter couplings are equal to the corresponding vertex correction in Eq. (3.4). This form can be always obtained, without any loss of generality, starting from an arbitrary dimension-6 Lagrangian provided the 2-derivative $hV_\mu\partial_\nu V_{\nu\mu}$ are kept in the Lagrangian.

Next, couplings of the Higgs boson to a gauge field and two fermions, which are not present in the SM Lagrangian, may be generated by dimension-6 operators. We define the following vertex-like contact interactions between the Higgs, electroweak gauge bosons, and fermions:

$$\begin{aligned}\mathcal{L}_{hvff} &= \sqrt{2}g\frac{h}{v}W_\mu^+\left(\bar{u}_L\gamma_\mu\delta g_L^{hWq}V_{\rm CKM}d_L + \bar{u}_R\gamma_\mu\delta g_R^{hWq}d_R + \bar{\nu}_L\gamma_\mu\delta g_L^{hW\ell}e_L\right) + \text{h.c.}\\ &+ 2\frac{h}{v}\sqrt{g^2+g'^2}Z_\mu\left[\sum_{f=u,d,e,\nu}\bar{f}_L\gamma_\mu\delta g_L^{hZf}f_L + \sum_{f=u,d,e}\bar{f}_R\gamma_\mu\delta g_R^{hZf}f_R\right], \qquad (3.9)\end{aligned}$$

As indicated before, we demand the coefficients of these interaction to be equal to the corresponding vertex correction in Eq. (3.4):

$$\delta g^{hZf} = \delta g^{Zf}, \qquad \delta g^{hWf} = \delta g^{Wf}. \qquad (3.10)$$

In addition, we also define the following dipole-type contact interactions of the Higgs boson:

$$\begin{aligned}\mathcal{L}_{\rm hdvff}^{D=6} &= -\frac{h}{4v^2}\left[g_s\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a d_{hGf}fG_{\mu\nu}^a + e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{hAf}fA_{\mu\nu}\right.\\ &\left.+\sqrt{g_L^2+g_Y^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{hZf}fZ_{\mu\nu} + \sqrt{2}g_L\left(\bar{d}\sigma_{\mu\nu}d_{hWq}uW_{\mu\nu}^- + \text{h.c.}\right)\right]\\ &-\frac{h}{4v^2}\left[\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a\tilde{d}_{hGf}f\widetilde{G}_{\mu\nu}^a + e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{hAf}f\widetilde{A}_{\mu\nu}\right.\\ &\left.+\sqrt{g_L^2+g_Y^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{hZf}f\widetilde{Z}_{\mu\nu} + \sqrt{2}g_L\left(\bar{d}\sigma_{\mu\nu}\tilde{d}_{hWq}u\widetilde{W}_{\mu\nu}^- + \text{h.c.}\right)\right] \qquad (3.11)\end{aligned}$$

3.5 Couplings of two or more Higgs bosons

To describe double Higgs production via gluon fusion ($gg \to hh$) at the LHC we need, apart from a subset of the single Higgs couplings introduced in Section 3.4, the following interactions with two or more Higgs bosons:

$$\Delta\mathcal{L}_{hh}^{D=6} = -\delta\lambda_3 vh^3 + \frac{h^2}{v^2}\frac{g_s^2}{8}\left(c_{gg}^{(2)}G_{\mu\nu}^aG_{\mu\nu}^a + \tilde{c}_{gg}^{(2)}G_{\mu\nu}^a\tilde{G}_{\mu\nu}^a\right) - \frac{h^2}{2v^2}\sum_{f;ij}\sqrt{m_{f_i}m_{f_j}}\left[\bar{f}_{i,R}[y_f^{(2)}]_{ij}f_{j,L} + \text{h.c.}\right]. \qquad (3.12)$$

Other couplings with two Higgs bosons are present in ΔL. Specifically, these are the couplings h^2VV to the SM electroweak gauge bosons, and h^2ffV contact interactions. As these do not play the role in the double Higgs production processes currently studied at the LHC, we do not display them here.

3.6 Other terms

In the subsections above we wrote down interactions terms in the effective Lagrangian that are relevant for SM precisions tests and for Higgs searches at the LHC. The remaining terms, which are not explicitly displayed in this note, are contained in $\mathcal{L}_{\rm other}$. The include 4-fermion terms, corrections quartic and higher Higgs boson self-interactions, self-interactions of more than 4 vector bosons, interactions of 2 or more Higgs bosons with SM matter, couplings of a single Higgs boson to 3 or more gauge bosons. Currently, these terms are relevant neither for SM precision tests nor for single and double Higgs production and decay at the LHC. If there's phenomenological interest, any of the terms in $\mathcal{L}_{\rm other}$ can be explicitly written down in this note.

4 Mapping Effective Lagrangian to Warsaw Basis of Dimension-6 Operators

We turn to discussing the map between the couplings of the effective Lagrangian introduced in Section 3 and Wilson coefficients of dimension-6 operators in the electroweak basis before electroweak symmetry breaking. The complete set of dimension-6 operators can be written in many different equivalent bases which are related by the use of equations of motion and integration by parts. Here we work with the so-called *Warsaw basis* of Ref. [5, 10], which is distinguished by the simplest tensor structure of the higher-dimensional operators. The analogous procedure can be applied to other bases: see Appendix A.1 for the map between the effective Lagrangian and the SILH basis.

The Lagrangian in the Warsaw basis is given by[2]

$$\mathcal{L}_{\rm warsaw} = \mathcal{L}^{\rm SM} + \frac{1}{\Lambda^2}\sum_i \hat{c}_i O_i, \tag{4.1}$$

where the SM Lagrangian $\mathcal{L}^{\rm SM}$ was introduced in Section 2, Λ is the mass scale of new particles, O_i are the dimension-6 operators in the Warsaw basis summarized in Table 1, can $\hat{c}_i$ is the Wilson coefficient multiplying the operator O_i. The scale Λ appears explicitly to emphasize this is the EFT expansion parameter, and Eq. (4.1) contains the zeroth- and the first-order term in this expansion. However, observables calculated in the EFT depend only on the combination $\hat{c}_i/\Lambda^2$. Therefore, working with the low-energy EFT, it is more convenient to redefine $\hat{c}_i \to c_i \Lambda^2/v^2$. In the following we will display all the formula using the redefine Wilson coefficients c_i.

To map the Wilson coefficients of dimension-6 operators in the Warsaw basis to the couplings in the effective Lagrangian we need first to bring $\mathcal{L}_{\rm warsaw}$ into the same form as $\mathcal{L}_{\rm eff}$ in Eq. (3.1). This can be achieved by a series of transformations using equations of motion, integration by parts, and rescaling of the fields and couplings. To begin with,

[2] We use a different notation than the original reference. We also replaced the operator $|H^\dagger D_\mu H|^2$ by $(H^\dagger D_\mu H - D_\mu H^\dagger H)^2$. For Yukawa-type operators O_f we subtracted v^2 so that these operators do not contribute to off-diagonal mass terms. This way we avoid tedious rotations of the fermion fields to bring them back to the mass eigenstate basis. Starting with the Yukawa couplings $-H \bar{f}'_R (Y'_f + c'_f H^\dagger H/v^2) f'_L$ we can bring them to the form in Eq. (2.1) and Table 1 by defining $f'_{L,R} = U_{L,R} f_{L,R}$, $c_f = U_R^\dagger c'_f U_L$, $Y_f = U_R^\dagger (Y'_f + c'_f/2) U_L$, where $U_{L,R}$ are unitary rotations to the mass eigenstate basis.

the operator O_{WB} leads to a kinetic mixing between the hypercharge and SU(2) gauge bosons, $O_{WB} \to -1/2gg'W^3_{\mu\nu}B_{\mu\nu}$. To get rid of it, we use the equations of motion:

$$\begin{aligned}\partial_\nu B_{\nu\mu} &= g'\frac{(v+h)^2}{4}\left(gW^3_\mu - g'B_\mu\right) - g'j^Y_\mu,\\ \partial_\nu W^3_{\nu\mu} &= -g\frac{(v+h)^2}{4}\left(gW^3_\mu - g'B_\mu\right) - gj^3_\mu - g\epsilon^{3jk}W^j_\nu W^k_{\nu\mu},\end{aligned} \tag{4.2}$$

where $j^Y_\mu = \sum_f Y_f \bar{f}\gamma_\mu f$, and $j^3_\mu = \bar{q}\gamma_\mu T^3 P_L q + \bar{\ell}\gamma_\mu T^3 P_L \ell$. Using this,

$$\begin{aligned}-c_{WB}\frac{gg'}{2}W^3_{\mu\nu}B_{\mu\nu} &\to c_{WB}e^2\left[\frac{(v+h)^2}{4}\left(gW^3_\mu - g'B_\mu\right)^2 - gW^3_\mu j^Y_\mu - g'B_\mu j^3_\mu\right.\\ &\quad\left.-\frac{g^2}{2g'}\epsilon^{3jk}W^j_\mu W^k_\nu B_{\mu\nu} - g'\epsilon^{3jk}B_\mu W^j_\nu W^k_{\nu\mu}\right]\\ &= c_{WB}e^2\left[\frac{(g^2+g'^2)(v+h)^2}{4}Z^2_\mu - eA_\mu j^{\rm em}_\mu + \sqrt{g^2+g'^2}Z_\mu\left(j^3_\mu - c^2_\theta j^{\rm em}_\mu\right)\right]\\ &+ ic_{WB}\frac{g^2g'}{(g^2+g'^2)^{3/2}}\left[g^2(gA_{\mu\nu} - g'Z_{\mu\nu})W^+_\mu W^-_\nu\right.\\ &\quad\left.-g'^2(gA_\mu - g'Z_\mu)(W^+_{\mu\nu}W^-_\nu - W^-_{\mu\nu}W^+_\nu)\right],\end{aligned} \tag{4.3}$$

where $j^{\rm em}_\mu = j^3_\mu + j^Y_\mu$ is the electromagnetic current. Next, the operators O_{BB}, O_{WW}, and O_{GG} change the normalization of the kinetic terms of the gauge bosons. To recover the canonical normalization we redefine the gauge fields as

$$B_\mu \to B_\mu\left(1 + \frac{c_{BB}g'^2}{4}\right), \quad W^i_\mu \to W^i_\mu\left(1 + \frac{c_{WW}g^2}{4}\right), \quad G^a_\mu \to G^a_\mu\left(1 + \frac{c_{GG}g^2_s}{4}\right). \tag{4.4}$$

We ignore here the contribution of the operator $\tilde{O}_{GG}$ to the QCD θ-term (we can always assume it cancels agains the θ-term in the SM Lagrangian, or is dynamically removed by an axion field). The operator O_H changes the normalization of the Higgs boson kinetic term, and also induces Higgs boson self-interactions that contain two derivatives. To recover the canonical normalization and remove the 2-derivative self-interactions we redefine the Higgs field as

$$h \to h\left(1 - c_H - \frac{h}{v}c_H - \frac{h^2}{3v^2}c_H\right). \tag{4.5}$$

The relation between the Higgs VEV v_0 and the mass parameter in the SM Lagrangian is affected by the O_{6H} operator:

$$v_0^2 = \frac{\mu_H^2}{\lambda}\left(1 + \frac{3}{4\lambda}c_{6H}\right), \tag{4.6}$$

while the relation between Higgs boson mass and the quartic coupling in the SM Lagrangian is affected by both O_{6H} and O_H:

$$m_h^2 = 2v_0^2\left(\lambda - 2c_H\lambda - \frac{3}{2}c_{6H}\right). \tag{4.7}$$

We have to make sure that the gauge couplings and the Higgs VEV have the same meaning as in the SM. In other words, the relation between the couplings and the observables employed to determine them This is a non-trivial requirement, because dimension-6 operators affect the observables used to extract these parameters. We have seen that the operator O_{WB} shifts the electric charge and the Z boson mass. Similarly, the operator O_T shifts the Z boson mass term. Furthermore, one of the $O_{\ell\ell}$ operators leads to the 4-fermion coupling $v^{-2}[c_{\ell\ell}]_{1221}(\bar{\nu}_{\mu,L}\gamma_\rho\nu_{e,L})(\bar{e}_L\gamma_\rho\mu_L)$ that contributes to the muon decay at the linear level and thus shifts the Fermi constant. Finally, the leptonic vertex operator $O_{H\ell}$ also shifts the Fermi constant. To undo these effects, we need to ensure that the photon and the gluon couple to the electromagnetic and strong currents as in Eq. (2.3). Furthermore, the Z boson mass term in the Lagrangian should be as in Eq. (2.2), and the tree-level $\mu \to e\bar{\nu}_e\nu_\mu$ decay width should be given by $\Gamma = \frac{m_\mu^5}{384\pi^3 v^4}$. This is achieved by the following redefinition of the coupling constants and the VEV:

$$
\begin{aligned}
g_s &\to g_s \left(1 - c_{GG}\frac{g_s^2}{4}\right), \\
g &\to g \left(1 - c_{WW}\frac{g^2}{4} - c_{WB}\frac{g^2 g'^2}{g^2 - g'^2} + (c_T - \delta v)\frac{g^2}{g^2 - g'^2}\right), \\
g' &\to g' \left(1 - c_{BB}\frac{g'^2}{4} + c_{WB}\frac{g^2 g'^2}{g^2 - g'^2} - (c_T - \delta v)\frac{g'^2}{g^2 - g'^2}\right), \\
v_0 &\to v\,(1 + \delta v)\,, \qquad (4.8)
\end{aligned}
$$

where $\delta v = ([c'_{H\ell}]_{11} + [c'_{H\ell}]_{22})/2 - [c_{\ell\ell}]_{1221}/4$.

One last transformation is needed to match the Higgs basis. At this point, the coefficients of the contact interactions in Eq. (3.9) differ from the vertex corrections by flavor universal terms depending only on the electric charge and the isospin of the fermions. It is possible to get rid of the latter using equations of motion for the gauge bosons, so as to traded them into zero- and two-derivative Higgs boson interactions with gauge bosons of the form $hV_\mu V_\mu$ and $hV_\mu\partial_\nu V_{\mu\nu}$.

After all these transformations the Lagrangian takes the same form as $\mathcal{L}_{\text{Higgs Basis}}$. The dictionary between the coefficients of dimension-6 operators and the independent and dependent couplings in $\mathcal{L}_{\text{Higgs Basis}}$ goes as follows. The shift of the W boson mass is given by

$$
\delta m = \frac{1}{g^2 - g'^2}\left[-g^2 g'^2 c_{WB} + g^2 c_T - g'^2 \delta v\right]. \qquad (4.9)
$$

The shift of W and Z boson couplings to leptons are given by

$$
\begin{aligned}
\delta g_L^{W\ell} &= c'_{H\ell} + f(1/2, 0) - f(-1/2, -1), \\
\delta g_L^{Z\nu} &= \frac{1}{2}c'_{H\ell} - \frac{1}{2}c_{H\ell} + f(1/2, 0), \\
\delta g_L^{Ze} &= -\frac{1}{2}c'_{H\ell} - \frac{1}{2}c_{H\ell} + f(-1/2, -1), \\
\delta g_R^{Ze} &= -\frac{1}{2}c_{He} + f(0, -1), \qquad (4.10)
\end{aligned}
$$

where

$$
f(T^3, Q) = I_3\left[-Qc_{WB}\frac{g^2 g'^2}{g^2 - g'^2} + (c_T - \delta v)\left(T^3 + Q\frac{g'^2}{g^2 - g'^2}\right)\right], \qquad (4.11)
$$

and I_3 is the 3×3 identity matrix. Vertex corrections to W and Z boson couplings to quarks are given by

$$
\begin{aligned}
\delta g_L^{Wq} &= c'_{Hq} + f(1/2, 2/3) - f(-1/2, -1/3), \\
\delta g_R^{Wq} &= -\frac{1}{2} c_{Hud}, \\
\delta g_L^{Zu} &= \frac{1}{2} c'_{Hq} - \frac{1}{2} c_{Hq} + f(1/2, 2/3), \\
\delta g_L^{Zd} &= -\frac{1}{2} c'_{Hq} - \frac{1}{2} c_{Hq} + f(-1/2, -1/3), \\
\delta g_R^{Zu} &= -\frac{1}{2} c_{Hu} + f(0, 2/3), \\
\delta g_R^{Zd} &= -\frac{1}{2} c_{Hd} + f(0, -1/3).
\end{aligned}
\tag{4.12}
$$

The coefficients of vertex-like contact interactions between the Higgs boson, W or Z boson, and two fermions in Eq. (3.9) are given by

$$
c^{Vf} = \delta g^{Vf}. \tag{4.13}
$$

The shifts of the Higgs couplings to W and Z are given by

$$
\begin{aligned}
\delta c_w &= -c_H - c_{WB} \frac{4g^2 g'^2}{g^2 - g'^2} + 4 c_T \frac{g^2}{g^2 - g'^2} - \delta v \frac{3g^2 + g'^2}{g^2 - g'^2}, \\
\delta c_z &= -c_H - 3\delta v.
\end{aligned}
\tag{4.14}
$$

The two-derivative Higgs couplings to gauge bosons are given by

$$
\begin{aligned}
c_{gg} &= c_{GG}, \qquad c_{gg}^{(2)} = c_{GG}, \\
c_{\gamma\gamma} &= c_{WW} + c_{BB} - 4c_{WB}, \\
c_{zz} &= \frac{g^4 c_{WW} + g'^4 c_{BB} + 4g^2 g'^2 c_{WB}}{(g^2 + g'^2)^2}, \\
c_{z\Box} &= -\frac{2}{g^2} \left(c_T - \delta v \right), \\
c_{z\gamma} &= \frac{g^2 c_{WW} - g'^2 c_{BB} - 2(g^2 - g'^2) c_{WB}}{g^2 + g'^2}, \\
c_{\gamma\Box} &= \frac{2}{g^2 - g'^2} \left((g^2 + g'^2) c_{WB} - 2c_T + 2\delta v \right), \\
c_{ww} &= c_{WW}, \\
c_{w\Box} &= \frac{2}{g^2 - g'^2} \left(g'^2 c_{WB} - c_T + \delta v \right).
\end{aligned}
\tag{4.15}
$$

and the same for the CP-odd couplings $\tilde{c}_{gg}$, $\tilde{c}_{\gamma\gamma}$, $\tilde{c}_{z\gamma}$, $\tilde{c}_{zz}$, $\tilde{c}_{ww}$, with $c \to \tilde{c}$ on the right hand side. The Yukawa interactions are given by

$$
\begin{aligned}
[\delta y_f]_{ij} \cos \phi_{ij}^f &= \frac{v \mathrm{Re}[c_f]_{ij}}{\sqrt{2 m_{f_i} m_{f_j}}} - \delta_{ij} \left(c_H + \delta v \right), \\
[\delta y_f]_{ij} \sin \phi_{ij}^f &= \frac{v \mathrm{Im}[c_f]_{ij}}{\sqrt{2 m_{f_i} m_{f_j}}}.
\end{aligned}
\tag{4.16}
$$

The coefficients of Yukawa-type interactions of two Higgs bosons with fermions in Eq. (3.12) are given by

$$[y_f^{(2)}]_{ij} = 3[\delta y_f]_{ij} e^{i\phi_{ij}} + (c_H + 3\delta v)\delta_{ij}. \tag{4.17}$$

The anomalous triple gauge couplings of electroweak gauge bosons are given by

$$\begin{aligned}
\delta g_{1,z} &= \frac{g^2+g'^2}{g^2-g'^2}\left(-g'^2 c_{WB} + c_T - \delta v\right), \\
\delta\kappa_\gamma &= g^2 c_{WB}, \\
\delta\kappa_z &= -2c_{WB}\frac{g^2 g'^2}{g^2-g'^2} + \frac{g^2+g'^2}{g^2-g'^2}\left(c_T - \delta v\right), \\
\lambda_\gamma &= -\frac{3}{2}g^4 c_{3W}, \\
\lambda_z &= -\frac{3}{2}g^4 c_{3W}, \\
\tilde\kappa_\gamma &= g^2 \tilde c_{WB}, \\
\tilde\kappa_z &= -g'^2 \tilde c_{WB}, \\
\tilde\lambda_\gamma &= -\frac{3}{2}g^4 \tilde c_{3W}, \\
\tilde\lambda_z &= -\frac{3}{2}g^4 \tilde c_{3W}.
\end{aligned} \tag{4.18}$$

The Higgs cubic interaction is given by

$$\delta\lambda_3 = -\lambda\left(3c_H + \delta v\right) - c_{6H}. \tag{4.19}$$

To summarize, in the Warsaw basis the Higgs boson couplings to matter and itself depend on linear combinations of the following Wilson coefficients:

$$\begin{gathered}
c_H,\ c_T,\ c_{GG},\ c_{WW},\ c_{BB},\ c_{WB},\ \tilde c_{GG},\ \tilde c_{WW},\ \tilde c_{BB},\ \tilde c_{WB},\ c_u,\ c_d,\ c_e,\ c_{6H} \\
c'_{H\ell},\ c_{H\ell},\ c_{He},\ c'_{Hq},\ c_{Hq},\ c_{Hu},\ c_{Hd},\ c_{Hud}.
\end{gathered} \tag{4.20}$$

In the limit the Wilson coefficients are flavor blind this makes 22 parameters affecting the processes of Higgs production and decay. All these coefficients are necessary to describe the results of LHC searches in a general EFT approach. At the same time, electroweak precision tests constrain (often stringently) linear combinations of the following Wilson coefficients:

$$c_T,\ c_{WB},\ c'_{H\ell},\ c_{H\ell},\ c_{He},\ c'_{Hq},\ c_{Hq},\ c_{Hu},\ c_{Hd},\ c_{Hud},\ c_{3W},\ \tilde c_{3W},\ [c_{\ell\ell}]_{12;21}. \tag{4.21}$$

In principle, there is not any theoretical obstacle to present the results of LHC Higgs analyses as constraints on the Wilson coefficients in Eq. (4.20). The practical difficulty is that some linear combinations of these parameters are already stringently constrained by electroweak precisions tests, such that they cannot yield observables effects at the LHC. In the next section we propose a more convenient parametrization where the strongly and weakly constrained combinations of Wilson coefficients are separated.

5 Higgs Basis

In this section we propose another parametrization of the effective dimension-6 Lagrangian in the linear realization of electroweak symmetry. The formalism is similar to Ref. [1], however the parametrization we propose here is slightly different. The goal is to choose a particular basis of operators that can be more directly connected (at least at tree-level) to observable quantities in Higgs physics. The basis, which we call the *Higgs basis*, is spanned by particular combinations of dimension-6 operators. Each of these combinations maps to a simple interaction term of the SM mass-eigenstate fields that can be probed by experiment. In fact, we will define the Higgs basis by a subset of the couplings in the effective Lagrangian Eq. (3.1). We will refer to this subset as the *independent couplings*.

We stress that the Higgs basis should be regarded as one of many possible bases of the dimension-6 Lagrangian beyond the SM. In particular, the independent couplings can be related by a linear transformation to parameters defining any other such basis in the literature; the linear transformation to the Warsaw basis [5] can be extracted from Section 4, and the transformation to the SILH [6] basis will be given in Appendix A.1. At the same time, the independent couplings can be easily connected to Higgs *pseudo-observables* at the amplitude level, as defined e.g. in Ref. [7].

The number of couplings in the effective Lagrangian of Eq. (3.1) is larger than the number of Wilson coefficients in a dimension-6 EFT basis. Therefore, some of the couplings can be expressed by the independent couplings; we call them the *dependent* couplings. The relations between dependent and independent couplings can be inferred from the matching between the effective Lagrangian and the Warsaw basis in Section 4. These relations *hold at the level of the dimension-6 Lagrangian*, and they are in general not respected in the presence of dimension-8 and higher operators. Of course, the choice which couplings are independent and which are dependent is a subjective choice dictated by convenience. In our case, the choice of the independent couplings was motivated by their direct connection to observables constrained by electroweak precision tests and Higgs searches. However, other choices can be envisaged and may be more convenient for other applications.

5.1 Independent Couplings

We select a subset of coupling in the effective Lagrangian of Eq. (3.1) that has a 1-to-1 mapping to the Wilson coefficients in the Warsaw basis (or any other dimension-6 basis). This subset of independent couplings defines the Higgs basis. It can be used on par with any other basis to describe the effect of dimension-6 operators on physical observables.

The first group of independent couplings are the ones affecting W boson mass and the Z and W boson couplings to fermions:

$$\begin{gathered}\delta m,\ \delta g_L^{Ze},\ \delta g_R^{Ze},\ \delta g_L^{W\ell},\ \delta g_L^{Zu},\ \delta g_R^{Zu},\ \delta g_L^{Zd},\ \delta g_R^{Zd},\ \delta g_R^{Wq},\\ d_{Gu},\ d_{Gd},\ d_{Ae},\ d_{Au},\ d_{Ad},\ d_{Ze},\ d_{Zu},\ d_{Zd},\ \tilde{d}_{Gu},\ \tilde{d}_{Gd},\ \tilde{d}_{Ae},\ \tilde{d}_{Au},\ \tilde{d}_{Ad},\ \tilde{d}_{Ze},\ \tilde{d}_{Zu},\ \tilde{d}_{Zd}.\end{gathered} \tag{5.1}$$

Here the mass correction δm is defined in Eq. (3.3), the vertex corrections δg's are defined in Eq. (3.4), and the dipole moments d_i are defined in Eq. (3.5). While they are free parameters from the EFT point of view, precision measurements constrain them to be small. In particular, most of the parameters in the first line are constrained to be $\lesssim 10^{-2} - 10^{-4}$ [12]. The remaining parameters are constrained by measurements of the magnetic and electric dipole moments. Therefore, even if combinations of dimension-6 operators defined the independent couplings in Eq. (5.1) affect the Higgs observables, it is a well-motivated assumption to neglect them in LHC Higgs analyses whose precision is worse than the existing constraints.

The second group of independent couplings are the ones describing the interactions of the Higgs boson with the SM gauge boson, fermions, and with itself:

$$\begin{aligned} & c_{gg},\ \delta c_z,\ c_{\gamma\gamma},\ c_{z\gamma},\ c_{zz},\ c_{z\Box},\ \tilde{c}_{gg},\ \tilde{c}_{\gamma\gamma},\ \tilde{c}_{z\gamma},\ \tilde{c}_{zz}, \\ & \delta y_u,\ \delta y_d,\ \delta y_e,\ \sin\phi_u,\ \sin\phi_d,\ \sin\phi_\ell,\ \delta\lambda_3. \end{aligned} \tag{5.2}$$

They are defined by Eq. (3.8), except for the last one which is defined in Eq. (3.12). As opposed to the ones in Eq. (5.1), the combinations of Wilson coefficients corresponding to the independent couplings in Eq. (5.2) are weakly constrained by SM precision tests. In fact, the strongest limits on these couplings typically come from Higgs searches. An important task of the LHC collaborations is to provide model-independent limits on the parameters in Eq. (5.2).

The third group of independent couplings are related gauge bosons self-couplings:

$$\lambda_z,\ \tilde{\lambda}_z,\ c_{3G},\ \tilde{c}_{3G}. \tag{5.3}$$

They are defined in Eq. (3.6). These couplings do not affect Higgs searches, and they are only weakly constrained by SM precision tests.

To complete the definition of the Higgs basis, one has to include the independent couplings corresponding to 4-fermion operators. We choose to parametrize them by the same set of Wilson coefficients as in the Warsaw basis:

$$\begin{aligned} & c_{\ell\ell},\ c_{qq},\ c'_{qq},\ c_{\ell q},\ c'_{\ell q},\ c_{quqd},\ c'_{quqd},\ c_{\ell equ},\ c'_{\ell equ},\ c_{\ell edq}, \\ & c_{\ell e},\ c_{\ell u},\ c_{\ell d},\ c_{qe},\ c_{qu},\ c'_{qu},\ c_{qd},\ c'_{qd},\ c_{ee},\ c_{uu},\ c_{dd},\ c_{eu},\ c_{ed},\ c_{ud},\ c'_{ud}. \end{aligned} \tag{5.4}$$

The parameters c_{ff} have 4 flavor indices. The non-trivial question which subset of all possible combinations of flavor indices constitute an independent set is worked out in Ref. [10]. In the Higgs basis we take the same choice of independent 4-fermion couplings as in that reference, with one exception. As explained in the next subsection, in the Higgs basis the coupling $[c_\ell]_{1221}$ is a dependent coupling that can be expressed by δm and δg's. Therefore $[c_\ell]_{1221}$ is not among the independent couplings defining the Higgs basis.

5.2 Dependent Couplings

The remaining couplings in the effective Lagrangian are called the dependent couplings because, at the level of a dimension-6 EFT Lagrangian, they can be expressed by the independent couplings defining the Higgs basis. To obtain the relations between the dependent and independent couplings one can use the matching between the Warsaw basis and the effective Lagrangian worked out in Section 4. The procedure is to solve for the Warsaw basis Wilson coefficients in terms of the independent couplings and eliminating the former from the expressions for the dependent couplings.

We start with the dependent couplings in Eq. (3.8) describing the single Higgs boson interactions with matter. They can be expressed by the independent couplings as[3]

$$\begin{aligned}
\delta c_w &= \delta c_z + 4\delta m, \\
c_{ww} &= c_{zz} + 2s_\theta^2 c_{z\gamma} + s_\theta^4 c_{\gamma\gamma}, \\
\tilde{c}_{ww} &= \tilde{c}_{zz} + 2s_\theta^2 \tilde{c}_{z\gamma} + s_\theta^4 \tilde{c}_{\gamma\gamma}, \\
c_{w\Box} &= \frac{1}{g^2 - g'^2}\left[g^2 c_{z\Box} + g'^2 c_{zz} - e^2 s_\theta^2 c_{\gamma\gamma} - (g^2 - g'^2) s_\theta^2 c_{z\gamma}\right], \\
c_{\gamma\Box} &= \frac{1}{g^2 - g'^2}\left[2g^2 c_{z\Box} + (g^2 + g'^2) c_{zz} - e^2 c_{\gamma\gamma} - (g^2 - g'^2) c_{z\gamma}\right].
\end{aligned} \tag{5.5}$$

Next, all the couplings with two higgs bosons in Eq. (3.12) can be expressed by the independent couplings:

$$\begin{aligned}
c_{gg}^{(2)} &= c_{gg}, \qquad \tilde{c}_{gg}^{(2)} = \tilde{c}_{gg}, \\
[y_f^{(2)}]_{ij} &= 3[\delta y_f]_{ij} e^{i\phi_{ij}} - \delta c_z\, \delta_{ij},
\end{aligned} \tag{5.6}$$

The dependent vertex corrections are expressed by the independent ones as

$$\delta g_L^{Z\nu} = \delta g_L^{Ze} + \delta g_L^{W\ell}, \qquad \delta g_L^{Wq} = \delta g_L^{Zu} - \delta g_L^{Zd}. \tag{5.7}$$

Note that we choose the W couplings to leptons (rather than the Z couplings to neutrinos) as our independent couplings, because in the flavor non-universal case the former are more directly constrained by experiment (in particular, in leptonic W decays measured at LEP).

Next, all but two triple gauge couplings in Eq. (3.6) are dependent couplings expressed by the independent couplings as

$$\begin{aligned}
\delta g_{1,z} &= \frac{1}{2(g^2 - g'^2)}\left[c_{\gamma\gamma} e^2 g'^2 + c_{z\gamma}(g^2 - g'^2) g'^2 - c_{zz}(g^2 + g'^2) g'^2 - c_{z\Box}(g^2 + g'^2) g^2\right] \\
\delta\kappa_\gamma &= -\frac{g^2}{2}\left(c_{\gamma\gamma}\frac{e^2}{g^2 + g'^2} + c_{z\gamma}\frac{g^2 - g'^2}{g^2 + g'^2} - c_{zz}\right), \\
\tilde{\kappa}_\gamma &= -\frac{g^2}{2}\left(\tilde{c}_{\gamma\gamma}\frac{e^2}{g^2 + g'^2} + \tilde{c}_{z\gamma}\frac{g^2 - g'^2}{g^2 + g'^2} - \tilde{c}_{zz}\right), \\
\delta\kappa_z &= \delta g_{1,z} - t_\theta^2 \delta\kappa_\gamma, \qquad \tilde{\kappa}_z = -t_\theta^2 \tilde{\kappa}_\gamma, \\
\lambda_\gamma &= \lambda_z, \quad \tilde{\lambda}_\gamma = \tilde{\lambda}_z.
\end{aligned} \tag{5.8}$$

Note that $\delta g_{1,z}$, $\delta\kappa_\gamma$, and $\tilde{\kappa}_\gamma$ are *dependent* couplings here, unlike in Ref. [1]. Our motivation is that the Higgs basis should be parametrized such that the connection with Higgs observables is the simplest. However, for the sake of studying WW and WZ production a different set of independent couplings would be more convenient. For example, one could choose the independent couplings as $\delta g_{1,z}$, $\delta\kappa_\gamma$, λ_z, $\tilde{\kappa}_\gamma$, $\tilde{\lambda}_z$, and consider $c_{z\Box}$, c_{zz}, and $\tilde{c}_{zz}$ as dependent couplings expressed by this set.

[3]The relation between c_{ww}, $\tilde{c}_{ww}$ and other parameters can also be viewed as a consequence of the accidental custodial symmetry at the level of the dimension-6 operators [8].

Finally, we discuss how the Wilson coefficient $[c_{\ell\ell}]_{1221}$ of the 2-electron-2-muon operator is expressed by the independent couplings. One feature of the effective Lagrangian Eq. (3.1) is that the tree-level relations between the SM electroweak parameters and input observables are not affected by new physics. On the other hand, one of the four-fermion couplings in the Lagrangian,

$$\mathcal{L}_{4f}^{D=6} \supset [c_{\ell\ell}]_{1221}(\bar{\ell}_{1,L}\gamma_\rho \ell_{2,L})(\bar{\ell}_{2,L}\gamma_\rho \ell_{1,L}) \tag{5.9}$$

does affect the relation between the parameter v and the muon decay width from which $G_F = 1/\sqrt{2}v^2$ is determined:

$$\frac{\Gamma(\mu \to e\nu\nu)}{\Gamma(\mu \to e\nu\nu)_{\rm SM}} \approx 1 + 2[\delta g_L^{We}]_{11} + 2[\delta g_L^{We}]_{22} - 4\delta m - [c_{\ell\ell}]_{1221}. \tag{5.10}$$

Therefore, the muon decay width is unchanged with respect to the SM when $[c_{\ell\ell}]_{1221}$ is related to δm and δg as

$$[c_{\ell\ell}]_{1221} = 2\delta[g_L^{We}]_{11} + 2[\delta g_L^{We}]_{22} - 4\delta m. \tag{5.11}$$

In other words, due to the fact that we defined δm as an independent coupling in the Higgs basis , $[c_{\ell\ell}]_{1221}$ has to be a dependent coupling. Of course, one could equivalently choose $[c_{\ell\ell}]_{1221}$ to define the Higgs basis, and remove δm from the list of independent couplings.

5.3 Final comments

In summary, the Higgs basis is parametrized by the independent couplings in Eqs. (5.1), 5.2), (5.3), (5.4). In total, the Higgs basis, much as any complete basis at the dimension-6 level, is parametrized by 2499 independent real couplings [10]. One should not, however, be intimidated by this number. The point is that a much smaller subset in Eq. (5.2) is adequate for EFT analyses of Higgs data at the leading order in new physics parameters. For example, to describe single Higgs production and decay processes in full generality one needs 10 bosonic and $2 \times 3 \times 3 \times 3 = 54$ fermionic couplings. Furthermore, 31 of these couplings are CP-odd, therefore they affect the Higgs signal strength measurement only at the quadratic level, while flavor off-diagonal Yukawa couplings only affect exotic Higgs decays. In the limit where fermionic couplings are flavor blind, 9 parameters are enough to describe leading order EFT corrections to the existing Higgs signal strength measurements at the LHC.

We conclude with a number of comments.

- The relations between independent and dependent couplings in Eqs. (5.5), (5.6), (5.7), (5.8), Eq. (5.11) are consequences of the *linear* realization of electroweak symmetry breaking at the level of dimension-6 EFT operators. *They are an essential part of the definition of the Higgs basis.* If the independent and dependent couplings were unrelated, then $\mathcal{L}_{\rm Higgs\,Basis}$ would not be a dimension-6 basis but would belong to a more general class of theories. Such theories are outside of the scope of this note.

- The independent couplings in Eq. (5.1) are probed by precision measurements of Z and W production and decays at LEP. In particular, assuming vertex corrections are flavor blind, all the independent couplings in Eq. (5.1) are constrained to be smaller than $O(10^{-3})$ (for the leptonic vertex corrections and $\delta m \equiv \delta m_W/m_W$), or $O(10^{-2})$ (for the quark vertex corrections) [2, 4, 11]. Dropping the assumption of flavor blindness, all the leptonic, bottom and charm quark vertex corrections are still constrained, in a model-independent way, at the level of $O(10^{-2})$ or better [12]. These constraints imply these couplings are too small to have any measurable effects at the LHC, therefore we recommend to impose the electroweak bounds on such constraints before analyzing LHC data. The 1st generation quark vertex corrections are less constrained in a model-independent way, though one combination of them is tightly constrained by measurements of the hadronic Z decays at LEP. Furthermore, the top quark vertex corrections are poorly constrained (at the $O(1)$ level) by experiment, especially the right-handed top couplings to Z. If feasible, the light quark and top couplings should be considered as free parameters in experimental analyses at the LHC, as this may provide new valuable information to constrain these couplings.

- The Higgs basis is convenient for extracting constraints on dimension-6 operators from Higgs and electroweak precision data. However, it may not be the optimal basis for some other applications. In particular, computing renormalization group running of the couplings or matching to concrete BSM model may be more straightforward in the language of $SU(3) \times SU(2) \times U(1)$ invariant operators.

- Customarily, the SM electroweak parameters are extracted from $\alpha(0)$, m_Z and G_F. One could also use m_W instead of G_F, as suggested in Ref. [2]. This formalism leads to the same relations between the independent and dependent couplings as written down here, except that $\delta m = 0$ by definition, and that $[c_{\ell\ell}]_{1221}$ becomes an independent couplings. The downside of this formalism is that the SM predictions for all observables would have to be recalculated, as all existing high-precision calculations use G_F as an input.

- The number of independent couplings in Eq. (5.2) relevant for Higgs observables is still large. At the early stages of the LHC run-2 it may be reasonable to employ simplified analyses with a smaller number of parameters. There are several motivated assumptions about the underlying UV theory that reduce the number of parameters:

  - *Flavor universality*, in which case the matrices $m_f \delta y_f$ and $\sin\phi_f$ reduce to a single number for each $f = u, d, e$.
  - *Minimal flavor violation*, in which case the dominant entries in δy_f are $[\delta y_u]_{33}$ and $[\delta y_d]_{33}$, while other diagonal entries are suppressed by the respective mass square ratio.
  - *CP conservation*, in which case all CP-odd couplings vanish: $\tilde{c}_i = 0 = \sin\phi_f$.
  - *Custodial symmetry*, in which case $\delta m = 0$.[4]

[4]Custodial symmetry implies several relations between Higgs couplings to gauge bosons: $\delta c_w = \delta c_z$,

We stress that independent couplings should not be arbitrarily set to zero without an underlying symmetry assumption. Furthermore, the relations between the dependent and independent couplings should be consistently imposed, so as to preserve the weak $SU(2)$ local symmetry.

- The independent couplings are formally of order v^2/Λ^2, where Λ is the scale of new physics. For completeness, it is important to define the range of independent couplings such that the EFT description is valid. The rule of thumb is that this is the case when the independent couplings are $\lesssim 1$; more sophisticated criteria will be worked out in the future when specific Higgs processes are discussed.

A Dictionary

In this section we give a translation between the Higgs basis parameters and several other bases of dimension-6 operators proposed in the literature. On request, translation to other bases may be added in the future.

A.1 SILH basis

Another $D = 6$ basis choice commonly used in the literature is the SILH basis [6, 8].[5] In this section we present the translation between the couplings in the Higgs basis and Wilson coefficients of dimension-6 operator in the SILH basis.

The SILH Lagrangian is written as

$$\mathcal{L}_{\rm SILH} = \mathcal{L}^{\rm SM} + \frac{1}{v^2}\sum_i s_i O_i. \tag{A.1}$$

Compared to the Warsaw basis defined in Section 4, the SILH basis of dimension-6 operators introduces the following nine new operators:

$$\begin{aligned}
O_W &= \frac{ig}{2}\left(H^\dagger \sigma^i \overleftrightarrow{D_\mu} H\right) D_\nu W^i_{\mu\nu}, \\
O_B &= \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D_\mu} H\right) \partial_\nu B_{\mu\nu}, \\
O_{HW} &= ig\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^i_{\mu\nu}, \\
O_{HB} &= ig'\left(D_\mu H^\dagger D_\nu H\right) B_{\mu\nu}, \\
O_{\widetilde{HW}} &= ig\left(D_\mu H^\dagger \sigma^i D_\nu H\right) \widetilde{W}^i_{\mu\nu}, \\
O_{\widetilde{HB}} &= ig'\left(D_\mu H^\dagger D_\nu H\right) \widetilde{B}_{\mu\nu}, \\
O_{2W} &= D_\mu W^i_{\mu\nu} D_\rho W^i_{\rho\nu}, \\
O_{2B} &= \partial_\mu B_{\mu\nu} \partial_\rho B_{\rho\nu}, \\
O_{2G} &= D_\mu G^a_{\mu\nu} D_\rho G^a_{\rho\nu}.
\end{aligned} \tag{A.2}$$

$c_{w\Box} = c_\theta^2 c_{z\Box} + s_\theta^2 c_{\gamma\Box}$, $c_{ww} = c_{zz} + 2s_\theta^2 c_{z\gamma} + s_\theta^4 c_\gamma$, and $\tilde{c}_{ww} = \tilde{c}_{zz} + 2s_\theta^2 \tilde{c}_{z\gamma} + s_\theta^4 \tilde{c}_\gamma$. The last three are satisfied automatically at the level of dimension-6 Lagrangian, while the first one is true for $\delta m = 0$, see Eq. (5.5).

[5] In this note, the SILH basis is understood simply as a particular choice of a non-redundant set of D=6 operators whose Wilson coefficients are arbitrary. We do not assume any hierarchy of the Wilson coefficients motivated by particular strongly coupled UV completions that was discussed in Refs. [6, 8].

Consequently, in order to have a non-redundant set of operators, 9 operators present in the Warsaw basis must be absent in the SILH basis. The absent ones are 4 bosonic operators O_{WW}, $O_{\widetilde{WW}}$, O_{WB}, $O_{\widetilde{WB}}$, 2 vertex operators $[O_{H\ell}]_{11}$, $[O'_{H\ell}]_{11}$, and 3 four-fermion operators $[O_{\ell\ell}]_{12;21}$, $[O_{\ell\ell}]_{11;22}$, $[O'_{uu}]_{33;33}$. The remaining operators are the same as in the Warsaw basis, and we use the normalizations in Table 1, which are often different than in Refs. [6, 8].[6]

One way to derive the translation is to first transform the operators in Eq. (A.2) to the Warsaw basis using integration by parts, Fierz transformations, and the equations of motion:

$$\begin{aligned}
\partial_\nu B_{\mu\nu} &= \frac{ig'}{2} H^\dagger \overleftrightarrow{D_\mu} H + g' \sum_{f=q,\ell} Y_f \bar{f}_L \gamma_\mu f_L + g' \sum_{f=u,d,e} Y_f \bar{f}_R \gamma_\mu f_R, \\
D_\nu W^i_{\mu\nu} &= \frac{ig}{2} H^\dagger \sigma^i \overleftrightarrow{D_\mu} H + \frac{g}{2} \sum_{f=q,\ell} \bar{f}_L \sigma^i \gamma_\mu f_L, \\
D_\nu G^a_{\mu\nu} &= g_s \bar{q}_L T^a \gamma_\mu q_L + g_s \sum_{f\in u,d} \bar{q}_R T^a \gamma_\mu q_R.
\end{aligned} \tag{A.3}$$

[6] The original references do not discuss the flavor structure explicitly, and the flavor indices of the absent operators are not specified. Here, for concreteness, we made a particular though somewhat arbitrary choice of these indices.

Using these, one can obtain:

$$
\begin{aligned}
O_{HB} &= O_B - \frac{1}{4} O_{WB} - O_{BB}, \\
O_{HW} &= O_W - \frac{1}{4} O_{WB} - O_{WW}, \\
O_{\widetilde{HB}} &= -\frac{1}{4} O_{\widetilde{WB}} - O_{\widetilde{BB}}, \\
O_{\widetilde{HW}} &= -\frac{1}{4} O_{\widetilde{WB}} - O_{\widetilde{WW}}, \\
O_B &= g'^2 \left[-\frac{1}{4} O_T + \frac{1}{2} \sum_{f \in q,u,d,\ell,e} Y_f \sum_i [O_{Hf}]_{ii} \right], \\
O_W &= g^2 \left[-\frac{1}{4} O_H + O_{HD} + \frac{1}{4} \sum_{f \in q,\ell} \sum_i [O'_{Hf}]_{ii} \right], \\
O_{2B} &= g'^2 \left[-\frac{1}{4} O_T + \sum_{f \in q,u,d,\ell,e} Y_f \sum_i [O_{Hf}]_{ii} + \sum_{f_1 f_2 \in q,u,d,\ell,e} Y_{f_1} Y_{f_2} \sum_{i,j} [O_{f_1 f_2}]_{ii;jj} \right], \\
O_{2W} &= g^2 \left[-\frac{1}{4} O_H + O_{HD} + \frac{1}{2} \sum_{f \in q,\ell} \sum_i [O'_{Hf}]_{ii} \right. \\
&+ \left. \sum_{ij} \left(\frac{1}{2} [O_{\ell\ell}]_{ij;ji} - \frac{1}{4} [O_{\ell\ell}]_{ii;jj} + \frac{1}{2} [O_{\ell q}]_{ii;jj} + \frac{1}{4} [O_{qq}]_{ii;jj} \right) \right], \\
O_{2G} &= g_s^2 \sum_{i,j} \left[\frac{1}{4} [O'_{qq}]_{ij;ji} + \frac{1}{4} [O_{qq}]_{ij;ji} - \frac{1}{6} [O_{qq}]_{ii;jj} + 2 [O'_{qu}]_{ii;jj} + 2 [O'_{qd}]_{ii;jj} \right. \\
&+ \left. 2 [O'_{ud}]_{ii;jj} + \frac{1}{2} [O'_{uu}]_{ij;ji} - \frac{1}{6} [O'_{uu}]_{ii;jj} + \frac{1}{2} [O'_{dd}]_{ij;ji} - \frac{1}{6} [O'_{dd}]_{ii;jj} \right].
\end{aligned}
\tag{A.4}
$$

The operator $O_{HD} = |H|^2 |D_\mu H|^2$ appearing above is present neither in the Warsaw nor in the SILH basis. One can remove it from the Lagrangian by rescaling the Higgs field and the Yukawa couplings as $H \to H(1 + \epsilon |H|^2/v^2)$, $y_f \to y_f(1 - \epsilon/2)$. To lowest order in ϵ, this rescaling generates the following terms in the Lagrangian

$$
\Delta \mathcal{L} = \epsilon \left(2 O_{HD} + O_H - 4\lambda O_{6H} + \sum_{f \in u,d,e} \sum_i [y_f]_{ii} [O_f]_{ii} \right). \tag{A.5}
$$

Thus, to get rid of the O_{HD} operator generated by the transformation from the SILH to the Warsaw basis we need to choose $\epsilon = -g^2(s_W + s_{HW} + s_{2W})/2$. Effectively, this amount to replacing in Eq. (A.4):

$$
O_{HD} \to -\frac{1}{2} O_H + 2\lambda O_{6H} - \frac{1}{2} \sum_{f \in u,de} \sum_i [y_f]_{ii} [O_f]_{ii}. \tag{A.6}
$$

We are ready to give the translation between the Wilson coefficient in the SILH and

Warsaw basis:

$$
\begin{aligned}
c_H &= s_H - \frac{3g^2}{4}\left(s_W + s_{HW} + s_{2W}\right), \\
c_T &= s_T - \frac{g'^2}{4}\left(s_B + s_{HB} + s_{2B}\right), \\
c_{6H} &= s_{6H} + 2\lambda g^2\left(s_W + s_{HW} + s_{2W}\right), \\
c_{WB} &= -\frac{1}{4}\left(s_{HB} + s_{HW}\right), \\
c_{BB} &= s_{BB} - s_{HB}, \\
c_{WW} &= -s_{HW}, \\
\tilde{c}_{WB} &= -\frac{1}{4}\left(\tilde{s}_{HB} + \tilde{s}_{HW}\right), \\
\tilde{c}_{BB} &= \tilde{s}_{BB} - \tilde{s}_{HB}, \\
\tilde{c}_{WW} &= -\tilde{s}_{HW},
\end{aligned} \tag{A.7}
$$

$$
\begin{aligned}
[c_{Hf}]_{ij} &= [s_{Hf}]_{ij} + \frac{g'^2 Y_f}{2}\left(s_B + s_{HB} + 2s_{2B}\right)\delta_{ij}, \\
[c'_{Hf}]_{ij} &= [s'_{Hf}]_{ij} + \frac{g^2}{4}\left(s_W + s_{HW} + 2s_{2W}\right)\delta_{ij},
\end{aligned} \tag{A.8}
$$

$$
[c_f]_{ij} = [s_f]_{ij} - \delta_{ij} g^2 [y_f]_{ii} \frac{s_W + s_{HW} + s_{2W}}{2}, \tag{A.9}
$$

$$
\begin{aligned}
[c_{\ell\ell}]_{ii;ii} &= [s_{\ell\ell}]_{ii;ii} + \frac{1}{4}\left(g'^2 s_{2B} + g^2 s_{2W}\right), \\
[c_{\ell\ell}]_{ii;jj} &= [s_{\ell\ell}]_{ii;jj} + \frac{1}{2}\left(g'^2 s_{2B} - g^2 s_{2W}\right), \qquad i < j, \\
[c_{\ell\ell}]_{ij;ji} &= [s_{\ell\ell}]_{ij;ji} + g^2 s_{2W}, \qquad i < j,
\end{aligned} \tag{A.10}
$$

where it is implicit that $[s_{H\ell}]_{11} = [s'_{H\ell}]_{11} = [s_{\ell\ell}]_{12;21} = [s_{\ell\ell}]_{11;22} = 0$. For the 4-lepton operators one should take into account that $[O_{\ell\ell}]_{ji;ij} \equiv [O_{\ell\ell}]_{ij;ji}$ and $[O_{\ell\ell}]_{jj;ii} \equiv [O_{\ell\ell}]_{ii;jj}$. The translation of other 4-fermion Wilson coefficients apart from the one in Eq. (A.10) can be easily derived from Eq. (A.4), but it will not be needed in the following. For the Wilson coefficients not listed above the translation is trivial: $c_i = s_i$.

Given these relations between the Warsaw and SILH basis Wilson coefficients and using the results of Section 4, we can derive the translation between the Higgs basis couplings and the SILH basis Wilson coefficients:

$$
\delta m = -\frac{g^2 g'^2}{4(g^2 - g'^2)}\left(s_W + s_B + s_{2W} + s_{2B} - \frac{4}{g'^2} s_T + \frac{2}{g^2}[s'_{H\ell}]_{22}\right), \tag{A.11}
$$

$$
\begin{aligned}
\hat{f}(T^3, Q) &\equiv \frac{1}{4}\left[g^2 s_{2W} + g'^2 s_{2B} + 4 s_T - 2[s'_{H\ell}]_{22}\right] T^3 \\
&+ \frac{g'^2}{4(g^2 - g'^2)}\left[-(2g^2 - g'^2)s_{2B} - g^2(s_{2W} + s_W + s_B) + 4 s_T - 2[s'_{H\ell}]_{22}\right] Q,
\end{aligned} \tag{A.12}
$$

$$
\begin{aligned}
\delta g_L^{Z\nu} &= \frac{1}{2}s'_{H\ell} - \frac{1}{2}s_{H\ell} + \hat{f}(1/2,0),\\
\delta g_L^{Ze} &= -\frac{1}{2}s'_{H\ell} - \frac{1}{2}s_{H\ell} + \hat{f}(-1/2,-1),\\
\delta g_R^{Ze} &= -\frac{1}{2}s_{He} + \hat{f}(0,-1),\\
\delta g_L^{Zu} &= \frac{1}{2}s'_{Hq} - \frac{1}{2}s_{Hq} + \hat{f}(1/2,2/3),\\
\delta g_L^{Zd} &= -\frac{1}{2}s'_{Hq} - \frac{1}{2}s_{Hq} + \hat{f}(-1/2,-1/3),\\
\delta g_R^{Zu} &= -\frac{1}{2}s_{Hu} + \hat{f}(0,2/3),\\
\delta g_R^{Zd} &= -\frac{1}{2}s_{Hd} + \hat{f}(0,-1/3),\\
\delta g_L^{W\ell} &= s'_{H\ell} + \hat{f}(1/2,0) - \hat{f}(-1/2,-1),\\
\delta g_L^{Wq} &= s'_{Hq} + \hat{f}(1/2,2/3) - \hat{f}(-1/2,-1/3),\\
\delta g_R^{Wq} &= -\frac{1}{2}s_{Hud},
\end{aligned}
\tag{A.13}
$$

$$
c^{Vf} = \delta g^{Vf}, \tag{A.14}
$$

$$
\begin{aligned}
\delta c_w &= -s_H - \frac{g^2 g'^2}{g^2 - g'^2}\left[s_W + s_B + s_{2W} + s_{2B} - \frac{4}{g'^2}s_T + \frac{3g^2 + g'^2}{2g^2 g'^2}[s'_{H\ell}]_{22}\right],\\
\delta c_z &= -s_H - \frac{3}{2}[s'_{H\ell}]_{22},\\
c_{gg} &= s_{GG},\\
c_{\gamma\gamma} &= s_{BB},\\
c_{zz} &= -\frac{1}{g^2 + g'^2}\left[g^2 s_{HW} + g'^2 s_{HB} - g'^2 s_\theta^2 s_{BB}\right],\\
c_{z\Box} &= \frac{1}{2g^2}\left[g^2(s_W + s_{HW} + s_{2W}) + g'^2(s_B + s_{HB} + s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right],\\
c_{z\gamma} &= \frac{s_{HB} - s_{HW}}{2} - s_\theta^2 s_{BB},\\
c_{\gamma\Box} &= \frac{s_{HW} - s_{HB}}{2} + \frac{1}{g^2 - g'^2}\left[g^2(s_W + s_{2W}) + g'^2(s_B + s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right],\\
c_{ww} &= -s_{HW},\\
c_{w\Box} &= \frac{s_{HW}}{2} + \frac{1}{2(g^2 - g'^2)}\left[g^2(s_W + s_{2W}) + g'^2(s_B + s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right],
\end{aligned}
\tag{A.15}
$$

$$
\begin{aligned}
[\delta y_f]_{ij}\cos\phi_{ij}^f &= \frac{v\mathrm{Re}[c_f]_{ij}}{\sqrt{2m_{f_i}m_{f_j}}} - \delta_{ij}\left[s_H + \frac{3g^2}{4}(s_W + s_{HW} + s_{2W}) + \frac{1}{2}[s'_{H\ell}]_{22}\right],\\
[\delta y_f]_{ij}\sin\phi_{ij}^f &= \frac{v\mathrm{Im}[s_f]_{ij}}{\sqrt{2m_{f_i}m_{f_j}}}.
\end{aligned}
\tag{A.16}
$$

$$\delta\lambda_3 = -\lambda\left(3s_H + \frac{1}{2}[s'_{H\ell}]_{22}\right) - s_{6H}, \tag{A.17}$$

$$\begin{aligned}
\delta g_{1z} &= -\frac{g^2+g'^2}{4(g^2-g'^2)}\left[(g^2-g'^2)s_{HW} + g^2(s_W+s_{2W}) + g'^2(s_B+s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right], \\
\delta\kappa_\gamma &= -\frac{g^2}{4}\left[s_{HW}+s_{HB}\right], \\
\delta\kappa_z &= -\frac{1}{4}\left(g^2 s_{HW} - g'^2 s_{HB}\right) - \frac{g^2+g'^2}{4(g^2-g'^2)}\left[g^2(s_W+s_{2W}) + g'^2(s_B+s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right], \\
\lambda_z &= -\frac{3}{2}g^4 s_{3W}, \qquad \lambda_\gamma = \lambda_z, \\
\delta\tilde\kappa_\gamma &= -\frac{g^2}{4}\left[\tilde s_{HW}+\tilde s_{HB}\right], \\
\delta\tilde\kappa_z &= \frac{g'^2}{4}\left[\tilde s_{HW}+\tilde s_{HB}\right], \\
\tilde\lambda_z &= -\frac{3}{2}g^4 \tilde s_{3W}, \qquad \tilde\lambda_\gamma = \tilde\lambda_z.
\end{aligned} \tag{A.18}$$

A.2 SILH' basis

to be completed

A.3 HISZ basis

To describe the di-boson production, Ref. [13] proposes to use the following 5 operators:

$$\begin{aligned}
\hat O_{WW} &= \mathrm{Tr}\left[W_{\mu\nu}W_{\nu\rho}W_{\rho\mu}\right], \\
\hat O_{W} &= D_\mu H^\dagger W_{\mu\nu} D_\mu H, \\
\hat O_{B} &= D_\mu H^\dagger B_{\mu\nu} D_\mu H, \\
\hat O_{\widetilde{WW}} &= \mathrm{Tr}\left[W_{\mu\nu}W_{\nu\rho}\widetilde W_{\rho\mu}\right], \\
\hat O_{\widetilde W} &= D_\mu H^\dagger \widetilde W_{\mu\nu} D_\mu H.
\end{aligned} \tag{A.1}$$

This is a subset of operators considered by Hagiwara et al. (HISZ) in Ref. [9]. The dimension-6 Lagrangian contains

$$\mathcal{L}^{\mathrm{D}=6} \supset \frac{1}{\Lambda^2}\left(d_{WW}\hat O_{WW} + d_W \hat O_W + d_B \hat O_B + \tilde d_{WW}\hat O_{\widetilde{WW}} + \tilde d_W \hat O_{\tilde W}\right). \tag{A.2}$$

These 5 operators contribute to the TGCs and Higgs couplings, but they do not contribute to oblique or vertex corrections. Thus, they are not strongly constrained by electroweak precision tests, and therefore represent a perfectly fine parameterization of EFT new physics in di-boson production.

One should remember that the covariant derivatives in Refs. [9, 13] are defined with the opposite sign than here. This amounts to rescaling the gauge fields as $W_\mu \to -W_\mu$,

$B_\mu \to -B_\mu$ in the translation. Then the electroweak field strength tensors defined in Ref. [13] are related to the ones used here by

$$B_{\mu\nu} \to -\frac{i}{2} g' B_{\mu\nu}, \qquad W_{\mu\nu} \to -\frac{i}{2} g \sigma^i W^i_{\mu\nu}. \tag{A.3}$$

This allows us to relate

$$\begin{aligned} \hat{O}_{WW} &= -\frac{1}{4} O_{3W}, \quad \hat{O}_W = -\frac{1}{2} O_{HW}, \quad \hat{O}_B = -\frac{1}{2} O_{HB}, \\ \hat{O}_{\widetilde{WW}} &= -\frac{1}{4} O_{\widetilde{3W}}, \quad \hat{O}_{\widetilde{W}} = -\frac{1}{2} O_{\widetilde{HW}}. \end{aligned} \tag{A.4}$$

where O_i on the right-hand side are operators in the SILH basis in the normalization of Section ??. Thus, the map between the HISZ and SILH coefficients is the following:

$$\begin{aligned} s_{3W} &= -\frac{1}{4}\frac{v^2}{\Lambda^2} d_{WW}, \quad s_{HW} = -\frac{1}{2}\frac{v^2}{\Lambda^2} d_W, \quad s_{HB} = -\frac{1}{2}\frac{v^2}{\Lambda^2} d_B, \\ \tilde{s}_{3W} &= -\frac{1}{4}\frac{v^2}{\Lambda^2} \tilde{d}_{WW}, \quad \tilde{s}_{HW} = -\frac{1}{2}\frac{v^2}{\Lambda^2} \tilde{d}_W. \end{aligned} \tag{A.5}$$

The anomalous TGCs and the HISZ basis Wilson coefficients are related by:

$$\begin{aligned} \delta g_{1z} &= \frac{g^2 + g'^2}{8}\frac{v^2}{\Lambda^2} d_W \\ \delta \kappa_\gamma &= \frac{g^2}{8}\frac{v^2}{\Lambda^2}\left(d_W + d_B\right), \qquad \delta\tilde{\kappa}_\gamma = \frac{g^2}{8}\frac{v^2}{\Lambda^2}\tilde{d}_W \\ \lambda_z &= \frac{3g^4}{8}\frac{v^2}{\Lambda^2} d_{WW}, \qquad \tilde{\lambda}_z = \frac{3g^4}{8}\frac{v^2}{\Lambda^2}\tilde{d}_{WW}. \end{aligned} \tag{A.6}$$

Inverting these formulas, the relation between the Wilson coefficients in the HISZ basis and the Higgs basis parameters reads

$$\begin{aligned} d_{WW} &= \frac{8\Lambda^2}{3g^4 v^2}\lambda_z, \\ d_W &= -\frac{4\Lambda^2}{(g^2 - g'^2)v^2}\left[g^2 c_{z\Box} + g'^2 c_{zz} - s_\theta^2 e^2 c_{\gamma\gamma} - s_\theta^2 (g^2 - g'^2) c_{z\gamma}\right], \\ d_B &= \frac{4\Lambda^2}{(g^2 - g'^2)v^2}\left[g^2 c_{z\Box} + g^2 c_{zz} - c_\theta^2 e^2 c_{\gamma\gamma} - c_\theta^2 (g^2 - g'^2) c_{z\gamma}\right], \\ \tilde{d}_{WW} &= \frac{8\Lambda^2}{3g^4 v^2}\tilde{\lambda}_z, \\ \tilde{d}_W &= \frac{8\Lambda^2}{g^2 v^2}\delta\tilde{\kappa}_\gamma. \end{aligned} \tag{A.7}$$

B Goldstone bosons and gauge fixing

In the main body of this note we worked in the unitary gauge where the Goldstone boson degrees of freedom in the Higgs doublet are set to zero. This is enough for the sake of tree-level EFT calculations. However, if the necessity arises to extend the calculations to a loop level, retrieving the Goldstone degrees of freedom is convenient, as this allows one to perform the standard gauge fixing procedure. This is done in this appendix.

We parametrize the Higgs doublet as

$$H = \begin{pmatrix} iG_+ \\ \frac{1}{\sqrt{2}}(v + h - iG_3) \end{pmatrix} \tag{B.1}$$

where $G_\pm$ and G_3 are three Goldstone fields, that will be eaten by the W and Z bosons. In the Higgs basis, derivation of the Goldstone boson couplings follows exactly the same algorithm as the one applied before to derive the Lagrangian for physical fields: we first derive these couplings in the Warsaw basis, and then perform the field and coupling redefinitions that take us to the Higgs basis. Of course, all the Goldstone boson couplings are dependent ones, that is they can be expressed by the independent couplings defining the Higgs basis. As an illustration, below we display a subset of these couplings that are relevant for the 1-loop calculation of $h \to VV^*$. These are

1. Goldstone kinetic terms and their mixing with the electroweak gauge fields.
2. Cubic interactions with one Higgs boson and one or two Goldstone fields.
3. Cubic interactions with one or two Goldstone fields and one electroweak gauge field.
4. Quartic interactions with one or two Goldstone fields and two electroweak gauge fields.

The relevant part of the Lagrangian is parametrized as

$$\mathcal{L}_G = \mathcal{L}_G^{\rm kin} + \mathcal{L}_G^{\rm S^3} + \mathcal{L}_G^{\rm S^2V} + \mathcal{L}_G^{\rm SV^2} + \mathcal{L}_G^{\rm SVdV} + \mathcal{L}_G^{\rm S^2V^2} + \mathcal{L}_G^{\rm S^2dV^2}. \tag{B.2}$$

where

$$\mathcal{L}_G^{\rm kin} = \partial_\mu G_+ \partial_\mu G_- + \frac{1}{2}(\partial_\mu G_3)^2 - \beta_{cW}\frac{gv}{2}\left(\partial_\mu G_+ W_\mu^- + {\rm h.c.}\right) - \frac{\sqrt{g^2+g'^2}v}{2}\partial_\mu G_3 Z_\mu, \tag{B.3}$$

$$\mathcal{L}_G^{\rm S^3} = -\frac{m_h^2}{v}\beta_{hcc} h G_+ G_- - \frac{m_h^2}{2v}\beta_{h33} h G_3 G_3 \tag{B.4}$$

$$\begin{aligned}
\mathcal{L}_G^{\rm S^2V} &= \beta_{hcW}\frac{g}{2}\partial_\mu h\left(G_+ W_\mu^- + {\rm h.c.}\right) + \beta_{h3z}\frac{\sqrt{g^2+g'^2}}{2}\partial_\mu h G_3 Z_\mu \\
&+ i\beta_{3cW}\frac{g}{2}\partial_\mu G_3\left(G_+ W_\mu^- - {\rm h.c.}\right) - \beta_{3hz}\frac{\sqrt{g^2+g'^2}}{2}\partial_\mu G_3 h Z_\mu \\
&+ ie\left(\partial_\mu G_+ G_- - {\rm h.c.}\right)A_\mu + i\beta_{ccZ}\frac{g^2-g'^2}{2\sqrt{g^2+g'^2}}\left(\partial_\mu G_+ G_- - {\rm h.c.}\right)Z_\mu \\
&- \beta_{chW}\frac{g}{2}\left(\partial_\mu G_+ W_\mu^- + {\rm h.c.}\right)h - i\beta_{c3W}\frac{g}{2}\left(\partial_\mu G_+ W_\mu^- - {\rm h.c.}\right)G_3,
\end{aligned} \tag{B.5}$$

$$\mathcal{L}_G^{\rm SV^2} = i\beta_{cWA}\frac{egv}{2}\left(G_+ W_\mu^- - {\rm h.c.}\right)A_\mu - i\beta_{cWZ}\frac{c_\theta g'^2 v}{2}\left(G_+ W_\mu^- - {\rm h.c.}\right)Z_\mu, \tag{B.6}$$

$$\mathcal{L}_G^{\rm SVdV} = i\eta_{cWA}\frac{eg}{2v}\left(G_+W^-_{\mu\nu} - {\rm h.c.}\right)A_{\mu\nu} - i\eta_{cWA}\frac{eg'}{2v}\left(G_+W^-_{\mu\nu} - {\rm h.c.}\right)Z_{\mu\nu} + ({\rm CP-odd}). \tag{B.7}$$

$$\begin{aligned}
\mathcal{L}_G^{\rm S^2V^2} &= G_+G_-\left(e^2A_\mu A_\mu + \beta_{ccAZ}\frac{e(g^2-g'^2)}{\sqrt{g^2+g'^2}}A_\mu Z_\mu + \beta_{ccZZ}\frac{(g^2-g'^2)^2}{4(g^2+g'^2)}Z_\mu Z_\mu + \beta_{ccWW}\frac{g^2}{2}W^+_\mu W^-_\mu\right) \\
&+ G_3G_3\left(\beta_{33WW}\frac{g^2}{4}W^+_\mu W^-_\mu + \beta_{33ZZ}\frac{g^2+g'^2}{8}Z_\mu Z_\mu\right) \\
&+ i\beta_{chWA}\frac{eg}{2}\left(G_+W^-_\mu - {\rm h.c.}\right)hA_\mu - \beta_{c3WA}\frac{eg}{2}\left(G_+W^-_\mu + {\rm h.c.}\right)G_3A_\mu \\
&- i\beta_{chWZ}\frac{eg'}{2}\left(G_+W^-_\mu - {\rm h.c.}\right)hZ_\mu + \beta_{c3WZ}\frac{eg'}{2}\left(G_+W^-_\mu + {\rm h.c.}\right)G_3Z_\mu \\
&+ \eta'_{ccWW}g_L^2\left(G_+G_+W^-_\mu W^-_\mu + {\rm h.c.}\right),
\end{aligned} \tag{B.8}$$

$$\begin{aligned}
\mathcal{L}_G^{\rm S^2dV^2} &= G_+G_-\left(\eta_{ccA^2}e^2A_{\mu\nu}A_{\mu\nu} + \eta_{ccAZ}gg'A_{\mu\nu}Z_{\mu\nu} + \eta_{ccZ^2}(g^2+g'^2)Z_{\mu\nu}Z_{\mu\nu} + \eta_{ccW^2}g^2W^+_{\mu\nu}W^-_{\mu\nu}\right) \\
&+ G_3G_3\left(\eta_{33AA}e^2A_{\mu\nu}A_{\mu\nu} + \eta_{33AZ}gg'A_{\mu\nu}Z_{\mu\nu} + \eta_{33ZZ}(g^2+g'^2)Z_{\mu\nu}Z_{\mu\nu} + \eta_{33WW}g^2W^+_{\mu\nu}W^-_{\mu\nu}\right) \\
&+ \eta_{c3WA}eg\left(G_+W^-_{\mu\nu} + {\rm h.c.}\right)G_3A_{\mu\nu} + \eta_{c3WZ}eg'\left(G_+W^-_\mu + {\rm h.c.}\right)G_3Z_{\mu\nu} + ({\rm CP-odd}).
\end{aligned} \tag{B.9}$$

Above, "CP-odd" stands for analogous terms with $V_{\mu\nu} \to \tilde{V}_{\mu\nu}$, and $\eta \to \tilde{\eta}$. Note the Goldstone kinetic terms in Eq. (B.3) are assumed to be canonically normalized. To achieve this, one needs to rescale the neutral Goldstone field as

$$G_3 \to G_3\left(1 + c_T + 2c_T\frac{h}{v}\right). \tag{B.10}$$

Moreover, the Lagrangian in Eq. (B.2) does not contain 2-derivative cubic scalar self-interactions. To ensure this feature, the Higgs boson field redefinition in Eq. (4.5) has to be generalized to

$$h \to h\left(1 - c_H - c_H\frac{h}{v} - c_H\frac{h^2}{3v^2}\right) - c_H\frac{2G_+G_- + G_3G_3}{v} - 2c_T\frac{G_3G_3}{v}. \tag{B.11}$$

The above field redefinitions are in addition to the steps described in Section 4. These include the gauge coupling rescaling and the use of the equations of motion (that are modified to include the Goldstone fields). The final step is to transform the couplings from the Warsaw to the Higgs basis using the dictionary provided in Section 4. At the end of the day, the coefficients in the Goldstone Lagrangian of Eq. (B.2) take the form

$$\beta_{cW} = 1 + \delta m, \tag{B.12}$$

$$\begin{aligned}
\beta_{hcc} &= 1 + g^2c_{w\Box} + \delta c_z + 2\delta m, \\
\beta_{h33} &= 1 + g^2c_{z\Box} + \delta c_z,
\end{aligned} \tag{B.13}$$

$$
\begin{aligned}
\beta_{hcW} &= 1 + g^2 c_{w\Box} + \delta c_z + 3\delta m, \\
\beta_{h3Z} &= 1 + g^2 c_{z\Box} + \delta c_z, \\
\beta_{3cW} &= 1 - 2g^2 c_{w\Box} + \frac{3}{2} g^2 c_{z\Box} - 3\delta m, \\
\beta_{3hZ} &= 1 + \delta c_z, \\
\beta_{ccZ} &= 1 + \frac{g^2 + g'^2}{2(g^2 - g'^2)} \left(-g^2 c_{z\Box} + 4\delta m \right), \\
\beta_{chW} &= 1 + \delta c_z + 3\delta m, \\
\beta_{c3W} &= 1 - \frac{g^2}{2} c_{z\Box} + \delta m,
\end{aligned} \tag{B.14}
$$

$$
\begin{aligned}
\beta_{cWA} &= 1 + \delta m, \\
\beta_{cWZ} &= 1 + \frac{g^2(g^2 + g'^2)}{2g'^2} \left(c_{z\Box} - c_{w\Box} \right) - \frac{2g^2 + g'^2}{g'^2} \delta m,
\end{aligned} \tag{B.15}
$$

$$
\eta_{cWA} = \eta_{cWZ} = c_{zz} - \frac{g^2 - g'^2}{g^2 + g'^2} c_{z\gamma} - e^2 c_{\gamma\gamma}, \tag{B.16}
$$

$$
\begin{aligned}
\beta_{ccAZ} &= 1 + \frac{g^2 + g'^2}{2(g^2 - g'^2)} \left(-g^2 c_{z\Box} + 4\delta m \right), \\
\beta_{ccZZ} &= 1 + \frac{(g^2 + g'^2)^2}{(g^2 - g'^2)^2} \left(-\frac{g^2(g^2 - g'^2)}{g^2 + g'^2} c_{z\Box} + 3g^2 c_{w\Box} + 2\delta c_z + 2\frac{5g^4 + 6g^2 g'^2 + g'^4}{(g^2 + g'^2)^2} \delta m \right), \\
\beta_{ccWW} &= 1 + 2g^2 c_{z\Box} + 2\delta c_z + 2\delta m, \\
\beta_{33ZZ} &= 1 + 2g^2 c_{z\Box} + 2\delta c_z, \\
\beta_{33WW} &= 1 + g^2 (c_{w\Box} + c_{z\Box}) + 2\delta c_z + 4\delta m, \\
\beta_{chWA} &= 1 + \delta c_z + 3\delta m, \\
\beta_{c3WA} &= 1 - \frac{g^2}{2} c_{z\Box} + \delta m, \\
\beta_{chWZ} &= 1 + \frac{3}{2} \frac{g^2(g^2 + g'^2}{g'^2} \left(c_{z\Box} - c_{w\Box} \right) + \delta c_z - 3\frac{2g^2 + g'^2}{g'^2} \delta m, \\
\beta_{c3WZ} &= 1 + \frac{g^4}{2g'^2} c_{z\Box} - \frac{g^2(g^2 + g'^2)}{2g'^2} c_{w\Box} - \frac{2g^2 + g'^2}{g'^2} \delta m, \\
\eta'_{ccWW} &= \frac{g^2}{2} \left(c_{w\Box} - c_{z\Box} \right) + \delta m,
\end{aligned} \tag{B.17}
$$

$$
\begin{aligned}
\eta_{ccAA} &= c_{zz} - \frac{g^2-g'^2}{g^2+g'^2}c_{z\gamma} + \frac{(g^2-g'^2)^2}{4(g^2+g'^2)}c_{\gamma\gamma}, \\
\eta_{33AA} &= \frac{1}{8}c_{\gamma\gamma}, \\
\eta_{ccAZ} &= \frac{g^2-g'^2}{g^2+g'^2}c_{zz} - \frac{g^4-6g^2g'^2+g'^4}{2(g^2+g'^2)^2}c_{z\gamma} - \frac{e^2(g^2-g'^2)}{(g^2+g'^2)^2}c_{\gamma\gamma}, \\
\eta_{33AZ} &= \frac{c_{z\gamma}}{4}, \\
\eta_{ccZZ} &= \frac{(g^2-g'^2)^2}{4(g^2+g'^2)^2}c_{zz} - \frac{e^2(g^2-g'^2)}{(g^2+g'^2)^2}c_{z\gamma} + \frac{e^4}{(g^2+g'^2)^2}c_{\gamma\gamma}, \\
\eta_{33ZZ} &= \frac{c_{zz}}{8}, \\
\eta_{ccWW} &= \frac{1}{2}c_{zz} + s_\theta^2 c_{z\gamma} + \frac{s_\theta^4}{2}c_{\gamma\gamma}, \\
\eta_{33WW} &= \frac{1}{4}c_{zz} + \frac{s_\theta^2}{2}c_{z\gamma} + \frac{s_\theta^4}{4}c_{\gamma\gamma}, \\
\eta_{c3WA} &= -\frac{1}{2}c_{zz} + \frac{g^2-g'^2}{2(g^2+g'^2)}c_{z\gamma} + \frac{e^2}{2(g^2+g'^2)}c_{\gamma\gamma}, \\
\eta_{c3WZ} &= \frac{1}{2}c_{zz} - \frac{g^2-g'^2}{2(g^2+g'^2)}c_{z\gamma} - \frac{e^2}{2(g^2+g'^2)}c_{\gamma\gamma}.
\end{aligned}
\tag{B.18}
$$

As soon as the Goldstone bosons are retrieved, gauge fixing can be implemented as in any gauge theory. Below we work with the linear R_ξ gauge. For the electroweak sector, we introduce the following gauge fixing Lagrangian

$$
\mathcal{L}_{\rm gf} = \frac{1}{2\xi}\left[F_A^2 + F_Z^2 + 2F_+F_-\right], \tag{B.19}
$$

where

$$
\begin{aligned}
F_A &= \partial_\mu A_\mu, \\
F_Z &= \partial_\mu Z_\mu - \xi\frac{\sqrt{g^2+g'^2}v}{2}G_3\left(1 - 2c_T + e^2 c_{WB}\right), \\
F_\pm &= \partial_\mu W_\mu^\pm - \xi\frac{gv}{2}G_\pm.
\end{aligned}
\tag{B.20}
$$

Above, the electroweak parameters g, g', v and the Goldstone fields $G_\pm$, G_3 are the ones before the rescaling in Eq. (4.8) and Eq. (B.10). After the rescaling and going to the Higgs basis the gauge fixing Lagrangian becomes

$$
\mathcal{L}_{\rm gf} = \frac{1}{2\xi}\left[(\partial_\mu A_\mu)^2 + \left(\partial_\mu Z_\mu - \xi\frac{\sqrt{g^2+g'^2}v}{2}G_3\right)^2 + 2\left|\partial_\mu W_\mu^+ - \xi\frac{gv}{2}(1+\delta m)G_+\right|^2\right]. \tag{B.21}
$$

This way, the kinetic mixing between the Goldstone bosons and massive vector bosons cancels after introducing the gauge fixing term. At the same time, the Goldstone bosons acquire the gauge dependent masses;

$$m_{G_\pm} = \sqrt{\xi}\frac{gv}{2}(1+\delta m) \equiv \sqrt{\xi} m_W, \qquad m_{G_3} = \sqrt{\xi}\frac{\sqrt{g^2+g'^2}v}{2} \equiv \sqrt{\xi} m_Z. \tag{B.22}$$

Finally, the ghost Lagrangian is given by

$$\mathcal{L}_{\rm ghost} = \sum_n \left[\bar{c}_+ \frac{\partial \delta F_+}{\partial \alpha_n} + \bar{c}_- \frac{\partial \delta F_-}{\partial \alpha_n} + \bar{c}_Z \frac{\partial \delta F_Z}{\partial \alpha_n} + \bar{c}_A \frac{\partial \delta F_A}{\partial \alpha_n} \right] c_n \tag{B.23}$$

where δF is the variation of the gauge fixing term under the infinitesimal $SU(2)\times U(1)$ gauge symmetry transformations parametrized by α_n. Since the F's in Eq. (B.20) contain the original (unrescaled) gauge and Goldstone fields, their gauge transformations are the same as in the SM. After the field and coupling rescaling and going to the Higgs basis, Eq. (B.23) leads to the gauge dependent mass terms for the ghost fields:

$$m_{c_\pm} = \sqrt{\xi}\frac{gv}{2}(1+\delta m) \equiv \sqrt{\xi} m_W, \qquad m_{c_Z} = \sqrt{\xi}\frac{\sqrt{g^2+g'^2}v}{2} \equiv \sqrt{\xi} m_Z, \tag{B.24}$$

as well as the Higgs and electroweak gauge boson interactions with 2 ghost fields. This last step completes the list of ingredients necessary to compute the $h \to VV$ amplitudes in EFT at the 1-loop level.

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H^4D^2 and H^6	
O_H	$\left[\partial_\mu(H^\dagger H)\right]^2$
O_T	$\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$
O_{6H}	$(H^\dagger H)^3$

f^2H^3	
O_e	$-(H^\dagger H - \frac{v^2}{2})\bar{e}H^\dagger \ell$
O_u	$-(H^\dagger H - \frac{v^2}{2})\bar{u}\widetilde{H}^\dagger q$
O_d	$-(H^\dagger H - \frac{v^2}{2})\bar{d}H^\dagger q$

V^3D^3	
O_{3G}	$g_s^3 f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$
$O_{3\widetilde{G}}$	$g_s^3 f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$
O_{3W}	$g^3 \epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
$O_{3\widetilde{W}}$	$g^3 \epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$

V^2H^2	
O_{GG}	$\frac{g_s^2}{4} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$
$O_{\widetilde{GG}}$	$\frac{g_s^2}{4} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
O_{WW}	$\frac{g^2}{4} H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$
$O_{\widetilde{WW}}$	$\frac{g^2}{4} H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$
O_{BB}	$\frac{g'^2}{4} H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$
$O_{\widetilde{BB}}$	$\frac{g'^2}{4} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
O_{WB}	$g g' H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$
$O_{\widetilde{WB}}$	$g g' H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$

f^2H^2D	
$O_{H\ell}$	$i\bar{\ell}\gamma_\mu \ell H^\dagger \overleftrightarrow{D}_\mu H$
$O'_{H\ell}$	$i\bar{\ell}\sigma^i\gamma_\mu \ell H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
O_{He}	$i\bar{e}\gamma_\mu e H^\dagger \overleftrightarrow{D}_\mu H$
O_{Hq}	$i\bar{q}\gamma_\mu q H^\dagger \overleftrightarrow{D}_\mu H$
O'_{Hq}	$i\bar{q}\sigma^i\gamma_\mu q H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
O_{Hu}	$i\bar{u}\gamma_\mu u H^\dagger \overleftrightarrow{D}_\mu H$
O_{Hd}	$i\bar{d}\gamma_\mu d H^\dagger \overleftrightarrow{D}_\mu H$
O_{Hud}	$i\bar{u}\gamma_\mu d \tilde{H}^\dagger D_\mu H$

f^2VHD	
O_{eW}	$g\bar{\ell}\sigma_{\mu\nu} e \sigma^i H W^i_{\mu\nu}$
O_{eB}	$g'\bar{\ell}\sigma_{\mu\nu} e H B_{\mu\nu}$
O_{uG}	$g_s\bar{q}\sigma_{\mu\nu} T^a u \widetilde{H}\, G^a_{\mu\nu}$
O_{uW}	$g\bar{q}\sigma_{\mu\nu} u \sigma^i \widetilde{H}\, W^i_{\mu\nu}$
O_{uB}	$g'\bar{q}\sigma_{\mu\nu} u \widetilde{H}\, B_{\mu\nu}$
O_{dG}	$g_s\bar{q}\sigma_{\mu\nu} T^a d H\, G^a_{\mu\nu}$
O_{dW}	$g\bar{q}\sigma_{\mu\nu} d \sigma^i H\, W^i_{\mu\nu}$
O_{dB}	$g'\bar{q}\sigma_{\mu\nu} d H\, B_{\mu\nu}$

$(\bar{L}L)(\bar{L}L)$ and $(\bar{L}R)(\bar{L}R)$	
$O_{\ell\ell}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{\ell}\gamma_\mu \ell)$
O_{qq}	$(\bar{q}\gamma_\mu q)(\bar{q}\gamma_\mu q)$
O'_{qq}	$(\bar{q}\gamma_\mu \sigma^i q)(\bar{q}\gamma_\mu \sigma^i q)$
$O_{\ell q}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{q}\gamma_\mu q)$
$O'_{\ell q}$	$(\bar{\ell}\gamma_\mu \sigma^i \ell)(\bar{q}\gamma_\mu \sigma^i q)$
O_{quqd}	$(\bar{q}^j u)\epsilon_{jk}(\bar{q}^k d)$
O'_{quqd}	$(\bar{q}^j T^a u)\epsilon_{jk}(\bar{q}^k T^a d)$
$O_{\ell equ}$	$(\bar{\ell}^j e)\epsilon_{jk}(\bar{q}^k u)$
$O'_{\ell equ}$	$(\bar{\ell}^j \sigma_{\mu\nu} e)\epsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u)$
$O_{\ell edq}$	$(\bar{\ell}^j e)(\bar{d}q^j)$

$(\bar{R}R)(\bar{R}R)$	
O_{ee}	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma_\mu e)$
O_{uu}	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma_\mu u)$
O_{dd}	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma_\mu d)$
O_{eu}	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma_\mu u)$
O_{ed}	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma_\mu d)$
O_{ud}	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma_\mu d)$
O'_{ud}	$(\bar{u}\gamma_\mu T^a u)(\bar{d}\gamma_\mu T^a d)$

$(\bar{L}L)(\bar{R}R)$	
$O_{\ell e}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{e}\gamma_\mu e)$
$O_{\ell u}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{u}\gamma_\mu u)$
$O_{\ell d}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{d}\gamma_\mu d)$
O_{qe}	$(\bar{q}\gamma_\mu q)(\bar{e}\gamma_\mu e)$
O_{qu}	$(\bar{q}\gamma_\mu q)(\bar{u}\gamma_\mu u)$
O'_{qu}	$(\bar{q}\gamma_\mu T^a q)(\bar{u}\gamma_\mu T^a u)$
O_{qd}	$(\bar{q}\gamma_\mu q)(\bar{d}\gamma_\mu d)$
O'_{qd}	$(\bar{q}\gamma_\mu T^a q)(\bar{d}\gamma_\mu T^a d)$

Table 1: A complete, non-redundant set of baryon-and-lepton-number-conserving dimension-6 operators built from SM fields [5]. In this table, e, u, d are always right-handed fermions, while ℓ and q are left-handed. A flavor index is implicit for each fermion field. For complex operators the complex conjugate operator is implicit. Including the flavor structure and complex conjugates, this table contains 2499 distinct operators [10].