QCD for boosted top production

Ben Pecjak

IPPP Durham

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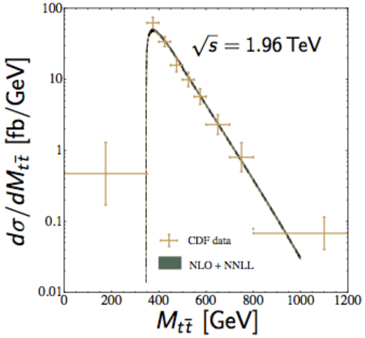
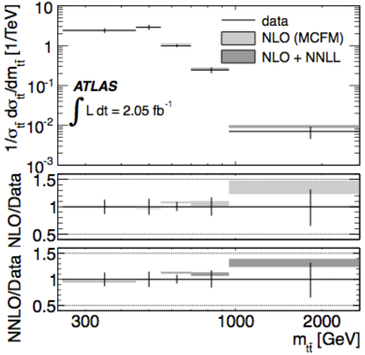
Boosted top production

Tevatron $\sqrt{s} \approx 2\ TeV$



LHC: $\sqrt{s} = 7\ TeV$



- more and more LHC results in "boosted regime" $M_{t\bar{t}}, p_T^t \gg m_t$
- not just "corner of phase space": important for new physics searches

Factorization for inclusive production

Factorization for $h_1 h_2 \to t\bar{t}X$:

$$d\sigma^{t\bar{t}X}_{h_1,h_2} = \sum_{i,j=q,\bar{q},g} \int dx_1 dx_2 f_i^{h_1}(x_1,\mu_F) f_j^{h_2}(x_2,\mu_F) d\hat{\sigma}_{ij}^{t\bar{t}\hat{X}}(\hat{s}, m_t, \ldots, \alpha_s(\mu_R), \mu_F, \mu_R) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_t}\right)$$

$$s = \left(p_{h_1} + p_{h_2}\right)^2 \, , \, \hat{s} = x_1 x_2 s$$

- PDFs from data
- partonic cross sections $d\hat{\sigma}_{ij}$ from perturbation theory
- fixed-order perturbation theory is default approach

NNLO CALCULATIONS

$$\hat{\sigma}^{\mathrm{NNLO}}_{t\bar{t}+X} = \hat{\sigma}^{\mathrm{VV}} + \hat{\sigma}^{\mathrm{RV}} + \hat{\sigma}^{\mathrm{RR}}$$

- the total $pp \to t\bar{t}X$ cross section is now known to NNLO [Czakon, Fiedler, Mitov '13]
- the FB asymmetry in $p\bar{p} \to t\bar{t}X$ cross section is now known to NNLO [Czakon, Fiedler, Mitov '14]
- soon arbitrary differential cross sections will also be known at NNLO

NNLO default predictions for years to come.

In this talk want to study refinements of NNLO relevant for boosted production: resummation.

Studying NNLO and resummation in tandem gives insights into both

Large corrections in boosted production

Consider very large pair invariant mass where $\tau = M_{t\bar{t}}^2/s \to 1$

$$\frac{d\sigma}{dM_{t\bar{t}}} = \sum_{i,j} \int_{\tau}^{1} \frac{dz}{z}\, ff_{ij}(\tau/z, \mu_f)\, \frac{d\hat{\sigma}_{ij}}{dM_{t\bar{t}}}(z, m_t, M_{t\bar{t}}, \mu_f)$$

$$ff_{ij}(y, \mu_f) = \int_{y}^{1} \frac{dx}{x}\, f_{i/h_1}(x, \mu_f)\, f_{j/h_2}(y/x, \mu_f)$$

Two kinds of large logarithms in $d\hat{\sigma}$:

- soft logs: $[\ln^n(1-z)/(1-z)]_+ \qquad (z \equiv M_{t\bar{t}}^2/\hat{s})$
- small-mass (collinear) logs: $\ln m_t/M_{t\bar{t}}$

Can resum logs by understanding partonic cross sections in certain limits

Two limits of partonic cross sections

$$\text{Soft Limit:} \quad \hat{s}, t_1, m_t^2 \gg \hat{s}(1-z)^2$$

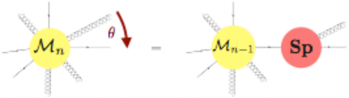
$$\text{Boosted Soft Limit:} \quad \hat{s}, t_1 \gg m_t^2 \gg \hat{s}(1-z)^2 \gg m_t^2(1-z)^2$$

- Resummation in soft limit for differential cross sections has been studied extensively , starting with [Sterman, Kidonakis '97], leading to NNLL calculations in [Ahrens et. al '10, Kidonakis '11]
- Resummation for boosted soft limit has been studied only recently: [Ferroglia, BP, Marzani, Yang '12, '13], [Ferroglia, BP, Scott, Yang , to appear]
- Goal of talk: explain formalism and give numerical results for resummation for high-p_T production, using $d\sigma/dM_{tt}$ as an example
- Resummation is technical, will start with the soft limit, then move to boosted soft limit

QCD MADE SIMPLER

Interplay of soft and collinear emissions is characteristic for high-energy processes. In both limits interactions simplify:

- Collinear limit, where multiple particles move in a similar directions



- Soft limit, in which particles with small energy and momentum are emitted. Eikonal interactions.

At the same time the cross sections are enhanced in these regions.

Factorization in the soft limit ($M \equiv M_{t\bar{t}}$)

Soft limit:

$$\hat{s}, \hat{t}_1, m_t^2 \gg \hat{s}(1-z)^2$$

Partonic cross section factorizes [Kidonakis, Sterman '97]

$$d\hat{\sigma}_{ij} = \mathrm{Tr}\left[\mathbf{H}_{ij}^m(M, m_t, \cos\theta, \mu_f)\,\mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), M, m_t, \cos\theta, \mu_f)\right] + \mathcal{O}(1-z)$$

- $\mathbf{H}_{ij}^m$ are color-space matrices related to virtual corrections
- $\mathbf{S}_{ij}^m$ are color-space matrices related to real emission in soft limit

  Soft corrections involve $\delta(1-z)$ or

$$\alpha_s^n \left[\frac{\ln^m(1-z)}{1-z}\right]_+ ; \quad m = 0, \cdots, 2n-1$$

Towards resummation

$$d\hat{\sigma}_{ij} = \mathrm{Tr}\left[\mathbf{H}^m_{ij}(M, m_t, \cos\theta, \mu_f)\,\mathbf{S}^m_{ij}(\sqrt{\hat{s}}(1-z), M, m_t, \cos\theta, \mu_f)\right] + \mathcal{O}(1-z)$$

- involves two one-scale functions, so large logs appear for any choice of μ_f
- standard EFT solution: derive and solve RG equations for $\mathbf{H}^m_{ij}$ and $\mathbf{S}^m_{ij}$ to resum large logarithms
- for technical reasons, will do this in Mellin space

Mellin transforms

- Under Mellin transforms

$$\tilde{f}(N) = \mathcal{M}[f](N) = \int_0^1 dx\, x^{N-1} f(x); \mathcal{M}^{-1}[\tilde{f}](x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N} f(N)$$

$$d\widetilde{\sigma}(N) = \sum_{ij} \widetilde{\mathcal{L}}_{ij}(N,\mu_f)_{ij}\, d\widetilde{\hat{\sigma}}_{ij}(N, M, m_t, \cos\theta, \mu_f)$$

- Factorization in Mellin space soft limit: $M, m_t \gg M/N$:

$$d\widetilde{\hat{\sigma}}_{ij} = \mathrm{Tr}\left[\mathbf{H}^m_{ij}(M, m_t, \cos\theta, \mu_f)\widetilde{\mathbf{s}}^m_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu_f^2}, M, m_t, \cos\theta, \mu_f\right)\right] + \mathcal{O}\left(\frac{1}{N}\right)$$

- Mellin-space soft function has simple RG equation

$$\frac{d}{d\ln\mu}\widetilde{\mathbf{s}}^m_{ij} = -\left[\Gamma_{\rm cusp}\ln\frac{M^2}{\bar{N}^2\mu^2} - \gamma_{ij}^{m,h\dagger}\right]\widetilde{\mathbf{s}}^m_{ij} - \widetilde{\mathbf{s}}^m_{ij}\left[\Gamma_{\rm cusp}\ln\frac{M^2}{\bar{N}^2\mu^2} - \gamma_{ij}^{m,h}\right]$$

- can solve with standard RG techniques to get resummed partonic cross section in Mellin space

Resummed partonic cross section

$$d\widetilde{\hat{\sigma}}_{ij}(\mu_f) = \exp\left[\frac{4\pi}{\alpha_s(\mu_h)} g_1^m(\lambda,\lambda_f) + g_2^m(\lambda,\lambda_f) + \frac{\alpha_s(\mu_h)}{4\pi} g_3^m(\lambda,\lambda_f) + \ldots\right]$$
$$\times \,\mathrm{Tr}\left[\widetilde{\mathbf{u}}_{ij}^m(\mu_h,\mu_s)\mathbf{H}_{ij}^m(M,m_t,\cos\theta,\mu_h)\widetilde{\mathbf{u}}_{ij}^{m\dagger}(\mu_h,\mu_s)\widetilde{\mathbf{s}}_{ij}^m\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2},M,m_t,\cos\theta,\mu_s\right)\right]$$

$$\lambda \equiv \beta_0 \frac{\alpha_s(\mu_h)}{2\pi}\ln(\mu_h/\mu_s), \qquad \lambda_f \equiv \beta_0 \frac{\alpha_s(\mu_h)}{2\pi}\ln(\mu_h/\mu_f)$$

$$\widetilde{\mathbf{u}}_{ij}^m(\mu_h,\mu_s) \equiv \mathcal{P}\exp\left\{\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu_s)} \frac{d\alpha}{\beta(\alpha)}\boldsymbol{\gamma}_{ij}^{m,h}(M,m_t,\cos\theta,\alpha)\right\}$$

- choosing $\mu_h \sim M$, $\mu_s \sim M/N$ resums all logs into exponentials
- inverse Mellin transforming (using Minimal prescription [Catani, Mangano, Nason, Trentadue '96]) gives resummed cross section

The boosted soft limit

Mellin space boosted soft limit: $M \gg m_t \gg \frac{M}{N} \gg \frac{m_t}{N}$

$$d\widetilde{\hat{\sigma}}_{ij} = \mathrm{Tr}\left[\mathbf{H}_{ij}^m(M, m_t, \cos\theta, \mu_f)\widetilde{\mathbf{s}}_{ij}^m\left(\ln\frac{M^2}{\bar{N}^2\mu_f^2}, M, m_t, \cos\theta, \mu_f\right)\right] + \mathcal{O}\left(\frac{1}{N}\right)$$

Can no longer use formula derived in soft limit, because both $\mathbf{H}_{ij}^m$ and $\widetilde{\mathbf{s}}_{ij}^m$ contain large logs of form $\ln(m_t/M)$ in the boosted soft limit

However, these small-mass logs are of collinear origin: can understand and factorize them

Factorization in the boosted soft limit

- Factorization of hard function [Ferroglia, BP, Yang '12], based on [Mitov/Moch '06] relation between massless and small-mass amplitudes

$$\mathbf{H}^m_{ij}(M, m_t, \cos\theta, \mu) = C_D^2(m_t, \mu)\mathbf{H}_{ij}(M, \cos\theta, \mu) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

  - $\mathbf{H}_{ij}$ related to virtual corrections to massless $gg, q\bar{q} \to \bar{Q}Q$ scattering
  - C_D related to virtual corrections to heavy-quark fragmentation function

- Factorization of soft-function relies on analysis of phase-space integrals using method of regions [Ferroglia, BP, Marzani, Yang '12,'13]

$$\widetilde{\mathbf{s}}^m_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu^2}, M, m_t, \cos\theta, \mu\right) = \widetilde{\mathbf{s}}_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu^2}, M, \cos\theta, \mu\right)\tilde{s}_D^2\left(\ln\frac{m_t}{\bar{N}\mu}, \mu\right) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

  - $\widetilde{\mathbf{s}}_{ij}$ related to wide-angle soft emission to massless $gg, q\bar{q} \to \bar{Q}Q$
  - $\widetilde{s}_D$ related to soft emission collinear to top or anti-top (soft part of heavy-quark fragmentation function)

The boosted soft limit at NNLO

$$d\hat{\sigma}_{ij} \sim \mathrm{Tr}[\mathbf{H}_{ij}(M,\mu)\mathbf{S}_{ij}(M(1-z),\mu)] \otimes C_D^2(m_t,\mu) S_D^2(m_t(1-z),\mu) + \mathcal{O}(1-z) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

All component functions known at NNLO

- C_D and S_D from fragmentation function for generic z [Melnikov, Mitov '04] (along with calculation of [Becher, Neubert '05])
- $\mathbf{H}_{ij}$ from virtual corrections to massless $gg, q\bar{q} \to \bar{Q}Q$ scattering [Glover et. al '00-'01] after IR renormalization procedure [Ferroglia, BP, Yang '13, '14]
- $\mathbf{S}_{ij}$ from double real emission corrections to massless $gg, q\bar{q} \to \bar{q}'q'$ in soft limit [Ferroglia, BP, Yang '12]

Allows for soft plus virtual approximation at NNLO in boosted soft limit, but all-orders resummation is more interesting.

Resummation in the boosted limit

Can derive and solve RG equations to get joint resummation formula
[Ferroglia, BP, Scott, Yang, to appear]

$$d\widetilde{\hat{\sigma}}_{ij}(\mu_f) = \mathrm{Tr}\Bigg[\widetilde{\mathbf{U}}_{ij}(\mu_f,\mu_h,\mu_s)\mathbf{H}_{ij}(M,\cos\theta,\mu_h)\widetilde{\mathbf{U}}^{\dagger}_{ij}(\mu_f,\mu_h,\mu_s)$$
$$\times\, \widetilde{s}_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2}, M, \cos\theta, \mu_s\right)\Bigg] \times \widetilde{U}_D^2(\mu_f,\mu_{dh},\mu_{ds})\, C_D^2(m_t,\mu_{dh})\, \tilde{s}_D^2\left(\ln\frac{m_t}{\bar{N}\mu_{ds}}, \mu_{ds}\right)$$
$$+\,\mathcal{O}\left(\frac{1}{N}\right) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

- remove large logs in matching functions through choices
  $\mu_h \sim M, \quad \mu_s \sim M/N, \quad \mu_{dh} \sim m_t \quad \mu_{ds} \sim m_t/N$
- $\tilde{U}_{ij}$ and $\widetilde{U}_D$ are RG factors which resum logs
- can estimate perturbative uncertainties by varying all scales independently

Logarithmic accuracy: the frontier

Resummation involves anomalous dimensions Γ, γ and matching functions

	$\Gamma^i_{\rm cusp}$	γ_i	$H, \widetilde{s}, c_D, \widetilde{s}_D$
NLL	2-loop	1-loop	0-loop
NNLL	3-loop	2-loop	1-loop
NNLL'	3-loop	2-loop	2-loop

Analytic frontier (differential cross sections):

- soft limit: NNLL [Ahrens, Ferroglia, Neubert, BP, Yang, Kidonakis]
- boosted soft limit: NNLL': [Ferroglia, BP, Yang]

Numerical implementations of resummation (not fixed-order approximations!)

- soft limit: NLL in Mellin space for A_{FB} [Almeida,Sterman,Vogelsang '08]
- soft limit: NNLL in momentum space [Ahrens, Ferroglia, Neubert, BP, Yang '10,'11]
- soft limit to NNLL and boosted soft limit to NNLL' in Mellin space [Ferroglia, BP, Scott, Yang, to appear soon]

Matching across kinematic limits and with fixed order

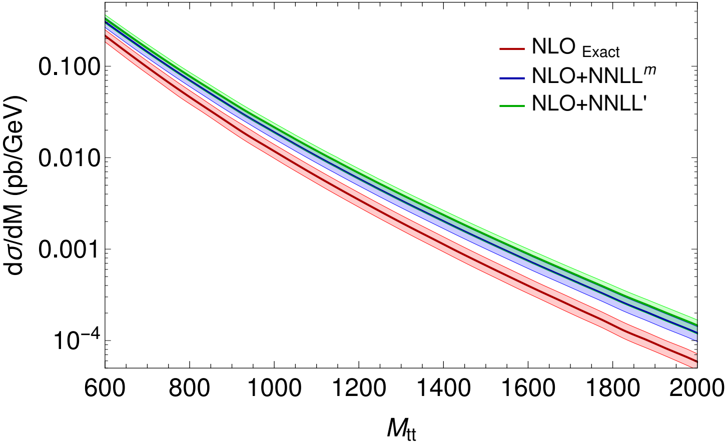
Best prediction matches NNLL'^{b} (boosted) with NNLL^{m} (soft) with NLO

$$
\begin{aligned}
d\sigma^{\mathrm{NLO+NNLL}'} = d\sigma^{\mathrm{NNLL}'^{b}} &+ \left(d\sigma^{\mathrm{NNLL}^{m}} - d\sigma^{\mathrm{NNLL}^{b}}\Big|_{\substack{\mu_{\mathrm{ds}}=\mu_s \\ \mu_{\mathrm{dh}=\mu_h}}} \right) \\
&+ \left(d\sigma^{\mathrm{NLO}} - d\sigma^{\mathrm{NNLL}^{m}}\Big|_{\substack{\mu_{\mathrm{s}}=\mu_f \\ \mu_{\mathrm{h}=\mu_f}}} \right)
\end{aligned}
$$

- first parenthesis vanishes as $m_t \to 0$, matches boosted soft and soft
- second line vanishes as $z \to 1$, matches resummed to NLO (can easily be modified to NNLO, once available)
- NLO+NNLL' contains all available information on soft-gluon in both limits

Results at high M_{tt} at LHC with $\sqrt{s} = 8$ TeV

MSTW2008NNLO PDFs, $m_t = 173.2$ GeV, $\mu_f \in [M_{tt}/2, 2M_{tt}]$



- bands from scale variations of μ_f, and μ_h, μ_s (NNLL^m), and μ_h, μ_s, μ_{dh}, μ_{ds} (NNLL$'$)
- NNLL^m resummation in soft limit is large effect compared to NLO (at $\mu_f = M$)
- boosted resummation included in NNLL$'$ produces mild, further enhancement

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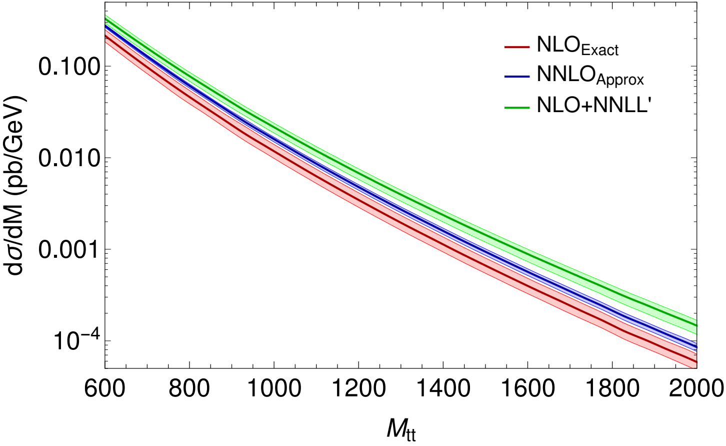

- bands from scale variations of μ_f and μ_h, μ_s, μ_{dh}, μ_{ds} (NNLL$'$)
- NNLO approximation of NNLL$'$ misses important effects

Conclusions

Summary:

- QCD for boosted top production is a complicated multi-scale problem
- presented formalism for joint resummation of soft and small-mass logs in Mellin space
- resummation effects significant at high M_{tt}
- can also apply formalism to p_T^t distributions (to appear soon)
- resummation adds information to NNLO but doesn't replace it: NNLL'+NNLO will be most interesting

Further issues:

- need to compare with NLO+parton shower-based resummations
- EW corrections important for boosted top