Interpretation of the top mass measurements: a theory overview

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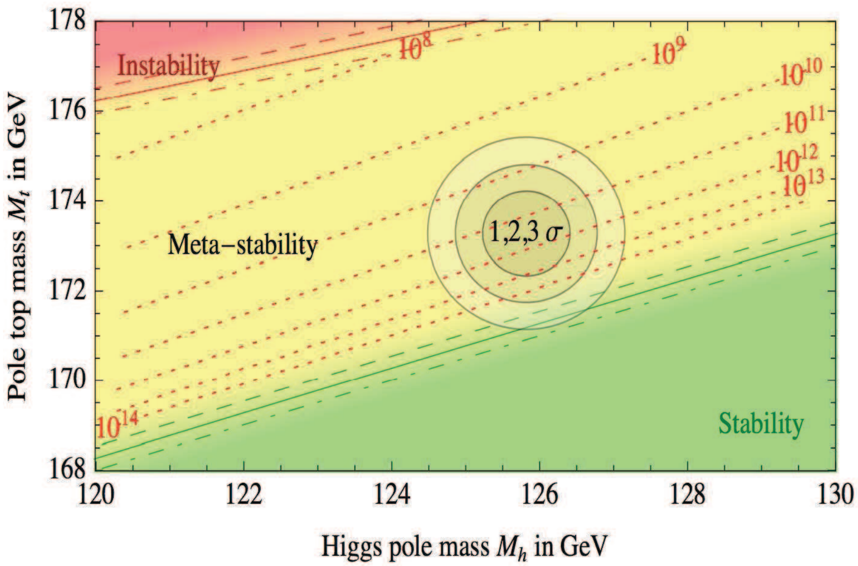
1. Introduction
2. Top mass definitions
3. Standard measurements and Monte Carlo mass
4. Ongoing work on understanding the MC mass and improving generators
5. Alternative methods for mass measurements and interpretations
6. Conclusions

Based on work and talks by G.C., A.Hoang, M.Mangano, M.Beneke, K.Ellis, E.Re, S.Moch, A.Mitov, M.Czakon, M.Dowling, P.Nason, M.Schulze, K.Melnikov, P.Marquard, S.Weinzierl, M.Steinhauser, Smirnovs, ATLAS/CMS working groups

See also Frascati workshop on 'Top mass: challenges in definition and determination', May 2015
https://agenda.infn.it/conferenceDisplay.py?confId=9202

The top quark mass plays a crucial role in the electroweak symmetry breaking

Stability of the SM vacuum depends on top and Higgs masses (Degrassi et al, JHEP'12)



All values in GeV	CDF	D0	ATLAS	CMS	Tevatron	LHC	WA
m_{top}	173.19	174.85	172.65	173.58	173.58	173.28	173.34
Stat	0.52	0.78	0.31	0.29	0.44	0.22	0.27
iJES	0.44	0.48	0.41	0.28	0.36	0.26	0.24
stdJES	0.30	0.62	0.78	0.33	0.27	0.31	0.20
flavourJES	0.08	0.27	0.21	0.19	0.09	0.16	0.12
bJES	0.15	0.08	0.35	0.57	0.13	0.44	0.25
MC	0.56	0.62	0.48	0.19	0.57	0.25	0.38
Rad	0.09	0.26	0.42	0.28	0.13	0.32	0.21
CR	0.21	0.31	0.31	0.48	0.23	0.43	0.31
PDF	0.09	0.22	0.15	0.07	0.12	0.09	0.09
DetMod	<0.01	0.37	0.22	0.25	0.09	0.20	0.10
b-tag	0.04	0.09	0.66	0.11	0.04	0.22	0.11
LepPt	<0.01	0.20	0.07	<0.01	0.05	0.01	0.02
BGMC	0.10	0.16	0.06	0.11	0.11	0.08	0.10
BGData	0.15	0.19	0.06	0.03	0.12	0.04	0.07
Meth	0.07	0.15	0.08	0.07	0.06	0.06	0.05
MHI	0.08	0.05	0.02	0.06	0.06	0.05	0.04
Total Syst	0.85	1.25	1.40	0.99	0.82	0.92	0.71
Total	1.00	1.48	1.44	1.03	0.94	0.94	0.76
χ^2/ndf	1.09 / 3	0.13 / 1	0.34 / 1	1.15 / 2	2.45 / 5	1.81 / 4	4.33 / 10
χ^2 probability [%]	78	72	56	56	78	77	93

SM vacuum at the border between stability and metastability regions

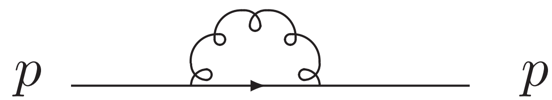
Top mass world average as pole mass in determination of Yukawa coupling

World average: $m_t^{\rm TeV+LHC} = [173.34 \pm 0.27({\rm stat}) \pm 0.71({\rm syst})]$ GeV (arXiv:1403.4427)

Theory uncertainty: Monte Carlo systematics (MC); modelling QCD radiation effects (Rad); colour reconnection (CR); parton distribution functions (PDF)

Issues: what mass is reconstructed; whether/why it is worth using any mass definition

Top-quark mass definitions - Subtraction of the UV divergences in the self energy $\Sigma(p)$



Renormalized propagator: $$S(p) = -\frac{i}{\not p - m_t^0 + \Sigma^R(p, m_t^0, \mu)}$$

$$\Sigma^R \sim \left[\frac{1}{\epsilon} - \gamma + \ln 4\pi + A(m_t^0, p, \mu)\right] \not p - \left[4\left(\frac{1}{\epsilon} - \gamma + \ln 4\pi\right) + B(m_t^0, p, \mu)\right] m_t^0 + (Z_2 - 1) \not p - (Z_2 Z_m - 1) m$$

On-shell renormalization (pole mass) - Z_2 and Z_m are determined by means of:

$$\Sigma^R(p) = 0 \ \text{ and } \ \frac{\partial \Sigma^R}{\partial \not p} = 0 \ \text{ for } \ \not p = m$$

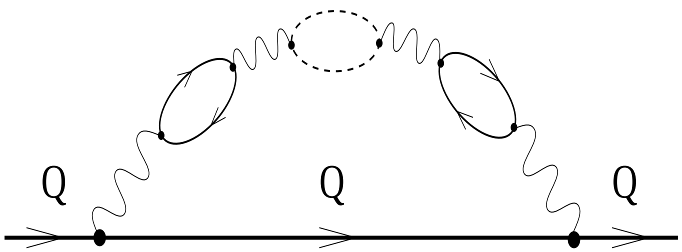
$\overline{\mathrm{MS}}$ renormalization: counterterm to subtract $(1/\epsilon + \gamma_E - \ln 4\pi)$

$$S^R_{o.s.}(p) \sim \frac{i}{\not p - m_{\mathrm{pole}}} \quad ; \quad S^R_{\overline{\mathrm{MS}}} \sim \frac{i}{\not p - m_{\mathrm{MS}} - (A - B) m_{\mathrm{MS}}}$$

Pole mass is the pole of the propagator; $\overline{\mathrm{MS}}$ mass is quite far from the pole

Pole mass is a physical mass for free particles like electrons, but for heavy quarks it exhibits an ambiguity $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ due to infrared renormalons (Braun,Beneke'94)

Higher-order corrections to the self energy



$$\Sigma(m,m) \sim m \sum_n \alpha_S^{n+1}\,(2\beta_0)^n\, n!$$

$$\delta m_{\rm pole} \approx \frac{2\pi C_F \Lambda}{\beta_0} e^{\frac{5}{6}} \left(\ln \frac{\bar{m}^2}{\Lambda^2}\right)^{-\frac{\beta_1}{2\beta_0^2}} \approx \ 300 \ {\rm MeV}$$

$\overline{\rm MS}$ mass suitable for off-shell quarks ($Z \to b\bar{b}$) at LEP, but $\sim (\alpha_S/v^2)^k$ at threshold

Relation pole/$\overline{\rm MS}$ mass at 4 loops [$\bar{m} = \bar{m}(\bar{m})$ and $\bar{\alpha}_S = \alpha_S(\bar{m})/\pi$] (P.Marquard et al, PRL'15)

$$m_{\rm pole} = \bar{m} \times \left[1 + c_1\bar{\alpha}_S + + c_2\bar{\alpha}_S^2 + c_3\bar{\alpha}_S^3 + c_4\bar{\alpha}_S^4 + \dots\right] \ ; \ c_4 \bar{m}_t \bar{\alpha}_S^4 \approx 200 \ {\rm MeV}$$

For top quarks: $m_{\rm pole} = \bar{m}[1 + 0.046 + 0.010 + 0.003 + 0.001 + \dots]$

MSR masses in terms of an infrared scale R, e.g. μ_F, $\bar{m}(\mu)$, etc. (A.Hoang et al)

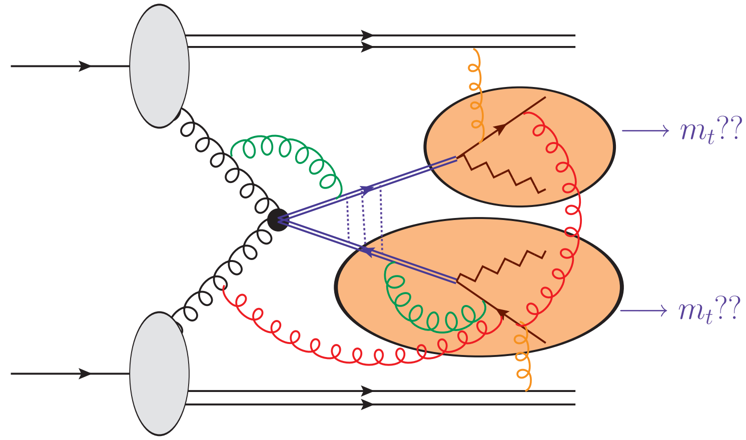
$m_t^{\rm MSR}(R) \to m_{\rm pole}$ for $R \to 0$; $m^{\rm MSR}(R) \to \bar{m}_t(\bar{m}_t)$ for $R \to \bar{m}_t(\bar{m}_t)$

$$m_{\rm pole} = m^{\rm MSR}(R,\mu) + \delta m(R,\mu) \ ; \ \delta m(R,\mu) = R \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk}\alpha_S(\mu)^n$$

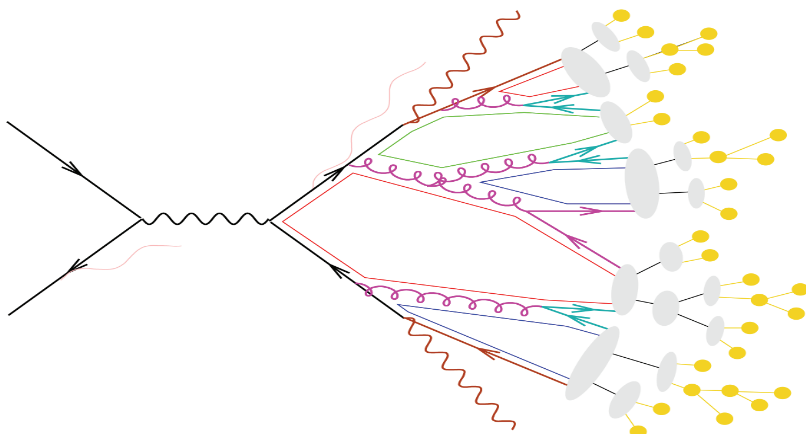
$$\frac{dm_{\rm pole}}{d\ln\mu} = 0 \Rightarrow \frac{dm^{\rm MSR}(R,\mu)}{d\ln\mu} = -R\gamma[\alpha_S(\mu)]$$

NLO top production+decays necessary to have a consistent top mass definition

Figure by A. Signer



Parton showers: LO + soft/collinear (N)LLs + non-perturbative models



$$dP = \frac{\alpha_S}{2\pi}\, P(z)dz\, \frac{dQ^2}{Q^2}\, \Delta_S(Q^2_{\rm max}, Q^2)$$

Δ_S captures leading-log virtual corrections

Width effects are neglected

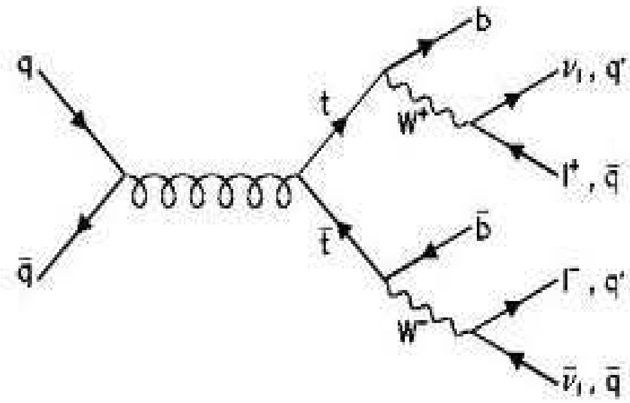
One calls 'Monte Carlo mass' the value of m_t based on standard reconstruction methods

aMC@NLO includes off-shell and non-resonant effects, not yet NLO decays

Improvement in POWHEG: NLO top decays and approximate treatment of top width

Both aMC@NLO and POWHEG use the top-quark pole mass

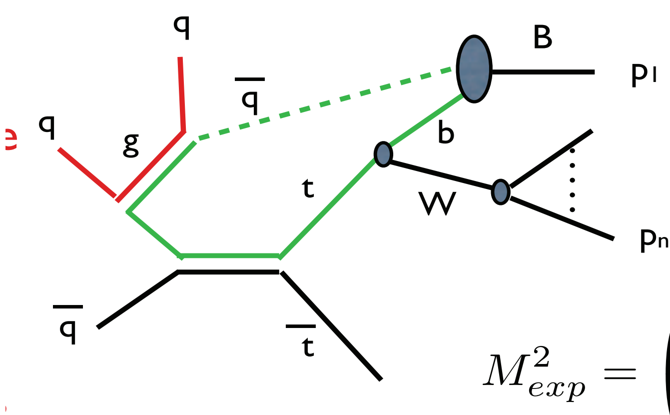
Measured mass must be close to $m_{\rm pole}$: top-decay kinematics is driven by $m_{\rm pole}$



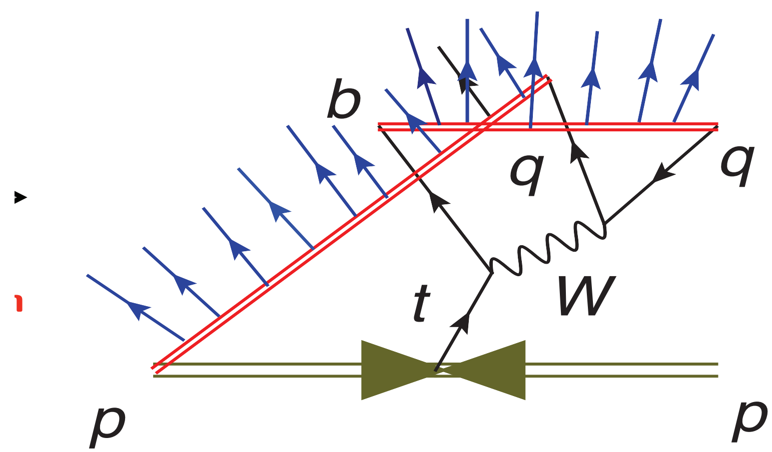
$$\frac{1}{[(p_W+p_b)^2-m_t^2]^2+m_t^2\Gamma_t^2}\sim\frac{1}{\pi}\,\delta[(p_W+p_b)^2-m_t^2]\ \mathrm{for}\ \Gamma_t\ll m_t$$

Reconstructed mass $p^2=(p_{b-\rm jet}+p_\nu+p_\ell)^2$ (with cuts on jets and leptons) with on-shell tops should be close to the pole mass, up to widths and higher-order corrections

Colour-reconnection effects can spoil this picture



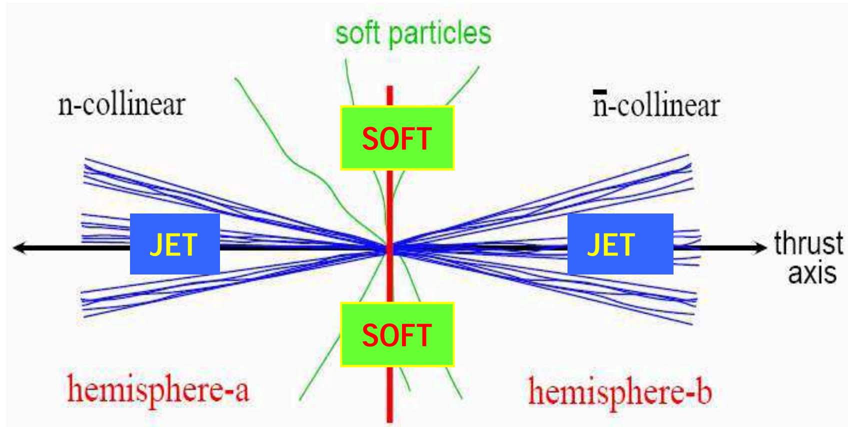
$$M^2_{exp}=\left(\sum_{i=1,\dots,n}p_i\right)^2$$



Left: M.L.Mangano, TOP 2013 workshop, Right: S.Argyropoulos, LNF'15 workshop

Leptonic observables without reconstructing top decay products minimize such effects

Attempt to address the MC mass using the SCET formalism $Q \gg m_t \gg \Gamma_t \gg \Lambda_{\rm QCD}$



Factorization theorem $(e^+e^- \to t\bar{t})$: A.H. Hoang and I.Stewart '08

$$\frac{d\sigma}{dM_t^2 dM_{\bar{t}}^2} \sim H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \int d\ell^+ d\ell^- B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_+\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S(\ell^+, \ell)$$

H_Q, H_m: hard scattering; $B_\pm$: jet function; S: soft function

Jet mass: MSR mass with $R \sim \Gamma_t$; $Q_0 \sim 1$ GeV shower cutoff

$$m_J(\mu) \sim \left[\frac{d \ln \tilde{B}(y,\mu)}{dy}\right]_{y=-ie^{-\gamma_E}/R} \Rightarrow m_{\rm pole} = m_J(\mu) + e^{\gamma_E}\Gamma_t \frac{\alpha_S(\mu) C_F}{\pi}\left(\ln\frac{\mu}{\Gamma_t} + \frac{1}{2}\right) + \mathcal{O}(\alpha_S^2)$$

$Q_0 \sim$ 1-2 GeV $\Rightarrow m_{\rm pole} - m_J(Q_0) \simeq 150 - 200$ MeV

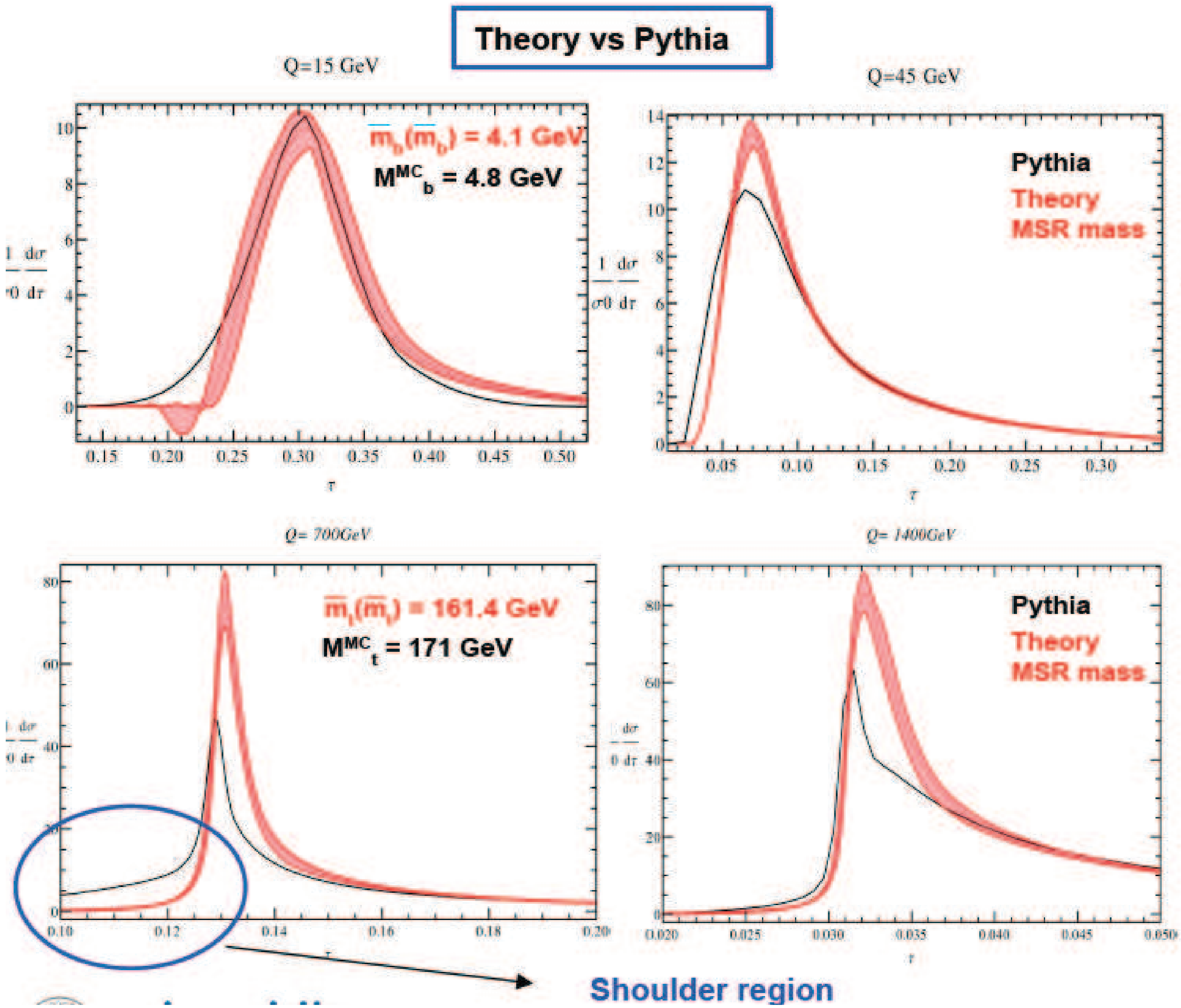
$m_t^{\rm MC} = m_t^{\rm MSR}(R) + \Delta_t(R)$ $\Rightarrow$ fix PYTHIA mass and tune $m_t^{\rm MSR}$

$e^+e^- \to b\bar{b}$

Thrust distribution

$e^+e^- \to t\bar{t}$

(A.Hoang)



SCET at NLO+(N)NLL+ power corrections: prospects to extend to hadron colliders

Other attempts to relate MC and pole masses: run HERWIG with top-hadron states

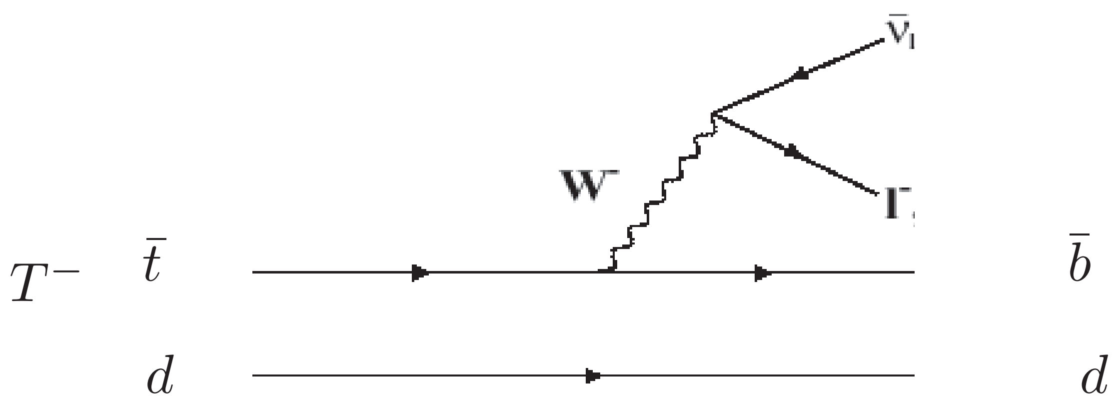
Pretend that top quarks hadronize and decay via the spectator model

From a given observable R extract the Monte Carlo mass $m_t^{T,\mathrm{MC}}$

Study the same observable R with standard top samples, get $m_t^{t\mathrm{MC}}$ and compare the extracted masses : $m_t^{T,\mathrm{MC}} = m_t^{t,\mathrm{MC}} + \Delta m$

In the hadronized samples, the Monte Carlo mass can be related to the T-meson mass M_T and ultimately to the pole or $\overline{\mathrm{MS}}$ top-quark masses by using lattice, potential models, NRQCD, etc.

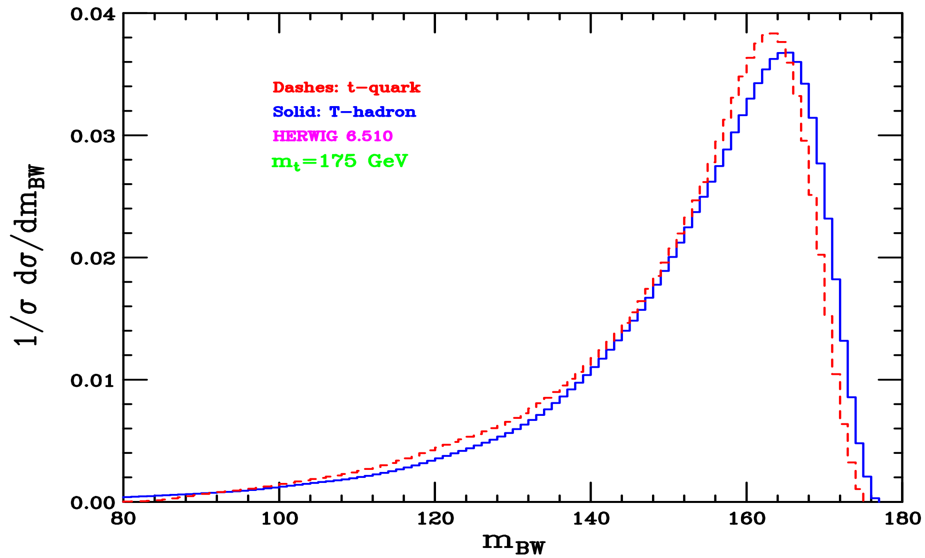
Spectator model decays: $T^- \to (\bar{b}d)\ell^-\bar{\nu}_\ell + X \ldots \quad p_T^2 = (p_{\bar{b}} + p_W + p_q + p_X)^2$

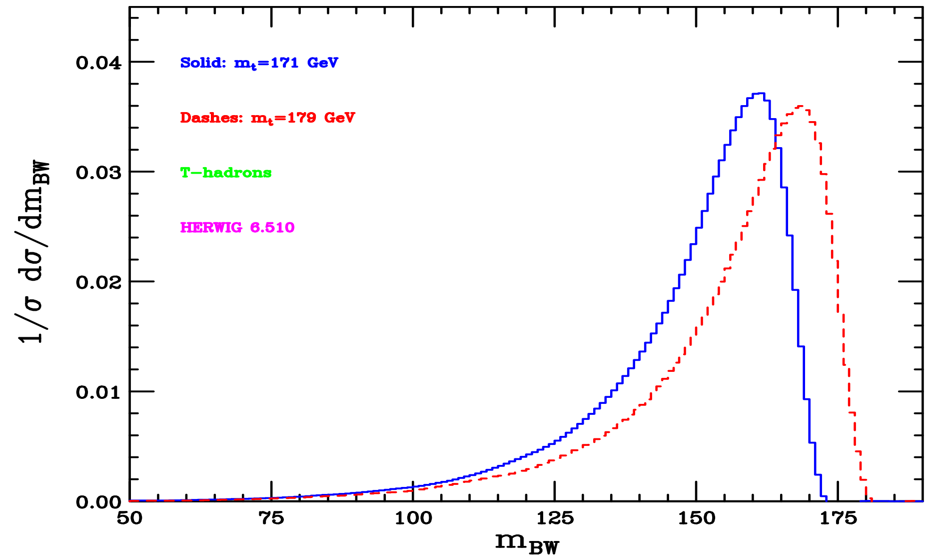


Spectator does not radiate, few events where the b quark does not emit

$\bar{b}$ tends to form clusters with spectator quarks with invariant mass closer to $(p_T - p_W)^2$ with respect to standard top decays

$e^+e^- \to T\bar{T} \to (bW^+\bar{q})(\bar{b}W^-q')$ collisions at $\sqrt{s} = 1$ TeV G.C.,'14





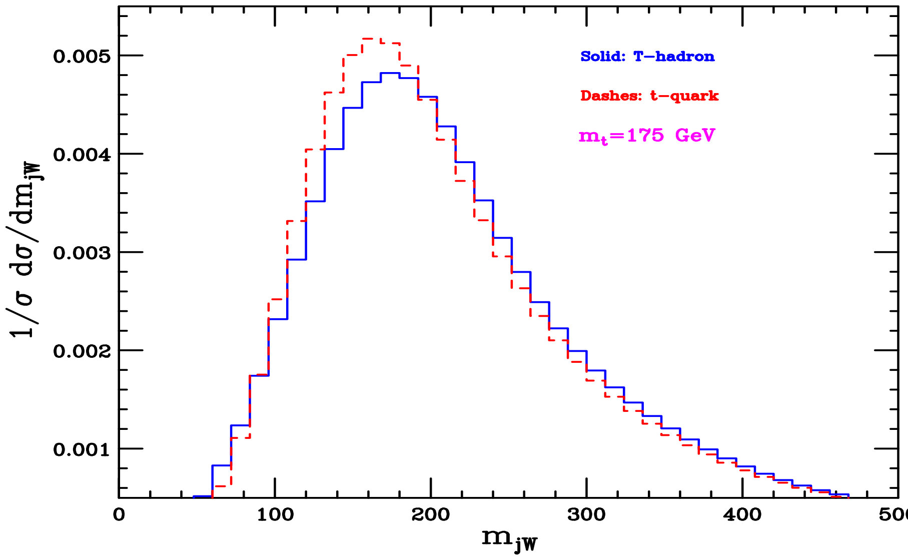
T-hadrons:

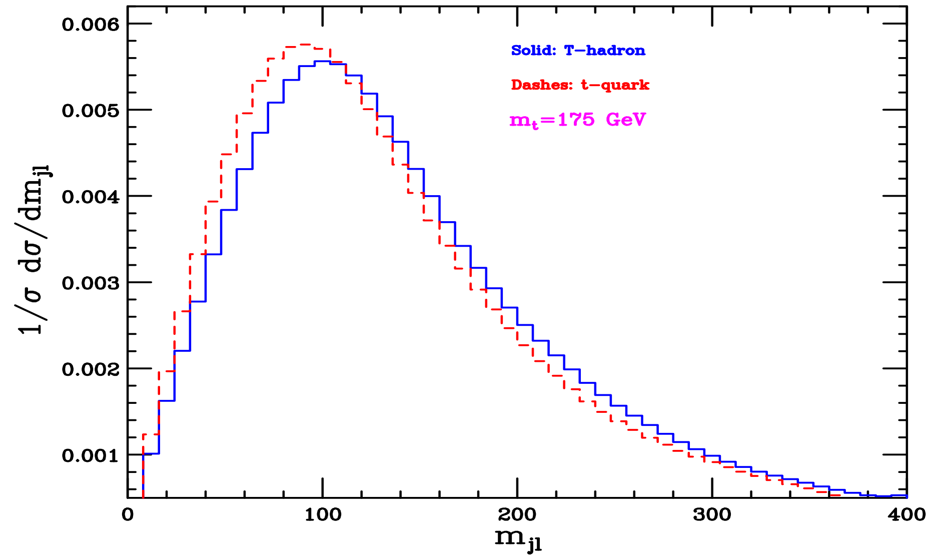
m_t (GeV)	$\langle m_{BW}\rangle$ (GeV)	$\langle m^2_{BW}\rangle$ (GeV^2)	$\langle m^3_{BW}\rangle$ (GeV^3)	$\langle m^4_{BW}\rangle$ (GeV^4)
171	148.76	2.24×10^4	3.41×10^6	5.24×10^8
173	150.44	2.29×10^4	3.53×10^6	5.48×10^8
175	152.18	2.35×10^4	3.66×10^6	5.74×10^8
177	153.80	2.40×10^4	3.77×10^6	5.99×10^8
179	155.61	2.45×10^4	3.91×10^6	6.28×10^8

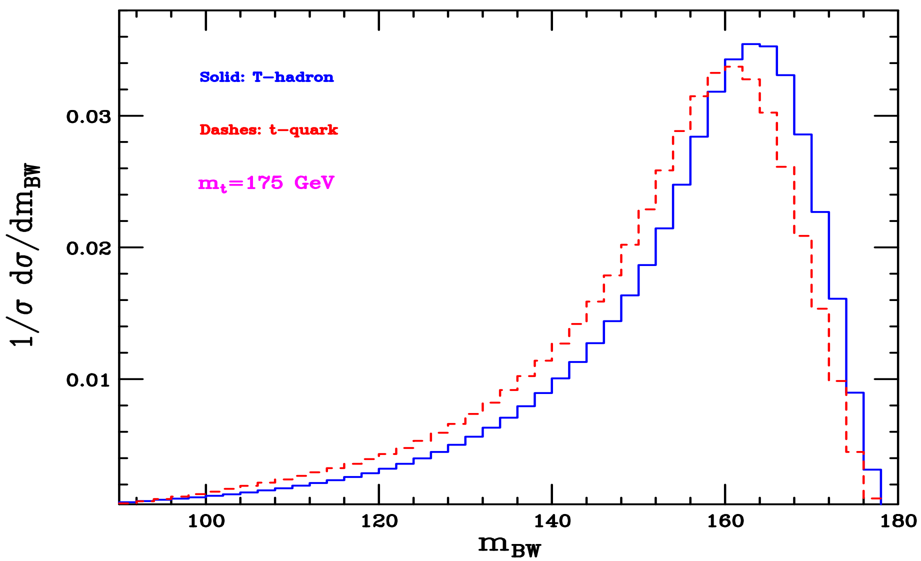
t-quarks:

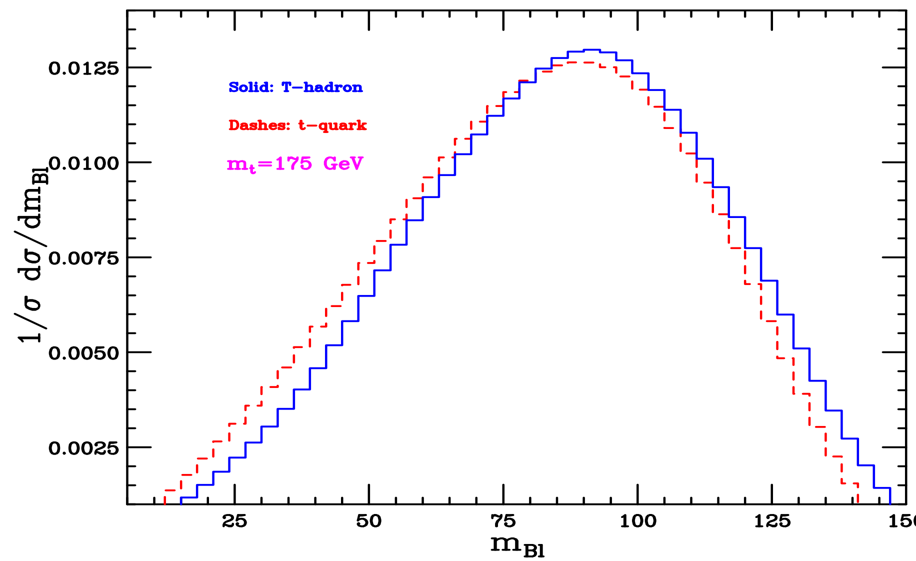
m_t (GeV)	$\langle m_{BW}\rangle$ (GeV)	$\langle m^2_{BW}\rangle$ (GeV^2)	$\langle m^3_{BW}\rangle$ (GeV^3)	$\langle m^4_{BW}\rangle$ (GeV^4)
171	148.08	2.21×10^4	3.35×10^6	5.11×10^8
173	149.56	2.26×10^4	3.46×10^6	5.32×10^8
175	151.00	2.30×10^4	3.56×10^6	5.54×10^8
177	152.60	2.36×10^4	3.67×10^6	5.78×10^8
179	153.97	2.40×10^3	3.78×10^6	6.00×10^8

pp collisions at $\sqrt{s} = 8$ TeV, dilepton channel, k_T algorithm, $R = 0.7$, $p_{T,j} > 30$ GeV, $p_{T,\ell} > 20$ GeV, MET> 20 GeV, $|\eta_{j,\ell,\nu}| < 2.5$ (HERWIG 6.510, preliminary)







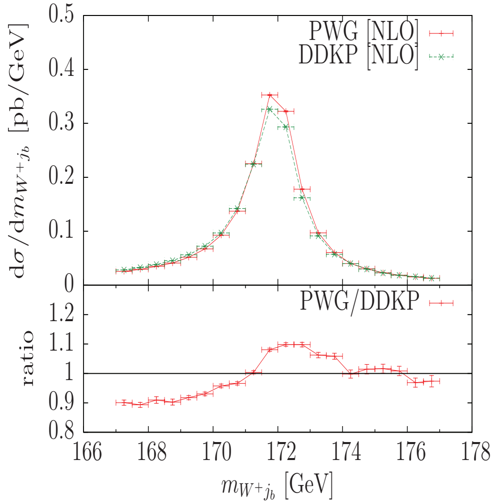


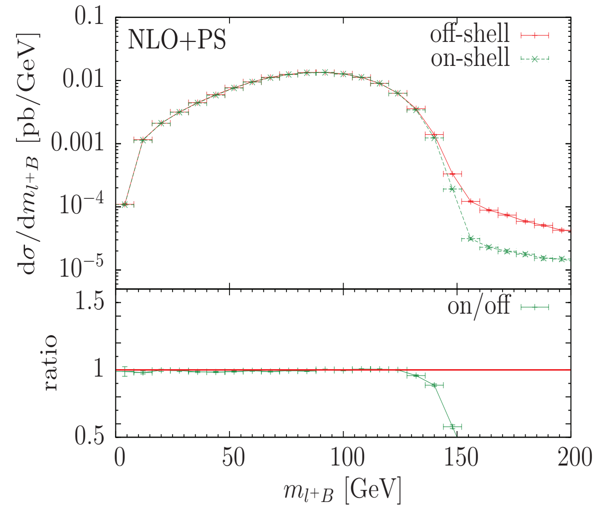
In progress: comparison with PYTHIA (cluster vs. string models)

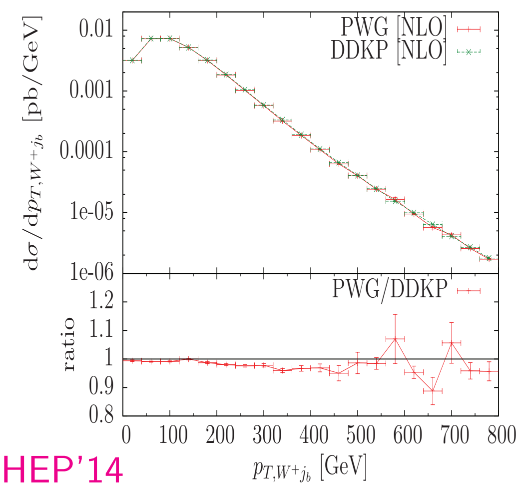
POWHEG implementation of NLO top decays: J.M. Campbell et al, JHEP'15

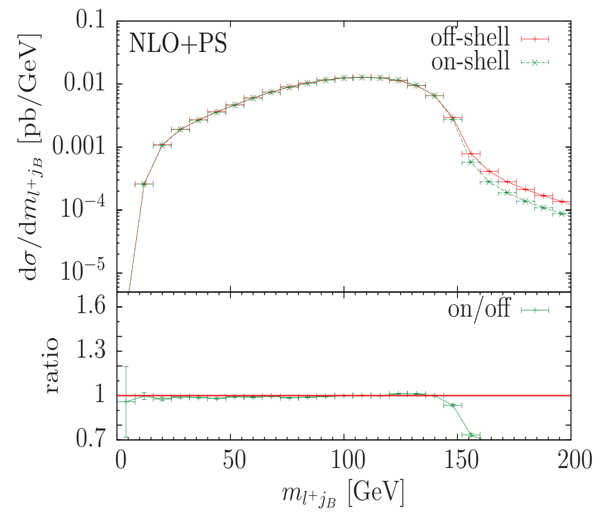
Off-shell effects: 1) Breit–Wigner reweighting; 2) reweighting by the ratio of double-resonant (DR) and LO amplitude; 3) DR method and reweighting by ratio of full off-shell/on-shell Born matrix elements

Comparison against NLO calculation: small discrepancy due to interference in DDKP



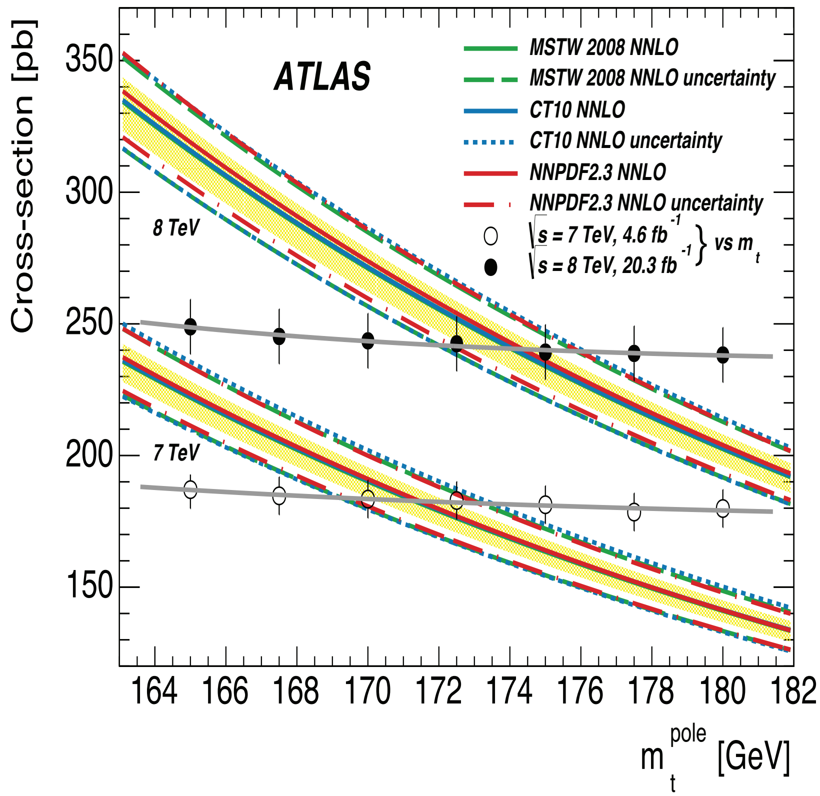


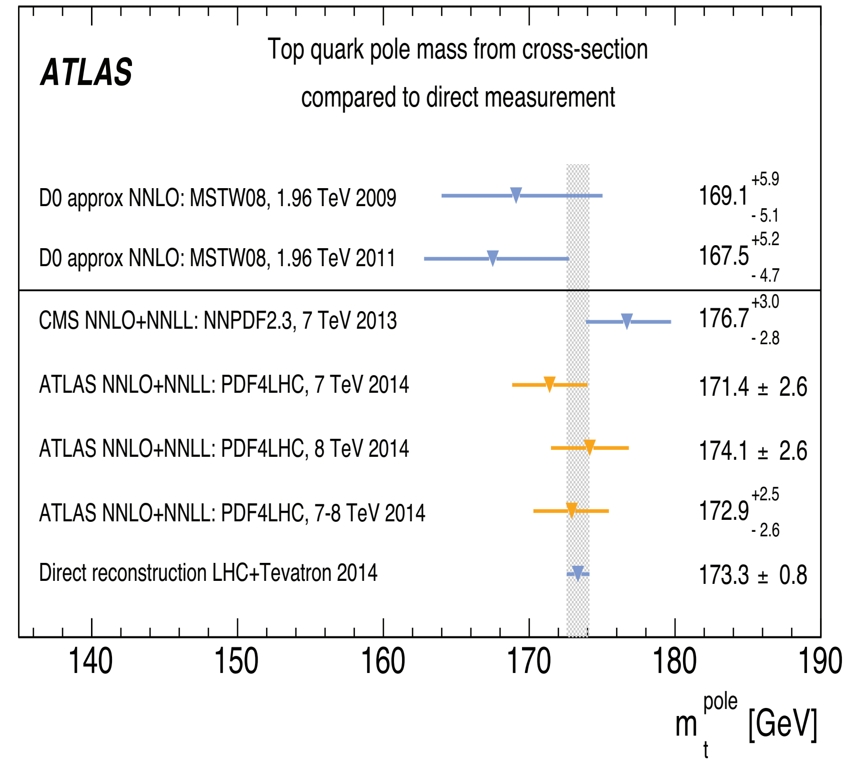




DDKP, A.Denner et al, JHEP'14

Alternative strategies: m_t from NNLO+NNLL $t\bar{t}$ cross section (Czakon, Fielder and Mitov, '13):



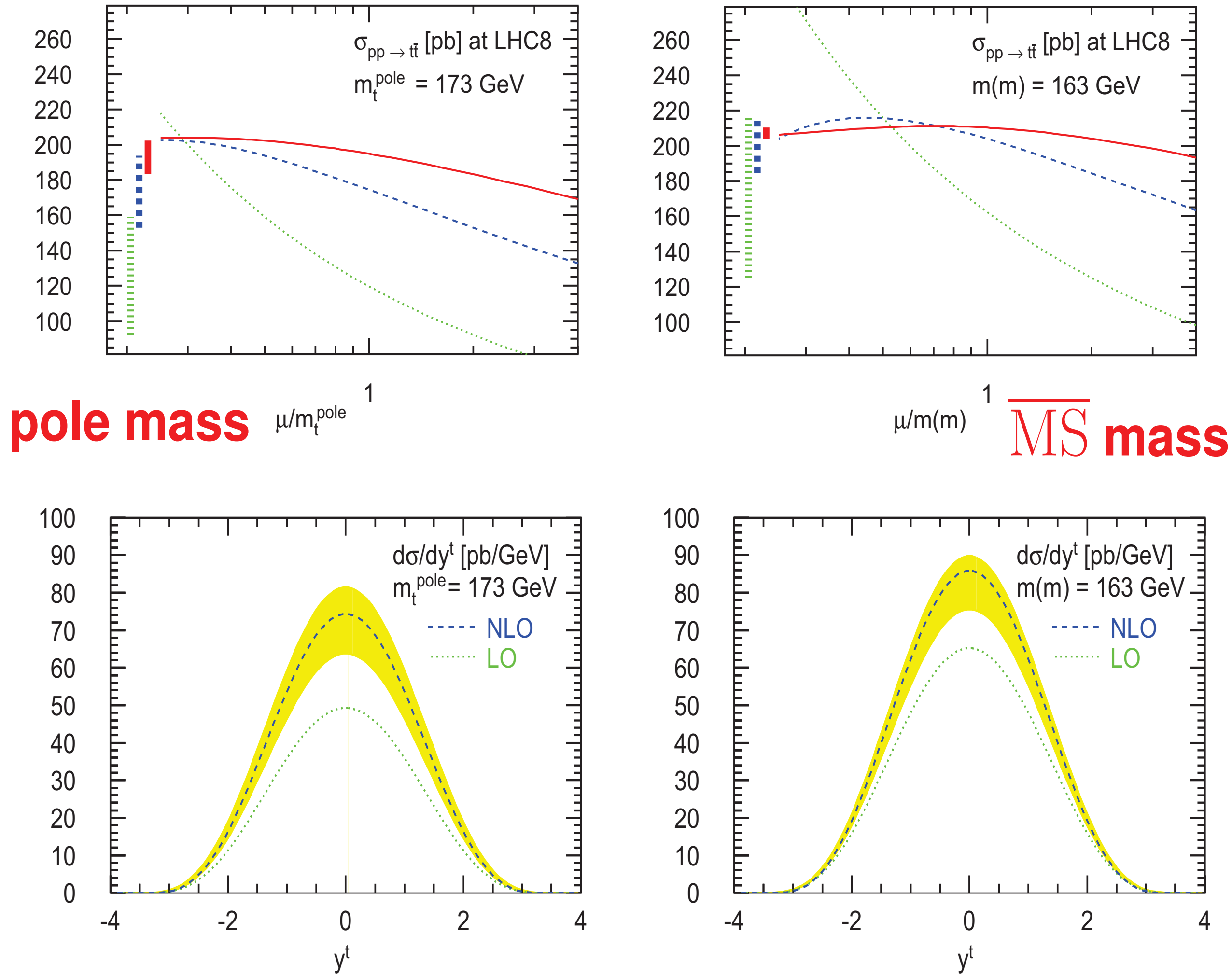


Extracted pole mass exhibits larger errors than matrix-element and template methods, but larger statistics are expected in Run 2

Mild dependence on MC m_t; $\Delta(\alpha_S + \mathrm{PDF}) \simeq 1.7$ GeV, $\Delta(\mu) \simeq^{+0.9}_{-1.3}$ GeV

Prospects to extend calculation to NNLO distributions and ultimately top decays

Extracting top running mass from NNLO $t\bar{t}$ cross section (M.Dowling and S.Moch'14)



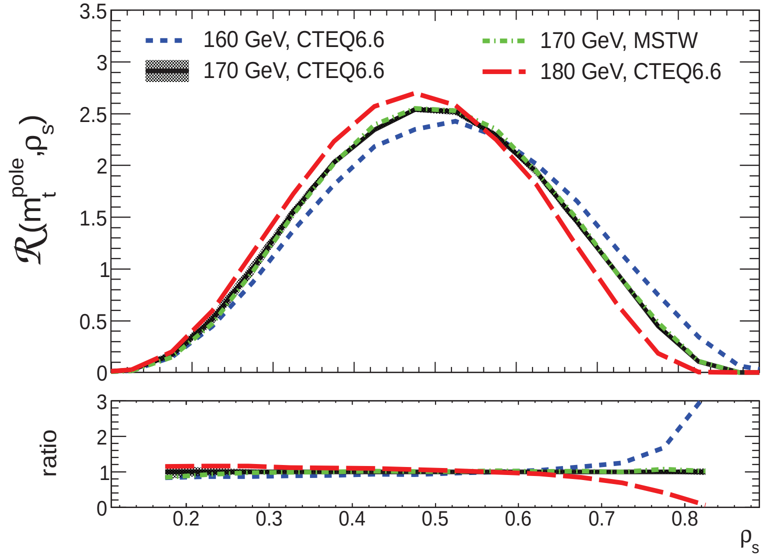
Using the $\overline{\rm MS}$ mass seems to yield a milder scale dependence

Scale variation: $1/2 < \mu/m_{\rm pole} < 2$; $1/2 < \mu/m(m) < 2$, $\mu = \mu_F = \mu_R$

ABM fit: $\bar{m}_t(\bar{m}_t) = 162.3 \pm 2.3$ GeV; $m_t^{\rm pole} = 171.2 \pm 2.4$ GeV

NLO calculation of $t\bar{t}$+jet cross section with the pole mass (S.Alioli et al.,'13)

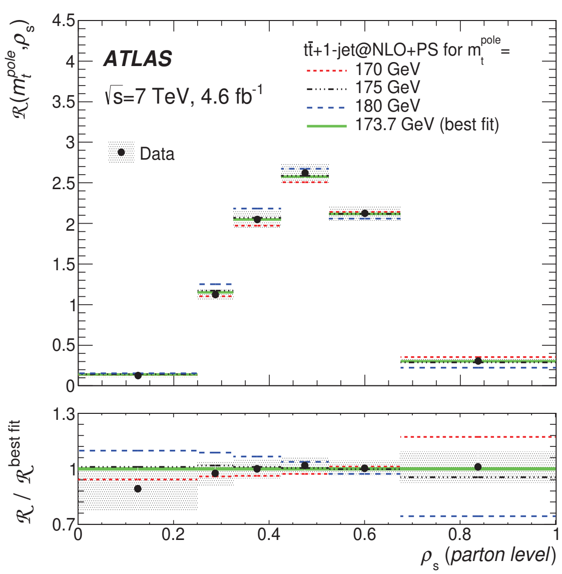
Matching with shower and hadronization through POWHEG+PYTHIA

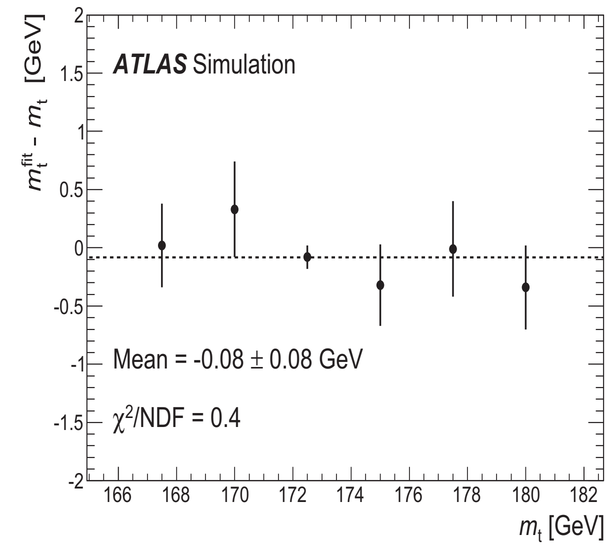


$$\mathcal{R} = \frac{1}{\sigma_{t\bar{t}j}} \frac{d\sigma_{t\bar{t}j}(m_t^{\rm pole})}{d\rho_S}$$

$$\rho_S = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}} \quad , \quad m_0 = 170 \text{ GeV}$$

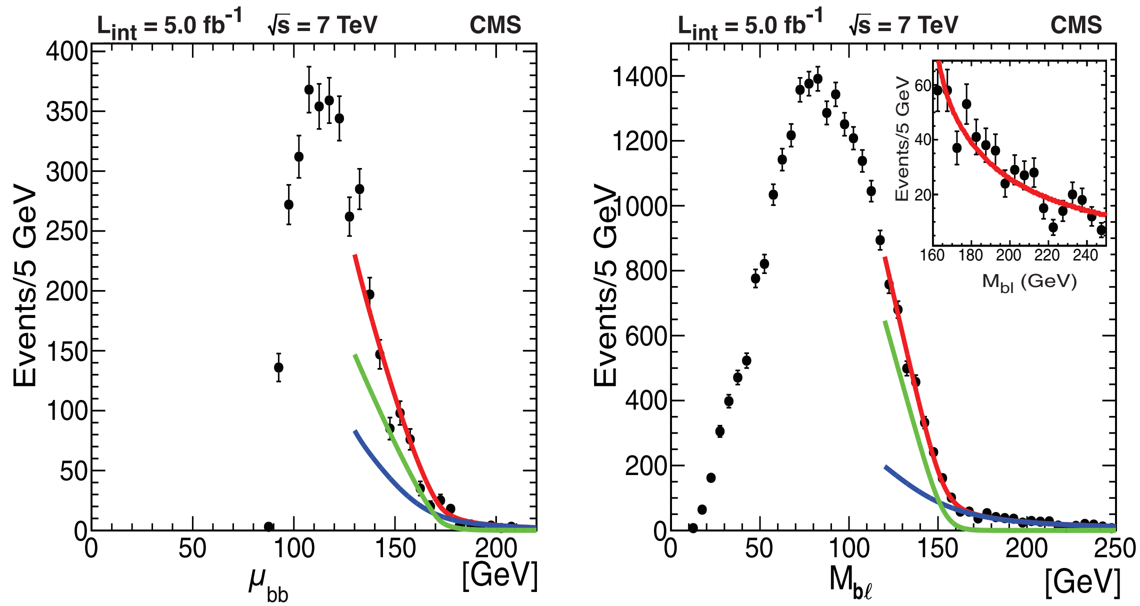
$$t\bar{t} + \text{jet (det.)} \rightarrow t\bar{t} + \text{jet (parton)}$$





$$m_t^{\rm pole} = [173.1 \pm 1.5(\text{stat}) \pm 1.4(\text{syst})^{+1.0}_{-0.5}(\text{theo})] \text{ GeV}$$

Endpoint method (CMS): in dilepton channels, the endpoint of ℓ+b-jet, '$\ell\ell$' or 'bb' invariant mass distributions are sensitive to m_t



Endpoint relations at LO (no radiative corrections) from energy-momentum conservation; shapes normalized to NLO cross sections

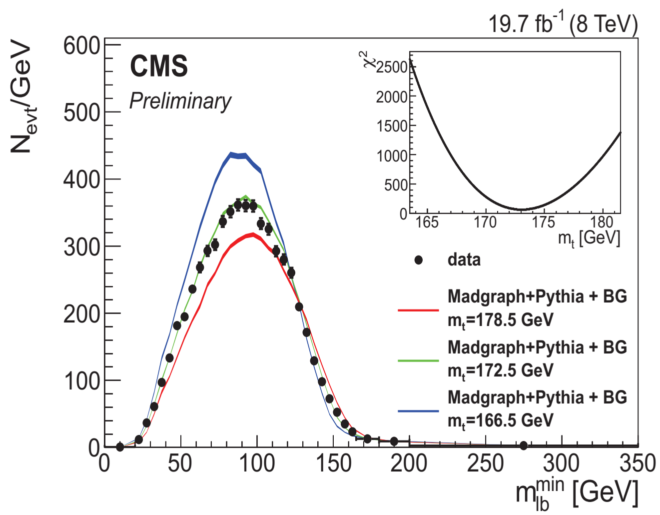
$$\mu_{bb}^{\max} = \frac{m_t}{2}\left(1 - \frac{m_W^2}{m_t^2}\right) + \sqrt{\frac{m_t^2}{4}\left(1 - \frac{m_W^2}{m_t^2}\right) + m_W^2}\ , \quad m_{b\ell}^{\max} = \sqrt{m_b^2 + \left(1 - \frac{m_\nu^2}{m_W^2}\right)(E_W^* + p^*)(E_b + p^*)}$$

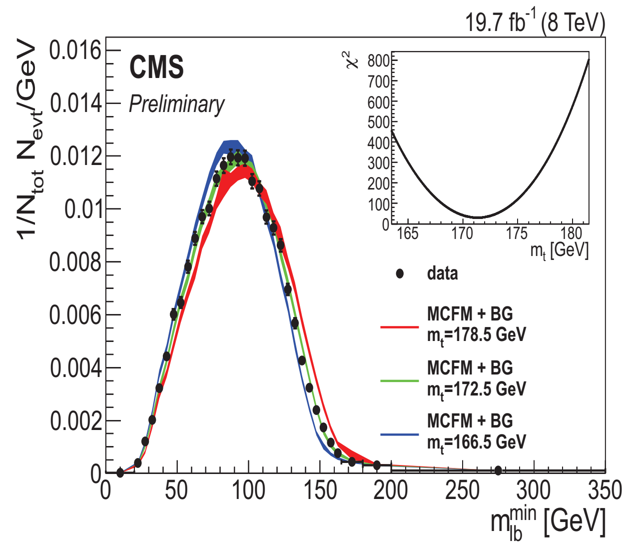
Minimizes generator impact since b-jet calibration uses data; theory error mostly due to colour reconnection effects

As it is based on kinematic constraints, the measured m_t must be close to the pole mass

Interesting to compare with recent POWHEG with NLO top decays and pole mass

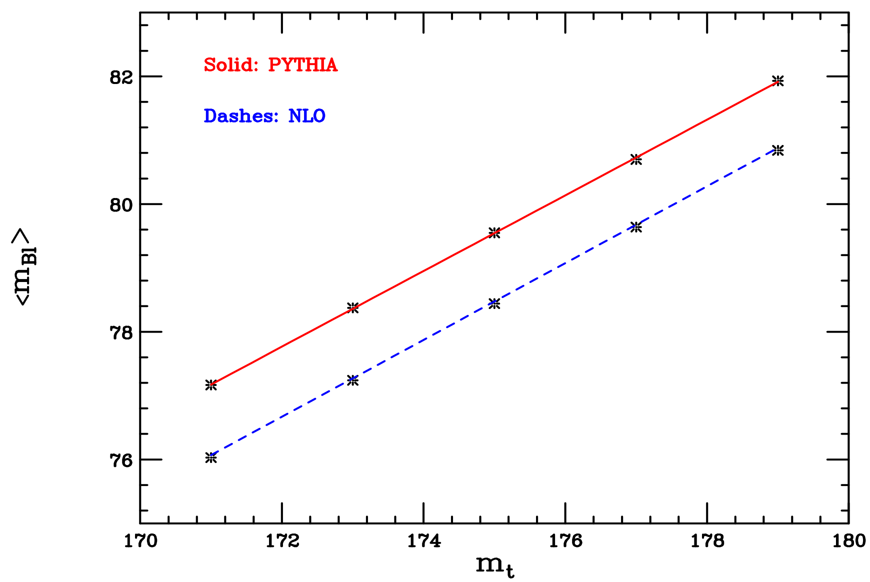
Invariant mass $m_{b\ell}$ using MadGraph+PYTHIA (MC mass) and MCFM (pole mass)





$m_t(\mathrm{Mad+PY}) = 172.3^{+1.3}_{-1.3}$ GeV ; $m_t(\mathrm{MCFM@LO}) = 171.4^{+1.0}_{-1.1}$ GeV

Comparison PYTHIA vs. NLO for $\langle m_{B\ell}\rangle$ after tuning hadronization to LEP data: $(\Delta m_t)_{\mathrm{NLO,PY}} \simeq 1$ GeV (S.Biswas et al,'10, G.C.,'14)



$$\langle m_{B\ell}\rangle_{\mathrm{PY}} \simeq -24.11\ \mathrm{GeV} + 0.59\ m_t$$

$$\langle m_{B\ell}\rangle_{\mathrm{NLO}} \simeq -26.70\ \mathrm{GeV} + 0.60\ m_t$$

Dilepton observables in moment space: Frixione, Mitov, JHEP'14

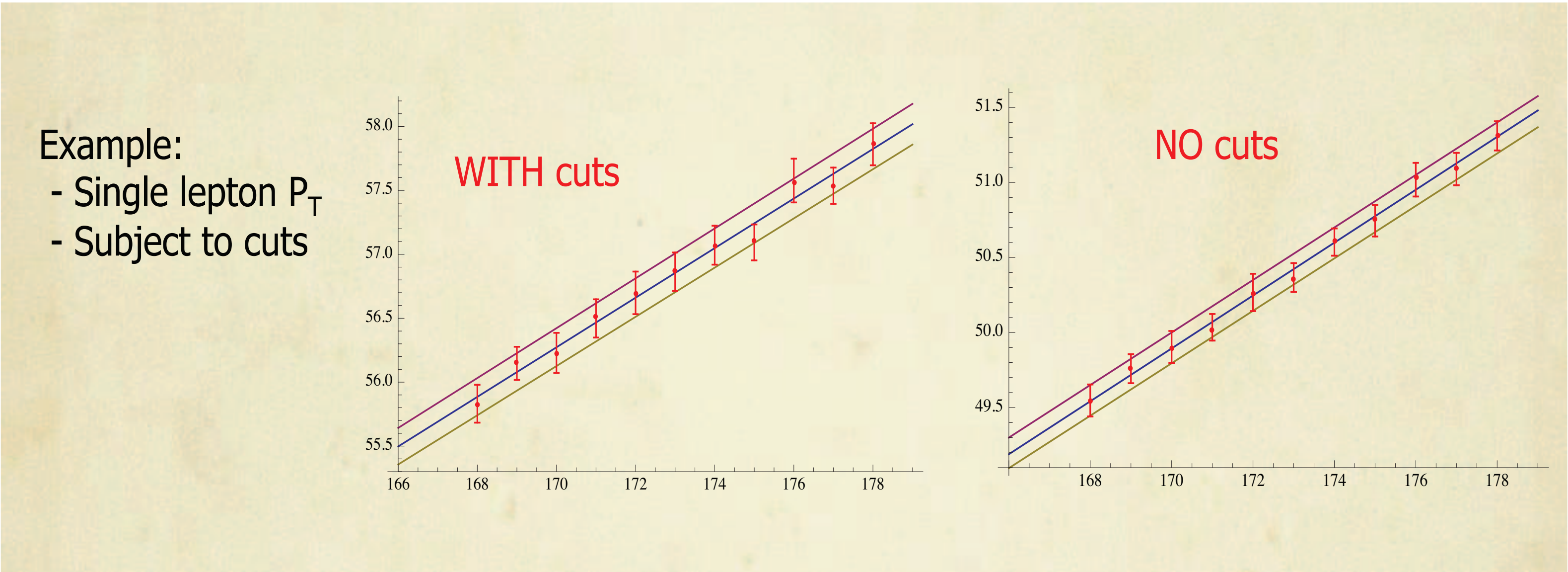
$p_T(\ell)$, $p_T(\ell^+\ell^-)$, $m(\ell^+\ell^-)$, $E(\ell^+)+E(\ell^-)$, $p_T(\ell^+)+p_T(\ell^-)$, $\mu_{O,i}=\frac{1}{\sigma}\int d\sigma O^i$

σ and $d\sigma$ total and differential $t\bar{t}$ cross sections, possibly including cuts

$\langle p_T\rangle$ without and with cuts: $|\eta_\ell|\leq 2.4$, $|\eta_b|\leq 2.4$, $p_{T,\ell}\geq 20$ GeV, $p_{T,b}\geq 30$ GeV

Linear fits:

$$\mu_{O,i}=\alpha_{O,i}(173\ \mathrm{GeV})^i+\beta_{O,i}m_t^i$$



Observables are inclusive with respect to strong interactions, very little dependence on hadronization corrections and no need to reconstruct the top quarks; dependence on the $t\bar{t}$ production phase

$\sqrt{s}=8$ TeV, with aMC@NLO (pole mass), HERWIG and MadSpin: $\Delta m_t\simeq 0.8$ GeV

Conclusions

Standard methods to reconstruct m_t yield a Monte Carlo mass which must be close to the pole mass

Alternative methods allow a determination of the pole mass, up to acceptance corrections and colour reconnection effects

Recent implementation of NLO+showers for top decays (POWHEG) should shed light on some reconstruction methods, like endpoint or B-lepton invariant mass

Ongoing work using SCET formalism with MSR mass and simulating fictitious T-hadrons may help the understanding of the Monte Carlo mass

Higher statistics at Run 2 will reduce the uncertainty on the pole mass extracted from $t\bar{t}$ and $t\bar{t}$+j

Investigating leptonic observables can be useful to get rid of systematic effects due to colour reconnection

Extension of SCET calculations to hadron collisions in progress

Prospects to fully include width and non-resonant effects in NLO+shower generators