

BSM primary effects

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\mathcal{L}_6 (dimension-six operators) \rightarrow **Leading BSM effects**

parametrized by **Wilson coefficients**: C_{WW} , C_{BB} , C_W , ...

Good for model-building (e.g. SILH), but...

not a clear connection with physics!

Then, better talk about **couplings** (interactions)!

couplings \approx observables

e.g. $g_{ff}^Z \leftrightarrow \Gamma(Z \rightarrow ff)$

\rightarrow **BSM primaries**

(a proposal to parametrize BSM effects)

$$\mathcal{O}_H = \frac{1}{2}(\partial_\mu |H|^2)^2$$

$$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)$$

$$\mathcal{O}_6 = \lambda |H|^6$$

$$\mathcal{O}_W = \frac{ig}{2} (H^\dagger \sigma_a D_\mu H) D^\nu W_{\mu\nu}^a$$

$$\mathcal{O}_B = \frac{ig'}{2} (H^\dagger D_\mu H) \partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_G = \frac{ig_s}{2} (D_\mu^\dagger H) \partial^\nu G_{\mu\nu}^a$$

$$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{O}_{GG} = g_s^2 |H|^2 G_{\mu\nu}^a G^{\mu\nu a}$$

$$\mathcal{O}_{BG} = g' g_s |H|^2 B_{\mu\nu} G^{\mu\nu a}$$

$$\mathcal{O}_{HW} = \frac{ig}{2} (H^\dagger \sigma_a D_\mu H) D^\nu W_{\mu\nu}^a$$

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$$\mathcal{O}_{HG} = \frac{ig_s}{2} (H^\dagger D_\mu H) \partial^\nu G_{\mu\nu}^a$$

$$\mathcal{O}_{3G} = \frac{1}{3!} g_s^3 f_{ABC} G_{\mu\nu}^a G_{\rho\sigma}^b G_{\lambda\tau}^c$$



arXiv:1405.0181

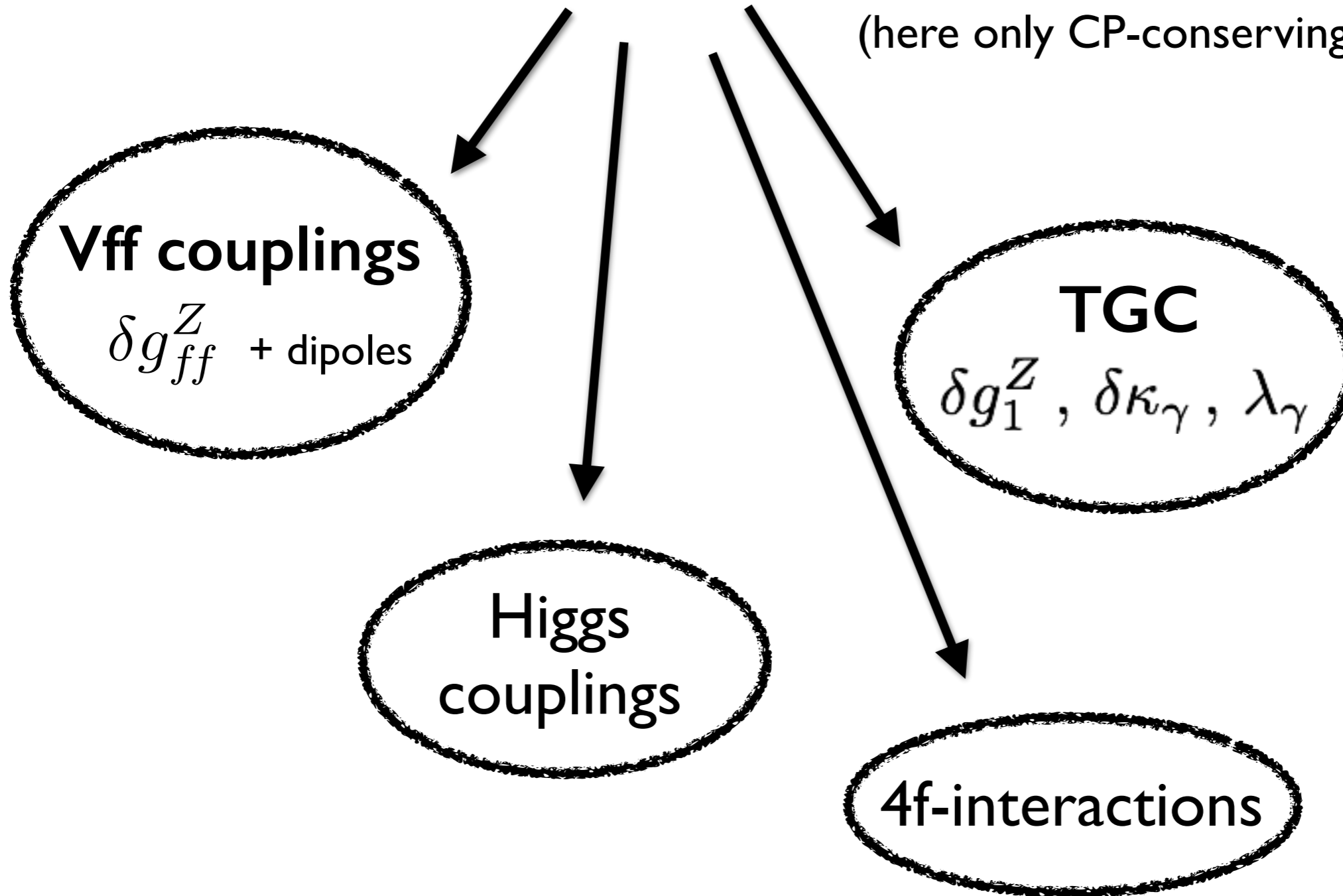
contributions to physical couplings!

- ➔ Not all type of interactions can arise from \mathcal{L}_6 !
- ➔ Plenty of correlations among possible interactions

(see also arXiv:1406.6376)

BSM primaries = Best-measured independent couplings

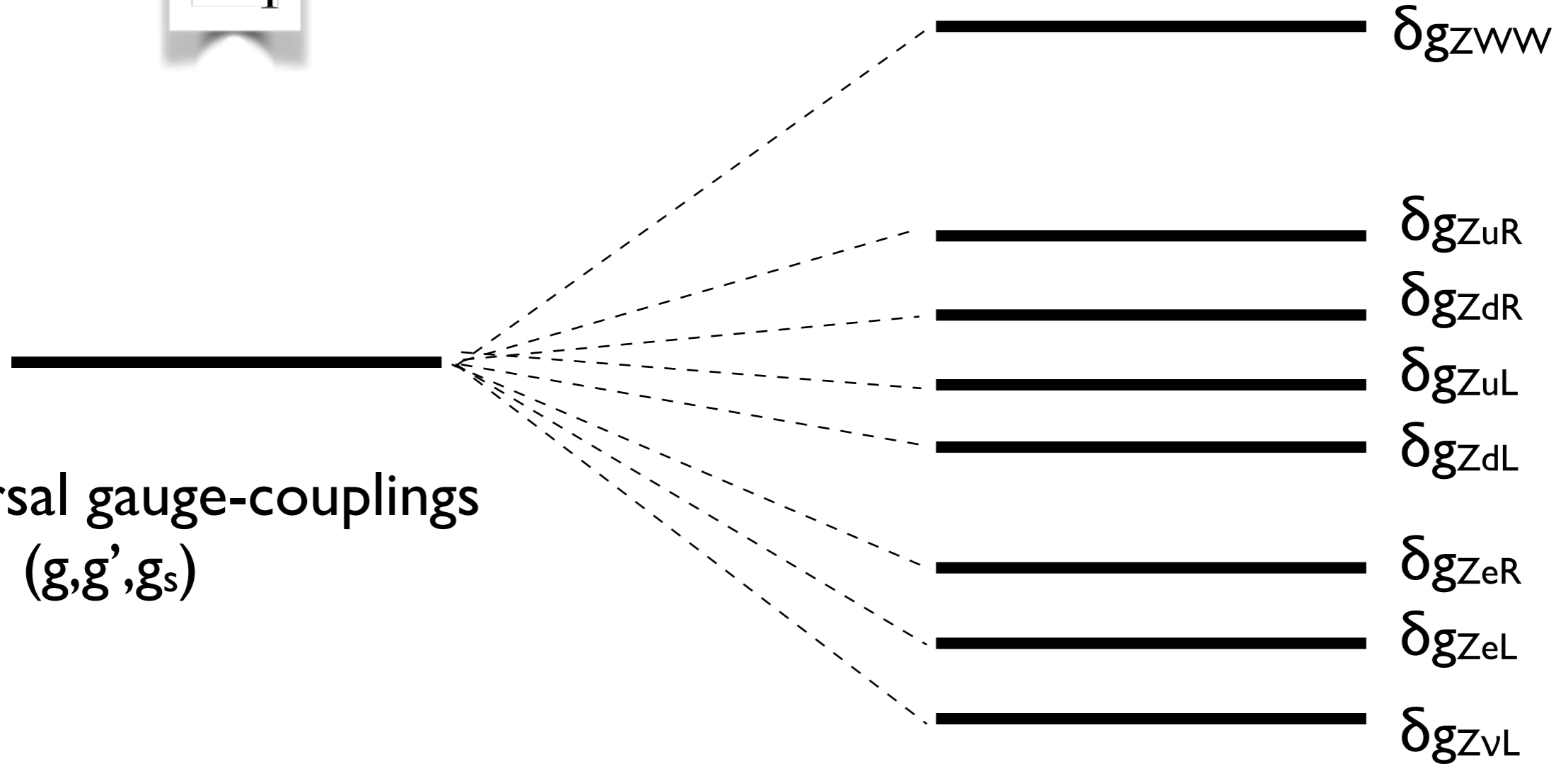
(here only CP-conserving for simplicity)



(non-Higgs) EWSB primaries:

\mathcal{L}_4

\mathcal{L}_6



universal gauge-couplings
 (g, g', g_s)

δg_{ZWW}

δg_{ZuR}

δg_{ZdR}

δg_{ZuL}

δg_{ZdL}

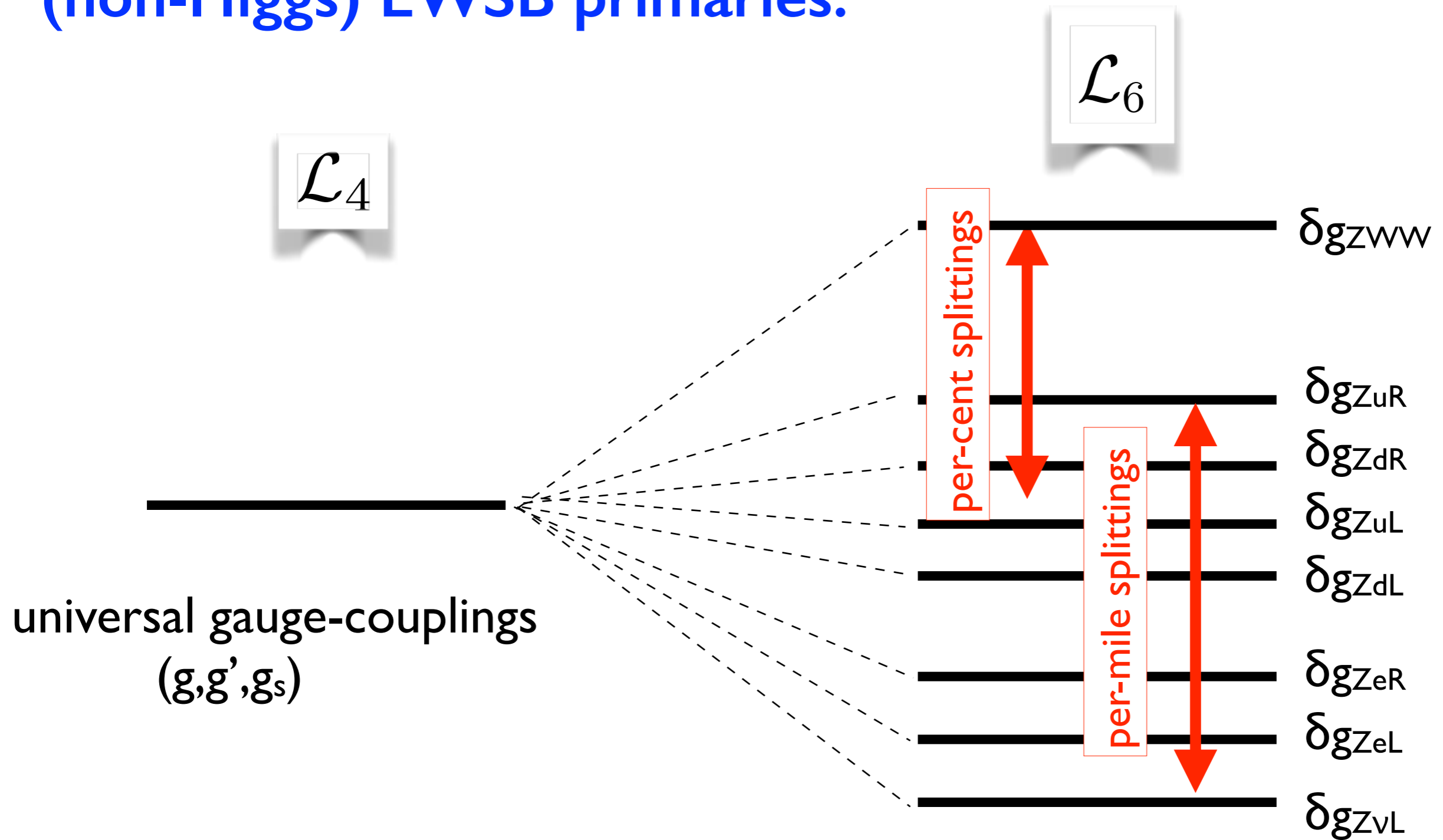
δg_{ZeR}

δg_{ZeL}

$\delta g_{Z\nu L}$

+ dipole-type interactions for W & f

(non-Higgs) EWSB primaries:



+ dipole-type interactions for W & f

Higgs couplings

$O(h^3)$, $O(h\partial^2 V^2)$ and $O(hV f^2)$

All relevant couplings for *single* Higgs physics:

$$\begin{aligned}\mathcal{L}_h^{\text{primary}} &= g_{VV}^h h \left[W^{+\mu} W_{\mu}^{-} + \frac{1}{2c_{\theta_W}^2} Z^{\mu} Z_{\mu} \right] + \frac{1}{6} g_{3h} h^3 + g_{ff}^h (h \bar{f}_L f_R + h.c.) \\ &+ \kappa_{GG} \frac{h}{2v} G^{A\mu\nu} G_{\mu\nu}^A + \kappa_{\gamma\gamma} \frac{h}{2v} A^{\mu\nu} A_{\mu\nu} + \kappa_{Z\gamma} \frac{h}{v} A^{\mu\nu} Z_{\mu\nu},\end{aligned}$$

$$\begin{aligned}\Delta\mathcal{L}_h &= \delta g_{ZZ}^h h \frac{Z^{\mu} Z_{\mu}}{2c_{\theta_W}^2} + g_{Zff}^h \frac{h}{2v} (Z_{\mu} J_N^{\mu} + h.c.) + g_{Wff'}^h \frac{h}{v} (W_{\mu}^{+} J_C^{\mu} + h.c.) \\ &+ \kappa_{WW} \frac{h}{v} W^{+\mu\nu} W_{\mu\nu}^{-} + \kappa_{ZZ} \frac{h}{2v} Z^{\mu\nu} Z_{\mu\nu},\end{aligned}$$

Higgs couplings

$O(h^3)$, $O(h\partial^2 V^2)$ and $O(hV f^2)$

independent from others

$$\begin{aligned}\mathcal{L}_h^{\text{primary}} &= g_{VV}^h h \left[W^{+\mu} W_{\mu}^{-} + \frac{1}{2c_{\theta_W}^2} Z^{\mu} Z_{\mu} \right] + \frac{1}{6} g_{3h} h^3 + g_{ff}^h (h \bar{f}_L f_R + h.c.) \\ &+ \kappa_{GG} \frac{h}{2v} G^{A\mu\nu} G_{\mu\nu}^A + \kappa_{\gamma\gamma} \frac{h}{2v} A^{\mu\nu} A_{\mu\nu} + \kappa_{Z\gamma} \frac{h}{v} A^{\mu\nu} Z_{\mu\nu},\end{aligned}$$

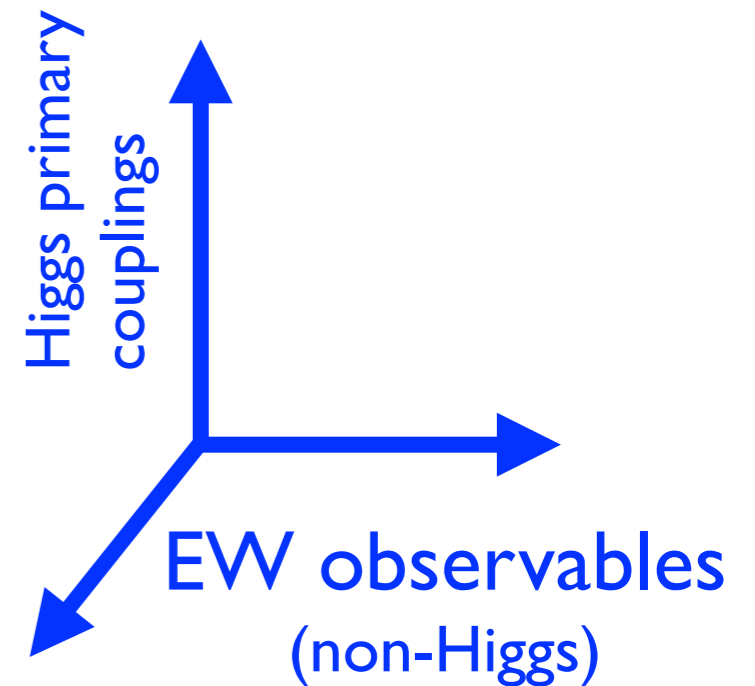
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correlated to other couplings

8 Primary Higgs couplings

(assuming CP-conservation)

$$\begin{aligned}\mathcal{L}_h^{\text{primary}} = & \quad g_{ff}^h h \bar{f}_L f_R + h.c. & \quad (f=b, \tau, t) \\ & + g_{VV}^h h \left[W^{+\mu} W_{\mu}^{-} + \frac{1}{2 \cos^2 \theta_W} Z^{\mu} Z_{\mu} \right] \\ & + \kappa_{GG} \frac{h}{2v} G^{\mu\nu} G_{\mu\nu} \\ & + \kappa_{\gamma\gamma} \frac{h}{2v} A^{\mu\nu} A_{\mu\nu} \\ & + \kappa_{Z\gamma} \frac{h}{v} A^{\mu\nu} Z_{\mu\nu} \\ & + \frac{1}{6} g_{3h} h^3\end{aligned}$$



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 \end{aligned}$$

6 measured
at the LHC
(the “kappas”)

bounds from $h \rightarrow Z\gamma$

Affects h^3 :

It can be measured
in the far future by
 $GG \rightarrow hh$

Expected largest corrections to Higgs couplings:


	hff	hVV	h $\gamma\gamma$	h γZ	hGG	h
MSSM	✓					✓
NMSSM	✓	✓	✓	✓	✓	✓
PGB Composite	✓	✓		✓		✓
SUSY Composite	✓	✓	✓	✓	✓	✓
SUSY partly-composite			✓	✓	✓	✓
“Bosonic TC”						✓
Higgs as a dilaton			✓	✓	✓	✓

Beyond the primary Higgs couplings

Beyond the primary Higgs couplings

$$\Delta\mathcal{L}_h = \delta g_{ZZ}^h h \frac{Z^\mu Z_\mu}{2c_{\theta_W}^2}$$

custodial breaking hVV



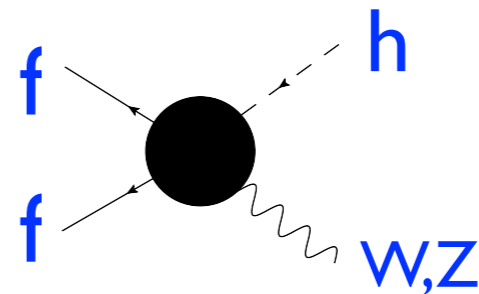
Beyond the primary Higgs couplings

$$\Delta\mathcal{L}_h = \delta g_{ZZ}^h h \frac{Z^\mu Z_\mu}{2c_{\theta_W}^2} + g_{Zff}^h \frac{h}{2v} (Z_\mu J_N^\mu + h.c.) + g_{Wff'}^h \frac{h}{v} (W_\mu^+ J_C^\mu + h.c.)$$

custodial breaking hVV



contact interactions



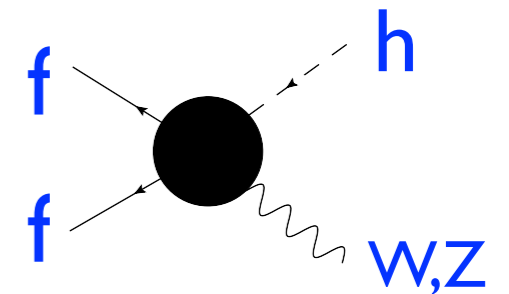
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 \end{aligned}$$

↙ custodial breaking hVV

} contact interactions

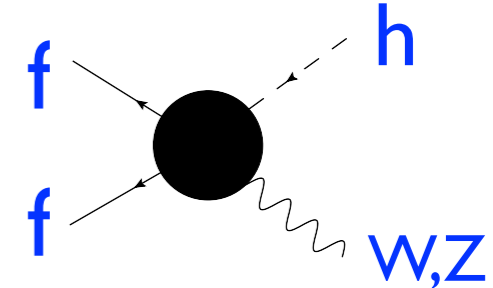
}
momentum-dependent
hVV couplings



Beyond the primary Higgs couplings

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 \end{aligned}$$

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} contact interactions
} momentum-dependent hVV couplings

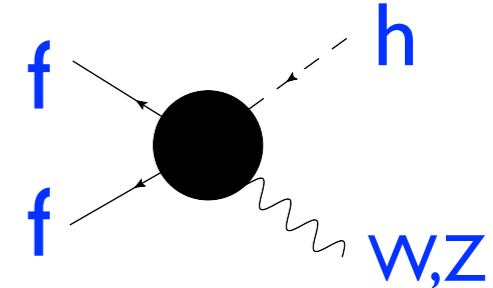


➡ I find this to be best parametrization to go beyond the “kappa’s approach”

Beyond the primary Higgs couplings

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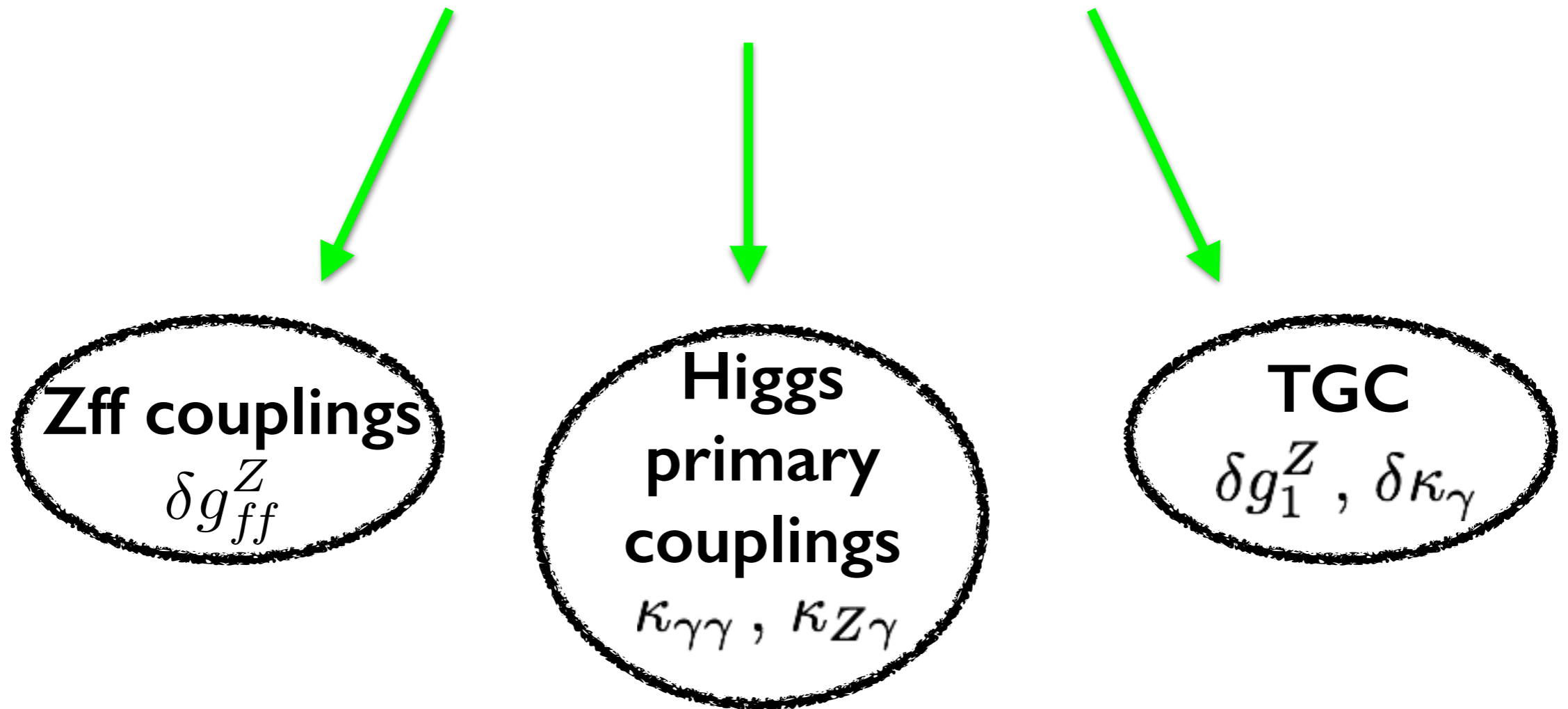
but remember BSM effects here are not independent from effects to other couplings!

All can be written as a function of contributions to other couplings:

$$\delta g_{ZZ}^h = 2gt_{\theta_W}^2 m_W (c_{\theta_W}^2 \delta g_1^Z - \delta \kappa_\gamma), \quad \delta g_{ff'}^W = \frac{c_{\theta_W}}{\sqrt{2}} (\delta g_{ff}^Z V_{\text{CKM}} - V_{\text{CKM}} \delta g_{f'f'}^Z) \text{ for } f = f_L$$

$$g_{Zff}^h = 2\delta g_{ff}^Z - 2\delta g_1^Z (g_{ff}^Z c_{2\theta_W} + g_{ff}^\gamma s_{2\theta_W}) + 2\delta \kappa_\gamma Y_f \frac{e s_{\theta_W}}{c_{\theta_W}^3}, \quad g_{Wff'}^h = 2\delta g_{ff'}^W - 2\delta g_1^Z g_{ff'}^W c_{\theta_W}^2,$$

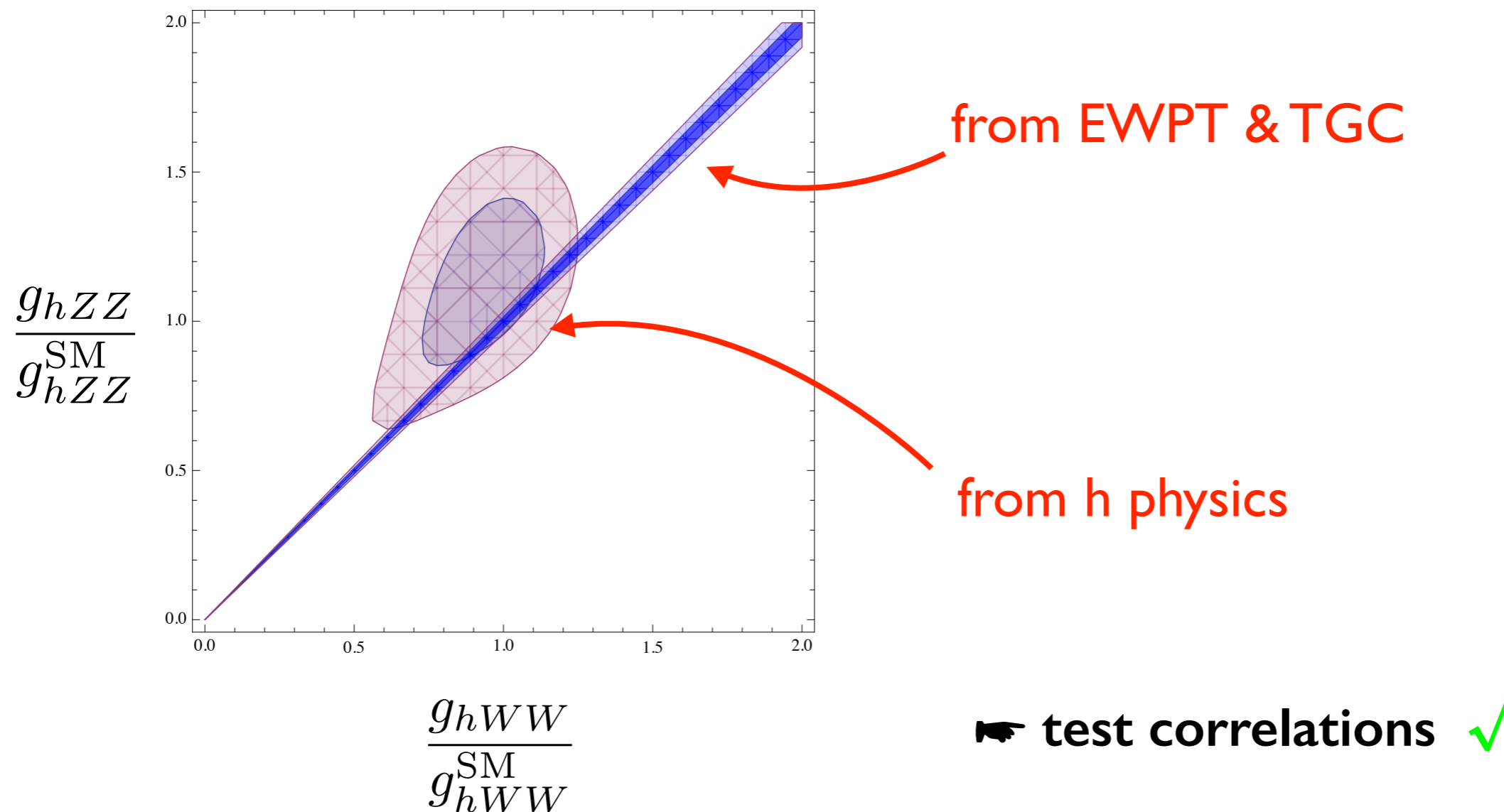
$$\kappa_{ZZ} = \frac{1}{c_{\theta_W}^2} \delta \kappa_\gamma + 2 \frac{c_{2\theta_W}}{s_{2\theta_W}} \kappa_{Z\gamma} + \kappa_{\gamma\gamma}, \quad \kappa_{WW} = \delta \kappa_\gamma + \kappa_{Z\gamma} + \kappa_{\gamma\gamma},$$



Proposal (for the future):

try to measure all Higgs couplings and compare with other non-Higgs “observables”

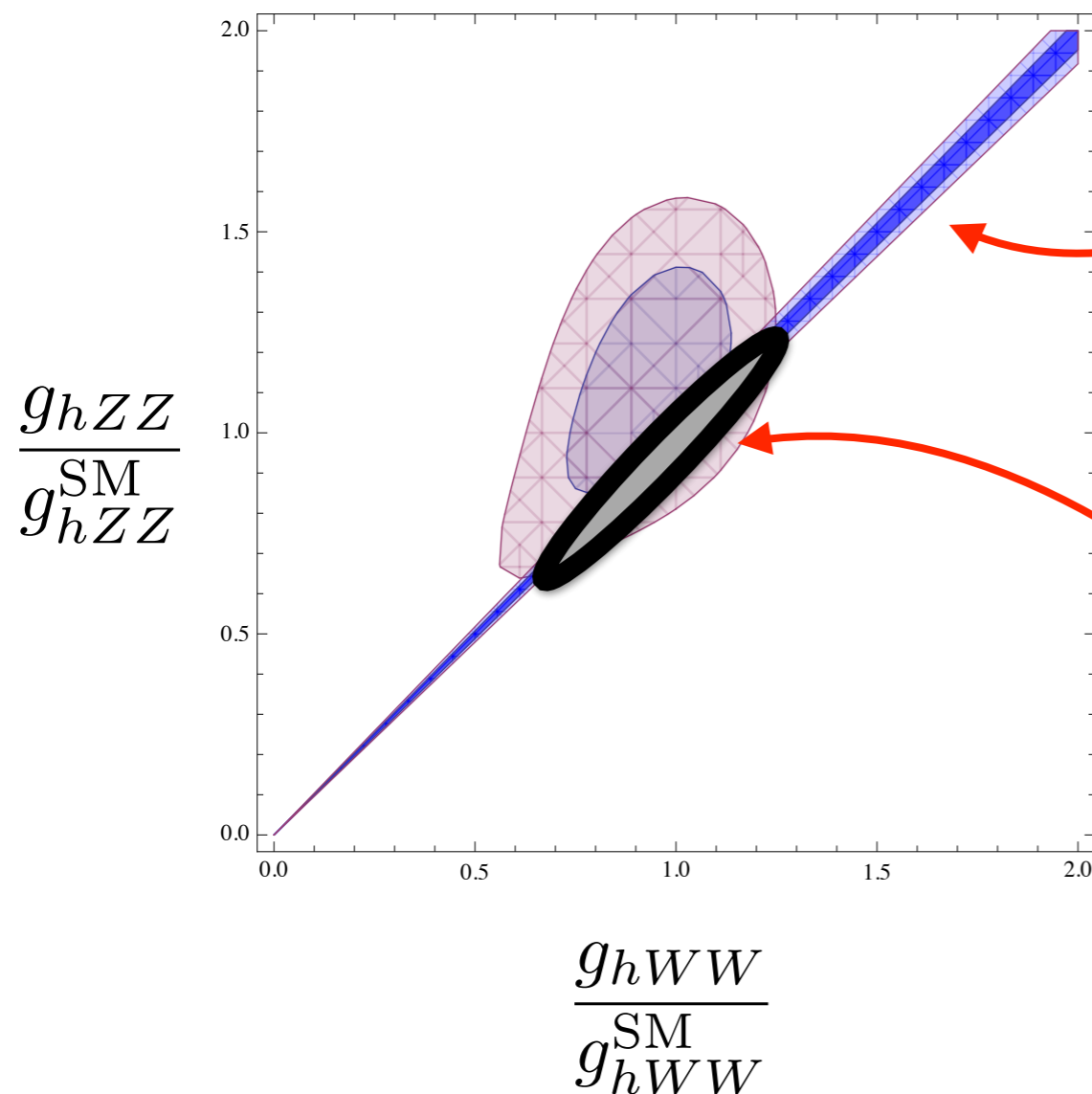
What I would like to see:



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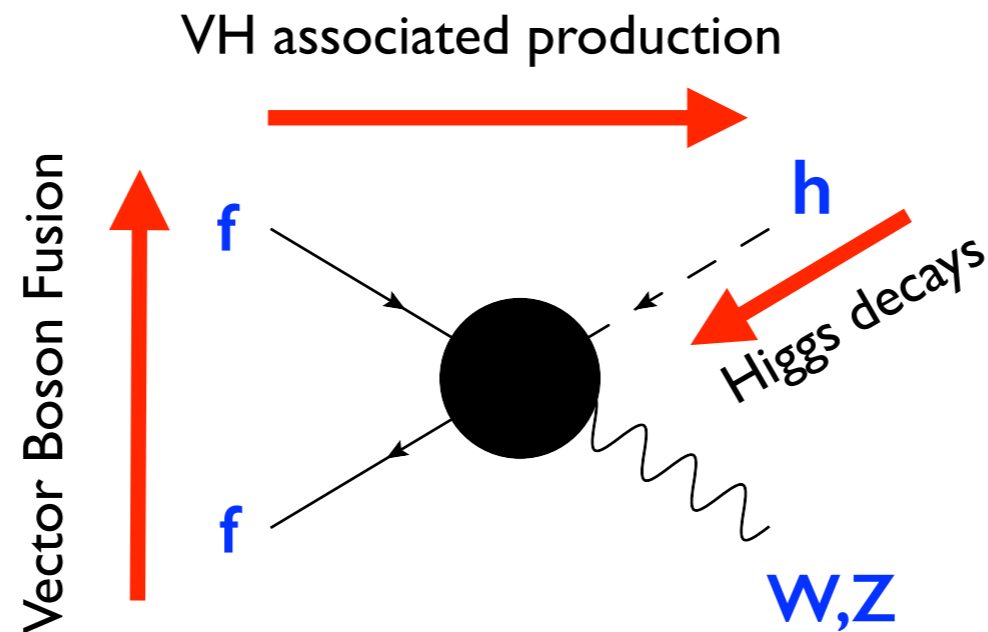
from EWPT & TGC

from h physics

☞ test correlations ✓

☞ improve global fit of primaries ✓

Non-primary Higgs couplings can be disentangled in distributions:



$$\mathcal{M}_{hVff}(q, p) = \frac{1}{v} \epsilon^{*\mu}(q) J_V^\nu(p) [A^V \eta_{\mu\nu} + B^V (p \cdot q \eta_{\mu\nu} - p_\mu q_\nu) + C^V \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma]$$

$$A^V = a^V + \hat{a}^V \frac{m_V^2}{p^2 - m_V^2}, \quad B^V = b^V \frac{1}{p^2 - m_V^2} + \hat{b}^V \frac{1}{p^2}$$

one-to-one correspondence
with Higgs couplings

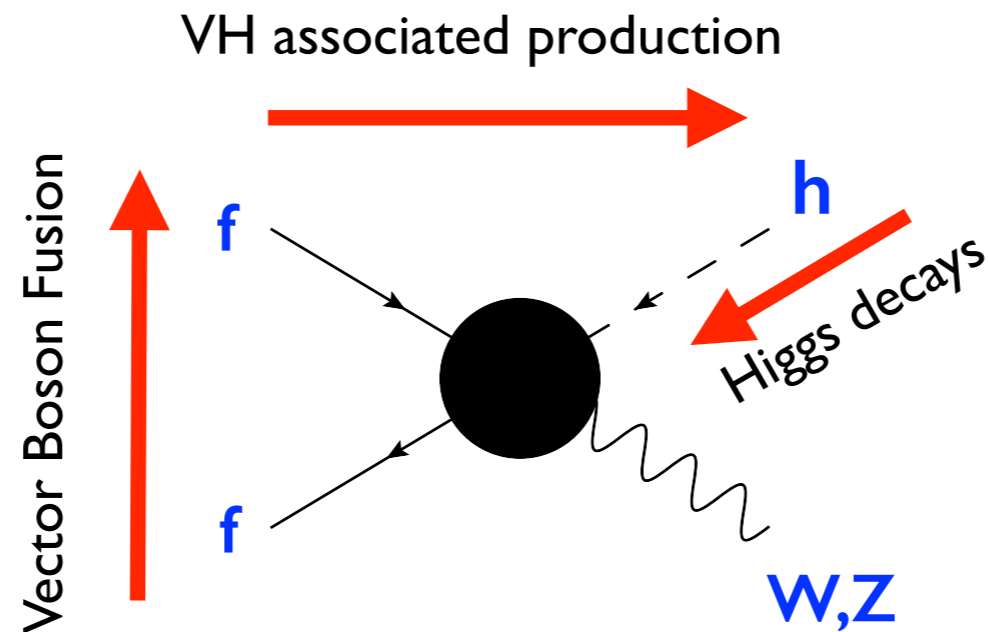
$$hV_\mu J^\mu$$

$$hV^\mu V_\mu$$

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$$hV_\mu J^\mu$$

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$$hV^{\mu\nu} V_{\mu\nu}$$

$$hZ^{\mu\nu} A_{\mu\nu}$$

dominant at high-energy!

Priorities (non-democratic approach: Not all couplings are equal(ly) interesting):

We must decide first which
are the best set of couplings to measure

Reduction of couplings must be either by

- 1) Symmetries
- 2) Dynamics (of the new-physics sector)

A proposal for non-primaries:

- ➡ Keep lowest-order in a q^2 -expansion
- ➡ Impose universality and custodial



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AREA**

**NO UNAUTHORIZED
PERSONNEL
*BEYOND THIS POINT***

Correlations between couplings

Explicit correlations between hZff and Zff:

arXiv:1405.0181
arXiv:1406.6376

$$\begin{aligned} \Delta\mathcal{L}_{qq}^V &= \delta g_{uR}^Z \frac{\hat{h}^2}{v^2} Z^\mu \bar{u}_R \gamma_\mu u_R + \delta g_{dR}^Z \frac{\hat{h}^2}{v^2} Z^\mu \bar{d}_R \gamma_\mu d_R \\ &+ \delta g_{dL}^Z \frac{\hat{h}^2}{v^2} \left[Z^\mu \bar{d}_L \gamma_\mu d_L - \frac{c_{\theta_W}}{\sqrt{2}} (W^{+\mu} \bar{u}_L \gamma_\mu d_L + \text{h.c.}) \right] \\ &+ \delta g_{uL}^Z \frac{\hat{h}^2}{v^2} \left[Z^\mu \bar{u}_L \gamma_\mu u_L + \frac{c_{\theta_W}}{\sqrt{2}} (W^{+\mu} \bar{u}_L \gamma_\mu d_L + \text{h.c.}) \right] \end{aligned}$$

$$\begin{aligned} \Delta\mathcal{L}_{ee}^V &= \delta g_{eR}^Z \frac{\hat{h}^2}{v^2} Z^\mu \bar{e}_R \gamma_\mu e_R \\ &+ \delta g_{eL}^Z \frac{\hat{h}^2}{v^2} \left[Z^\mu \bar{e}_L \gamma_\mu e_L - \frac{c_{\theta_W}}{\sqrt{2}} (W^{+\mu} \bar{\nu}_L \gamma_\mu e_L + \text{h.c.}) \right] \\ &+ \delta g_{\nu L}^Z \frac{\hat{h}^2}{v^2} \left[Z^\mu \bar{\nu}_L \gamma_\mu \nu_L + \frac{c_{\theta_W}}{\sqrt{2}} (W^{+\mu} \bar{\nu}_L \gamma_\mu e_L + \text{h.c.}) \right] \end{aligned}$$

$$\hat{h} \equiv v + h$$

appear combined
in the same
operators

Correlations with the primary Higgs couplings:

arXiv:1405.0181
arXiv:1406.6376

$$\Delta\mathcal{L}_{\gamma\gamma}^h = \kappa_{\gamma\gamma} \left(\frac{h}{v} + \frac{h^2}{2v^2} \right) \left[A_{\mu\nu} A^{\mu\nu} + Z_{\mu\nu} Z^{\mu\nu} + 2W_{\mu\nu}^+ W^{-\mu\nu} \right]$$

hVV form-factor
correlated with $h\gamma\gamma$

$$\Delta\mathcal{L}_{Z\gamma}^h = \kappa_{Z\gamma} \left(\frac{h}{v} + \frac{h^2}{2v^2} \right) \left[t_{\theta_W} A_{\mu\nu} Z^{\mu\nu} + \frac{c_{2\theta_W}}{2c_{\theta_W}^2} Z_{\mu\nu} Z^{\mu\nu} + W_{\mu\nu}^+ W^{-\mu\nu} \right]$$

$$\Delta\mathcal{L}_{GG}^h = \kappa_{GG} \left(\frac{h}{v} + \frac{h^2}{2v^2} \right) G_{\mu\nu}^A G^{A\mu\nu}$$

hVV form-factor
correlated with $h\gamma Z$

$$\Delta\mathcal{L}_{ff}^h = \delta g_{ff}^h \left(h \bar{f}_L f_R + \text{h.c.} \right) \left(1 + \frac{3h}{2v} + \frac{h^2}{2v^2} \right)$$

$$\Delta\mathcal{L}_{3h} = \delta g_{3h} h^3 \left(1 + \frac{3h}{2v} + \frac{3h^2}{4v^2} + \frac{h^3}{8v^3} \right)$$

$$\Delta\mathcal{L}_{VV}^h = \delta g_{VV}^h \left[h \left(W^{+\mu} W_{\mu}^- + \frac{Z^{\mu} Z_{\mu}}{2c_{\theta_W}^2} \right) + \Delta \right]$$

$$\begin{aligned} \Delta = & \left(W^{+\mu} W_{\mu}^- + \frac{Z^{\mu} Z_{\mu}}{2c_{\theta_W}^2} \right) \left(\frac{2h^2}{v} + \frac{4h^3}{3v^2} + \frac{h^4}{3v^3} \right) + \frac{m_h^2}{12m_W^2} \left(\frac{h^4}{v} + \frac{3h^5}{4v^2} + \frac{h^6}{8v^3} \right) \\ & + \frac{m_f}{4m_W^2} \left(\frac{h^2}{v} + \frac{h^3}{3v^2} \right) (\bar{f}_L f_R + \text{h.c.}), \end{aligned}$$

Correlations with triple gauge couplings (TGC):

$$\Delta\mathcal{L}_{\kappa\gamma} = \frac{\delta\kappa_\gamma}{v^2} \left[ie\hat{h}^2 (A_{\mu\nu} - t_{\theta_W} Z_{\mu\nu}) W^{+\mu} W^{-\nu} + Z_\nu \partial_\mu \hat{h}^2 (t_{\theta_W} A^{\mu\nu} - t_{\theta_W}^2 Z^{\mu\nu}) \right. \\ \left. + \frac{(\hat{h}^2 - v^2)}{2} \left(t_{\theta_W} Z_{\mu\nu} A^{\mu\nu} + \frac{c_{2\theta_W}}{2c_{\theta_W}^2} Z_{\mu\nu} Z^{\mu\nu} + W_{\mu\nu}^+ W^{-\mu\nu} \right) \right],$$

$$\hat{h} \equiv v + h$$

**hVV form-factor
correlated with ZWW**

$$\Delta\mathcal{L}_{g_1^Z} = \delta g_1^Z \left[igc_{\theta_W} \left(Z^\mu (W^{+\nu} W_{\mu\nu}^- - \text{h.c.}) + Z^{\mu\nu} W_\mu^+ W_\nu^- \right) + \frac{e^2 v}{2c_{\theta_W}^2} h Z_\mu Z^\mu \right. \\ \left. - 2c_{\theta_W}^2 \frac{h}{v} \left(g(W_\mu^- J_-^\mu + \text{h.c.}) + \frac{gc_{2\theta_W}}{c_{\theta_W}^3} Z_\mu J_Z^\mu + 2et_{\theta_W} Z_\mu J_{em}^\mu \right) \left(1 + \frac{h}{2v} \right) \right. \\ \left. - g^2 c_{\theta_W}^2 \left(W_\mu^+ W^{-\mu} + \frac{c_{2\theta_W}}{2c_{\theta_W}^4} Z_\mu Z^\mu \right) \left(\frac{5}{2} h^2 + 2\frac{h^3}{v} + \frac{h^4}{2v^2} \right) + g^2 c_{\theta_W}^2 v \Delta \right].$$

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$$\hat{h} \equiv v + h$$

custodial breaking hVV-coupling
correlated with ZWW

$$\Delta\mathcal{L}_{g_1^Z} = \delta g_1^Z \left[igc_{\theta_W} \left(Z^\mu (W^{+\nu} W_{\mu\nu}^- - \text{h.c.}) + Z^{\mu\nu} W_\mu^+ W_\nu^- \right) + \frac{e^2 v}{2c_{\theta_W}^2} h Z_\mu Z^\mu \right. \\ \left. - 2c_{\theta_W}^2 \frac{h}{v} \left(g(W_\mu^- J_-^\mu + \text{h.c.}) + \frac{gc_{2\theta_W}}{c_{\theta_W}^3} Z_\mu J_Z^\mu + 2et_{\theta_W} Z_\mu J_{em}^\mu \right) \left(1 + \frac{h}{2v} \right) \right. \\ \left. - g^2 c_{\theta_W}^2 \left(W_\mu^+ W^{-\mu} + \frac{c_{2\theta_W}}{2c_{\theta_W}^4} Z_\mu Z^\mu \right) \left(\frac{5}{2} h^2 + 2\frac{h^3}{v} + \frac{h^4}{2v^2} \right) + g^2 c_{\theta_W}^2 v \Delta \right].$$