Precise SM Estimates of $B \to l^+ l^-$ Decay Rates

(& the $b \to s\, l^+ l^-$ Weak Hamiltonian)

Zurich Phenomenology Workshop: “The flavour of new physics”
8 Jan 15

Based on works with Christoph Bobeth, Emanuel Stamou [PRD 89, 034023 (2014)]
Christoph Bobeth, Thomas Hermann, Mikolaj Misiak, Emanuel Stamou and Matthias Steinhauser [PRL 112, 101801 (2014)]

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Content

Introduction:

$B_s \to \mu^+ \mu^-$ in the Standard Model with QCD at NLO

What type of QED / EW corrections are there?

Status of the $\mathcal{L}_{eff}$ for $b \to s\, l^+ l^-$

Theory prediction for $B \to l^+ l^-$

Rare B Decays

FCNCs which are dominated by top-quark loops:

$b \to s:$ $|V_{tb}^* V_{ts}| \propto \lambda^2$

$b \to d:$ $|V_{tb}^* V_{td}| \propto \lambda^3$

$s \to d:$ $|V_{ts}^* V_{td}| \propto \lambda^5$

B decays do not show the CKM suppression of K decays

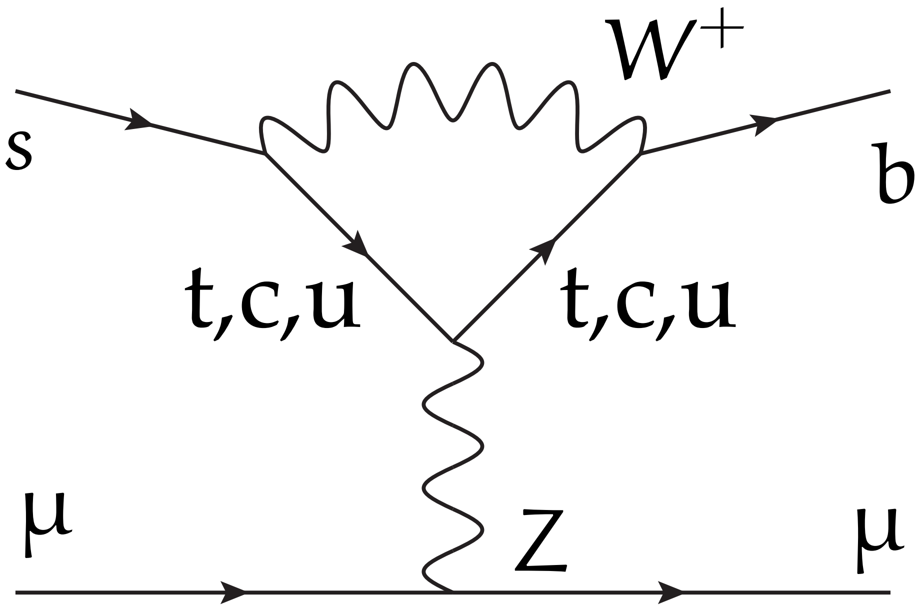
2 photon pollution is much smaller in $b \to s\, l^+ l^-$ decays

We can test helicity suppressed modes and more operators

$$Q_7 = (\bar{b}_L \sigma_{\mu\nu} s_L) F^{\mu\nu},\ Q_V = (\bar{b}_L \gamma_\mu s_L)(\bar{l} \gamma_\mu l),\ Q_A = (\bar{b}_L \gamma_\mu s_L)(\bar{l} \gamma_\mu \gamma_5 l)$$

E.g. $B_{(s)} \to l^+ l^-$, $B \to K^{(*)} l^+ l^-$, $B \to X_s \gamma$, ...

$B_s \to \mu^+ \mu^-$ in the Standard Model



+ Box diagrams

B_s is (pseudo)scalar – no photon penguin

$$Q_A = (\bar{b}_L \gamma_\mu s_L)(\bar{l}\gamma_\mu \gamma_5 l)$$

Dominant operator in the SM

helicity suppression $\left(\propto \frac{m_l^2}{M_B^2}\right)$

$$\propto |V_{tb}^* V_{ts}| \simeq \left|1 - \lambda^2 \left(\tfrac{1}{2} - i\eta - \rho\right)\right| V_{cb}$$

Effective Lagrangian in the SM:

$$\mathcal{L}_{\rm eff} = G_F^2 M_W^2 \, V_{tb}^* V_{ts} \left(C_A Q_A + C_S Q_S + C_P Q_P\right) + h.c.$$

Scalar operators: $Q_S = (\bar{b}_R q_L)(\bar{l} l)$ $\quad Q_P = (\bar{b}_R q_L)(\bar{l}\gamma_5 l)$

Standard Model: C_S & C_P are highly suppressed

$B_s \rightarrow \mu^+ \mu^-$ and New Physics

Contribution of Q_S and Q_P are not helicity suppressed

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Yet, only if contribution to ΔM_s is suppressed,
i.e. type 2 Higgs potential, $\lambda_5 \ll 1$ and type 3 Yukawas

which is the MSSM at $\tan\beta \gg 1$, with the Branching Ratio

$$BR \propto (\tan\beta)^6 \; M_A^{-4}$$

Non-zero $\Delta\Gamma_s$ allows for another untagged observable
beyond the BR via an effective lifetime measurement.
[Bruyn, Fleischer, Knegjens et.al. `12]

Experimental Status

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = \left(2.8\,^{+0.7}_{-0.6}\right) \times 10^{-9} \text{ and}$$
$$\mathcal{B}(B^0 \to \mu^+\mu^-) = \left(3.9\,^{+1.6}_{-1.4}\right) \times 10^{-10},$$

CMS & LHCb ArXiv:1411.4413v1

For $B_{(s)} \to \mu^+ \mu^-$ experiment and theory consistent within present accuracy (2 σ).

Reduce the (theory) uncertainty:

Either $B_{(s)} \to \mu^+ \mu^-$ will result in a signal of new physics or in a precision test of the standard model.

Either way we will get additional information on C_A, C_S and C_P (+ flipped Operators ...)

Theory Status at NLO

C_S & C_P can be neglected within the Standard Model

$C_A(m_t\ /\ M_W)^{NLO} = 1.0113\ C_A(m_t\ /\ M_W)^{LO}$
– for QCD MS-bar $m_t = m_t(m_t)$ [Buras, Buchalla; Misiak, Urban `99]

For pure QCD determine $< \mu^- \mu^+ | Q_A | B_s >$ from
$< 0\ | \bar{b}\, \gamma^\mu\, \gamma_5\, s\, | B_s > = i\, p^\mu f_{Bs}$ (f_{Bs}= 227.7(4.5)MeV [FLAG])

QED & Electroweak were so far only known at LO –
this leads to a ±2% & ±7% uncertainty

QED corrections I

B_s decay into a 2 lepton final state always helicity suppressed

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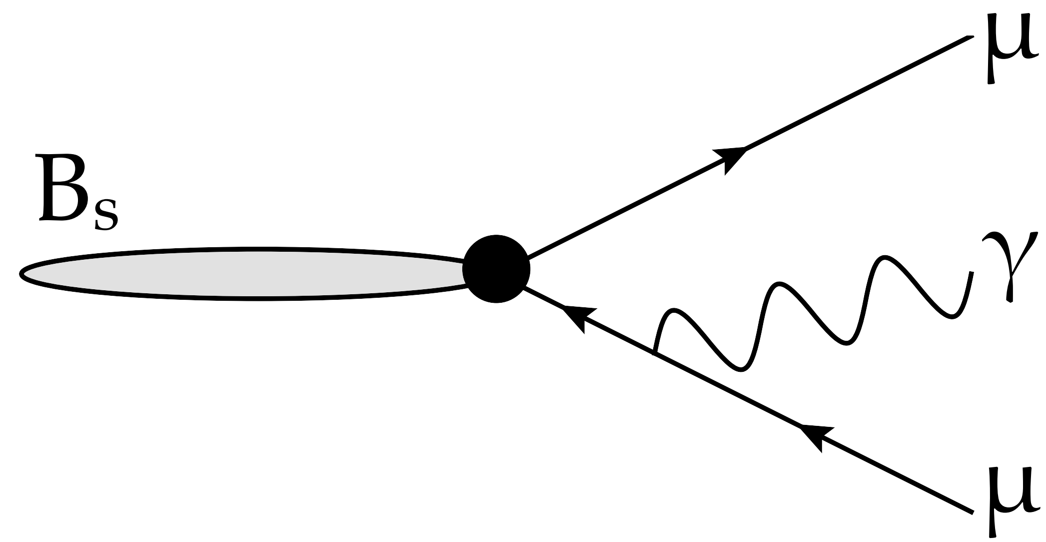
Soft photon radiation from muons: Theoretical branching ratio is fully inclusive of bremsstrahlung. There would be sizeable corrections otherwise [Buras, Girrbach, Guadagnoli, Isidori] arXiv:1208.0934.



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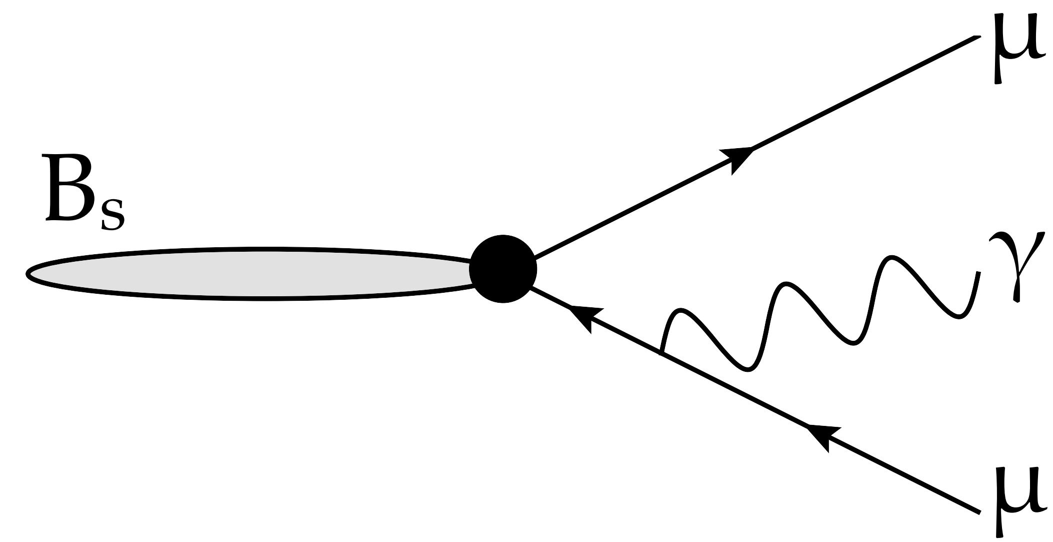
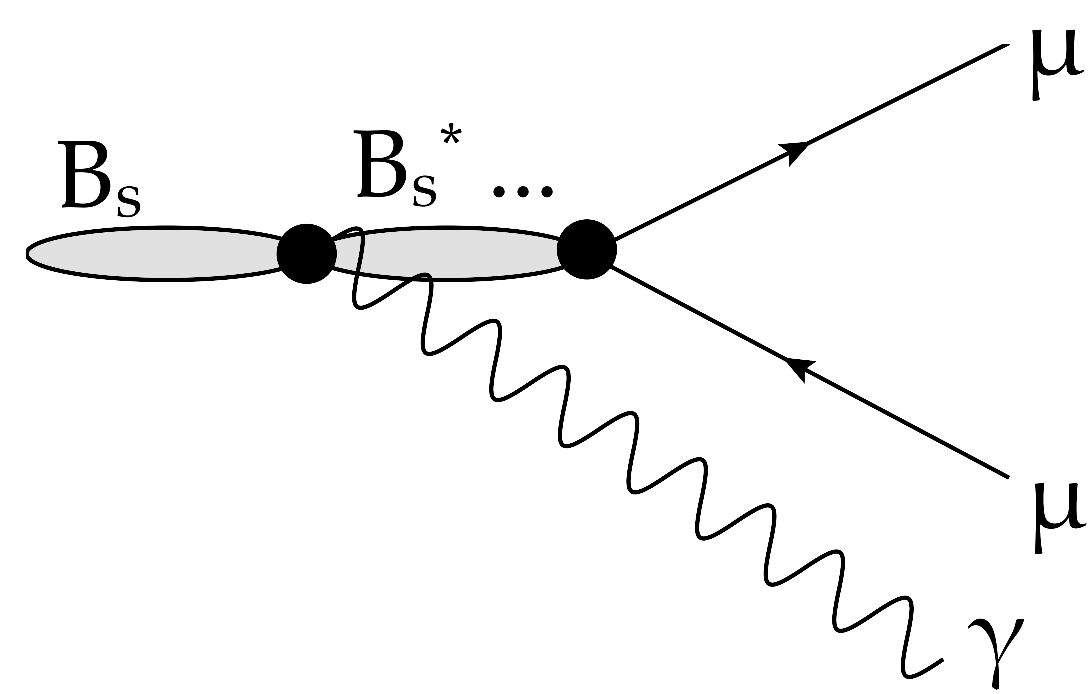


Direct emission is IR safe (B_s is neutral) and phase space suppressed for invariant mass $m_{\mu\mu}$ close to M_{Bs}. [Aditya, Healey, Petrov] arXiv: 1212.4166



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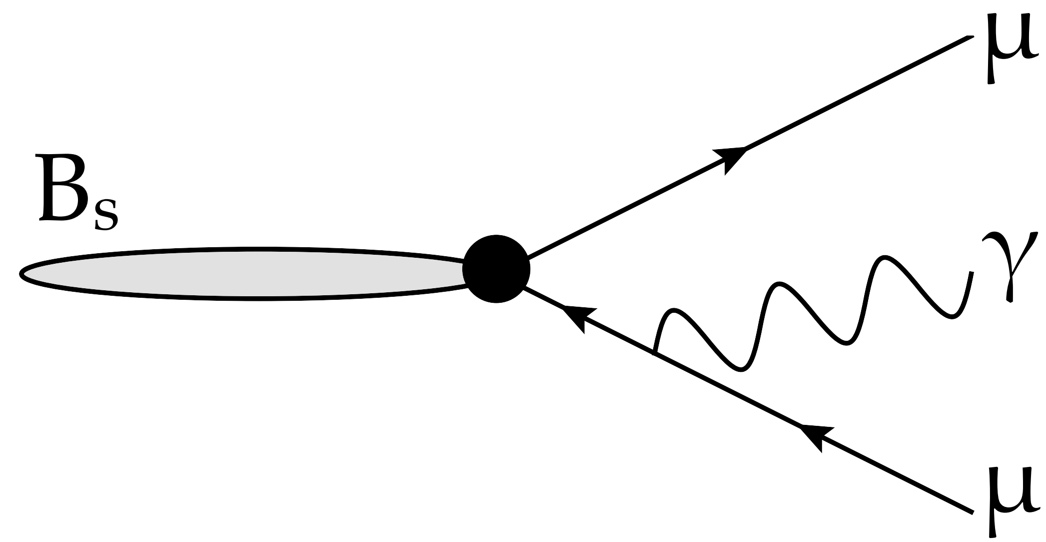

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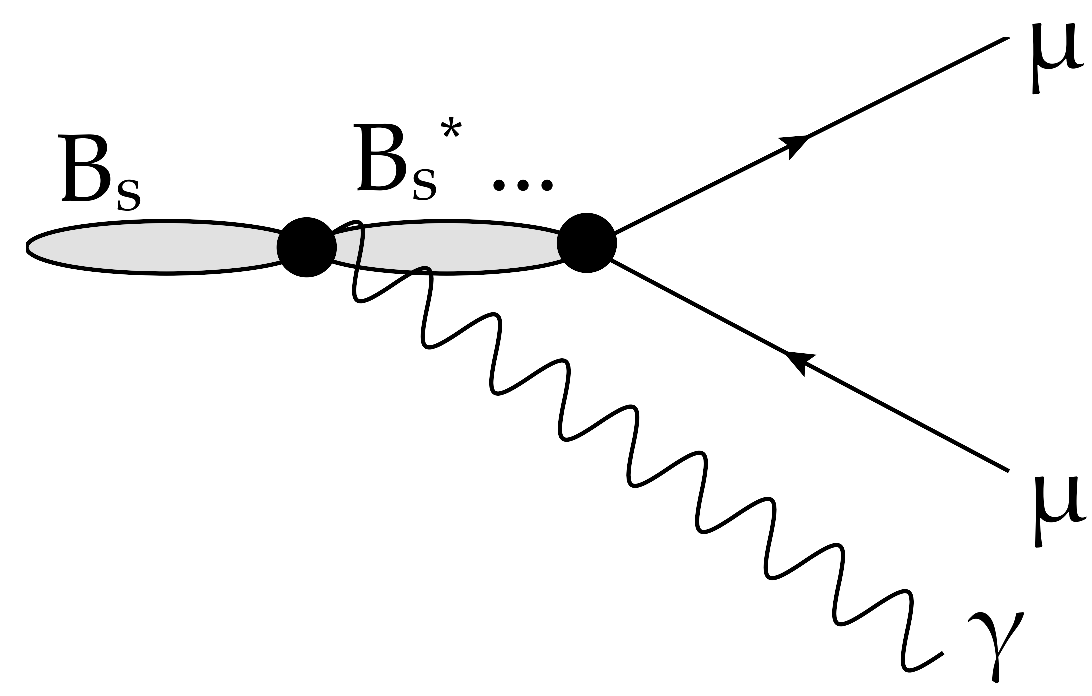
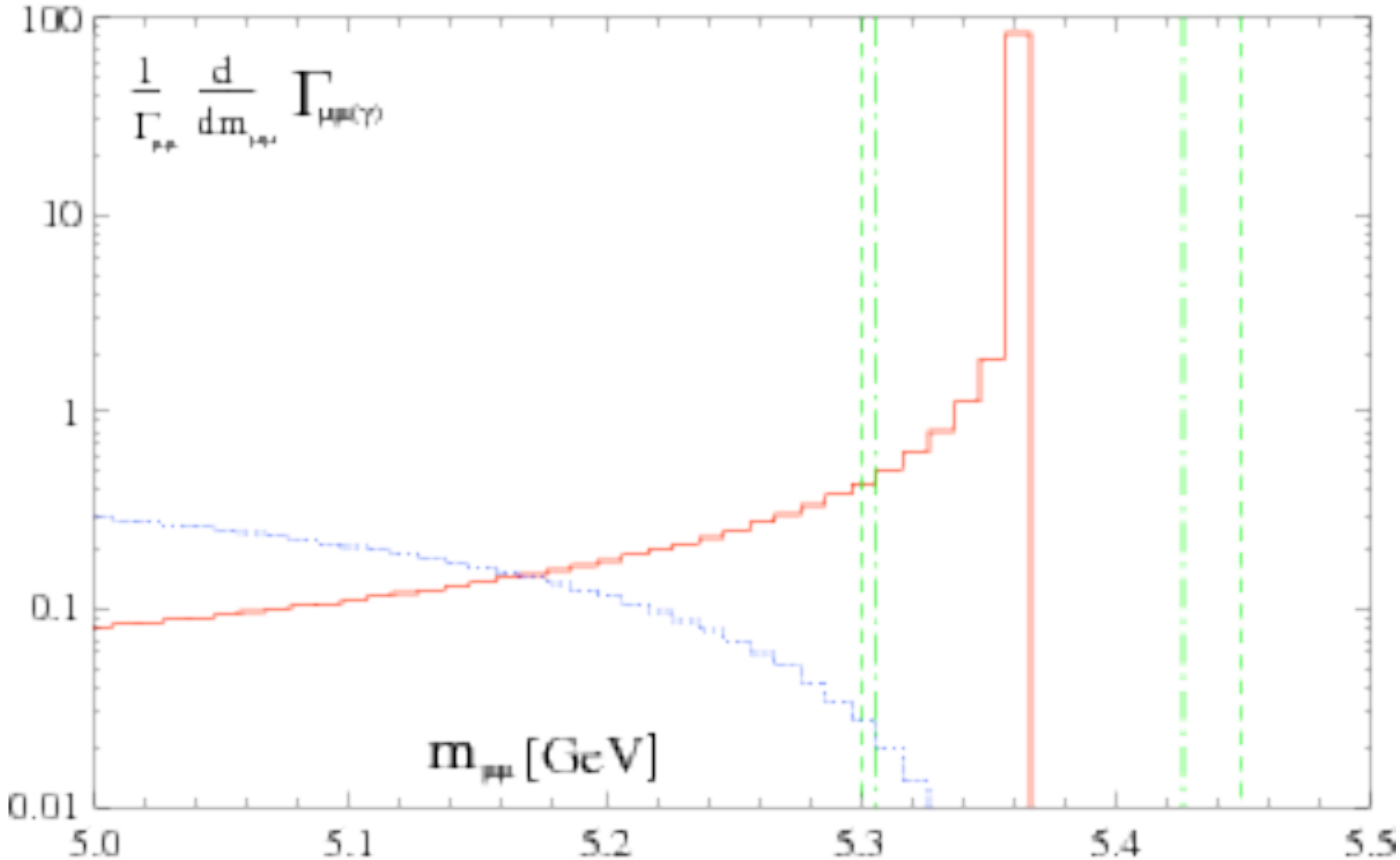


Next correction would be $O(\alpha^3)$

Illustration

Consider an experimental signal window for the invariant mass of the muon pair $m_{\mu\mu}$



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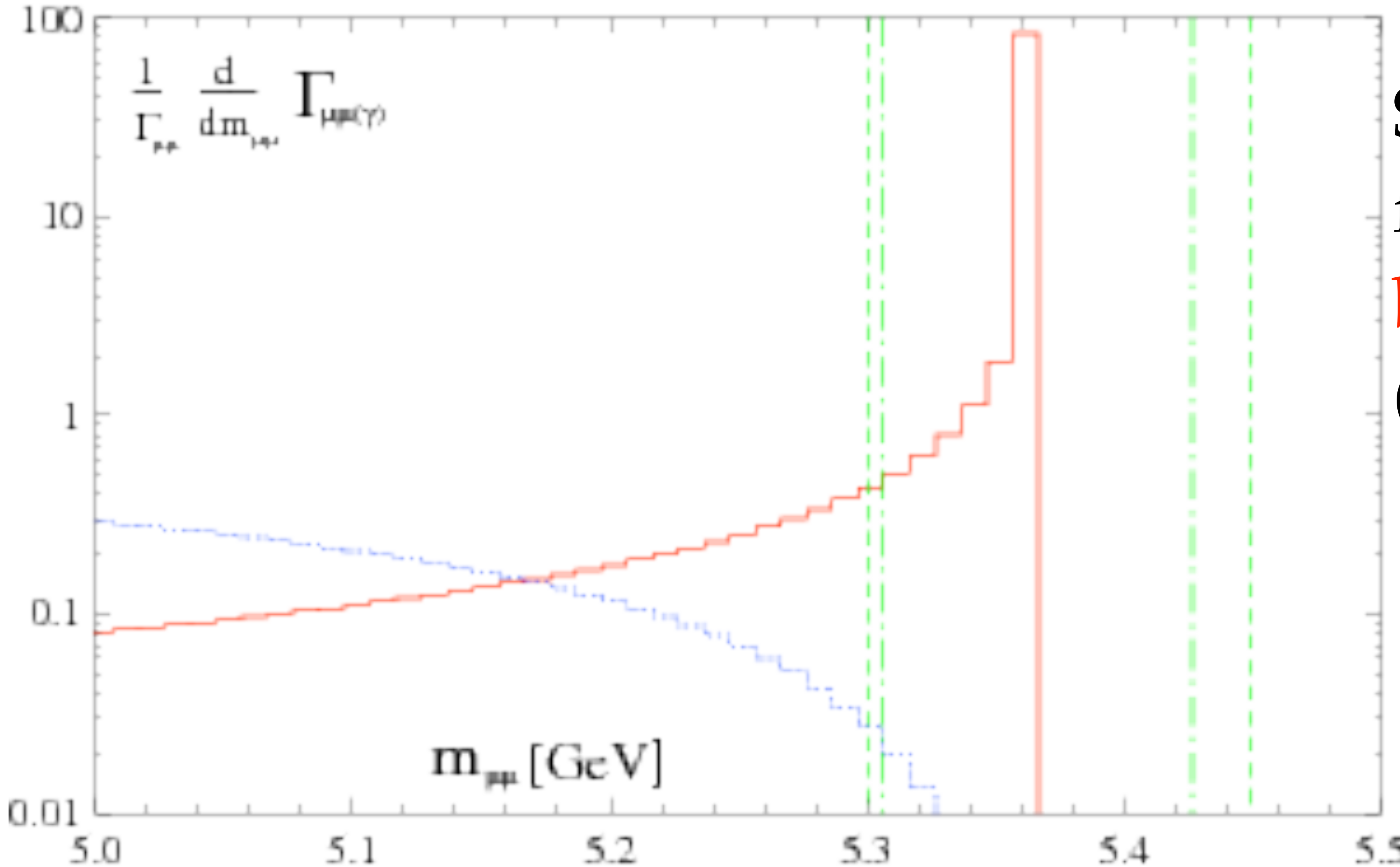

Simulate signal fully inclusive of bremsstrahlung (PHOTOS)

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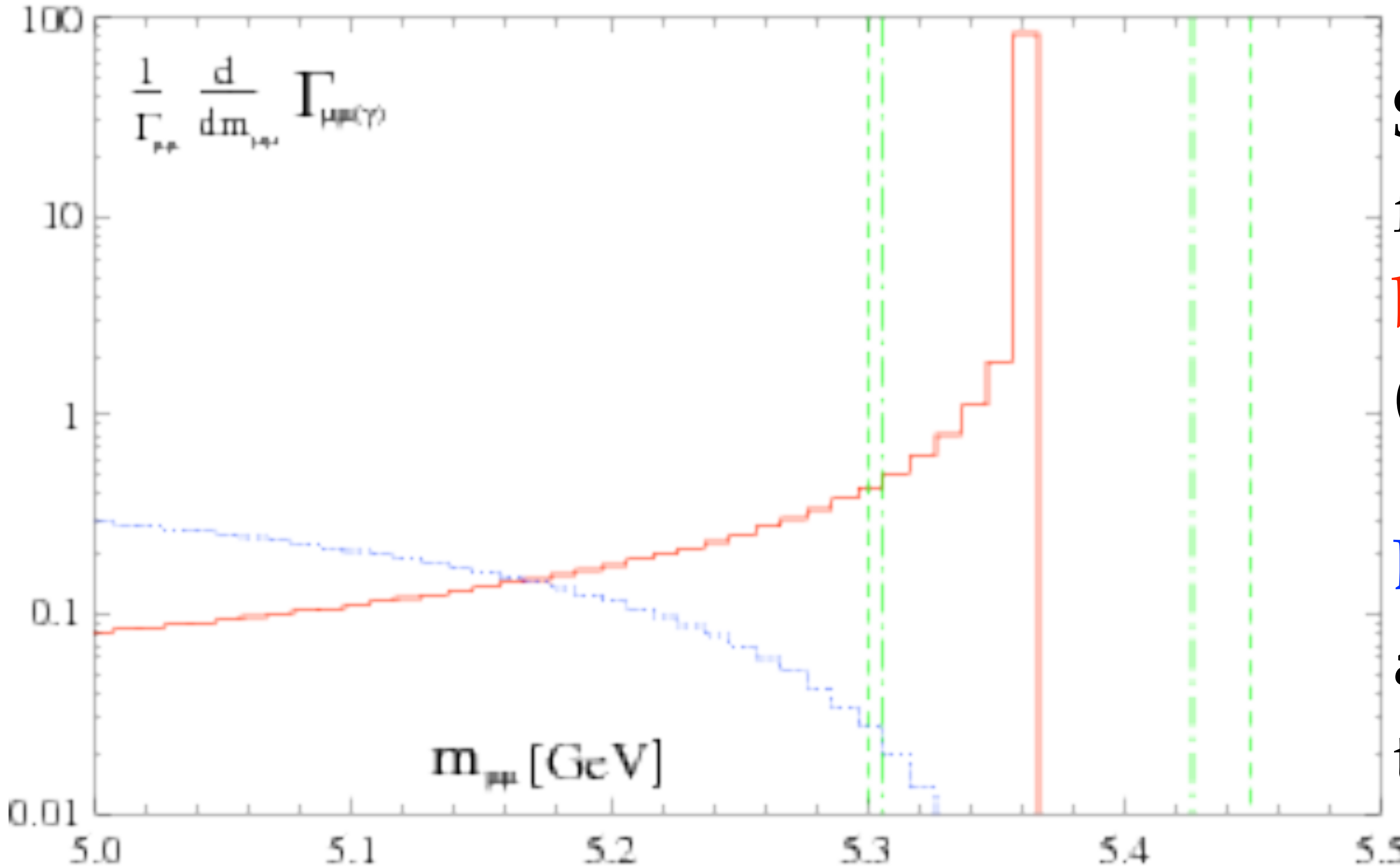

Simulate signal fully inclusive of bremsstrahlung (PHOTOS)

Direct emission is a background in the signal window

Comparing Theory and Experiment

Bremsstrahlung taken into account by the experiment and direct emission treated as background.

The B_s system has a non-zero decay width difference:
$\rightarrow$ instantaneous $\neq$ time integrated branching ratio
[de Bruyn, Fleischer et. al. `12] This correction is precisely known.

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$\rightarrow$ Only electroweak corrections and QED to $C_A(\mu_b)$ are potentially large – enhanced by m_{top}/M_W, $1/s_W$, $\alpha_e \log^2(M_W/m_b)$. NNLO is important to remove the scale uncertainty.

Electroweak Corrections

Consider $\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha\,\pi\,V^*_{tb} V_{ts}}{\sin^2\theta_W} C_A Q_A + \text{h.c.}$

$G_F\,\alpha/\sin^2\theta_W$ does not renormalise under QCD: can be factored out for QCD calculation

Only $G_F\,\alpha/\sin^2\theta_W\,C_A(m_t/M_W)$ invariant under electroweak scheme change

This combination should always give the same result if we use the same input (G_F, α, M_Z, M_t, M_H) up to higher order corrections

Electroweak Scheme Uncertainties

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha \pi V_{tb}^* V_{ts}}{\sin^2 \theta_W} C_A(\frac{m_t}{M_W}) Q_A + \text{h.c.}$$

	MS-bar	OS	unct. $B_s\ \mu^+\ \mu^-$
$\sin \theta_W$	0,231	0,223	± 4 %
m_t(QCD-MS-bar)	163,5 GeV	164,8 GeV	± 1 %

These scheme uncertainties should be canceled by the 2-loop electroweak matching corrections!

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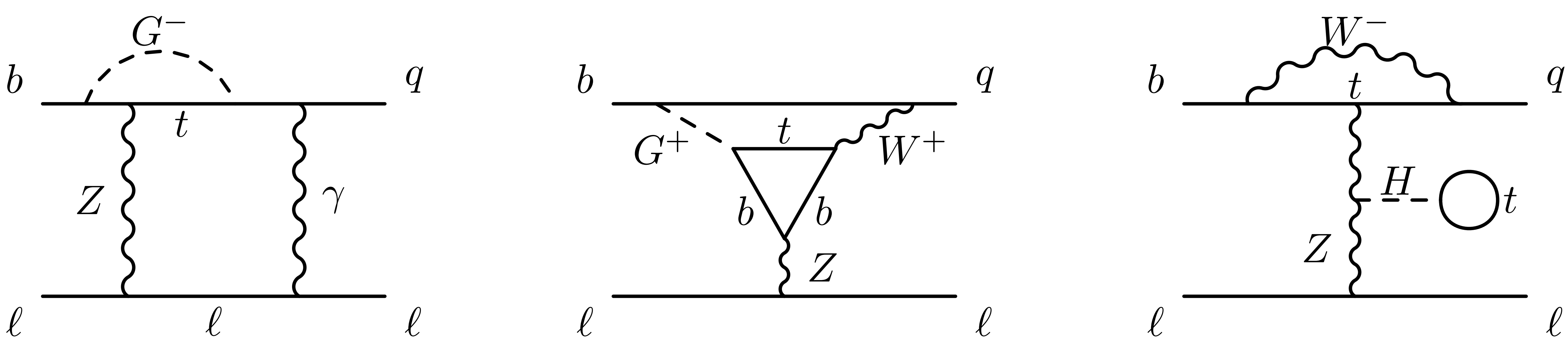

Renormalisation Schemes

1. On-shell scheme: Determine M_W including loop corrections from input: results in $\sin\theta_W$, m_t and M_W counterterms to $C_A^{(EW)}$.

2. MS-bar scheme: Fit g_1, g_2, v, λ, m_t from data i.e. from G_F, α, M_Z, M_t, M_H

3. Hybrid scheme: Masses on-shell couplings MS-bar

4. OS2: Use $G_F^2 M_W^2$ normalisation and on-shell scheme

Note: QCD is MS-bar renormalised for all schemes i.e. we use a QCD MS-bar top mass at a fixed scale

Matching Correction for C_A

There are sizeable shifts and reduction of scale dependence
if we go from 1-loop to 2-loop

Note: α(nq=6) used for plot

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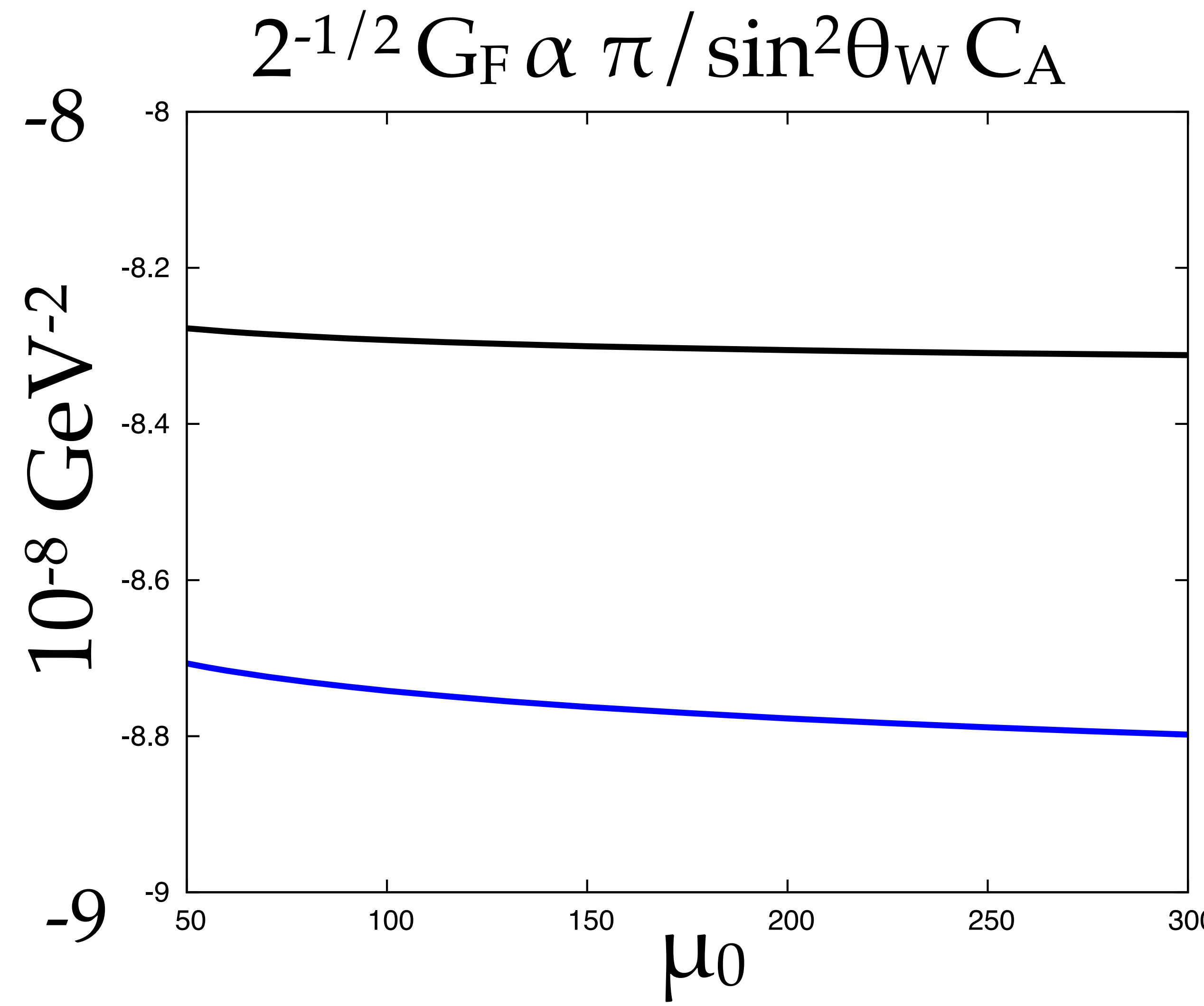
1. We find largest shift in the on-shell scheme,



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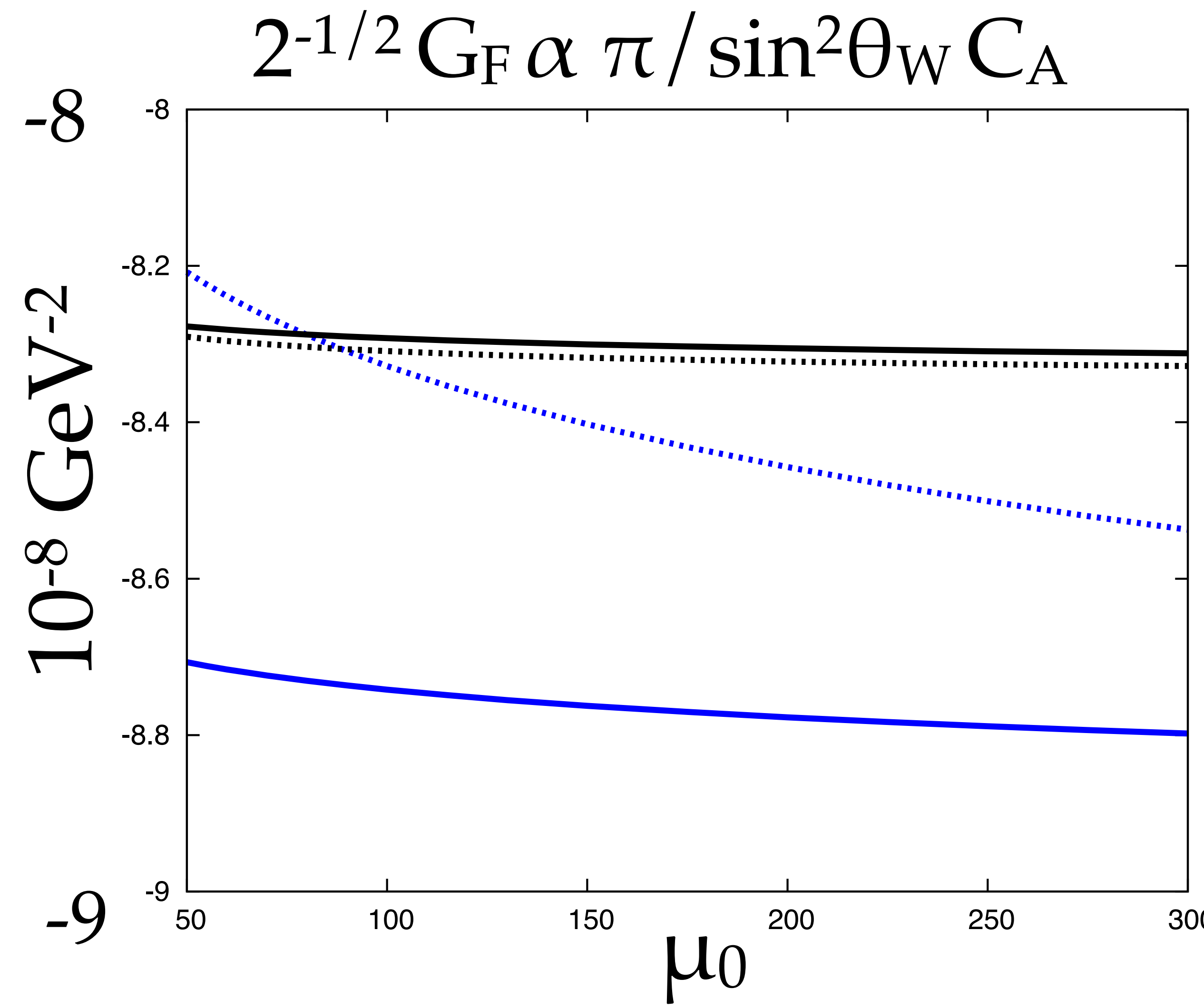

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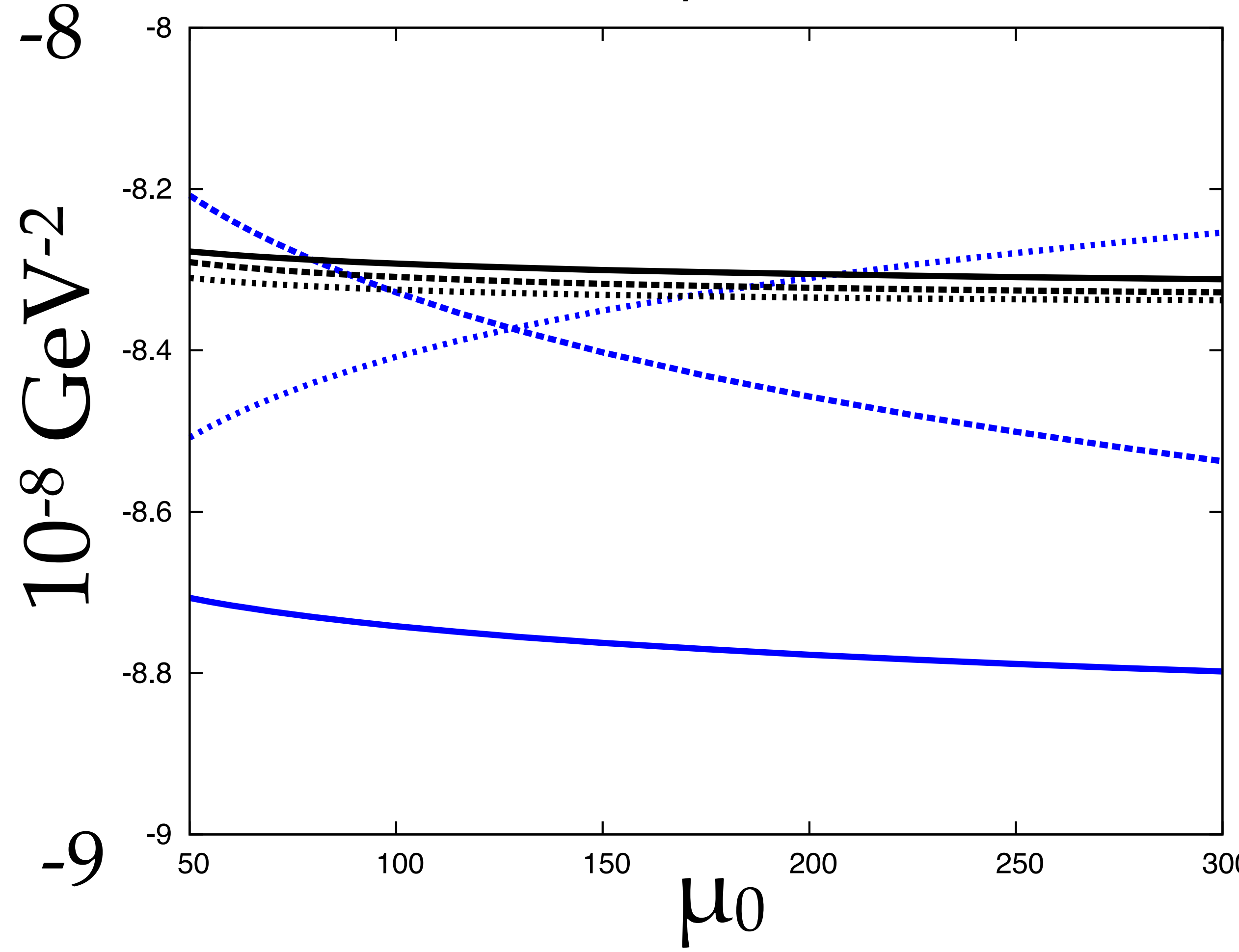
3. and significant shift for the hybrid scheme at MZ.

$2^{-1/2} G_F \alpha \pi / \sin^2\theta_W C_A$



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4. $G_F^2 M_W^2$ normalisation removes `artificial´ scale and parameter dependence

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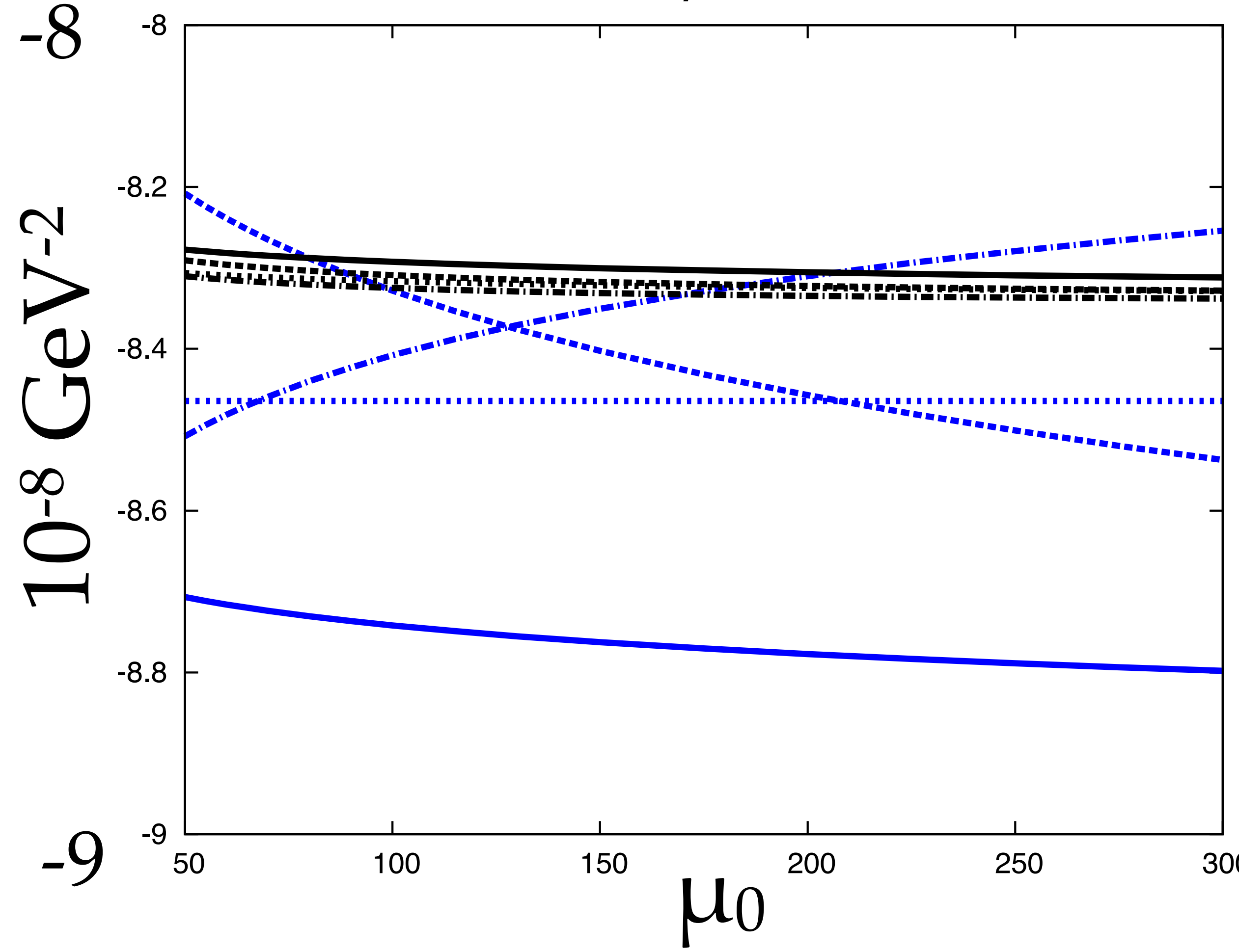
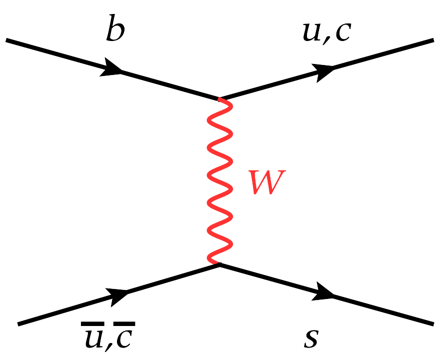
$2^{-1/2} G_F \alpha \pi / \sin^2\theta_W C_A$



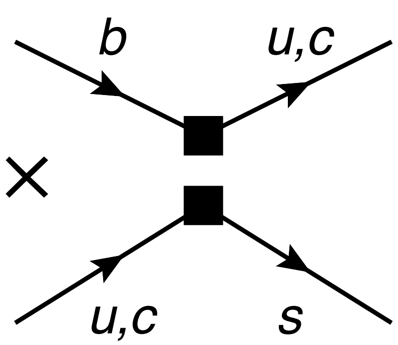
EW corrections reduce modulus of Wilson Coefficient and remove 7 % scale uncertainty in the BR

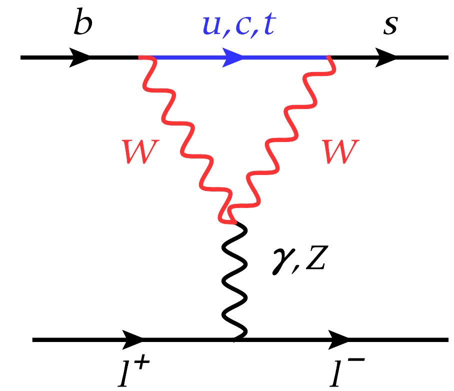
Status of $\mathcal{L}_{\rm eff}$ for $b \to s\, l^+ l^-$

SM Wilson coefficients: Matching at $\mu \approx M_W$

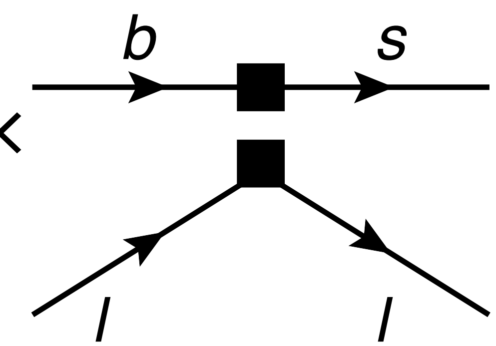


$\to C_2 \times$



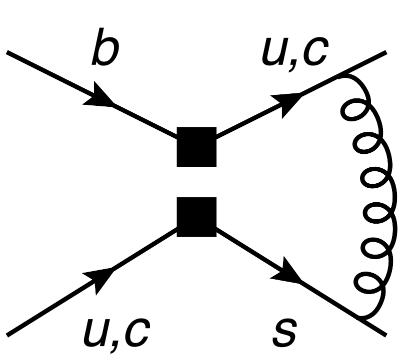


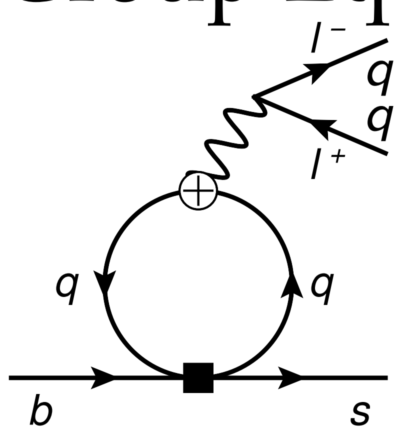
$\to C_{A,V} \times$ $(C_{10,9})$



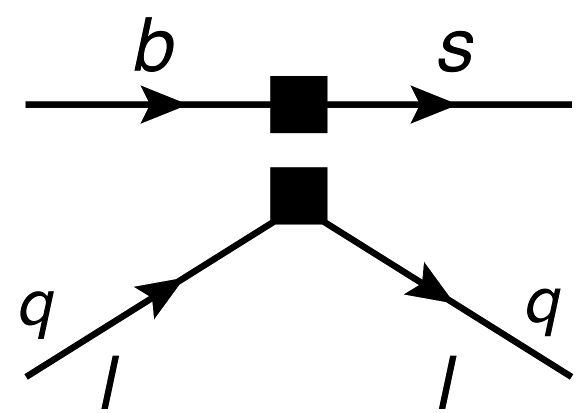
Known at two-loops in QCD for NNLL [Bobeth, Misiak, Urban, `99]

Renormalisation Group Equation $\to \mu \approx M_W$





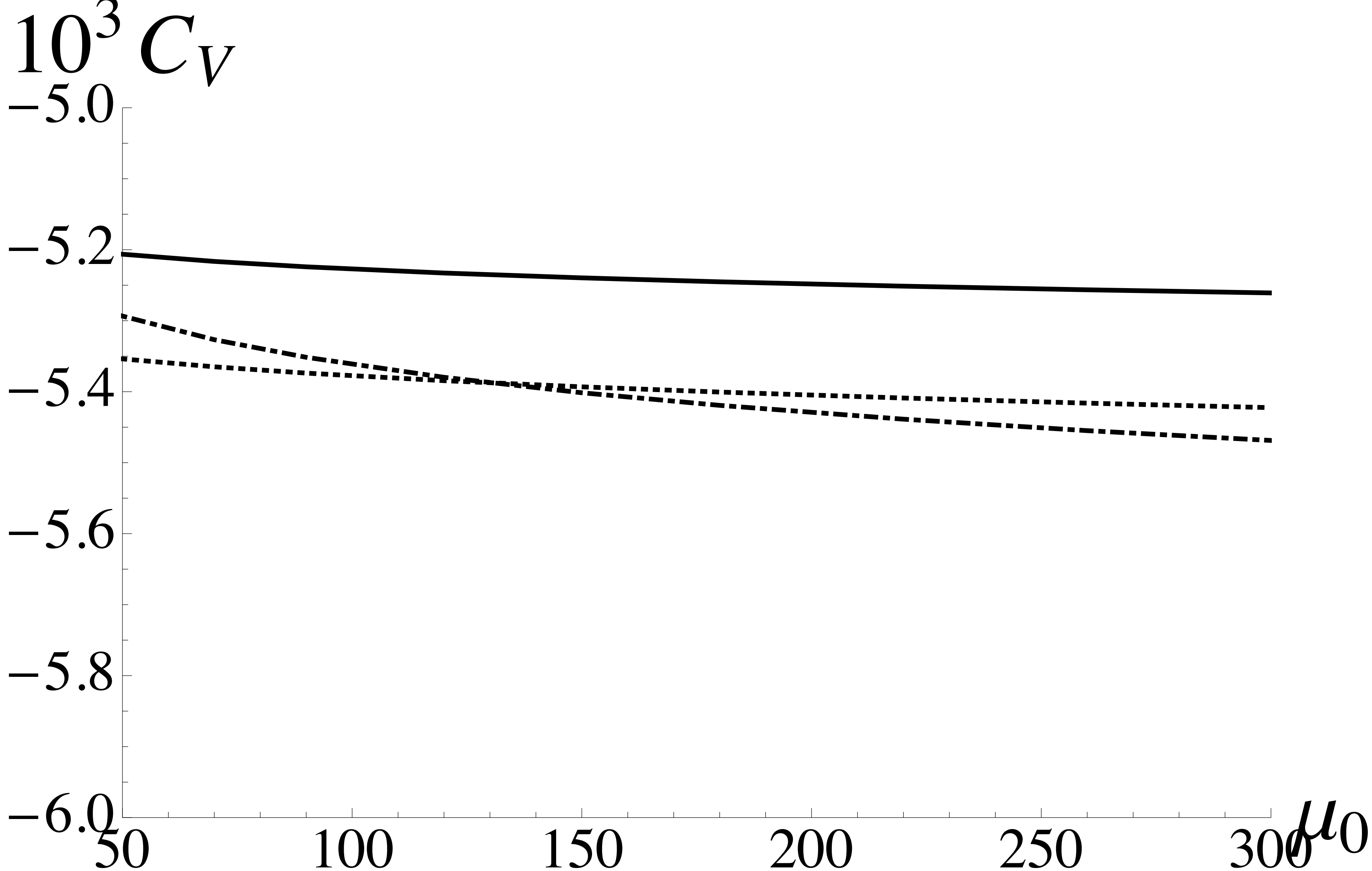
$\to C_{1\ldots10} \times$



$\mathcal{L}_{\rm eff}$ @ NNLL in QCD and NLL EW for all but C_9 & C_{10} EW matching [Gambino Haisch`01; Haisch `05, Bobeth, Gambino, MG, Haisch `04, MG, Haisch `05, Huber et. al. `05]

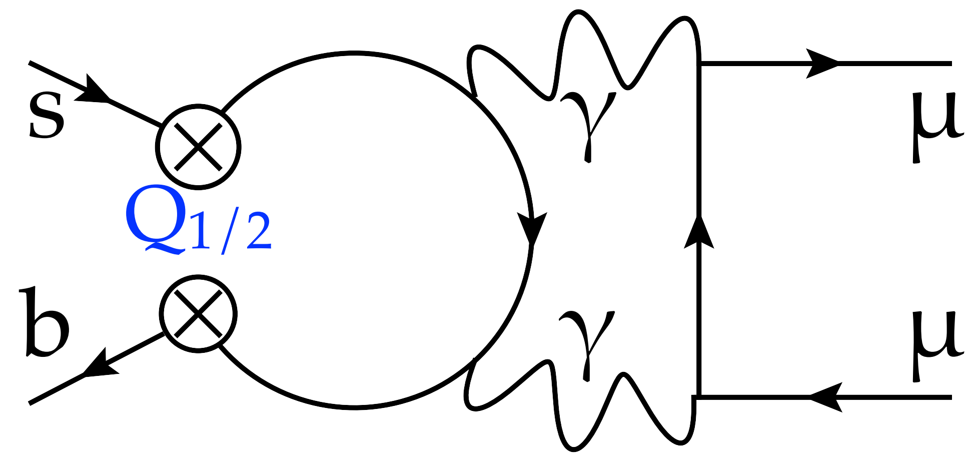
EW corrections for Q_V ?

EW Uncertainties for LO matching below 5% level



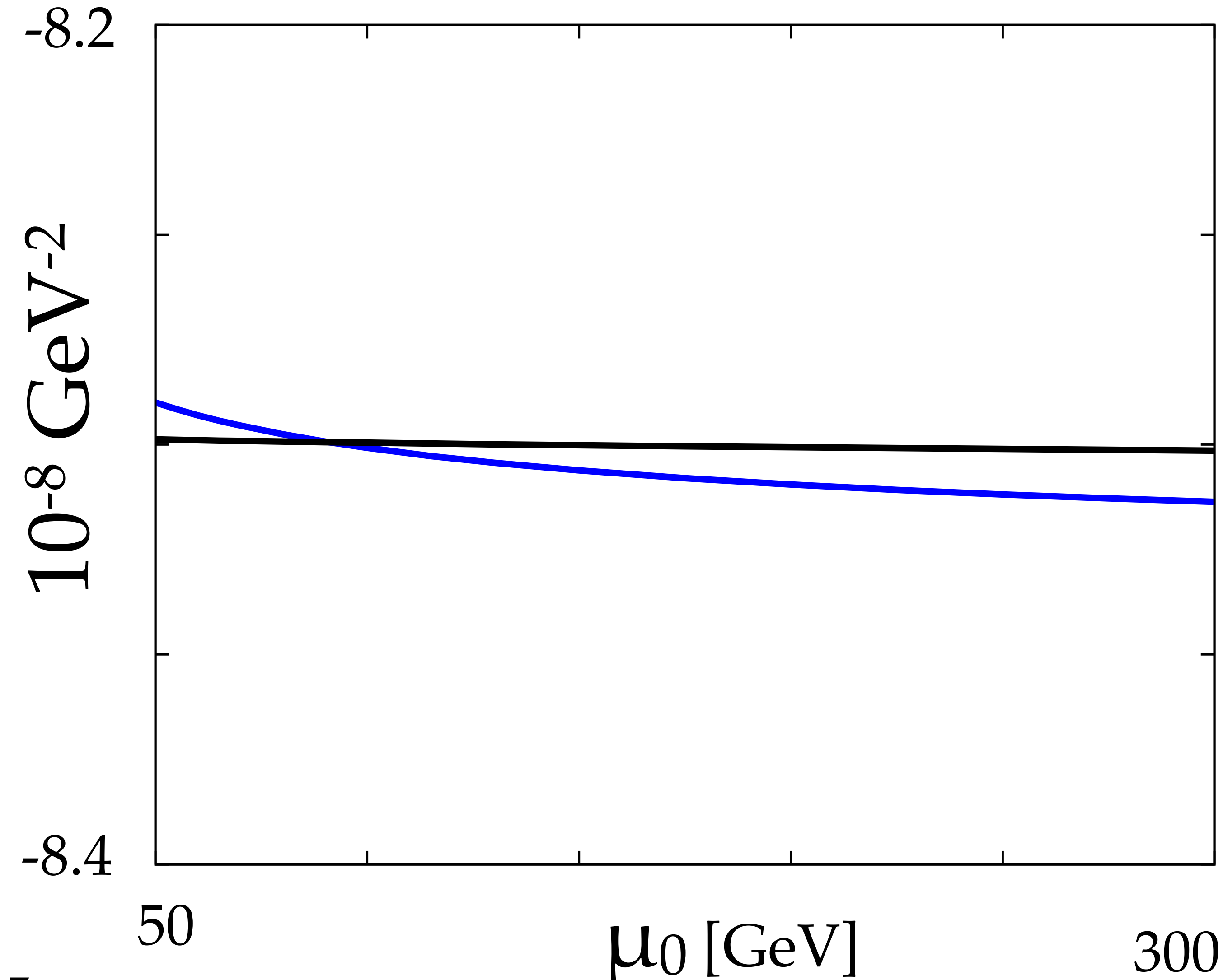
Only the electroweak scheme dependence is plotted, while the effect of operator mixing is switched off

QED RGE for C_A



NLL running cancels matching scale dependence in Q_A

$G^2_F M^2_W C_A(M_Z)$



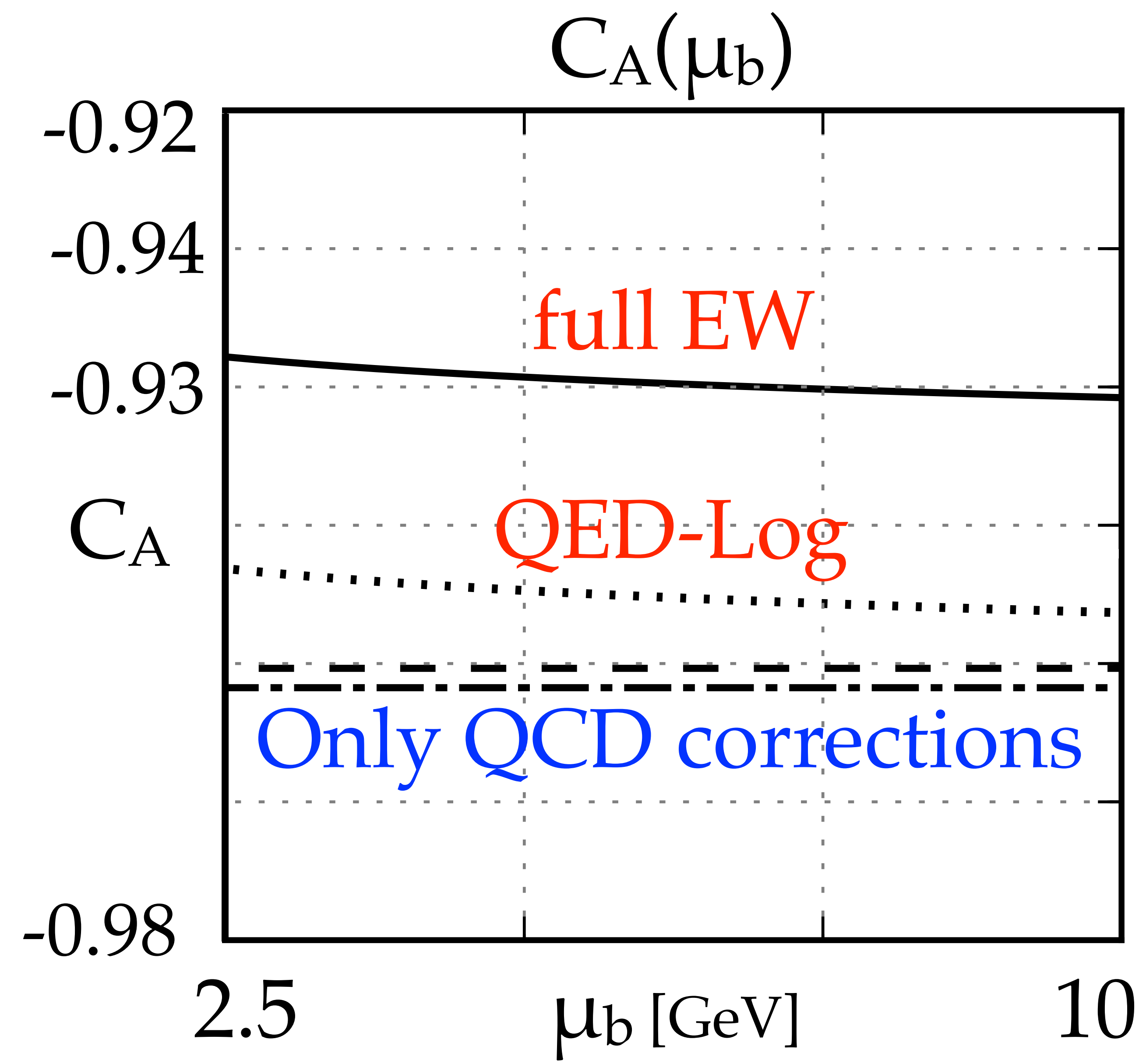
Study residual scale dependence for the $G_F^2 M_W^2$ normalised results

$G_F^2 M_W^2 C(\mu_0)$ is scale dependent, while $U(M_Z, \mu_0)\, G_F^2 M_W^2 C(\mu_0)$ is only residually scale dependent.

Wilson Coefficient at m_b

The log enhanced QED corrections further reduce the modulus of the Wilson coefficient further.

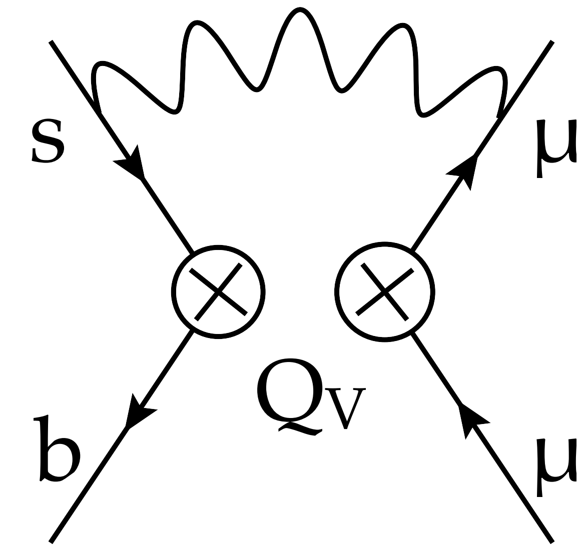
Varying μ_b in $U(\mu_b, m_t)\, G_F^2 M_W^2\, C(m_t)$ gives a measure of uncertainty regarding the contributions of virtual QED corrections at m_b.



The 0.3% scale dependence is not canceled at the scale μ_b

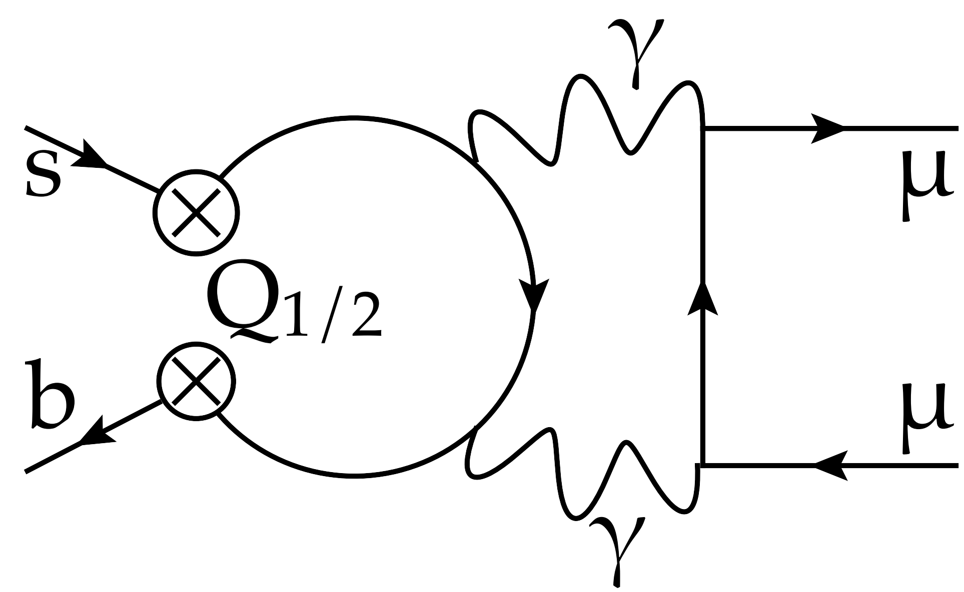
Remaining QED uncertainty

The remaining 0.3% μ_b scale dependence will only be removed after non-perturbative QED corrections are included.



I.e. QED⊗QCD Matrix elements of

$$\begin{aligned} Q_1 &= (\bar{b}\gamma_\mu T^a q_L)(\bar{q}\gamma_\mu T^a s_L) \\ Q_2 &= (\bar{b}\gamma_\mu q_L)(\bar{q}\gamma_\mu s_L) \\ Q_V &= (\bar{b}\gamma_\mu s_L)(\bar{l}\gamma_\mu l) \end{aligned}$$

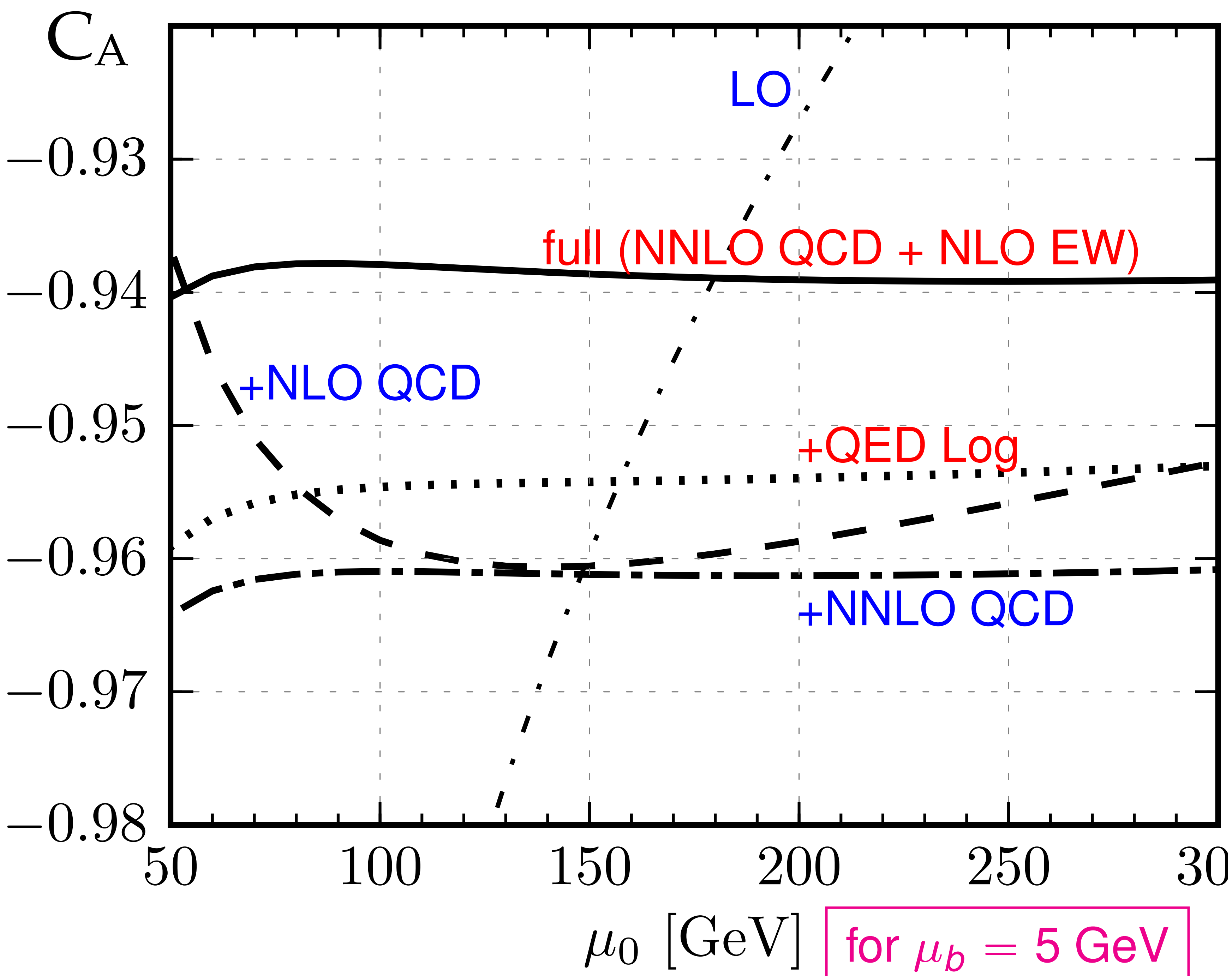


could be considered, but they are $O(\alpha/\pi) \lesssim 0.3\%$ – our error estimate

No relevant lifting of Helicity suppression

Combine with NNLO QCD

Three loop QCD matching, i.e. NNLO, removes scale ambiguities – fixes top mass [Hermann, Misiak, Steinhauser `14]



Theory Prediction $B_s \to \mu^+\mu^-$

We find for the time integrated BR @ NNLO & EW

[Bobeth MG, Hermann, Misiak, Steinhauser, Stamou `13]

$\mathrm{Br}_{\mathrm{the}} = (3.65 \pm 0.23)\, 10^{-9}$

$\mathrm{Br}_{\mathrm{exp}} = (2.8 + 0.7 - 0.6)\, 10^{-9}$

LHCb CMS Combination

	f_{B_q}	CKM	τ_H^q	M_t	α_s	other param.	non-param.	$\sum$
$\overline{\mathcal{B}}_{s\ell}$	4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%

f_{Bs} [MeV]	τ_{Bs} [ps^{-1}]	$\lvert V_{tb} V_{ts} \rvert$	M_t [GeV]
227.7(45)	1.516(11)	0.0415(13)	173.1(9)

where we have used $V_{cb} = 0.0424(9)$ [Gambino, Schwanda `13]

Remaining $B_{(s)} \to l^+ l^-$ decays

Br_{the} $(B_{d\mu}) = (1.06 \pm 0.09)\ 10^{-10}$

Br_{exp} $(B_{d\mu}) = (3.9\ ^{+1.6}_{-1.4})\ 10^{-10}$

$$
\begin{aligned}
\overline{\mathcal{B}}_{se} \times 10^{14} &= 8.54 \pm 0.55 \\
\overline{\mathcal{B}}_{s\tau} \times 10^{7} &= 7.73 \pm 0.49 \\
\overline{\mathcal{B}}_{de} \times 10^{15} &= 2.48 \pm 0.21 \\
\overline{\mathcal{B}}_{d\mu} \times 10^{10} &= 1.06 \pm 0.09 \\
\overline{\mathcal{B}}_{d\tau} \times 10^{8} &= 2.22 \pm 0.19
\end{aligned}
$$

	f_{B_q}	CKM	τ_H^q	M_t	α_s	other param.	non-param.	$\sum$
$\overline{\mathcal{B}}_{s\ell}$	4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%
$\overline{\mathcal{B}}_{d\ell}$	4.5%	6.9%	0.5%	1.6%	0.1%	$< 0.1\%$	1.5%	8.5%

Conclusions

7% electroweak scheme ambiguity in $B_s \to \mu+ \mu-$ is removed

Largest theory uncertainty (@ NLO EW and NNLO) :
- from f_{Bs} (4%), which will be reduced in the future
- rest (<2 %)

But dependence on V_{cb} results in parametric uncertainty, might be reduced in the future or removed by normalising to ΔM_s

Significantly smaller than experimental uncertainty

Only EW corrections to C_V missing to $\mathcal{L}_{\text{eff}}$