# $B \rightarrow K^{(*)} \mu^{+} \mu^{-}:$SM versus New Physics 

Joaquim Matias<br>Universitat Autònoma de Barcelona

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## PLAN of the TALK

- Motivation and theoretical description of $B \rightarrow K^{*}(\rightarrow K \pi) I^{+} I^{-}$at large recoil.
- Analysis of LHCb data on $P_{i}^{(/)}$and model independent understanding of the anomaly.
- Possible explanations of the pattern of deviations and most updated SM predictions.
- New symmetry results and S-wave.
- Conclusions

Many of us thought that the "scalar particle" found at CERN was going to be ALSO $\Rightarrow$ the PORTAL for NEW PHYSICS.

## Motivation

Many of us thought that the "scalar particle" found at CERN was going to be ALSO $\Rightarrow$ the PORTAL for NEW PHYSICS.


BUT the "scalar particle" found resembles very much the SM Higgs particle, with SM-like couplings up to the present precision $\Rightarrow$ it will be a long term task...

Many of us thought that the "scalar particle" found at CERN was going to be ALSO $\Rightarrow$ the PORTAL for NEW PHYSICS.


HOWEVER, there are OTHER PORTALS:
RARE B DECAYS (FCNC)

- New Physics same footing as SM
- They allow you to explore higher scales $\Lambda$
- A promising golden handle: $B \rightarrow K^{*} \mu^{+} \mu^{-}$
$\Rightarrow$ In this portal the best paradigm to unveil New Physics in Flavour Physics will be an accurate determination of Wilson coefficients. In particular those associated to operators:

$$
\mathcal{O}_{\mathbf{7}}=\frac{e}{16 \pi^{2}} m_{b}\left(\overline{\mathbf{s}} \sigma_{\mu \nu} P_{R} b\right) F^{\mu \nu}, \quad \mathcal{O}_{\mathbf{9}}=\frac{\mathbf{e}^{\mathbf{2}}}{\mathbf{1 6} \pi^{2}}\left(\overline{\mathbf{s}} \gamma_{\mu} \mathbf{P}_{\mathbf{L}} \mathbf{b}\right)\left(\bar{\ell} \gamma^{\mu} \ell\right), \quad \mathcal{O}_{\mathbf{1 0}}=\frac{e^{2}}{16 \pi^{2}}\left(\overline{\mathbf{s}} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right)
$$

and chiral counterparts $\mathcal{O}_{7,9,10}^{\prime}(\mathrm{L} \leftrightarrow \mathrm{R})$

$\bullet$ Wilson Coefficients are tested $C_{i}=C_{i}^{S M}+\mathbf{C}_{\mathrm{i}}^{N P}\left\{\begin{array}{l}\text { different levels of accuracy } \\ \text { allow different ranges of NP }\end{array}\right.$
Wilson coefficients $\left[\mu_{b}=\mathcal{O}\left(m_{b}\right)\right]$

$$
\begin{array}{lcr}
\mathrm{C}_{7}^{\text {eff }}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(\overline{\mathbf{B}} \rightarrow \mathbf{X}_{\mathbf{s}} \gamma\right), A_{l}\left(B \rightarrow K^{*} \gamma\right), S_{K^{*} \gamma}, A_{F B}, F_{L}, & -0.292 \\
\mathbf{C}_{9}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L}, & 4.075 \\
\mathrm{C}_{10}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(\mathrm{B}_{\mathbf{s}} \rightarrow \mu^{+} \mu^{-}\right), \mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L}, & -4.308 \\
\mathbf{C}_{7}^{\prime}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(\bar{B} \rightarrow X_{\mathbf{s}} \gamma\right), A_{l}\left(B \rightarrow K^{*} \gamma\right), S_{K^{*} \gamma}, A_{F B}, F_{L} & -0.006 \\
\mathbf{C}_{9}^{\prime}\left(\mu_{\mathbf{b}}\right) & \mathcal{B}\left(B \rightarrow X_{\mathbf{s}} \ell \ell\right), A_{F B}, F_{L} & 0 \\
\left.\mathbf{C}_{10}^{\prime} \mu_{\mathbf{b}}\right) & \mathcal{B}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right), A_{F B}, F_{L}, & 0
\end{array}
$$

More Precision Observables are necessary to overconstrain the deviations $\mathbf{C}_{\mathbf{i}}^{\text {NP }}$
$\Rightarrow$ In this portal the best paradigm to unveil New Physics in Flavour Physics will be an accurate determination of Wilson coefficients. In particular those associated to operators:

$$
\mathcal{O}_{\mathbf{7}}=\frac{e}{16 \pi^{2}} m_{b}\left(\bar{s} \sigma_{\mu \nu} P_{R} b\right) F^{\mu \nu}, \quad \mathcal{O}_{\mathbf{9}}=\frac{\mathbf{e}^{\mathbf{2}}}{\mathbf{1 6} \pi^{2}}\left(\overline{\mathbf{s}} \gamma_{\mu} \mathbf{P}_{\mathbf{L}} \mathbf{b}\right)\left(\bar{\ell} \gamma^{\mu} \ell\right), \quad \mathcal{O}_{\mathbf{1 0}}=\frac{e^{2}}{16 \pi^{2}}\left(\overline{\mathbf{s}} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right),
$$

and chiral counterparts $\mathcal{O}_{7,9,10}^{\prime}(\mathrm{L} \leftrightarrow \mathrm{R})$

$\bullet$ Wilson Coefficients are tested $C_{i}=C_{i}^{S M}+\mathrm{C}_{\mathrm{i}}^{\text {NP }}\left\{\begin{array}{l}\text { different levels of accuracy } \\ \text { allow different ranges of NP }\end{array}\right.$
Wilson coefficients [ $\left.\mu_{b}=\mathcal{O}\left(m_{b}\right)\right]$

## Observables

$\underline{S M \text { values }}$

$$
\begin{align*}
& \mathbf{C}_{7}^{e f f}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{9}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{10}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{7}^{\prime}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{9}^{\prime}\left(\mu_{\mathbf{b}}\right) \\
& \mathbf{C}_{10}^{\prime}\left(\mu_{\mathbf{b}}\right)
\end{align*}
$$

$$
\begin{array}{cr}
\mathcal{B}\left(\overline{\mathrm{B}} \rightarrow \mathrm{X}_{\mathrm{s}} \gamma\right), A_{l}\left(B \rightarrow K^{*} \gamma\right), S_{K^{*} \gamma}, A_{F B}, F_{L}, P_{2}, P_{4,5}^{\prime} & -0.292 \\
\mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L}, P_{2}, P_{4,5}^{\prime} & 4.075 \\
\mathcal{B}\left(\mathrm{~B}_{s} \rightarrow \mu^{+} \mu^{-}\right), \mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L}, P_{4}^{\prime} & -4.308 \\
\mathcal{B}\left(\bar{B} \rightarrow X_{s} \gamma\right), A_{l}\left(B \rightarrow K^{*} \gamma\right), S_{K^{*} \gamma}, A_{F B}, F_{L}, P_{1} & -0.006 \\
\mathcal{B}\left(B \rightarrow X_{s} \ell \ell\right), A_{F B}, F_{L}, P_{1} & 0 \\
\mathcal{B}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right), A_{F B}, F_{L}, P_{1}, P_{4}^{\prime} & 0
\end{array}
$$

$\Rightarrow B \rightarrow K^{*}(\rightarrow K \pi) \mu^{+} \mu^{-}$can fulfill this requirement providing a large set of clean observables that can test in an unprecedented way $C_{9}$ and $C_{7,9,10}^{\prime}$.

All those new observables $P_{i}^{(\prime)}$ come from the angular distribution $\overline{\mathbf{B}}_{\mathbf{d}} \rightarrow \overline{\mathbf{K}}^{* 0}\left(\rightarrow \mathbf{K}^{-} \pi^{+}\right) \mathbf{I}^{+} \mathbf{I}^{-}$with the $K^{* 0}$ on the mass shell. It is described by $\mathbf{s}=\mathbf{q}^{2}$ and three angles $\theta_{\ell}, \theta_{\mathbf{K}}$ and $\phi$

$$
\frac{d^{4} \Gamma\left(\bar{B}_{d}\right)}{d q^{2} d \cos \theta_{\ell} d \cos \theta_{K} d \phi}=\frac{9}{32 \pi} \mathbf{J}\left(\mathbf{q}^{2}, \theta_{\ell}, \theta_{K}, \phi\right)
$$


$\theta_{\ell}$ : Angle of emission between $\bar{K}^{* 0}$ and $\mu^{-}$in di-lepton rest frame.
$\theta_{\mathrm{K}}$ : Angle of emission between $\bar{K}^{* 0}$ and $K^{-}$in di-meson rest frame. $\phi$ : Angle between the two planes.
$\mathbf{q}^{2}$ : dilepton invariant mass square.

Notice LHCb uses $\theta_{\ell}^{L H C b}=\pi-\theta_{\ell}^{\text {us }}$

- large recoil for $K^{*}: E_{K^{*}} \gg \Lambda_{Q C D}$ or $4 m_{\ell}^{2} \leq q^{2}<9 \mathrm{GeV}^{2}$

Three regions in $q^{2}$ : $\quad$ resonance region $\left(q^{2}=m_{J / \Psi}^{2}, \ldots\right)$ betwen $9<q^{2}<14 \mathrm{GeV}^{2}$.

- low-recoil for $K^{*}: E_{K^{*}} \sim \Lambda_{Q C D}$ or $14<q^{2} \leq\left(m_{B}-m_{K^{*}}\right)^{2}$.

The differential distribution splits in $J_{i}$ coefficients:

$$
\begin{gathered}
J\left(q^{2}, \theta_{l}, \theta_{K}, \phi\right)= \\
J_{1 s} \sin ^{2} \theta_{K}+J_{1 c} \cos ^{2} \theta_{K}+\left(J_{2 s} \sin ^{2} \theta_{K}+J_{2 c} \cos ^{2} \theta_{K}\right) \cos 2 \theta_{l}+J_{3} \sin ^{2} \theta_{K} \sin ^{2} \theta_{l} \cos 2 \phi \\
+J_{4} \sin 2 \theta_{K} \sin 2 \theta_{l} \cos \phi+J_{5} \sin 2 \theta_{K} \sin \theta_{l} \cos \phi+\left(J_{6 s} \sin ^{2} \theta_{K}+J_{6 c} \cos ^{2} \theta_{K}\right) \cos \theta_{l} \\
+J_{7} \sin 2 \theta_{K} \sin \theta_{l} \sin \phi+J_{8} \sin 2 \theta_{K} \sin 2 \theta_{l} \sin \phi+J_{9} \sin ^{2} \theta_{K} \sin ^{2} \theta_{l} \sin 2 \phi .
\end{gathered}
$$

The coefficients $\mathbf{J}_{\mathbf{i}}$ of the distribution can be reexpressed now in terms of this basis of clean observables:

Correspondence $\mathbf{J}_{\mathbf{i}} \leftrightarrow \mathbf{P}_{\mathbf{i}}^{(\prime)}$ :
BROWN: LO FF-dependent observables ( $F_{L}$ Longitudinal Polarization Fraction of $K^{*}$ )

RED: LO FF-independent observables at large-recoil (defined from these eqs.)

$$
\begin{aligned}
& \left(\mathbf{J}_{2 \mathrm{~s}}+\bar{J}_{2 \mathrm{~s}}\right)=\frac{1}{4} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq} \boldsymbol{q}^{2}} \quad\left(\mathrm{~J}_{2 \mathrm{c}}+\overline{\mathrm{J}}_{2 \mathrm{c}}\right)=-\mathrm{F}_{\mathrm{L}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq} \boldsymbol{q}^{2}} \\
& \mathrm{~J}_{3}+\bar{J}_{3}=\frac{1}{2} \mathrm{P}_{1} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \Gamma+\mathrm{d} \bar{\Gamma} \overline{\mathrm{~T}}}{\mathrm{dq} q^{2}} \quad \mathrm{~J}_{3}-\bar{J}_{3}=\frac{1}{2} \mathrm{P}_{1}^{\mathrm{CP}} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \Gamma+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}}{ }^{2} \\
& \mathbf{J}_{6 \mathrm{~s}}+\overline{\mathbf{J}}_{6 \mathrm{~s}}=2 \mathrm{P}_{2} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \overline{\boldsymbol{\Gamma}}}{\mathrm{dq}}{ }^{2} \quad \quad \mathrm{~J}_{6 \mathrm{~s}}-\overline{\mathbf{J}}_{6 \mathrm{~s}}=2 \mathrm{P}_{2}^{\mathrm{CP}} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \overline{\boldsymbol{\Gamma}}}{\mathrm{dq}} \\
& \mathrm{~J}_{9}+\bar{J}_{9}=-\mathrm{P}_{3} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \bar{\Gamma} \overline{\mathrm{~T}}}{\mathrm{dq} \mathbf{q}^{2}} \quad \mathrm{~J}_{9}-\bar{J}_{9}=-\mathrm{P}_{3}^{\mathrm{CP}} \mathrm{~F}_{\mathrm{T}} \frac{\mathrm{~d} \boldsymbol{\Gamma}+\mathrm{d} \bar{\Gamma}}{\mathrm{dq}}{ }^{2} \\
& \mathrm{~J}_{4}+\bar{J}_{4}=\frac{1}{2} \mathrm{P}_{4}^{\prime} \sqrt{\mathrm{F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}} \frac{\mathrm{~d} \mathrm{\Gamma}+\mathrm{d} \bar{\Gamma}}{d q^{2}} \quad \mathrm{~J}_{4}-\bar{J}_{4}=\frac{1}{2} P_{4}^{\prime C P} \sqrt{\mathrm{~F}_{\mathrm{T}} F_{\mathrm{L}}} \frac{\mathrm{~d} \Gamma+\mathrm{d} \bar{\Gamma}}{d q^{2}} \\
& J_{5}+\bar{J}_{5}=P_{5}^{\prime} \sqrt{F_{T} F_{L}} \frac{d \Gamma+d \bar{\Gamma}}{d q^{2}} \quad J_{5}-\bar{J}_{5}=P_{5}^{\prime C P} \sqrt{F_{T} F_{L}} \frac{d \Gamma+d \bar{\Gamma}}{d q^{2}} \\
& J_{7}+\bar{J}_{7}=-P_{6}^{\prime} \sqrt{F_{T} F_{L}} \frac{d \Gamma+d \bar{\Gamma}}{d q^{2}} \quad J_{7}-\bar{J}_{7}=-P_{6}^{\prime C P} \sqrt{F_{T} F_{L}} \frac{d \Gamma+d \bar{\Gamma}}{d q^{2}}
\end{aligned}
$$

$P_{i}, P_{i}^{\prime}$ defines an Optimal Basis of observables, a compromise between:

- Excellent experimental accessibility and simplicity of the fit.
- Reduced FF dependence (in the large-recoil region: $0.1 \leq q^{2} \leq 8 \mathrm{GeV}^{2}$ ).

$$
\mathbf{P}_{5}^{\prime}=\sqrt{2} \frac{\operatorname{Re}\left(A_{0}^{L} A_{\perp}^{L *}-A_{0}^{R *} A_{\perp}^{R}\right)}{\sqrt{\left|A_{0}\right|^{2}\left(\left|A_{\|}\right|^{2}+\left|A_{\perp}\right|^{2}\right)}}=c_{1}+\mathcal{O}\left(\alpha_{s} \xi_{\perp, \|}\right) \quad \mathbf{S}_{5}=\sqrt{2} \frac{\operatorname{Re}\left(A_{0}^{L} A_{\perp}^{L *}-A_{0}^{R *} A_{\perp}^{R}\right)}{\left|A_{\|}\right|^{2}+\left|A_{\perp}\right|^{2}+\left|A_{0}\right|^{2}}=\frac{c_{1} \xi_{\perp} \xi_{\|}}{c_{2} \xi_{\perp}^{2}+c_{3} \xi_{\|}^{2}}
$$

Our proposal for CP-conserving basis:

$$
\left\{\frac{\mathbf{d \Gamma}}{\mathbf{d} \mathbf{q}^{2}}, \mathbf{A}_{\mathbf{F B}}, \mathbf{P}_{\mathbf{1}}, \mathbf{P}_{\mathbf{2}}, \mathbf{P}_{\mathbf{3}}, \mathbf{P}_{\mathbf{4}}^{\prime}, \mathbf{P}_{\mathbf{5}}^{\prime}, \mathbf{P}_{6}^{\prime}\right\} \text { or } \mathbf{P}_{\mathbf{3}} \leftrightarrow \mathbf{P}_{\mathbf{8}}^{\prime} \text { and } \mathbf{A}_{\mathrm{FB}} \leftrightarrow \mathbf{F}_{\mathrm{L}}
$$

where $P_{1}=A_{T}^{2}$ [Kruger, J.M'05],
$P_{2}=\frac{1}{2} A_{T}^{\mathrm{re}}, P_{3}=-\frac{1}{2} A_{T}^{\mathrm{im}}$ [Becirevic, Schneider'12]
$P_{4,5,6}^{\prime}$ [Descotes, JM, Ramon, Virto'13]).
The corresponding CP-violating basis $\left(J_{i}+\bar{J}_{i} \rightarrow J_{i}-\bar{J}_{i}\right.$ in numerators $)$ :

$$
\left\{\mathbf{A}_{\mathrm{CP}}, \mathbf{A}_{\mathrm{FB}}^{\mathrm{CP}}, \mathbf{P}_{1}^{\mathrm{CP}}, \mathbf{P}_{2}^{\mathrm{CP}}, \mathbf{P}_{3}^{\mathrm{CP}}, \mathbf{P}_{4}^{\prime \mathrm{CP}}, \mathbf{P}_{5}^{\prime \mathrm{CP}}, \mathbf{P}_{6}^{\prime \mathrm{CP}}\right\} \text { or } \mathbf{P}_{3}^{\mathrm{CP}} \leftrightarrow \mathbf{P}_{8}^{\prime \mathrm{CP}} \text { and } \mathbf{A}_{\mathrm{FB}}^{\mathrm{CP}} \leftrightarrow \mathbf{F}_{\mathrm{L}}^{\mathrm{CP}}
$$

"Barcelona/Aachen" approach: QCDF+exploit the symmetry relations at large-recoil among FF:

$$
\begin{gathered}
\frac{m_{B}}{m_{B}+m_{K^{*}}} V\left(q^{2}\right)=\frac{m_{B}+m_{K^{*}}}{2 E} A_{1}\left(q^{2}\right)=T_{1}\left(q^{2}\right)=\frac{m_{B}}{2 E} T_{2}\left(q^{2}\right)=\xi_{\perp}(E) \\
\frac{m_{K^{*}}}{E} A_{0}\left(q^{2}\right)=\frac{m_{B}+m_{K^{*}}}{2 E} A_{1}\left(q^{2}\right)-\frac{m_{B}-m_{K^{*}}}{m_{B}} A_{2}\left(q^{2}\right)=\frac{m_{B}}{2 E} T_{2}\left(q^{2}\right)-T_{3}\left(q^{2}\right)=\xi_{\|}(E)
\end{gathered}
$$

$\Rightarrow$ Transparent, valid for ANY FF parametrization (BZ, KMPW,...) and easy to reproduce.
$\Rightarrow$ Dominant correlations automatically implemented in a transparent way.
$\Rightarrow$ This allows you to construct clean observables from the observation that at LO in $1 / m_{b}, \alpha_{s}$ and large-recoil limit ( $E_{K}^{*}$ large):

$$
\begin{aligned}
& A_{\perp}^{L, R}=\sqrt{2} N m_{B}(1-\hat{s})\left[\left(\mathcal{C}_{9}^{\text {eff }}+\mathcal{C}_{9}^{\text {eff }}\right) \mp\left(\mathcal{C}_{10}+\mathcal{C}_{10}^{\prime}\right)+\frac{2 \hat{m}_{b}}{\hat{s}}\left(\mathcal{C}_{7}^{\text {eff }}+\mathcal{C}_{7}^{\text {eff }}\right)\right] \xi_{\perp}\left(E_{K^{*}}\right), \quad A_{\|}^{L, R} \propto \xi_{\perp}\left(E_{K^{*}}\right) \\
& A_{0}^{L, R}=-\frac{N m_{B}(1-\hat{s})^{2}}{2 \hat{m}_{K^{*}} \sqrt{\hat{s}}}\left[\left(\mathcal{C}_{9}^{\text {eff }}-\mathcal{C}_{9}^{\text {eff }}\right) \mp\left(\mathcal{C}_{10}-\mathcal{C}_{10}^{\prime}\right)+2 \hat{m}_{b}\left(\mathcal{C}_{7}^{\text {eff }}-\mathcal{C}_{7}^{\text {eff }}\right)\right] \xi_{\| \|}\left(E_{K^{*}}\right) .
\end{aligned}
$$

$\Rightarrow$ Symmetry Breaking corrections ( $\alpha_{s}$ and P.C.) are added in our computation:

- known $\alpha_{s}$ factorizable and non-factorizable corrections from QCDF.
- factorizable power corrections (using a systematic procedure for each FFp, see later)
- non-factorizable power corrections including charm-quark loops.


## Analysis of LHCb data On

$$
B \rightarrow K^{*} \mu^{+} \mu^{-}
$$

Present bins: $[0.1,2],[2,4.3],[4.3,8.68],[1,6],[14.18,16],[16,19] \mathrm{GeV}^{2}$.

| Observable | Experiment | SM prediction | Pull |
| :--- | ---: | ---: | ---: |
| $\left\langle P_{1}\right\rangle_{[0.1,2]}$ | $-0.19_{-0.35}^{+0.40}$ | $0.000_{-0.044}^{+0.043}$ | -0.5 |
| $\left\langle P_{1}\right\rangle_{[2,4.3]}$ | $-0.29_{-0.46}^{+0.65}$ | $-0.051_{-0.046}^{+0.046}$ | -0.4 |
| $\left\langle P_{1}\right\rangle_{[4.38 .68]}$ | $0.36_{-0.30}^{+0.31}$ | $-0.117_{-0.056}^{+0.052}$ | +1.5 |
| $\left\langle P_{1}\right\rangle_{[1,6]}$ | $0.15_{-0.41}^{+0.39}$ | $-0.055_{-0.043}^{+0.041}$ | +0.5 |
| $\left\langle P_{2}\right\rangle_{[0.1,2]}$ | $0.03_{-0.15}^{+0.14}$ | $0.172_{-0.021}^{+0.020}$ | -1.0 |
| $\left\langle P_{2}\right\rangle_{[2,4.3]}$ | $0.50_{-0.00}^{+0.07}$ | $0.234_{-0.080}^{+0.060}$ | +2.9 |
| $\left\langle P_{2}\right\rangle_{[4.3,8.68]}$ | $-0.25_{-0.08}^{+0.07}$ | $-0.400_{-0.037}^{+0.049}$ | +1.7 |
| $\left\langle P_{2}\right\rangle_{[1,6]}$ | $0.33_{-0.12}^{+0.11}$ | $0.084_{-0.078}^{+0.060}$ | +1.8 |
| $\left\langle A_{\mathrm{FB}}\right\rangle_{[0.1,2]}$ | $-0.02_{-0.13}^{+0.13}$ | $-0.136_{-0.048}^{+0.051}$ | +0.8 |
| $\left\langle A_{\mathrm{FB}}\right\rangle_{[2,4.3]}$ | $-0.20_{-0.08}^{+0.08}$ | $-0.081_{-0.055}^{+0.059}$ | -1.1 |
| $\left\langle A_{\mathrm{FB}}\right\rangle_{[4.3,8.68]}$ | $0.16_{-0.05}^{+0.06}$ | $0.220_{-0.113}^{+0.138}$ | -0.5 |
| $\left\langle A_{\mathrm{FB}}\right\rangle_{[1,6]}$ | $-0.17_{-0.06}^{+0.06}$ | $-0.035_{-0.034}^{+0.037}$ | -2.0 |

- $\mathbf{P}_{1}$ : No substantial deviation (large error bars).
- $\mathbf{A}_{\mathrm{FB}}-\mathbf{P}_{2}$ : A slight tendency for a lower value of the second and third bins of $A_{\mathrm{FB}}$ is consistent with a $2.9 \sigma(1.7 \sigma)$ deviation in the second (third) bin of $P_{2}$.
- Zero: Preference for a slightly higher $q^{2}$-value for the zero of $A_{\text {FB }}$ (same as the zero of $P_{2}$ ).

Both effects can be accommodated with $\mathcal{C}_{7}^{\mathrm{NP}}<0$ and/or $\mathcal{C}_{9}^{\mathrm{NP}}<0$.

## Connection via $\mathcal{H}_{\text {eff }}$

| Observable | Experiment | SM prediction | Pull |
| :---: | :---: | :---: | :---: |
| $\left\langle P_{4}^{\prime}\right\rangle_{[0.1,2]}$ | $0.00_{-0.52}^{+0.52}$ | $-0.342_{-0.026}^{+0.031}$ | +0.7 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[2,4.3]}$ | $0.74{ }_{-0.60}^{+0.54}$ | $0.569_{-0.063}^{+0.073}$ | +0.3 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[4.3,8.68]}$ | $1.18{ }_{-0.32}^{+0.26}$ | $1.003_{-0.032}^{+0.028}$ | +0.6 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[1,6]}$ | $0.58{ }_{-0.36}^{+0.32}$ | $0.555_{-0.058}^{+0.067}$ | +0.1 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[0.1,2]}$ | $0.45{ }_{-0.24}^{+0.21}$ | $0.533_{-0.041}^{+0.033}$ | -0.4 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[2,4.3]}$ | $0.29{ }_{-0.39}^{+0.40}$ | $-0.334_{-0.113}^{+0.097}$ | +1.6 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[4.3,8.68]}$ | $-0.19_{-0.16}^{+0.16}$ | $-0.872_{-0.041}^{+0.053}$ | +4.0 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[1,6]}$ | $0.21_{-0.21}^{+0.20}$ | $-0.349_{-0.100}^{+0.088}$ | +2.5 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[14.18,16]}$ | $-0.18_{-0.70}^{+0.54}$ | $1.161_{-0.332}^{+0.190}$ | -2.1 |
| $\left\langle P_{4}^{\prime}\right\rangle_{[16,19]}$ | $0.70_{-0.52}^{+0.44}$ | $1.263_{-0.248}^{+0.119}$ | -1.1 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[14.18,16]}$ | $-0.79_{-0.22}^{+0.27}$ | $-0.779_{-0.363}^{+0.328}$ | +0.0 |
| $\left\langle P_{5}^{\prime}\right\rangle_{[16,19]}$ | $-0.60{ }_{-0.18}^{+0.21}$ | $-0.601_{-0.367}^{+0.282}$ | +0.0 |

## Definition of the anomaly:

- $\mathbf{P}_{\mathbf{5}}^{\prime}$ : There is a striking $4.0 \sigma(1.6 \sigma)$ deviation in the third (second) bin of $P_{5}^{\prime}$.

Consistent with large negative contributions in $\mathcal{C}_{7}^{\mathrm{NP}}$ and/or $\mathcal{C}_{9}^{\mathrm{NP}}$.

- $\mathbf{P}_{4}^{\prime}$ : in agreement with the SM , but within large uncertainties, and it has future potential to determine the sign of $\mathcal{C}_{10}^{\mathrm{NP}}$.
- $\mathbf{P}_{6}^{\prime}$ and $\mathbf{P}_{8}^{\prime}$ : show small deviations with respect to the SM, but such effect would require complex phases in $\mathcal{C}_{9}^{\mathrm{NP}}$ and/or $\mathcal{C}_{10}^{\mathrm{NP}}$.

Us: $(-0.19-(-0.872)) / \sqrt{0.16^{2}+0.053^{2}}=4.05$ and Exp: $(-0.19-(-0.872+0.053)) / \sqrt{0.16^{2}+0.053^{2}}=3.73$

## Our SM predictions+LHCb data



Figure: Experimental measurements and SM predictions for some $B \rightarrow K^{*} \mu^{+} \mu^{-}$observables. The black crosses are the experimental LHCb data. The blue band corresponds to the SM predictions for the differential quantities, whereas the purple boxes indicate the corresponding binned observables.

Goal: Determine the Wilson coefficients $\mathcal{C}_{7,9,10}, \mathcal{C}_{7,9,10}^{\prime}: \mathcal{C}_{i}=\mathcal{C}_{i}^{S M}+\mathcal{C}_{i}^{N P}$
Standard $\chi^{2}$ frequentist approach: Asymmetric errors included, estimate theory uncertainties for each set of $\mathcal{C}_{i}^{N P}$ and all uncertainties are combined in quadrature.

IMPORTANT: Experimental correlations are included in the updated plot
We do three analysis: a) large-recoil data b) large+low-recoil data c) [1-6] bin
Observables:

- $B \rightarrow K^{*} \mu^{+} \mu^{-}$: We take observables $P_{1}, P_{2}, P_{4}^{\prime}, P_{5}^{\prime}, P_{6}^{\prime}$ and $P_{8}^{\prime}$ in the following binning:
-large-recoil: $[0.1,2],[2,4.3],[4.3,8.68] \mathrm{GeV}^{2}$.
-low recoil: $[14.18,16],[16,19] \mathrm{GeV}^{2}$
-wide large-recoil bin: $[1,6] \mathrm{GeV}^{2}$.
- Radiative and dileptonic $B$ decays: $\mathcal{B}\left(B \rightarrow X_{s} \gamma\right)_{E_{\gamma}>1.6 \mathrm{GeV}}, \mathcal{B}\left(B \rightarrow X_{s} \mu^{+} \mu^{-}\right)_{[1,6]}$ and $\mathcal{B}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right), A_{l}\left(B \rightarrow K^{*} \gamma\right)$ and the $B \rightarrow K^{*} \gamma$ time-dependent CP asymmetry $S_{K^{*} \gamma}$

Updated result using $\mathbf{P}_{\mathbf{i}}, \mathbf{P}_{\mathbf{i}}^{\prime}, \mathbf{A}_{\text {FB }}$ and experimental correlations.
2013 Data favours clearly contributions inside $C_{7}$ and $C_{9}$.


From the analysis of the set
$\mathbf{P}_{\mathbf{i}}, \mathbf{P}_{\mathbf{i}}^{\prime}, \mathbf{A}_{\mathrm{FB}}+\mathbf{B R}+\exp$. correlations we get:
4.3 $\sigma$ (large-recoil)
3.6 $\sigma$ (large + low recoil)
$2.8 \sigma$ for [1-6] bin.

Colored: large-recoil and dashed: large+low recoil orange: [1-6] bin

- We checked (for completeness) that we find same significance using $\mathbf{P}_{\mathbf{i}}, \mathbf{P}_{\mathbf{i}}^{\prime}, \mathbf{F}_{\mathbf{L}}$ instead of $\mathbf{A}_{\mathrm{FB}}$. Positive: Our SM $\mathrm{F}_{\mathrm{L}}$ fully compatible with all data (not only LHCb ) and less correlated. Negative: Result using $F_{L}$ is less solid than using $A_{F B}$ since it depends on choice of FF.

Result of our analysis (large+low recoil data+rad) if we allow all Wilson coefficients to vary freely:

| Coefficient | $1 \sigma$ | $2 \sigma$ | $3 \sigma$ |
| :--- | :---: | :---: | :---: |
| $\mathcal{C}_{7}^{\mathrm{NP}}$ | $[-0.05,-0.01]$ | $[-0.06,0.01]$ | $[-0.08,0.03]$ |
| $\mathcal{C}_{9}^{\mathrm{NP}}$ | $[-1.6,-0.9]$ | $[-1.8,-0.6]$ | $[-2.1,-0.2]$ |
| $\mathcal{C}_{10}^{\mathrm{NP}}$ | $[-\mathbf{0 . 4 , \mathbf { 1 . 0 }}$ | $[-1.2,2.0]$ | $[-2.0,3.0]$ |
| $\mathcal{C}_{7}^{\mathrm{NP}}$ | $[-\mathbf{0 . 0 4}, \mathbf{0 . 0 2 ]}$ | $[-0.09,0.06]$ | $[-0.14,0.10]$ |
| $\mathcal{C}_{9^{\prime}}^{\mathrm{NP}}$ | $[-0.2,0.8]$ | $[-0.8,1.4]$ | $[-1.2,1.8]$ |
| $\mathcal{C}_{10^{\prime}}^{\mathrm{NP}}$ | $[-\mathbf{0 . 4 , 0 . 4 ]}$ | $[-1.0,0.8]$ | $[-1.4,1.2]$ |

Table: $68.3 \%(1 \sigma), 95.5 \%(2 \sigma)$ and $99.7 \%(3 \sigma)$ confidence intervals for the NP contributions to WC.

In conclusion our pattern of [PRD88 (2013) 074002] obtained from an $\mathcal{H}_{\text {eff }}$ approach is

$$
\mathrm{C}_{9}^{N P} \sim[-1.6,-0.9], \quad \mathrm{C}_{7}^{N P} \sim[-0.05,-0.01], \quad \mathrm{C}_{9}^{\prime} \sim \pm \delta \quad \mathbf{C}_{10}, \mathrm{C}_{7,10}^{\prime} \sim \pm \epsilon
$$

where $\delta$ is small and $\epsilon$ is smaller.

Other groups later on confirmed independently the same finding of $C_{9}^{N P}<0$ :

- different observables $S_{i}\left([1,6]\right.$ bins and low recoil from $\left.B^{+} \rightarrow K^{+} \mu^{+} \mu^{-}\right)$, other techniques (lattice) and statistical approaches (bayesian)


However, all those groups also claimed $C_{9}^{N P}+C_{9}^{\prime} \simeq 0 \Rightarrow C_{9}^{\prime}=-C_{9}^{N P}$, i.e., POSITIVE $\Rightarrow$ based mainly on $1 \mathrm{fb}^{-1}$ data at on $B^{-} \rightarrow K^{-} \mu^{+} \mu^{-}$

## BUT

We showed in [1307.5683] that:

- 3 rd bin of $P_{5}^{\prime}$ prefers clearly a $C_{9}^{\prime}$ NEGATIVE, i.e., $C_{9}^{N P}+C_{9}^{\prime}<0$.


There was TENSION between $B \rightarrow K^{*} \mu^{+} \mu^{-}$data and $B^{-} \rightarrow K^{-} \mu \mu\left(\right.$ not with $\left.B^{0} \rightarrow K^{0} \mu^{+} \mu^{-}\right)$

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... till the new $3 \mathrm{fb}^{-1}$ data from LHCB on $B^{+} \rightarrow K^{+} \mu^{+} \mu^{-}$CAME OUT

New $3 \mathbf{f b}^{-1}$ data shows excellent consistency between anomaly in $P_{5}^{\prime}\left(B \rightarrow K^{*}\right)$ and $B \rightarrow K$ modes:



CONFIRMS a deficit in the $3 \mathrm{fb}^{-1} B^{+} \rightarrow K^{+} \mu^{+} \mu^{-}$and $B^{0} \rightarrow K^{0} \mu^{+} \mu^{-}$data


Confirms $C_{9}^{N P}+C_{9}^{\prime}<0$ from D.M.V. 1307.5683


$$
C_{9}^{N P}+C_{9}^{\prime} \simeq 0
$$

Independent cross-check (Wingate) from lattice low-recoil.

# Possible Explanations of the Anomaly 

## and

## Updated SM predictions

- Factorizable or non-factorizable power corrections?
$\rightarrow$ under control
- Effect from charm resonances? [Lyon,Zwicky] versus [Khodjamirian, Mannel, Pivovarov, Wang] KMPW says positive contribution to $C_{9}^{\text {eff }}$
Controversial LZ says negative (easy to test by checking other observables, i.e, $P_{1}$ )
- Statistical fluctuation of data?
$\rightarrow$ perform consistency checks [Matias,Serra]
$\Rightarrow$ New physics explanation within a 'model"
- Possible model: $Z^{\prime}$ respecting $\Delta M$ constrain. [Descotes,JM,Virto'13]
- $R_{K}$ deficit: Consistent with $C_{9}^{N P \mu}=-1.5$ but with Universal LFV.

General idea: (Jäger,Camalich): Parametrize power corrections to form factors:

$$
F\left(q^{2}\right)=F^{\text {soft }}\left(\xi_{\perp, \|}\left(q^{2}\right)\right)+\Delta F^{\alpha_{s}}\left(q^{2}\right)+a_{F}+b_{F} \frac{q^{2}}{m_{B}^{2}}+\ldots
$$

$\Rightarrow$ fit $a_{F}, b_{F}, \ldots$ to the full form factor $F$ (taken e.g. from LCSR)
BUT two CRUCIAL POINTS not to miss:
I. Power corrections are constrained from

- exact kinematic FF relations at $q^{2}=0$. Example $a_{T 1}=a_{T 2}$ from $T_{1}(0)=T_{2}(0)$
- definition of input scheme to fix $\xi_{\perp, \|}$. Example $a_{A 2}=\frac{m_{B}+m_{K_{*}}}{m_{B}-m_{K *}} a_{A 1}$ from $\xi_{\|} \equiv c_{1} A_{1}\left(q^{2}\right)+c_{2} A_{2}\left(q^{2}\right)$
$\Rightarrow$ Correlations among $a_{F_{i}}, b_{F_{i}}, \ldots$ that cannot be VIOLATED.
II. Freedom to choose the most appropriate scheme to reduce the impact of power corrections:
- input: $\left\{T_{1}, A_{0}\right\}$ to define $\left\{\xi_{\perp}, \xi_{\|}\right\} \Rightarrow$ power corrections eliminated in $T_{1}$ and $A_{0}$
- our input: $\left\{V, c_{1} A_{1}+c_{2} A_{2}\right\} \Rightarrow$ power corrections eliminated in $V$ and minimized in $A_{1}, A_{2}$

Philosophy of [Jäger\& Camalich'12 and '14]: No Form Factor computation (LCSR, DSE,...) is trustable $\Rightarrow$ For this reason they need to focus on observables less sensitive to FF like the $P_{i}$ and they do not give predictions for the $S_{i}$ (in any paper), because with their approach the errors on the $S_{i}$ would be huge.

We disagree with this point of view: good to reduce dependence on FF but up to a compromise.

## Jaeger-Camalich 2012

- $a_{F}, b_{F}$ and $\Delta a_{F}, \Delta b_{F}$ estimated from average of central values of different FF parametrizations:
$\Rightarrow$ Lost fundamental correlations
$\Rightarrow$ Central values of $P_{i}$ from SFF
- Definition of $\xi_{\perp, \|}$ from $T_{1}, A_{0}$ : Non-optimal scheme chosen $\times 2$ errors size. ( $P_{i}$ indep. of $A_{0}$ )
- $q^{2}$-dependence for $\xi_{\perp, \|}$ : old HQET limit prediction, $\Rightarrow$ Transfer known info artificially inflated unknown power corrections.
- Identification $\xi_{\perp}(0)=T_{1}^{\text {exp }}(0)$ from $B \rightarrow K^{*} \gamma$ assumes $S M$, and inconsistently includes non-factorizable PC inside $T_{1}$.
- ALL Form Factors in helicity basis.
- only $P_{i}$ considered.


## Our paper JHEP12(2014)125

- Work consistently within one FF parametrisation at a time (KMPW, BZ) compute $a_{F}, b_{F}$. $\Rightarrow$ Respect correlations:
(central values and errors)
$\Rightarrow$ Central values of $P_{i}$ from
SFF + PC reproduce exactly FF.
- $\Delta a_{F}, \Delta b_{F}=\mathcal{O}\left(\Lambda / m_{B}\right) \times F$
- Definition of $\xi_{\perp, \|}$ from $V, A_{1}+A_{2}$ like Beneke et al.: choose the most appropriate scheme.
- $\mathrm{q}^{2}$-dependence of $\xi_{\perp, \|}$ : $\frac{\xi_{i}(0) m_{F}^{2}}{m_{F}^{2}-s}\left(1+b_{F}\left[z\left(s, \tau_{0}\right)-z\left(0, \tau_{0}\right)\right]+\ldots\right.$
- We do a flat scan of power correction parameters and provide each error separately.
- We include non-factorizable PC.
- ALL Form Factors always consistently in Transversity Basis.


## Jaeger-Camalich 2014

- Soft FF are undervalutated:
$\xi_{\perp}(0)=0.31 \pm 0.04$
meaning of this error unclear!:
Average of LCSR ONLY c.v.!!!
$\xi_{\perp}(0)=0.31_{-0.10}^{+0.20}($ our KMPW)
$\Rightarrow F_{L}$ error smaller than us!
$\Rightarrow$ Central values of $P_{i}$ from SFF
- $\Delta a_{F}, \Delta b_{F}=10 \% \times \xi_{\perp, \|}(0)$
(our same approach) BUT some Helicity FF: $T_{+}, V_{+} \simeq 0$
- Definition of $\xi_{\perp, \|}$ :
- Still BAD scheme used $\times 2$
- Wrong: our scheme is $\xi_{\perp}\left(q^{2}\right) \propto V\left(q^{2}\right)$ not $V_{-}\left(q^{2}\right)!!$ $\Rightarrow P_{5}^{\prime}$ IS scheme dependent
- They do also flat scan but do not provide errors that are added linearly.
- ALL Form Factors in helicity basis.
- only $P_{i}$ considered.

It is a well known fact in QFT the problem of scheme dependence and
$\rightarrow$ the convenience to choose the most appropriate scheme.

- one should choose the renormalisation scheme in such a way that effects of unknown power corrections get absorbed as much as possible into the soft form factors (input parameters taken from LCSR calculations or from experiment.)
$\rightarrow$ complete analogy to the case of perturbative loop calculations.
- one can always construct a scheme that artificially blows up uncertainties from power corrections: Consider an observable depending on only one single form factor.
- good scheme: Take this FF directly as input and power corrections would not appear at all.
- bad scheme: Instead one could choose a scheme where this FF is related to a different input parameter up to unknown power corrections, but obviously this increases the uncertainty of the result artificially.

In summary: In the $P_{5}^{\prime}$ case the combination of a bad scheme choice to define $\xi_{\perp, \|}$ together with a change of FF basis from transversity (where they are computed) to helicity (J.\&C choice) blow up factorizable power correction errors (x 3-5)

Jaeger\&Camalich'14: $\mathrm{S}_{5}^{[1,6]}=-0.13_{-0.19}^{+0.22}$ (only error from $P_{5}^{\prime}$ ): They added errors linearly. (but $\xi_{\perp}(0)$ is clearly undervalutated so the error is possibly larger)

## On the contrary, two very different methods gets very good agreement:

Our computation'14: Model-independent (applicable to different LCSR), dimens. arguments for p.c.

$$
\mathrm{S}_{5}^{[1,6]}=-0.18_{-0.06-0.05}^{+0.05+0.05} \text { CASE BZ par. (cv. use of } m_{c}^{M S} \text { or } m_{c}^{\text {pole }} \text { ) }
$$

Errors: Param+Hadronic+ Factorizable p.c. + non-factorizable p.c. + charm-loop effects: Flat scan p.c.
Altmannshofer\&Straub'13: Full form factors with correlations using BZ (factorizable p.c. included)

$$
\mathrm{S}_{5}^{[1,6]}=-0.14 \pm 0.02 \text { (non-factorizable p.c. }+ \text { charm not included) }
$$

Error gaussian to flat scan $\times 2$ approx. $\rightarrow+0.04$ (good agreement with our +0.05 )
$\rightarrow$ The error in J\&C +0.22 based on an estimated of p.c. is $>200 \%$ larger when compared to us. Bad scheme used in J\&C induced a factor of 2 in some bins.

Besides some FF errors in J\&C like $V_{+}$has duplicate error size from 2012 to 2014? and no complete set of FF are presented in 2014 to compare with 2012.

## Non-factorizable contributions and charm-loop effects

We add to this:

- non-factorizable power corrections: power corrections that are not part of form factors
$\Rightarrow$ We single out the pieces not associated to FF $\mathcal{T}_{i}^{\text {had }}=\left.\mathcal{T}_{i}\right|_{C_{7}^{(1)} \rightarrow 0}$ entering $\left\langle K^{*} \gamma^{*}\right| H_{\text {eff }}|B\rangle$ and multiply each of them with a complex $q^{2}$-dependent factor:

$$
\mathcal{T}_{i}^{\text {had }} \rightarrow\left(1+r_{i}\left(q^{2}\right)\right) \mathcal{T}_{i}^{\text {had }},
$$

with

$$
r_{i}(s)=r_{i}^{a} e^{i \phi_{i}^{a}}+r_{i}^{b} e^{i \phi_{i}^{b}}\left(s / m_{B}^{2}\right)+r_{i}^{c} e^{i \phi_{i}^{c}}\left(s / m_{B}^{2}\right)^{2} .
$$

$r_{i}^{a, b, c} \in[0,0.1]$ and $\phi_{i}^{a, b, c} \in[-\pi, \pi]$ : random scan and take the maximum deviation from the central values $r_{i}\left(q^{2}\right) \equiv 0$ to each side, to obtain asymmetric error bars.

(a)

(c)

(b)

(d)

Charm loop: Insertion of 4-quark operators $\left(\mathcal{O}_{1,2}^{c}\right)$ or penguin operators $\left(\mathcal{O}_{3-6}\right)$ induces a positive contribution in $C_{9}^{\text {eff }}$. - We followed LCSR computation and prescription from KMPW to recast the effect inside $C_{9}^{\text {eff }}$.

$$
\mathcal{C}_{9} \rightarrow \mathcal{C}_{9}+s_{i} \delta C_{9}^{K M P W}\left(q^{2}\right)
$$

even if KMPW says $s_{i}=1$, we allow $s_{i}$ in a range $[-1,1]$.

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In [Lyon,Zwicky'14] a $350 \%$ "correction" to the FA to explain the anomaly in $P_{5}^{\prime}$ instead of NP.

- Many model-dependent assumptions: resonance model extrapolated far from resonances, constant fudge factors $\eta_{c}, \eta_{c}^{\prime}$ are valid everywhere?

$$
C_{9}^{\text {eff }}=C_{9}+\eta_{c} h_{c}\left(q^{2}\right)+h_{\text {rest }}\left(q^{2}\right) \quad C_{9}^{\prime \text { eff }}=C_{9}^{\prime}+\eta_{c}^{\prime} h_{c}\left(q^{2}\right)
$$

same for $B \rightarrow K \mu \mu$ than for $B \rightarrow K^{*} \mu \mu$ ? can a $350 \%$ correction be accommodated within QCD? constraints on new $\bar{b} s c \bar{c}$ structures??
We propose different tests to disprove it:

- The proposal should survive a global analysis of all $P_{i}$. Indeed NONE of the illustrative examples selected works for all observables in all bins, either fail for some bin of $P_{2}$ and/or $P_{5}^{\prime}$.
- $B^{+} \rightarrow \pi^{+} \mu^{+} \mu^{-}: b \rightarrow d$ transition assume no NP. Similar charm contribution with few changes ( $1-\frac{R_{b}}{R_{t}} e^{i \alpha}$ ) prefactor infront of charm loop and presence of annihilation contributions.

$$
\text { At } 8 \mathrm{GeV}^{2} \quad\left|C_{9}^{+}\right|^{2} \sim 32.1 \text { with } \eta_{c}+\eta_{c}^{\prime}=1(F A) \quad\left|C_{9}^{+}\right|^{2} \sim 2.5 \text { with } \eta_{c}+\eta_{c}^{\prime}=-2.5(L Z)
$$

where $C_{9}^{+}=C_{9}^{\text {eff }}+C_{9}^{\text {eff }}$.
$\Rightarrow$ Test: If no suppression is seen in the measured BR w.r.t. SM the L\&Z proposal is in trouble. However one can play with the phase to pass the test, assuming a huge $\operatorname{SU}(3)$ breaking.

- Finally if $R_{K}$ deviation is confirmed increasing its significance the proposed charm pollution cannot explain it while on the contrary our pattern [see D. Ghosh et al.'14] can make it. This is probably one of the clearest discriminating method.


## Our final Predictions in SM [1407.8526].

The most complete prediction including all errors in KMPW parametrization for the relevant observables. Errors included: parametric, FF, factorizable and non-factorizable p.c. NOT charm loops.





Blue prediction in scheme $2\left(T_{1}, A_{0}\right)$. (see 1407.8526 for BZ and more observables). Summary: Power corrections cannot be the explanation of anomaly

The most complete prediction including all errors in KMPW parametrization for the relevant observables. Errors added in quadrature: parametric, FF, factorizable and non-factorizable p.c. including charm loops.





Orange band is all errors except charm. Green band is charm loop.

- Number of symmetries of S -wave and P -wave part is 4 (same as P -wave).
- Number of free parameters (observables)

$$
2 n_{\text {Amplitudes }}-n_{\text {symmetries }}=2(6+2)-4=12 \text { observables }
$$

8 P-wave observables and 4 S -wave observables. BUT S-wave part has 6 parameters:

$$
\begin{aligned}
\frac{\mathbf{W}_{S}}{\Gamma_{\text {full }}^{\prime}}= & \frac{3}{16 \pi}\left[\mathbf{F}_{\mathrm{S}} \sin ^{2} \theta_{\ell}+\mathbf{A}_{\mathbf{S}} \sin ^{2} \theta_{\ell} \cos \theta_{K}+\mathbf{A}_{\mathrm{S}}^{4} \sin \theta_{K} \sin 2 \theta_{\ell} \cos \phi\right. \\
& \left.+\mathbf{A}_{\mathrm{S}}^{5} \sin \theta_{K} \sin \theta_{\ell} \cos \phi+\mathbf{A}_{S}^{7} \sin \theta_{K} \sin \theta_{\ell} \sin \phi+\mathbf{A}_{\mathrm{S}}^{8} \sin \theta_{K} \sin 2 \theta_{\ell} \sin \phi\right]
\end{aligned}
$$

Only 4 parameters out of $F_{S}, A_{S}, A_{S}^{4,5,7,8}$ are independent!!! Two new constraints [L. Hofer, J.M'15]:

$$
\begin{aligned}
\bar{k}_{S} F_{T}\left[\bar{k}_{2}^{2}-\bar{P}_{1}^{2}-4 \bar{P}_{2}^{2}-4 \bar{P}_{3}^{2}\right]= & -\frac{8}{3} \bar{P}_{2}\left[\bar{A}_{S}^{4} \bar{A}_{S}^{5}+\bar{A}_{S}^{7} \bar{A}_{S}^{8}\right]+\frac{4}{3} \bar{P}_{3}\left[\bar{A}_{S}^{5} \bar{A}_{S}^{7}-4 \bar{A}_{S}^{4} \bar{A}_{S}^{8}\right] \\
& +\frac{1}{3}\left(\bar{k}_{2}+\bar{P}_{1}\right)\left[4\left(\bar{A}_{S}^{4}\right)^{2}+\left(\bar{A}_{S}^{7}\right)^{2}\right]+\frac{1}{3}\left(\bar{k}_{2}-\bar{P}_{1}\right)\left[\left(\bar{A}_{S}^{5}\right)^{2}+4\left(\bar{A}_{S}^{8}\right)^{2}\right], \\
\bar{A}_{S} \sqrt{\frac{F_{T}}{1-F_{T}}}\left[\bar{k}_{2}^{2}-\bar{P}_{1}^{2}-4 \bar{P}_{2}^{2}-4 \bar{P}_{3}^{2}\right]= & -4 \bar{P}_{2}\left[\bar{P}_{4}^{\prime} \bar{A}_{S}^{5}+2 \bar{P}_{5}^{\prime} \bar{A}_{S}^{4}-2 \bar{P}_{6}^{\prime} \bar{A}_{S}^{8}-\bar{P}_{8}^{\prime} \bar{A}_{S}^{7}\right] \\
& +4 \bar{P}_{3}\left[\bar{P}_{5}^{\prime} \bar{A}_{S}^{7}-\bar{P}_{6}^{\prime} \bar{A}_{S}^{5}-2 \bar{P}_{4}^{\prime} \bar{A}_{S}^{8}+2 \bar{P}_{8}^{\prime} \bar{A}_{S}^{4}\right] \\
& +2\left(\bar{k}_{2}+\bar{P}_{1}\right)\left[2 \bar{P}_{4}^{\prime} \bar{A}_{S}^{4}-\bar{P}_{6}^{\prime} \bar{A}_{S}^{7}\right]+2\left(\bar{k}_{2}-\bar{P}_{1}\right)\left[\bar{P}_{5}^{\prime} \bar{A}_{S}^{5}-2 \bar{P}_{8}^{\prime} \bar{A}_{S}^{8}\right] .
\end{aligned}
$$

where $\bar{k}_{2}=1+F_{T}^{C P} / F_{T}, \bar{k}_{S}=1+F_{S}^{C P} / F_{S}$ and $\bar{P}_{i}=P_{i}+P_{i}^{C P}, \bar{A}_{S}^{i}=\left(A_{S}^{i}+A_{S}^{i C P}\right) / \sqrt{F_{S}\left(1-F_{S}\right)}$

Consequences:

- 1st quadratic equation $\bar{A}_{S}^{5}=f\left(\bar{A}_{S}^{4}, \bar{A}_{S}^{7}, \bar{A}_{S}^{8}, \bar{P}_{1,2,3}, F_{T}\right)$
- 2on linear equation $\bar{A}_{S}=g\left(\bar{A}_{S}^{4}, \bar{A}_{S}^{5}, \bar{A}_{S}^{7}, \bar{A}_{S}^{8}, \bar{P}_{1,2,3}, \bar{P}_{4,5,6,8}^{\prime}, F_{T}\right)$

One obtains immediately the constraints:

$$
\begin{array}{lr}
\left|\bar{A}_{S}^{4}\right| \leq \frac{1}{2} \sqrt{3 \bar{k}_{S} F_{T}\left(\bar{k}_{2}-\bar{P}_{1}\right)}, & \left|\bar{A}_{S}^{5}\right| \leq \sqrt{3 \bar{k}_{S} F_{T}\left(\bar{k}_{2}+\bar{P}_{1}\right)}, \\
\left|\bar{A}_{S}^{7}\right| \leq \sqrt{3 \bar{k}_{S} F_{T}\left(\bar{k}_{2}-\bar{P}_{1}\right)}, & \left|\bar{A}_{S}^{8}\right| \leq \frac{1}{2} \sqrt{3 \bar{k}_{S} F_{T}\left(\bar{k}_{2}+\bar{P}_{1}\right) .}
\end{array}
$$

More interestingly at the maximum of $P_{2}$ namely $\mathbf{q}_{1}^{2}$ (taken no NP phases $O^{C P} \sim 0$ and $P_{3} \sim 0$ ):

$$
A_{S}^{4}\left(\mathbf{q}_{1}^{2}\right)=\frac{1}{2} A_{S}^{5}\left(\mathbf{q}_{1}^{2}\right) \quad \text { and } \quad A_{S}^{7}\left(\mathbf{q}_{1}^{2}\right)=2 A_{S}^{8}\left(\mathbf{q}_{1}^{2}\right)
$$

And at the zero of $P_{2}$ namely $\mathbf{q}_{0}^{2}$ two relations are fulfilled (under same hypothesis and $P_{6,8} \sim 0$ ):

$$
\begin{gathered}
{\left[\left(4 A_{S}^{42}+A_{S}^{72}\right)\left(1+P_{1}\right)+\left(A_{S}^{52}+4 A_{S}^{82}\right)\left(1-P_{1}\right)\right]_{\mathrm{q}_{0}^{2}}=3\left[\left(1-F_{S}\right) F_{S} F_{T}\left(1-P_{1}^{2}\right)\right]_{\mathrm{q}_{0}^{2}}} \\
A_{S}\left(\mathbf{q}_{0}^{2}\right)=\left[\frac{2 F_{L}\left(2 A_{S}^{4}\left(1+P_{1}\right) P_{4}^{\prime}+A_{S}^{5}\left(1-P_{1}\right) P_{5}^{\prime}\right)}{\left.\sqrt{F_{L} F_{T}\left(1-P_{1}^{2}\right)}\right]_{\mathbf{q}_{0}^{2}}}\right.
\end{gathered}
$$

From the symmetries of the distribution in absence of scalars [JM, N. Serra'14]

$$
\begin{gathered}
\bar{P}_{2}=+\frac{1}{2 \bar{k}_{1}}\left[\left(\bar{P}_{4}^{\prime} \bar{P}_{5}^{\prime}+\delta_{1}\right)+\frac{1}{\beta} \sqrt{\left(-1+\bar{P}_{1}+\bar{P}_{4}^{\prime 2}\right)\left(-1-\bar{P}_{1}+\beta^{2} \bar{P}_{5}^{\prime 2}\right)+\delta_{2}+\delta_{3} \bar{P}_{1}+\delta_{4} \bar{P}_{1}^{2}}\right] \\
\text { where } \bar{P}_{i}=P_{i}+P_{i}^{C P} \quad \beta=\sqrt{1-4 m_{\ell}^{2} / s}
\end{gathered}
$$

Assuming NP is real in WC it is an excellent approximation $\delta_{i} \sim\left(\operatorname{Im} A_{i}\right)^{2} \rightarrow 0, P_{i}^{C P} \rightarrow 0$.


- At the zero of $P_{2}$ called $q_{0}^{2}$

$$
P_{4}^{\prime 2}\left(q_{0}^{2}\right)+\beta^{2} P_{5}^{\prime 2}\left(q_{0}^{2}\right)=1+\eta\left(q_{0}^{2}\right)
$$

where $\eta\left(q_{0}^{2}\right) \rightarrow 0$ if $P_{1} \rightarrow 0$

- with $\eta=0$ if not fulfilled this equation is a test of presence of RHC.
- with $\eta$ included this equation establishes a relation between the zero of $A_{F B}$ and the anomaly in $P_{5}^{\prime}$
- At the maximum of $P_{2}$ called $q_{1}^{2}$

$$
P_{4}^{\prime}\left(q_{1}^{2}\right)=\beta P_{5}^{\prime}\left(q_{1}^{2}\right)
$$


** KMPW in BZ: $0.16 \pm 0.12$.

This bin is as interesting/important as the third bin of $P_{5}^{\prime}$. It contains three important infos:

- If $3 \mathrm{fb}^{-1}$ data confirms saturation $\Rightarrow$ shift of maximum of $P_{2}$ from $q_{1}^{2 S M}=2 \mathrm{GeV}^{2}$.
- At LO the position of the maximum (free from SFF) is:

$$
q_{1}^{2}=\frac{2 m_{b} M_{B} C_{7}^{\text {eff }}}{C_{10}-C_{9}^{\text {eff }}\left(q_{1}^{2}\right)}
$$

with $C_{7}^{\text {eff } \prime}=C_{9}^{\prime}=C_{10}^{\prime}=0$ and $P_{2}^{\max }\left(q_{1}^{2}\right)=1 / 2$

- We have established a new link between:

Maximum of $P_{2}$ and presence of RH currents:

$$
P_{2}^{\max }=1 / 2 \Rightarrow \text { NO RH currents }
$$

Intuitively,
At the maximum of $P_{2} \Rightarrow\left|n_{\perp}\right| \simeq\left|n_{\|}\right| \Rightarrow P_{1} \simeq 0$

- We proposed in [PRD88(2013)074002] a simple "model" a $\mathbf{Z}^{\prime}$ gauge boson contributing to $\mathcal{O}_{9}=e^{2} /\left(16 \pi^{2}\right)\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \ell\right)$ with couplings:

$\mathcal{L}^{q}=\left(\bar{s} \gamma_{\nu} P_{L} b \Delta_{L}^{s b}+\bar{s} \gamma_{\nu} P_{R} b \Delta_{R}^{s b}+\right.$ h.c. $) Z^{\prime \nu} \quad \mathcal{L}^{l e p}=\left(\bar{\mu} \gamma_{\nu} P_{L} \mu \Delta_{L}^{\mu \bar{\mu}}+\bar{\mu} \gamma_{\nu} P_{R} \mu \Delta_{R}^{\mu \bar{\mu}}+\ldots\right) Z^{\prime \nu}$
- $\Delta_{R}^{s b} \sim 0$ and $\Delta_{L}^{s b}$ with same phase as $V_{t b} V_{t s}^{*}$ (to avoid $\phi_{s}$ ), $\Delta_{L}^{\mu \mu}=\Delta_{R}^{\mu \mu}$ (to keep $C_{10}^{N P} \sim 0$ ).
- The model would contribute to $\Delta m_{S}$ ( $\Delta_{R}^{s b} \sim 0$ kills the largest contribution) bound on $\Delta_{L}^{s b}$.
- Considering the constraints from [Buras, de Fazio, Girrbach] our $Z^{\prime}$ with $M_{z}^{\prime}=1 \mathrm{TeV}$ (compatible with $\Delta m_{s}$ ) and couplings to muons of at least order 0.1-0.2 would yield $C_{9}^{N P} \sim \mathcal{O}(-1)$.
- Recent analysis on $R_{K}$ from [D. Ghosh, M. Nardecchia, S.A. Renner'14] points that our NP solution also works for $R_{K}$ with NP in muons and not electrons. Also our second scenario with NP in $C_{9}^{N P \mu}$ and $C_{9}^{\prime \mu}$ NEGATIVE is preferred.
Particular embeddings of a $Z^{\prime}$ inside models discussed by [R. Gauld et al'13, W. Altmannshofer et al.' 14 ].
- Our analysis of the LHCb data on $B \rightarrow K^{*} \mu^{+} \mu^{-}$based on the clean observables $P_{i}^{(\prime)}$ together with a set of radiative data shows the following pattern:

$$
\mathrm{C}_{9}^{N P} \sim[-1.6,-0.9], \quad \mathrm{C}_{7}^{\mathrm{NP}} \sim[-0.05,-0.01], \quad \mathrm{C}_{9}^{\prime} \sim \pm \delta \quad \mathrm{C}_{10}, \mathrm{C}_{7,10}^{\prime} \sim \pm \epsilon
$$

with $\delta$ and $\epsilon$ small.

- New $3 \mathrm{fb}^{-1}$ data on $B^{-} \rightarrow K^{-} \mu^{+} \mu^{-}$and $B^{0} \rightarrow K^{0} \mu^{+} \mu^{-}$confirms this pattern.
- Possible alternative explanations to NP to explain the anomaly: power corrections are indeed under control and huge charm loop effects can be easily tested.
- Using the symmetries of the distribution on the P and S -wave we found: a) the S -wave parameters are not independent, b) a connection between the zero of $A_{F B}$ and the anomaly in $P_{5}^{\prime}, \mathrm{c}$ ) we have established a new link between the value of the maximum of $P_{2}$ and the presence of RH currents.
- A simple model with a $Z^{\prime}$ can possibly explain the deviations observed. But we should wait for $3 \mathrm{fb}^{-1}$ data on $B \rightarrow K^{*} \mu^{+} \mu^{-}$to come soon.


## Back-up slides:

## The folding technique. S-wave pollution

## PROPOSAL for an ALTERNATIVE way to approach the full fit angular distribution

## Full fit of the angular distribution with a small dataset

Under the assumption of ABSENCE of NP: no new scalars and real Wilson coefficients one has

- Free parameters $F_{L}, P_{1}, P_{4,5}^{\prime}$.
- $P_{2}$ is a function of the other observables and $P_{6,8}^{\prime}$ are set to zero.


Figure: Residual distribution of $P_{5}^{\prime}$ when fitting with 100 events. The fit of a gaussian distribution is superimosed.

We find testing this fit for values around the measured values: convergence and unbiased pulls with as little as 50 events per bin. Gaussian pulls are obtained with only 100 events.

This opens the possibility to perform a full angular fit analysis with small bins in $q^{2}$

The main hypothesis (real WC) can be tested measuring $P_{i}^{C P}$.

Independent cross check from "Lattice": M. Wingate (private communication and preliminary result)
$\Rightarrow$ confirming our result with $C_{9}^{N P}+C_{9}^{\prime} \sim-1$







## The Folding Technique

- Full angular distribution: Difficult it requires more data. Possible way using symmetries N.Serra, JM'14.
- Uniangular distributions: - Integrates out the interesting observables • S-wave polluted in a bad way. JM'12.
- Breakthrough at LHCb: Substitute uniangular distributions $\rightarrow$ folded distributions.

A prototypical example: The identification of $\phi \leftrightarrow \phi+\pi$ (for $\phi<0$ ) produces a "folded" angle $\hat{\phi} \in[0, \pi]$ with $\theta_{K}, \theta_{\ell} \in[0, \pi]$ in terms of which a (folded) differential rate $d \hat{\Gamma}(\hat{\phi})=d \Gamma(\hat{\phi})+d \Gamma(\hat{\phi}-\pi)$ is:

$$
\begin{aligned}
& \frac{1}{\Gamma_{\text {full }}} \frac{d^{4} \Gamma}{d q^{2} d \cos \theta_{K} d \cos \theta_{l} d \hat{\phi}}=\frac{9}{16 \pi}\left[2 \mathrm{~F}_{\mathrm{L}} \cos ^{2} \theta_{K} \sin ^{2} \theta_{\ell}+\frac{1}{4} \mathbf{F}_{\mathrm{T}} \sin ^{2} \theta_{K}\left(3+\cos 2 \theta_{\ell}\right)\right. \\
& \left.\quad+\frac{1}{2} \mathrm{P}_{1} \mathbf{F}_{\mathrm{T}} \sin ^{2} \theta_{K} \sin ^{2} \theta_{\ell} \cos 2 \hat{\phi}+2 \mathrm{P}_{2} \mathbf{F}_{\mathrm{T}} \sin ^{2} \theta_{K} \cos \theta_{\ell}-\mathrm{P}_{3} \mathbf{F}_{\mathrm{T}} \sin ^{2} \theta_{K} \sin ^{2} \theta_{\ell} \sin 2 \hat{\phi}\right]\left(\mathbf{1}-\mathbf{F}_{\mathrm{S}}\right)+\frac{\mathbf{W}_{1}}{\Gamma_{\text {full }}}
\end{aligned}
$$

where the S-wave piece is

$$
\delta_{\mathrm{sw}}^{(1)}=\frac{\mathbf{W}_{1}}{\boldsymbol{\Gamma}_{\text {full }}}=\frac{3}{8 \pi}\left(\mathbf{F}_{\mathrm{S}}+\mathbf{A}_{\mathrm{S}} \cos \hat{\theta}_{K}\right) \sin ^{2} \theta_{\ell}
$$

This folded distribution is used to determine $P_{1,2,3}$. Generalization with lepton masses in [JM'12].

## Advantages of folding:

- It reduces the \# of coefficients (observables) to a manageable experimentally subset.

In this case: $11 \mathrm{~J}+8 \tilde{\jmath} \rightarrow 7 \mathrm{~J}+4 \tilde{\jmath}$

- It helps to disentangle the unwanted S-wave pollution due to its distinct angular dependence.
- An important remark is that at LHCb $\mathrm{P}_{1}$ is obtained in a folding in association with $P_{2,3}$. But $P_{1}\left(=A_{T}^{2}\right)$ who is called to play a relevant role in determining the presence of RH currents in Nature ( $C_{7,9,10}^{\prime}$ ) has large error bars.


## We propose 3 foldings (second, third and fourth in the list) that can disentangle $P_{1}$ from $P_{2,3}$.

| Obs. | S-wave | Folding | $\hat{\phi}$ range |
| :---: | :---: | :---: | :---: |
| $P_{1,2,3}$ | $A_{s}$ | $d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}-\pi, \hat{\theta}_{l}, \hat{\theta}_{K}\right)$ | $[0, \pi]$ |
| $P_{1}$ | $A_{s 5}, A_{s 8}$ | $d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \pi-\hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \pi-\hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)$ | $[0, \pi]$ |
| $P_{1}$ and $P_{2}$ | $A_{s 4}, A_{s 5}$ | $d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)$ | $[0, \pi]$ |
| $P_{1}$ and $P_{3}$ | $A_{s 5}, A_{s 7}$ | $d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \pi-\hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \pi-\hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)$ | $[0, \pi]$ |
| $P_{1}$ and $P_{4}^{\prime}$ | $A_{s 5}$ | $d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \pi-\hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \pi-\hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)$ | $[0, \pi]$ |
| $P_{1}$ and $P_{5}^{\prime}$ | $A_{s}, A_{s 5}$ | $d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \pi-\hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(-\hat{\phi}, \pi-\hat{\theta}_{l}, \hat{\theta}_{K}\right)$ | $[0, \pi]$ |
| $P_{1}$ and $P_{6}^{\prime}$ | $A_{s}, A_{s 7}$ | $d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\pi-\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \pi-\hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\pi-\hat{\phi}, \pi-\hat{\theta}_{l}, \hat{\theta}_{K}\right)$ | $[-\pi / 2, \pi / 2]$ |
| $P_{1}$ and $P_{8}^{\prime}$ | $A_{s 7}$ | $d \Gamma\left(\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\pi-\hat{\phi}, \hat{\theta}_{l}, \hat{\theta}_{K}\right)+d \Gamma\left(\hat{\phi}, \pi-\hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)+d \Gamma\left(\pi-\hat{\phi}, \pi-\hat{\theta}_{l}, \pi-\hat{\theta}_{K}\right)$ | $[-\pi / 2, \pi / 2]$ |

Table: Foldings needed to single out the interesting observables, with the corresponding remaining S-wave pollution. For all foldings, $\hat{\theta}_{\ell}$ and $\hat{\theta}_{K}$ lie within $[0, \pi / 2]$, whereas $\hat{\phi}$ has different ranges depending on the observables considered.

## S-wave pollution

- Another possible source of uncertainty is the $\mathbf{S}$-wave contribution coming from $B \rightarrow K_{0}^{*} I^{+} I^{-}$. [Becirevic, Tayduganov '13], [Blake et al.'13]
- We will assume that both $P$ and $S$ waves are described by $q^{2}$-dependent FF times a Breit-Wigner function.
- The distinct angular dependence of the S -wave terms in folded distributions allow to disentangle the signal of the P -wave from the S -wave: $P_{i}^{(\prime)}$ can be disentangled from $S$-wave pollution [JM'12].
Problem: Changing the normalization used for the distribution from

$$
\frac{d \Gamma_{K}^{*}}{d q^{2}} \equiv \Gamma_{K^{*}}^{\prime} \rightarrow \Gamma_{\text {full }}^{\prime}
$$

introduces a $\left(1-F_{S}\right)$ in front of the P -wave.

$$
\Gamma_{\text {full }}^{\prime}=\Gamma_{K^{*}}^{\prime}+\Gamma_{S}^{\prime}
$$

and the longitudinal polarization fraction associated to $\Gamma_{S}^{\prime}$ is

$$
\mathrm{F}_{\mathrm{S}}=\frac{\Gamma_{S}^{\prime}}{\Gamma_{\text {full }}^{\prime}} \quad \text { and } \quad 1-\mathrm{F}_{\mathrm{S}}=\frac{\Gamma_{K^{*}}^{\prime}}{\Gamma_{\text {full }}^{\prime}}
$$

The modified distribution including the S-wave and new normalization $\Gamma_{\text {full }}^{\prime}$ :

$$
\begin{aligned}
& \frac{1}{\Gamma_{\text {full }}^{\prime}} \frac{d^{4} \Gamma}{d q^{2} d \cos \theta_{K} d \cos \theta_{l} d \phi}=\frac{9}{32 \pi}\left[\frac{3}{4} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K}+\mathrm{F}_{\mathrm{L}} \cos ^{2} \theta_{K}\right. \\
& \quad+\left(\frac{1}{4} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K}-F_{L} \cos ^{2} \theta_{K}\right) \cos 2 \theta_{l}+\frac{1}{2} \mathrm{P}_{1} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K} \sin ^{2} \theta_{I} \cos 2 \phi \\
& \quad+\sqrt{\mathrm{F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}}\left(\frac{1}{2} \mathrm{P}_{4}^{\prime} \sin 2 \theta_{K} \sin 2 \theta_{I} \cos \phi+\mathrm{P}_{5}^{\prime} \sin 2 \theta_{K} \sin \theta_{I} \cos \phi\right) \\
& \quad-\sqrt{\mathrm{F}_{\mathrm{T}} \mathrm{~F}_{\mathrm{L}}}\left(\mathrm{P}_{6}^{\prime} \sin 2 \theta_{K} \sin \theta_{I} \sin \phi-\frac{1}{2} \mathrm{P}_{8}^{\prime} \sin 2 \theta_{K} \sin 2 \theta_{I} \sin \phi\right) \\
& \\
& \left.\quad+2 \mathrm{P}_{2} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K} \cos \theta_{l}-\mathrm{P}_{3} \mathrm{~F}_{\mathrm{T}} \sin ^{2} \theta_{K} \sin ^{2} \theta_{l} \sin 2 \phi\right]\left(1-\mathrm{F}_{\mathrm{S}}\right)+\frac{1}{\Gamma_{\text {full }}^{\prime}} W_{\mathrm{S}}
\end{aligned}
$$

in the massless case and where the polluting terms are

$$
\begin{aligned}
& \frac{\mathbf{W}_{S}}{\Gamma_{\text {full }}^{\prime}}=\frac{3}{16 \pi}\left[\mathbf{F}_{\mathrm{S}} \sin ^{2} \theta_{\ell}+\mathbf{A}_{\mathrm{S}} \sin ^{2} \theta_{\ell} \cos \theta_{K}+\mathbf{A}_{\mathrm{S}}^{4} \sin \theta_{K} \sin 2 \theta_{\ell} \cos \phi\right. \\
& \\
& \left.\quad+\mathbf{A}_{S}^{5} \sin \theta_{K} \sin \theta_{\ell} \cos \phi+\mathbf{A}_{\mathrm{S}}^{7} \sin \theta_{K} \sin \theta_{\ell} \sin \phi+\mathbf{A}_{S}^{8} \sin \theta_{K} \sin 2 \theta_{\ell} \sin \phi\right]
\end{aligned}
$$

We can get bounds on the size of the $S$-wave polluting terms.Let's take for instance $A_{S}$

$$
\mathbf{A}_{\mathbf{S}}=2 \sqrt{3} \frac{1}{\Gamma_{\text {full }}^{\prime}} \int \operatorname{Re}\left[\left(A_{0}^{\prime}{ }^{L} A_{0}^{L *}+A_{0}^{\prime R} A_{0}^{R *}\right) B W_{K_{0}^{*}}\left(m_{K \pi}^{2}\right) B W_{K^{*}}^{\dagger}\left(m_{K \pi}^{2}\right)\right] d m_{K \pi}^{2}
$$

where

$$
\mathrm{F}_{\mathrm{S}}=\frac{8}{3} \frac{\tilde{J}_{1 a}^{c}}{\Gamma_{\text {full }}^{\prime}}=\frac{\left|A_{0}^{\prime} L\right|^{2}+\left|A_{0}^{\prime} R\right|^{2}}{\Gamma_{\text {full }}^{\prime}} \mathbf{Y} \quad \mathbf{Y}=\int d m_{K \pi}^{2}\left|B W_{K_{0}^{*}}\left(m_{K \pi}^{2}\right)\right|^{2}
$$

$\mathbf{Y}$ factor included to take into account the width of scalar resonance $K_{0}^{*}$
A bound is obtained once we define the $S-P$ interference integral

$$
\mathbf{Z}=\int\left|B W_{K_{0}^{*}}\left(m_{K \pi}^{2}\right) B W_{K^{*}}^{\dagger}\left(m_{K \pi}^{2}\right)\right| d m_{K \pi}^{2}
$$

and use the bound from the Cauchy-Schwartz inequality

$$
\begin{gathered}
\left|\int(\mathrm{Re}, \mathrm{Im})\left[\left(A_{0}^{\prime}{ }^{L} A_{j}^{L *} \pm A_{0}^{\prime R} A_{j}^{R *}\right) B W_{K_{0}^{*}}\left(m_{K \pi}^{2}\right) B W_{K^{*}}^{\dagger}\left(m_{K \pi}^{2}\right)\right] d m_{K \pi}^{2}\right| \\
\leq \mathbf{Z} \times \sqrt{\left[\left|A_{0}^{\prime} L\right|^{2}+\left|A_{0}^{\prime R}\right|^{2}\right]\left[\left|A_{j}^{L}\right|^{2}+\left|A_{j}^{R}\right|^{2}\right]}
\end{gathered}
$$

From the definitions of $F_{S}$ and $F_{L}$ and $P_{1}$ one gets the following bound:

$$
\left|A_{S}\right| \leq 2 \sqrt{3} \sqrt{F_{S}\left(1-F_{S}\right) F_{L}} \frac{Z}{\sqrt{\mathbf{X Y}}}
$$

the factor $\left(1-F_{S}\right)$ in the bound arises due to the fact that $F_{\mathrm{L}}$ is defined with respect to $\Gamma_{K^{*}}^{\prime}$ rather than $\Gamma_{\text {full }}^{\prime}$.

$$
\begin{aligned}
\left|A_{S}^{4}\right| & \leq \sqrt{\frac{3}{2}} \sqrt{F_{S}\left(1-F_{S}\right)\left(1-F_{L}\right)\left(\frac{1-P_{1}}{2}\right)} \frac{\mathbf{Z}}{\sqrt{\mathbf{X Y}}} \sim[0.05-0.11,0.10-0.19] \\
\left|A_{S}^{5}\right| & \leq 2 \sqrt{\frac{3}{2}} \sqrt{F_{S}\left(1-F_{S}\right)\left(1-F_{L}\right)\left(\frac{1+P_{1}}{2}\right)} \frac{\mathbf{Z}}{\sqrt{\mathbf{X Y}}} \sim[0.11-0.22,0.11-0.23] \\
\left|A_{S}^{7}\right| & \leq 2 \sqrt{\frac{3}{2}} \sqrt{F_{S}\left(1-F_{S}\right)\left(1-F_{L}\right)\left(\frac{1-P_{1}}{2}\right)} \frac{\mathbf{Z}}{\sqrt{\mathbf{X Y}}} \sim[0.11-0.22,0.19-0.38] \\
\left|A_{S}^{8}\right| & \leq \sqrt{\frac{3}{2}} \sqrt{F_{S}\left(1-F_{S}\right)\left(1-F_{L}\right)\left(\frac{1+P_{1}}{2}\right)} \frac{\mathbf{Z}}{\sqrt{\mathbf{X Y}}} \sim[0.05-0.11,0.06-0.11]
\end{aligned}
$$

Large recoil and low recoil ranges with $F_{S} \sim 7 \%$.
Symmetries will add non-trivial correlations [L.Hofer, JM, N.Serra'14]

